

## TUNABLE TRAPPED MODE IN SYMMETRIC RESONATOR DESIGNED FOR METAMATERIALS

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**Abstract**—The excitation of an antisymmetric trapped mode on a symmetric metamaterial resonator is experimentally demonstrated. We use an active electronic device to break the electrical symmetry and therefore to generate this trapped mode on a symmetric split ring resonator. Even more, with such a tunable mode coupling resonator, we can precisely tune the resonant mode frequency. In this way, a shift of up to 15 percent is observed.

### 1. INTRODUCTION

At the beginning of this century, left-handed metamaterials have attracted considerable interest of scientists working in the field of microwave technology [1–4]. Since then, planar metamaterials realized in microstrip technology have been demonstrated [5, 6]. A compact lefthanded coplanar waveguide (CPW) design based on complementary split ring resonators (SRRs) was proposed afterwards [7]. Due to their inherent magnetic resonance, SRRs can advantageously be employed in microwave filter designs. They deliver a sharp cut-off at the lower band edge which corresponds to their resonance frequency. Moreover, the SRRs can be tuned using varactor diodes. By this way, tracking filters can be designed for multiband telecommunication systems, radiometers, and wide-band radar systems. Actually, these resonators can be tuned easily using varactor diodes [8].

Recently, a resonant response with a very high quality factor has been achieved in planar SRRs based metamaterials by introducing symmetry breaking in the shape of its structural elements [9, 10].

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This structural asymmetry enables excitation of the so-called trapped modes. These modes are normally inaccessible in symmetric split rings.

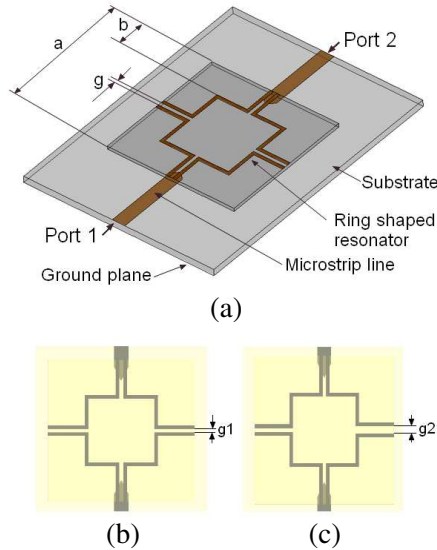
In this paper, we propose an original way to excite such a trapped mode in a metamaterial resonator. Instead of using an asymmetric resonator, we propose to use an active electronic device that breaks the electrical symmetry of the structure. With such a Tunable Mode Coupling Resonator (TMCR), it is possible to precisely tune the coupling between the symmetric and the antisymmetric modes. Especially, we can switch from a single resonance to two distinct resonances: a symmetric and an antisymmetric one.

To demonstrate this phenomenon, we use a multigap SRR [11] etched on a microstrip circuit with typical dimensions much smaller than the wavelength. In the first step, we checked that with asymmetric gaps, the trapped mode appears. In a second step, the TMCR is realized using two independently controlled varactor diodes on a symmetric multigap SRR. A trapped mode with a high  $Q$  factor is observed. Moreover, up to 15 percent tuning of the resonant trapped mode frequency is obtained. An equivalent circuit model is developed to interpret the results.

## 2. TRAPPED MODE EXCITATION IN A PLANAR METAMATERIAL RESONATOR

A schematic view of the proposed metamaterial resonator is presented in Fig. 1(a). It consists of planar square ring etched on a 0.5 mm thick epoxy substrate (FR4). The ring is made of a 0.5 mm wide copper strip. Each side of the square ring is cut off in its middle. A microstrip capacitance, made of two parallel strips, is added to each of the four created splits. The length of the added strips is  $b = 5$  mm. The width of the hole structure is  $a = 20$  mm. In this configuration, the width and length of the gap  $g$  set the value of the capacitance.

