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APPLICATION OF TAGUCHI'S OPTIMIZATION METHOD AND SELF-ADAPTIVE DIFFERENTIAL EVOLUTION TO THE SYNTHESIS OF LINEAR ANTENNA ARRAYS

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Abstract—In this paper, the problem of designing linear antenna arrays for specific radiation properties is dealt with. The design problem is modeled as a single optimization problem. The objectives of this work are to minimize the maximum side lobe level (SLL) and perform null steering for isotropic linear antenna arrays by controlling different parameters of the array elements (position, amplitude, and phase). The optimization is performed using two techniques: Taguchi's optimization method and the self-adaptive differential evolution (SADE) technique. The advantage of Taguchi's optimization technique is the ability of solving problems with a high degree of complexity using a small number of experiments in the optimization process Taguchi's method is easy to implement and converges to the desired goal quickly in comparison with gradient-based methods and particle swarm optimization (PSO) Results obtained using Taguchi's method are in very good agreement with those obtained using the SADE technique.

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1. INTRODUCTION

The synthesis of antenna arrays plays a very important role in communication systems. Several papers exist in the literature that address this design problem [1–5]. The increased use of such arrays creates more challenges upon the antenna engineers. More requirements, such as pattern shaping, low profile, wideband/narrowband, and interference cancellation; and more limitations such as power dissipation and antenna size, lead to the urgent need for simple, time saving and efficient optimization techniques. Several new synthesis and optimization techniques have emerged in the last two decades that mimic biological evolution, brain function, or the way biological entities communicate in nature. Although some of these new algorithms are still in their infancy, they have been used successfully in single and multiobjective synthesis and optimization problems with many constraints and nonlinear processes. Most of these new techniques make use of previous history, natural tendencies, training, memory updates and biologically inspired processes. Optimization algorithms can be categorized as evolutionary algorithms, machine learning/neural networks, traditional methods and hybrid methods. Each of these four categories is expanded according to the nature of specific techniques used: local, global, heuristic, deterministic, stochastic, evolutionary, hybrid, multiobjective ... etc.

Among these, the evolutionary algorithms (EAs) (e.g., genetic algorithms (GAs) [6], simulated annealing (SA) [7], particle-swarm optimizers (PSO) [8, 9], differential evolution (DE) [10], ant colony optimization (ACO) [11], and central force optimization (CFO) [12, 13]) have become widely used in electromagnetics due to their simplicity, versatility, and robustness. However, these methods present certain drawbacks usually related to the intensive computational effort they demand to achieve the global optimum and the possibility of premature convergence to a local optimum.

DE is a population based stochastic method [14]. Several DE variants or strategies exist. The classical DE algorithm has been applied to design problems in electromagentics [15, 16]. One of the DE advantages is the fact that very few parameters have to be adjusted in order to produce results. Short while ago, a new DE version that self-adapts these control parameters has been applied to numerical benchmark problems in [17]. This self-adaptive DE algorithm has been used for microwave absorber design in [18].

Recently, another global optimization technique, Taguchi's optimization method [19], has been introduced to the electromagnetics and antennas communities [20–22]. It was successfully used to optimize linear antenna arrays, ultra-wideband antenna, planar microwave filter design, and coplanar waveguide (CPW) slot antenna. In this paper, Taguchi's optimization method is further demonstrated by synthesizing linear antenna arrays. Taguchi's method is used to adjust the elements positions, amplitudes, and phases to achieve maximum side lobe level reduction and null steering. For comparison purposes, self-adaptive differential evolution (SADE) [17, 18] is applied to the same problems and its results are compared to those obtained using Taguchi's method. To the best of our knowledge, this is the first time that these methods are compared for linear array synthesis.

This paper is organized as follows. Taguchi's method is briefly described in Section 2, while the SADE technique is presented in Section 3. The formulation details of the array factor for linear arrays are given in Section 4. Section 5 has the numerical results while the conclusions are discussed in Section 6.

2. TAGUCHI'S OPTIMIZATION METHOD

Taguchi's optimization method will be briefly described here. The interested reader may consult [20–22] for more details. The steps taken in Taguchi's optimization can be summarized as follows [20]:

2.1. Problem Initialization

This step includes formulating a suitable fitness (or objective) function defining the solution space, and selecting an appropriate orthogonal array (OA). The fitness function and the solution space are chosen according to the optimization goal. The selection of the orthogonal array depends on many parameters such as the number of experiments (N), number of variables to be optimized (k), number of levels that the variables can select from (s), and the strength of the orthogonal array (t). The notation OA (N, k, s, t) is usually used to represent an orthogonal array. Here, we choose $s = 3$ and $t = 2$ which are found to be sufficient for most problems [20–22].

2.2. Input Parameters Design Using Orthogonal Array

In this step, the input parameters are selected to conduct the experiments (i.e., the evaluation of the fitness function). For an orthogonal array with $s = 3$, the value of level 2 for each parameter is chosen at the center of the optimization range corresponding to that parameter. Then, the values of the other levels (1 and 3) are evaluated by subtracting and adding a specific "level difference" (LD) to the value of level 2. The equation which determines the level difference in the first iteration is taken as [20]:

$$LD_1 = \frac{\max - \min}{s+1} \tag{1}$$

where "max" and "min" are the upper and lower bounds of the optimization range, respectively.

2.3. Conduct Experiments, Build a Response Table and Identify Optimal Level Values

After determining the value of each parameter according the selected level, they are be used to evaluate the fitness function (i.e., conduct the experiments). Then, the value of the fitness function is transformed to the signal-to-noise ratio using the following equation [20]:

$$\eta = -20 \log(Fitness) \tag{2}$$

where η is the signal-to-noise (S/N) ratio.

After conducting all the experiments and finding the fitness values and the corresponding S/N ratio, a response table is built by averaging the S/N for each parameter n and level m using [20]:

$$\bar{\eta}(m,n) = \frac{s}{N} \sum_{i, OA(i,n)=m} \eta_i \tag{3}$$

Next, the largest S/N ratio for each parameter in the response table is used to determine the optimal level for the next iteration. After determining the optimal levels, a confirmation experiment is performed using the optimal level for each parameter. The value of the fitness function resulting from such an experiment is considered as the fitness value for the current iteration.

2.4. Check the Termination Criteria and Reduce the Optimization Range

If the termination criterion is not satisfied, the optimal level for the current iteration will be the center of the next iteration:

$$x_n|_{i+1}^{2} = x_n|_{i}^{opt} \tag{4}$$

Also, the optimization range for the next iteration is reduced by multiplying the current level difference by the reducing rate (rr) which can be set between 0.5 and 1 depending on the problem [20]. So, for the $(i+1)$th iteration:

$$LD_{i+1} = (rr)\,(LD_i) = (rr^i)\,(LD_1) = RR(i)\; LD_1 \tag{5}$$

where $RR(i) = rr^i$ is called the reduced function [20].

Finally, the above algorithm is repeated until a specific termination criterion is achieved or a specific number of iterations is reached.

3. DIFFERENTIAL EVOLUTION OPTIMIZATION METHOD

A population in DE consists of NP vectors $\bar{x}_{iG}, i = 1, 2, \ldots, NP$, where G is the generation number. The population is initialized randomly from a uniform distribution. Each D-dimensional vector represents a possible solution. The initial population evolves in each generation with the use of three operators: mutation, crossover and selection. Depending on the form of these operators several DE variants or strategies exist in the literature [14]. The most popular is the one known as DE/rand/1/bin strategy. In this strategy a mutant vector $\bar{v}$ for every target vector $\bar{x}_{iG}$ is computed by:

$$\bar{v}_{i,G+1} = \bar{x}_{r_1,G} + F(\bar{x}_{r_2,G} - \bar{x}_{r_3,G}), \quad r_1 \neq r_2 \neq r_3 \tag{6}$$

where r_1, r_2, r_3 are randomly chosen indices from the population, and F is a mutation control parameter. After mutation the crossover operator is applied to generate a trial vector $\bar{u}_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \ldots, u_{Di,G+1})$ whose coordinates are given by:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1}, & \text{if rand}(j) \leq CR \text{ or } j = rn(i) \\ x_{ji,G}, & \text{if rand}(j) > CR \text{ and } j \neq rn(i) \end{cases} \tag{7}$$

where $j = 1, 2, \ldots, D$, rand(j) is a number from a uniform random distribution from the space $[0, 1]$, $rn(i)$ a randomly chosen index from $(1, 2, \ldots, D)$ and CR the crossover constant from the space $[0, 1]$. DE uses a greedy selection operator. According to this selection scheme for minimization problems:

$$\bar{x}_{i,G+1} = \begin{cases} \bar{u}_{i,G+1}, & \text{if } f(\bar{u}_{i,G+1}) < f(\bar{x}_{i,G}) \\ \bar{x}_{i,G}, & \text{otherwise} \end{cases} \tag{8}$$

where $f(\bar{u}_{i,G+1})$, $f(\bar{x}_{i,G})$ are the fitness values of the trial and the old vector respectively. Therefore the newly found trial vector $\bar{u}_{i,G+1}$ replaces the old vector $\bar{x}_{i,G}$ only when it produces a lower objective function value than the old one. Otherwise the old vector remains in the next generation. The stopping criterion for the DE is usually the generation number or the number of objective function evaluations. Compared with PSO, DE has been found to produce better results on numerical benchmark problems [23].