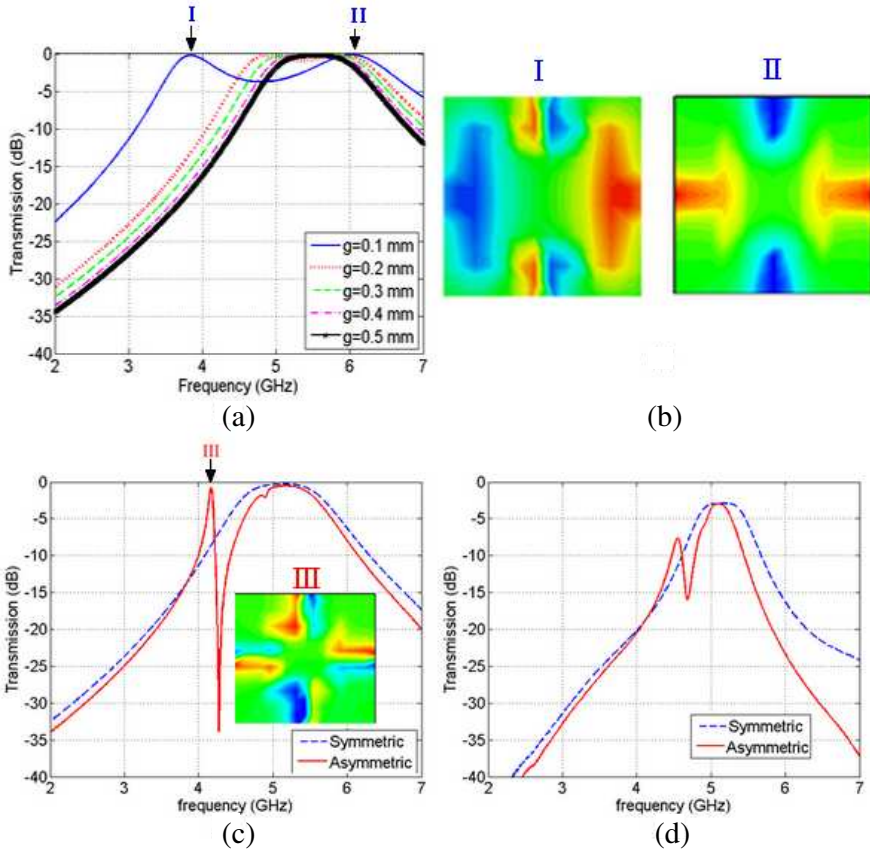
Afterwards, the metamaterial resonator has to be coupled to an external circuit. Fig. 1(a) shows the microstrip feeding system used to characterize the ring shaped resonator. It consists of two ( $50 \Omega$ )-microstrip lines etched on 1 mm thick epoxy substrate. In this system, the resonator is placed above a microstrip circuit. The parallel strips of the resonator are carefully aligned with respect to the end of each microstrip line. We studied two different types of structures designated as symmetric and asymmetric resonators. The layout of the symmetric ring resonator is presented in Fig. 1(b). This resonator has four similar metallic gaps ( $g_1 = g = 0.5$  mm). In the asymmetric resonator, the right side gap is wider than the three other gaps as shown in Fig. 1(c).



**Figure 1.** (a) Schematic view of the metamaterial resonator with the microstrip feeding system. (b) Layout of the symmetric resonator ( $g_1 = g$ ). (c) Layout of the asymmetric resonator ( $g_2 \neq g$ ).

In the case of microstrip line, the dominant EM mode behaves like a TEM mode. We characterized the proposed metamaterial resonator using a FDTD electromagnetic simulator (CST software). Fig. 2(a) shows the calculated transmission parameter for several gap sizes  $g$ . For the smallest values, we distinguish two peaks. Because of the symmetrical nature of the resonator, its resonant modes are either odd (electrical symmetry) or even (magnetic symmetry) [12]. The respective electric field distributions at the 2 resonant frequencies are shown in Fig. 2(b). We observe that the first mode is odd, and the second one is even. Moreover, the more the gap size decreases, the lower the resonant frequency is. However, the even mode keeps constant. Actually, due to the magnetic symmetry, this last mode is much less sensitive to the gap size. As discussed later, the even mode depends essentially on the electric length of the resonator.

The symmetric and asymmetric resonators are then characterized numerically and experimentally. In both cases, the transmission parameter is measured versus frequency (see Figs. 2(c) and 2(d)). Besides a slight difference in the width of the resonance peak, the agreement between simulation and experiment is good. The difference between the simulation and measurement is mainly due to the precision of fabrication process of the sample.



**Figure 2.** (a) The metamaterial resonator simulated response for several values of the gap size. (b) Electric field distribution of the odd and the even mode corresponding to resonant features I and II marked in Section (a). (c) Simulated transmission parameter of the symmetric and the asymmetric design. The insert shows the electric field distribution of the resonant mode marked as III. (d) Measured transmission parameter in the two configurations.

First, we observe a transmission band around 5 GHz that is common to the symmetric and asymmetric structures. We have seen that the resonant frequencies of the odd and even modes are almost identical when the gap size  $g$  equals 0.5 mm. Second, the asymmetrical structure shows an extra ultrasharp resonance at 4.2 GHz (marked as III), where transmission reaches 0 dB. A mode with very high  $Q$  factor ( $\sim 100$ ) is excited at this frequency. At this resonance, the arms of the four parallel strips have the opposite voltage potentials as indicated in

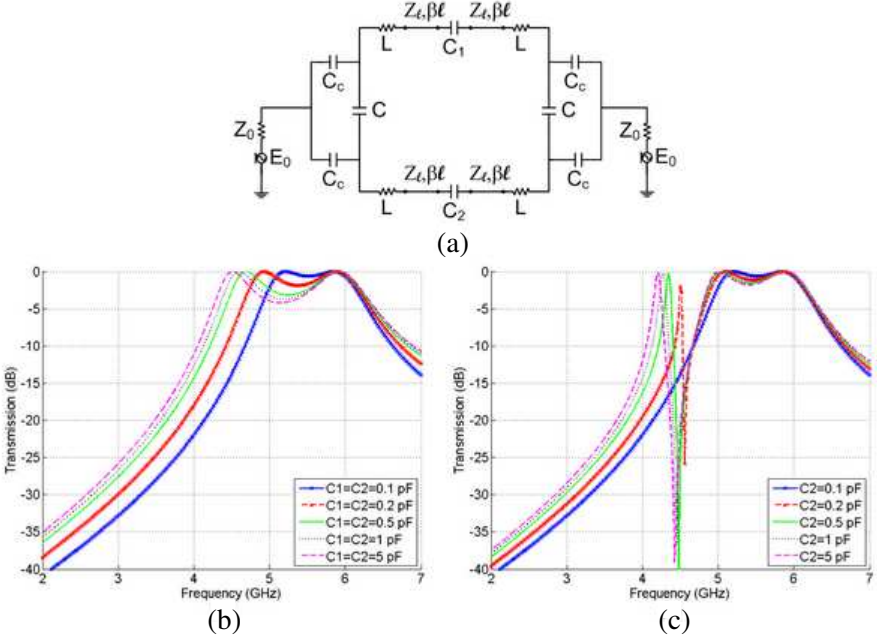
the insert of Fig. 2(c). An asymmetric extra mode is excited.

### 3. ELECTRICAL ANALYSIS

To understand the resonant nature of the asymmetric structure, analysis based upon transmission line theory is developed. The ring shaped structure is modeled using an equivalent electrical circuit. Actually, basic design theories of filters are based on lumped element circuit and, thus, make it possible to apply these results to the design of such resonators. Fig. 3(a) illustrates the equivalent electrical circuit of the metamaterial resonator. In this circuit, the dissipation effects of the resonator are not taken into account. The high-impedance strip lines of the ring act as series inductors ( $L$ ) and transmission lines of electrical lengths  $(2\pi l)/\lambda$ . Otherwise, the four parallel pairs of strip lines act as capacitors. The pairs of strip lines parallel to the microstrip lines are modeled by the capacitors  $C$ . The two other pairs of strip lines are represented by the capacitors  $C_1$  and  $C_2$ . The coupling to the feeding external circuit is represented by the coupling capacitors  $C_c$ .

For the symmetric resonator, the calculation using the equivalent electrical circuit yields the frequency response illustrated in Fig. 3(b). We recognize the two transmission peaks of the odd and even modes. The curves obtained for several values of the capacitors  $C_1$  and  $C_2$  show the shifting of the odd mode resonant frequency by increasing the capacitors values. Indeed, decreasing the gap size is equivalent to increasing the equivalent capacitance. We also note that the even mode peak is constant. Actually, we can show that the frequency of this mode depends mainly on the electric length of the strip lines of the ring.

Figure 3(c) shows the calculated frequency response of the asymmetric resonator. For this calculation, the value of the capacitor  $C_1$  is kept at 0.1 pF. By raising the second capacitor value, the ultrasharp resonance appears increasingly. The asymmetric extra mode is excited. The electrical symmetry breaking in the ring shaped resonator achieves a high  $Q$ . This anti-symmetric mode is a “trapped mode”. It can be also observed when transmitting plane wave through asymmetric split ring resonators based metamaterial [9, 10]. An antisymmetric mode cannot be excited with a source that is symmetric with respect to the resonator axis. That is why this mode is observed only when the geometrical symmetry [9, 10], or the electrical symmetry are broken. We also observed this behavior when the feeding is off-axis.