3.1. Self-adaptive DE (SADE)

Storn has suggested [14] that the differential evolution control parameters are adjusted in the following way: $F \in [0.5, 1]$, $CR \in [0.8, 1]$ and $NP = 10D$. In [17], a novel approach is proposed for the self-adapting of DE control parameters. This strategy is based on DE/rand/1/bin scheme. Each vector is extended with its own F and CR values. Therefore the control parameters are self-adjusted in every generation for each individual according to the following scheme:

$$F_{i,G+1} = \begin{cases} F_l + rnd_1 \times F_u & \text{if } rnd_2 < 0.1 \\ F_{i,G}, & \text{otherwise} \end{cases}$$
$$CR_{i,G+1} = \begin{cases} rnd_3 & \text{if } rnd_4 < 0.1 \\ CR_{i,G}, & \text{otherwise} \end{cases} \quad (9)$$

where $rnd_1, rnd_2, rnd_3, rnd_4$ are uniform random numbers $\in [0, 1]$ and F_l, F_u are the lower and the upper limits of F set to 0.1 and 0.9, respectively [17]. So, by using the self-adaptive algorithm the user does not have to adjust the F and CR parameters while the time complexity does not increase. More details about the self-adaptive DE algorithm can be found in [17, 18].

4. LINEAR ANTENNA ARRAY FACTOR

For a $2N$-element symmetrical array placed on the x-axis shown in Figure 1, the array factor (AF) can be written as:

$$AF(\vartheta) = \sum_{n=1}^{2N} I_n \exp(j[kx_n \sin(\vartheta) + \phi_n]) \quad (10)$$

where k is the wave number, and I_n, φ_n, and x_n are, respectively, the excitation amplitude, phase, and location of the array elements.

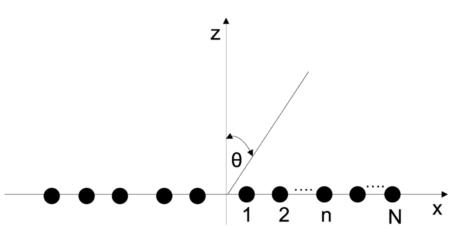

Figure 1. Geometry of $2N$-element symmetric linear array placed along the x-axis.

Assuming that the $2N$ isotropic radiators are placed symmetrically along the x-axis, Equation (10) can be simplified as:

$$AF(\vartheta) = 2\sum_{n=1}^{N} I_n \cos[kx_n \sin(\vartheta) + \phi_n] \tag{11}$$

From Equations (10) and (11), it is obvious that three parameters are controlling the AF: the amplitudes, the phases, and the positions of the elements. In this paper, Taguchi's optimization method and SADE are used to design linear antennas by optimizing these parameters individually. In all examples that follow, the SADE algorithm is executed 10 times. The best result is compared with Taguchi's optimization method. The population size was set equal to $10D$.

5. NUMERICAL RESULTS

The fitness function for maximum side lobe level (SLL) minimization is expressed as:

$$\begin{aligned} &\text{Minimize} \quad fitt = \max\{20\log|AF(\vartheta)|\} \\ &\text{subject to} \quad \vartheta = [14^\circ, 90^\circ]. \end{aligned} \tag{12}$$

where $AF(\theta)$ is given in Equation (11).

5.1. Optimize Element Amplitudes (I_n)

In order to optimize the amplitudes, the remaining control parameters x_n and φ_n are fixed, where φ_n is taken as zero and the spacing between adjacent elements is taken as $(\lambda/2)$, $n = 1, \ldots, N$. Assuming that the first element is placed at $x_1 = (\lambda/4)$, the array factor can be simplified as:

$$AF(\vartheta) = 2\sum_{n=1}^{N} I_n \cos[(n-0.5)\pi \sin(\vartheta)] \tag{13}$$

The excitation amplitudes of the $2N$-element array will be optimized in the range $[0, 1]$. Three linear array design cases of 10, 16 and 24 elements are optimized using Taguchi's and SADE methods.

It must be pointed out that the synthesis of excitations can be formulated as a Convex Programming problem or even as Linear Programming problem and solved more efficiently [24–27]. But, this case also represents an appropriate example to compare both Taguchi's and SADE methods. In other papers as well, excitation synthesis using global optimizers like PSO [28] has been used for comparison purposes between different algorithms.

5.1.1. 10 Elements AF Optimization

Using Equation (13), and applying both Taguchi's and SADE optimization methods on a 10-element linear array (LA), the obtained radiation patterns are shown in Figure 2, while Figure 3 shows the convergence of the fitness function versus the iteration number. Using 100 iterations for both algorithms, the obtained optimum values of the amplitudes are given in Table 1.

A Laptop with 2 GHz Intel Celeron CPU and 512 RAM was used for simulating the Taguchi's code, and the simulation time was only 17 seconds, while the simulation time for the SADE was about 15 seconds. The maximum SLL obtained using Taguchi's technique is −24.88 dB, while the maximum SLL of the uniform one is −12.97 dB. On the other hand, the maximum SLL found by SADE is −24.41 dB. The obtained SLL using Taguchi's method is less than the uniform

Table 1. Optimum amplitude values found by Taguchi's method and SADE for the 10-element array.

n	Taguchi's method	SADE
1	1.0000	1.0000
2	0.8999	0.9028
3	0.7228	0.7277
4	0.5077	0.5153
5	0.3994	0.4158

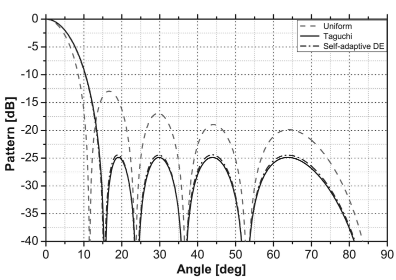

Figure 2. Radiation pattern of 10 elements $\lambda/2$ spaced array optimized with respect to amplitudes, compared with uniform array.

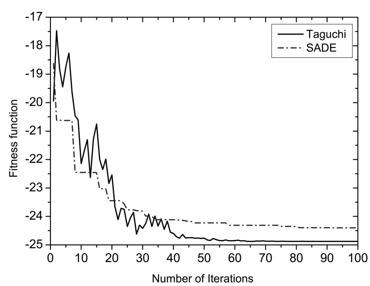

Figure 3. Convergence curves of the fitness value of the 10 elements $\lambda/2$ spaced LA.

one by 11.9 dB and is also better than the PSO results in [29] by about 0.3 dB. The beamwidth of the optimized array is slightly larger than the conventional one, but the uniform array is optimum in regard to the beamwidth.

5.1.2. 16 Elements AF Optimization

In this example, Taguchi's optimization and SADE methods will be applied on a 16-element linear array. Table 2 holds the optimum values of the amplitudes obtained using Taguchi's method (after 100 iterations) and SADE (after 500 iterations).

Table 2. Optimum amplitude values found by Taguchi's method and SADE for the 16-element array.

n	Taguchi's method	SADE
1	1.0000	1.0000
2	0.9500	0.9515
3	0.8575	0.8586
4	0.7317	0.7333
5	0.5861	0.5889
6	0.4381	0.4404
7	0.2988	0.3020
8	0.2552	0.2616

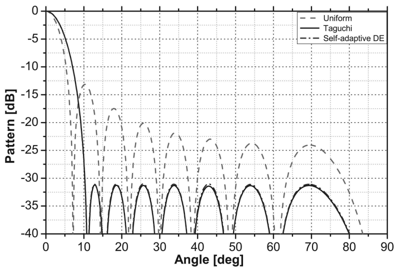

Figure 4. Radiation pattern of 16 elements $\lambda/2$ spaced array optimized with respect to amplitudes, compared with uniform array.

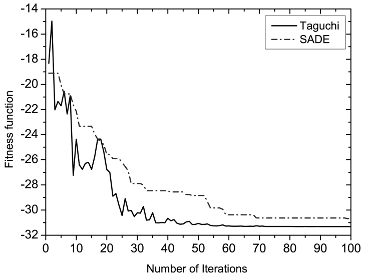

Figure 5. Convergence curves of the fitness value of the 16 elements $\lambda/2$ spaced LA.

Figure 4 shows the obtained radiation patterns compared with uniform array. The maximum SLL obtained using Taguchi's method is −31.31 dB, while the maximum SLL of the uniform one is −13.15 dB. On the other hand, the maximum SLL obtained using SADE is −31.06 dB. The maximum SLL obtained using Taguchi's method is less than the uniform one by about 18.1 dB and is also better than the PSO results in [29] by about 0.6 dB. Figure 5 shows the convergence of the fitness function versus the iteration number. Using Taguchi's technique, the goal of the optimization is obtained after 80 iterations only, while it takes around 500 iterations for the SADE to converge.

5.1.3. 24 Elements AF Optimization

The radiation pattern of an optimized 24-element LA is shown in Figure 6 and the convergence of the fitness function is shown in Figure 7. The goal of the optimization is obtained within only 50 iterations for Taguchi's method while it requires 500 iterations for SADE. The maximum SLL obtained using Taguchi's method is −35.25 dB (SADE −35.21 dB), while the maximum SLL of the uniform one is −13.95 dB. The obtained SLL is less than the uniform one by about 21.3 dB and is also better than the PSO results in [29] by about 0.45 dB. The optimum amplitude values found by both methods are shown in Table 3.