**Figure 3.** (a) Electrical circuit representation of the metamaterial resonator ( $Z_0 = 50 \Omega$ ,  $C = 0.05$  pF,  $L = 10$  nH,  $C_c = 0.4$  pF,  $Z_l = 5Z_0$  and  $l = 10$  mm). Calculated transmission responses of the resonator using the equivalent circuit analysis for several values of the capacitor  $C_2$  and for:  $C_1 = C_2$  (b) and  $C_1 = 0.1$  pF (c).

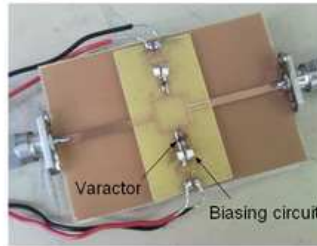
#### 4. TRAPPED MODE TUNING WITH A TUNABLE MODE COUPLING RESONATOR

We now use active components to control the trapped mode and shift it in frequency. A prototype of a TMCR is designed. It is made of a symmetric multigap SRR where all the gaps present the same size and capacitance. Varactors are then incorporated between the parallel strips that are perpendicular to the microstrip feeding line as shown in Fig. 4(a). Bias voltage is supplied through the biasing circuit. Fig. 4(b) shows the experimental results obtained by tuning simultaneously the two capacitances  $C_1$  and  $C_2$ . The 10% shift of the transmission bandwidth is due to the odd mode shifting. This experimental tuning bandwidth agrees very well with the theoretical prediction from the circuit model.

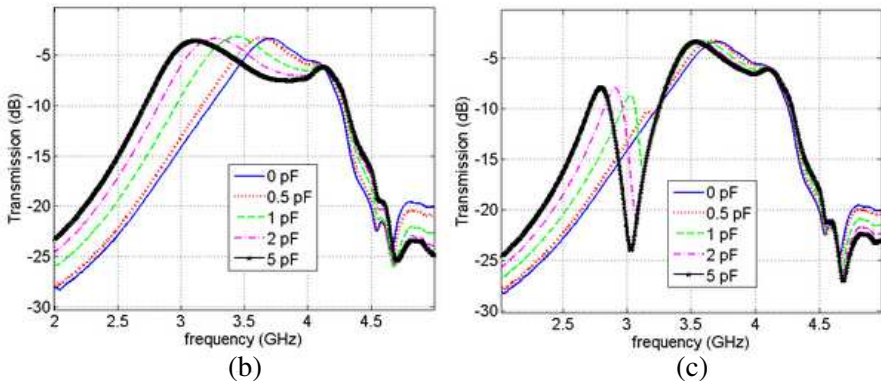
The 1 GHz frequency shift between the passive (Fig. 2(d)) and

active (Figs. 4(b),(c)) structure response can be explained with our equivalent circuit model. Indeed, due to biasing circuit for the active structure, the electric length is increased of 8mm which induces a roughly 1GHz frequency shift. However, the model does not explain the transmission dip at 4.7GHz (see Fig 4(b)). The decoupling inductances of the biasing circuit could be the origin of this phenomenon.

The varactor diode mounted on the bottom gap shown in Fig. 4(a) is now used to fix the capacitance  $C_1$  at  $\sim 0.3$  pF. In this case, the tuning of the varactor diode mounted on the top gap causes the desired asymmetry. Then, the trapped mode appears as shown in Fig. 4(c). The resonant frequency of this mode varies with the varactor capacitance  $C_2$  as predicted by the theoretical model. The tuning bandwidth is about 15 percent around 3 GHz.



(a)



**Figure 4.** (a) Photograph of the tuned metamaterial circuit. Varactor diodes are mounted on the top and the bottom gap of the resonator. (b) Transmission band widening by increasing simultaneously the tunable capacitances  $C_1$  and  $C_2$ . (c) Trapped mode tuning by varying the capacitance  $C_2$  of the varactor diode mounted on the bottom gap ( $C_1 = 0.3$  pF).

## 5. CONCLUSION

In summary, we have demonstrated experimentally, numerically and with an electric model that the proposed asymmetric resonator for metamaterials exhibits unusual resonances with a narrow transmission band. We show that it is equivalent to breaking the geometric or electric symmetry to excite the trapped mode. The TMCR enables to tune the trapped mode finally. This TMCR would be used as electronically tunable filters or varactor-tuned oscillators.

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