Table 3. Optimum amplitude values found by Taguchi's method and SADE for the 24-element array.

n	**Taguchi's method**	**SADE**
1	1.0000	1.0000
2	0.9731	0.9747
3	0.9283	0.9272
4	0.8585	0.8584
5	0.7745	0.7735
6	0.6758	0.6775
7	0.5772	0.5743
8	0.4686	0.4712
9	0.3719	0.3701
10	0.2764	0.2781
11	0.1995	0.1972
12	0.2026	0.2053

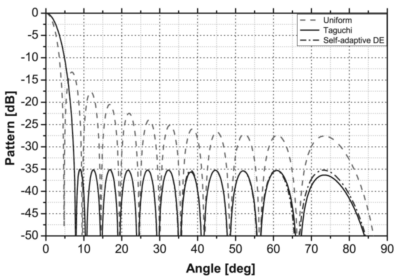

Figure 6. Radiation pattern of 24 elements $\lambda/2$ spaced array optimized with respect to amplitudes, compared with uniform array.

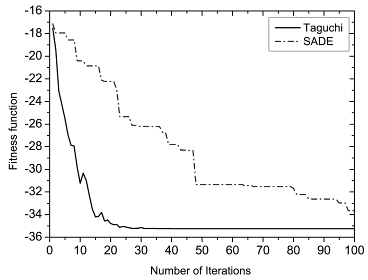

Figure 7. Convergence curves of the fitness value of the 24 elements $\lambda/2$ spaced LA.

5.2. Optimize Element Positions (x_n)

In this part, the amplitudes and phases are set to $I_n = 1$ and $\varphi_n = 0$, for $n = 1, \ldots, N$. The positions x_n's are adjusted by Taguchi's optimization method and the SADE technique to minimize the maximum side lobe level. As an example, a 10-element array is considered here. In order not to exceed the uniform array length, the last elements positions are set to $x_{\pm N} = \pm 2.25\lambda$. Thus, this problem is solved in four dimensional solution space. The minimum distance between two neighboring elements is set to 0.25λ, and the $\min(x_i)$ is set to 0.125λ. The simplified array factor is as follows:

$$AF(\vartheta) = 2\left[\sum_{n=1}^{N-1} \cos[kx_n \sin(\vartheta)] + \cos[4.5\pi \sin(\vartheta)]\right] \tag{14}$$

The radiation pattern of the 10 elements optimized LA is shown in Figure 8. The maximum SLL obtained using both techniques is -19.7 dB, while the maximum SLL of the uniform one is -12.96 dB. The obtained SLL is less than the uniform one by about 6.7 dB which exactly agrees with the PSO results presented in [29]. The convergence of the fitness function versus the number of iteration is shown in Figure 9. The goal of the optimization is achieved after only 100 iterations for both algorithms. The optimum values of the positions obtained using Taguchi's method and SADE are given in Table 4.

Several applications in wireless communications require the minimization of the close-in SLL (i.e., the first side lobe nearest to the

Table 4. Optimum position values (wavelengths) found by Taguchi's method and SADE for the 10-element uniform amplitude array (SLL suppression case).

n	Taguchi's method	SADE
1	0.2142	0.2145
2	0.5989	0.5995
3	1.0597	1.0605
4	1.5861	1.5865
5	2.2500	2.2500

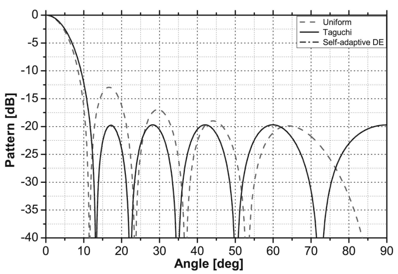

Figure 8. Radiation pattern of 10 elements LA optimized with respect to positions compared with uniform array.

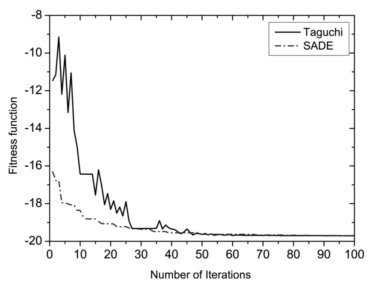

Figure 9. The convergence curves of the fitness value of the 10 elements LA optimized with respect to positions.

main beam). Therefore, the fitness function is modified as follows [29]:

$$\begin{aligned} &\text{Minimize } fitt = \alpha_1 \max\{20\log|AF(\vartheta_{AS})|\} + \alpha_2 \max\{20\log|AF(\vartheta_{NS})|\} \\ &\text{Subject to } \vartheta_{AS} \in [14^\circ, 90^\circ] \text{ and } \vartheta_{NS} \in [14^\circ, 21^\circ] \end{aligned} \quad (15)$$

In the above equation, the region of Θ_{AS} is weighted by α_1, and the close-in region of Θ_{NS} is weighted by α_2. Here we take $\alpha_1 = 1$ and $\alpha_2 = 2$. Table 5 has the optimum position values for this case, while the obtained radiation patterns are shown in Figure 10. The maximum SLL for the uniform array is -12.96 dB, while the maximum SLL for the Taguchi's optimized array and the SADE optimized array are -18.08 dB and -18.20 dB, respectively. The Taguchi's optimized array has a close-in SLL level of -30.44 dB, while the SADE gives -30.22 dB. The convergence of the fitness function versus the number of iterations is shown in Figure 11. Both algorithms converge to the optimum solution after 100 iterations.

Table 5. Optimum position values (wavelengths) found by Taguchi's method and SADE for the 10-element uniform amplitude array (close-in SLL suppression case).

n	Taguchi's method	SADE
1	0.1642	0.1745
2	0.5509	0.5445
3	0.9362	0.9395
4	1.5199	1.5165
5	2.2500	2.2500

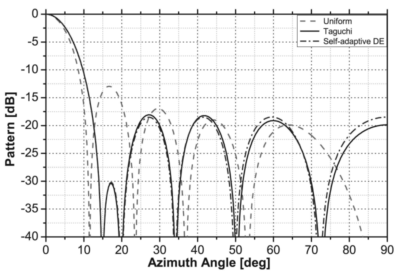

Figure 10. Radiation pattern of the 10 elements LA optimized with respect to positions (close-in SLL suppression case).

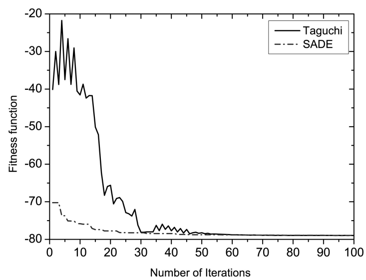

Figure 11. The convergence curves of the fitness value of the 10 elements position-optimized LA (close-in SLL suppression case).

5.3. Optimize Element Phases (φ_n)

In this section, the optimization problem is treated by assuming that all elements have the same exciting amplitudes and only phase control is allowable. The linear array has $2N$ equally spaced elements with spacing of 0.5λ. The array elements phases are assumed to be symmetric as:

$$\varphi_i = \varphi_{-i}, \quad i = 1, 2, 3, \ldots, N \tag{16}$$

where φ_i is the phase of the i-th element. For a symmetric array, the array factor is given as:

$$AF(\theta) = 2 \sum_{i=1}^{N} e^{j\varphi_i} \cos\left[(i - 0.5)\pi \sin\theta\right] \tag{17}$$

Equation (17) could be written in decibels as:

$$AF_{dB}(\theta) = 20 \cdot \log \left| \frac{AF(\theta)}{AF(\theta_o)} \right| \tag{18}$$

where θ_0 is the direction of the main beam which is taken as $\theta_o = 0°$. The goal of the optimization is to adjust the phases of the array elements to reduce the maximum side lobe level and to impose nulls at specific angles. The fitness function is given as [30]:

$$fit = k_1 \cdot f_{SL}(\theta) + k_2 \cdot f_{NS}(\theta) \tag{19}$$

where k_1 and k_2 are the weights of the two goals. $f_{SL}(\theta)$ and $f_{NS}(\theta)$ are given by:

$$f_{SL}(\theta) = \max\{AF_{dB}(\theta)\} \tag{20}$$

$$f_{NS}(\theta) = \sum_k AF_{dB}(\theta^k_{null}) \tag{21}$$

where SL is the feasible region of side lobes excluding the main beam, and θ^k_{null} denotes the direction of the k-th null. Here, for comparison purposes, we consider several examples which are similar to those presented in [30].

Figure 12 shows the optimum array patterns for the phase-optimized 20 elements LA, compared with uniform array. The optimum phase values (in degrees) obtained using Taguchi's method and SADE are presented in Table 6. The maximum SLL for the uniform array is −13.19 dB while the maximum SLL for the optimized array (obtained using both algorithms) is −16.24 dB. This shows that the maximum side lobe reduction is improved by 3 dB which is slightly better than that in [30] by about 0.1 dB.

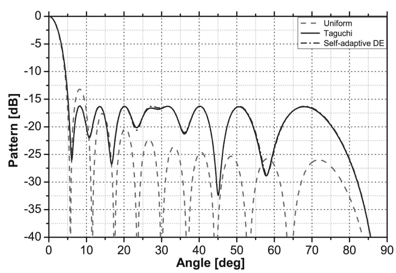

Figure 12. Optimum array pattern for the phase-optimized 20 elements LA, compared with uniform array.

Table 6. Optimum phase values (degrees) found by Taguchi's method and SADE for the 20-element uniform amplitude array.

n	**Taguchi's method**	**SADE**
1	34.296	−8.216
2	28.558	−1.345
3	25.851	1.475
4	27.328	0.023
5	20.520	5.39
6	25.816	0.71
7	−18.624	46.728
8	85.245	−58.136
9	44.900	−18.198
10	28.062	−1.904

5.3.1. Unidirectional Null Steering

Reaching the optimum side lobe reduction under the constraint of null steering will be the goal of the next example. The desired null level is less than −60 dB and the desired null direction is at 9°. After applying both Taguchi's optimization method and the SADE technique on a 32 elements linear array, the obtained radiation patterns are shown in Figure 13. The optimum values of the phases (in degrees) found by both methods are shown in Table 7. The Taguchi's null value is −78.33 dB (SADE −95.41 dB). The maximum SLL for the Taguchi's optimized array with null steering is −14.6792 dB (SADE −16.73 dB). For the same problem, the maximum SLL obtained in [30] (using the electromagnetics-like optimization technique) is −13.03 dB. It is worth mentioning that the maximum SLL for the uniform array is −13.24 dB, and the maximum SLL for the optimized array without null steering is −17.55 dB. This means that after imposing the null on the 32 elements array, the final Taguchi's optimum maximum SLL is reduced by 2.87 dB. For this problem, the SADE results are better than Taguchi's ones.

Figure 14 shows the results for the case of 40 elements array. The optimum values of the phases (in degrees) found by both methods are shown in Table 8. The Taguchi's null value is −100.15 dB (SADE −103.13 dB). The maximum SLL for the Taguchi's optimized array with null steering is −16.26 dB, while that for the SADE is −15.23 dB. For the same problem, the maximum SLL obtained in [30] is −16.17 dB. It is worth mentioning that the maximum SLL

Table 7. Optimum phase values (degrees) found by Taguchi's method and SADE for the 32-element uniform amplitude array with unidirectional null steering.

n	**Taguchi's method**	*SADE*
1	97.008	0.468
2	89.395	−1.000
3	84.816	−1.000
4	89.771	−1.127
5	89.476	0.000
6	85.127	−4.170
7	93.879	−1.000
8	90.626	−1.000
9	90.311	0.000
10	73.426	0.000
11	179.009	28.288
12	73.499	0.678
13	−22.226	83.517
14	127.411	−95.316
15	83.729	−32.137
16	121.048	0.000

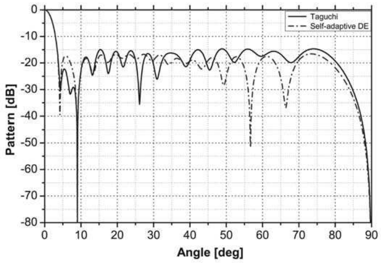

Figure 13. Optimum array pattern of 32 element array by the phase-only synthesis with unidirectional null steering.

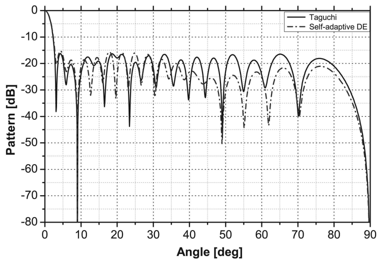

Figure 14. Optimum array patterns of 40 element array by the phase-only synthesis with unidirectional null steering.

Table 8. Optimum phase values (degrees) found by Taguchi's method and SADE for the 40-element uniform amplitude array with unidirectional null steering.

n	Taguchi's method	SADE
1	63.935	−2.152
2	52.596	−4.772
3	62.808	−1.000
4	55.933	6.145
5	73.719	−1.000
6	81.473	−1.000
7	86.684	−9.465
8	55.439	−1.000
9	19.538	−1.000
10	23.263	−5.280
11	59.938	0.000
12	77.890	0.000
13	62.579	−1.000
14	56.510	−1.000
15	66.814	12.082
16	112.846	20.134
17	−30.596	131.145
18	118.459	55.280
19	60.524	0.000
20	40.272	−27.990

for the uniform array is −13.24 dB, and the maximum SLL for the optimized array without null steering is −17.81 dB. This means that after imposing the null on the 40 elements array, the final Taguchi's optimum maximum SLL is reduced by 1.54 dB.

5.3.2. Bidirectional Null Steering

Bidirectional null steering is applied on a 20 elements LA. Two cases are considered: null steering at 14 and 20.5 degrees and null steering at 33.5 and 40 degrees. The obtained radiation patterns are shown in Figure 15, while the optimum phase values are presented in Tables 9 and 10. The results are as follows:

a. Null Steering at 14 and 20.5 Degrees:

The Taguchi's first null value is −75.22 dB (SADE −77.25 dB), and the Taguchi's second null value is −77.25 dB (SADE −81.97 dB). The maximum SLL for the Taguchi's optimized array is −12.27 dB, while SADE gives −12.60 dB. For the same problem, the maximum SLL obtained in [30] is −11.37 dB.

Table 9. Optimum phase values (degrees) found by Taguchi's method and SADE for the 20-element uniform amplitude array with bidirectional null steering at 14 and 20.5 degrees.

n	Taguchi's method	SADE
1	46.607	−1.000
2	26.703	−1.000
3	33.178	0.000
4	38.478	−1.000
5	42.276	0.000
6	53.586	−1.000
7	43.362	−19.125
8	−34.757	53.613
9	112.326	−7.236
10	33.945	−126.688

Table 10. Optimum phase values (degrees) found by Taguchi's method and SADE for the 20-element uniform amplitude array with bidirectional null steering at 33.5 and 40 degrees.

n	Taguchi's method	SADE
1	55.747	−1.000
2	59.298	2.346
3	69.506	−1.203
4	56.368	−1.008
5	29.602	21.694
6	90.837	−39.172
7	71.518	−16.064
8	46.722	10.626
9	44.301	0.999
10	−14.444	68.775

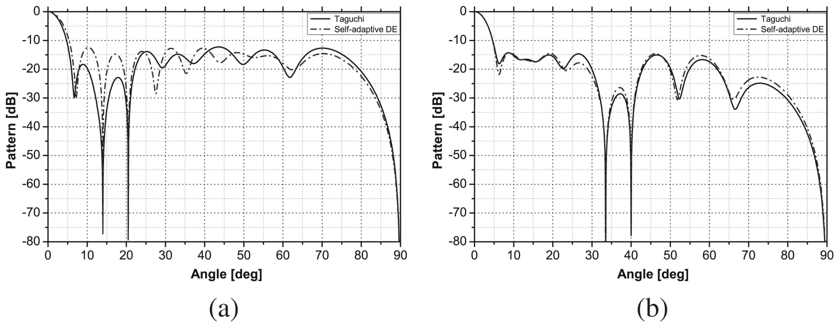

Figure 15. Optimum array patterns of 20 element array by the phase-only synthesis with bidirectional null steering. (a) Null steering at 14 and 20.5 degrees. (b) Null steering at 33.5 and 40 degrees.

b. Null Steering at 33.5 and 40 Degrees:

The Taguchi's first null value is −79.51 dB (SADE −105.82 dB), and the Taguchi's second null value is −77.82 dB (SADE −68.98 dB). The maximum SLL for the Taguchi's optimized array is −14.29 dB, while it is −14.30 dB for the SADE optimized array. For the same problem, the maximum SLL obtained in [30] is −12.41 dB.

6. CONCLUSIONS

Linear array synthesis using Taguchi's method and self-adaptive differential evolution has been presented. We have compared the above methods in several common linear array design cases like amplitude, position and phase synthesis. The results show that Taguchi's method is a powerful optimizer that converges fast and finds optimum values within 100 iterations. Self-adaptive DE is also a robust optimizer but in several cases requires more iterations than Taguchi's method. The main advantage of SADE is the fact that it requires only the adjustment of two parameters: the population size and the number of iterations. In amplitude synthesis, Taguchi's method finds better results than SADE. For position-only and phase-only synthesis, the best values found by both methods are quite similar. Both methods may also be used in conjunction with a numerical technique to optimize different antennas (e.g., microstrip patch antennas). In such cases where computational time plays an important role, fast convergence is an additional requirement.

REFERENCES

1. Oliveri, G. and L. Poli, "Synthesis of monopulse sub-arrayed linear and planar array antennas with optimized sidelobes," *Progress In Electromagnetics Research*, PIER 99, 109–129, 2009.
2. Perez Lopez, J. R. and J. Basterrechea, "Hybrid particle swarm-based algorithms and their application to linear array synthesis," *Progress In Electromagnetics Research*, PIER 90, 63–74, 2009.
3. Li, W. T., X. W. Shi, and Y. Q. Hei, "An improved particle swarm optimization algorithm for pattern synthesis of phased arrays," *Progress In Electromagnetics Research*, PIER 82, 319–332, 2008.
4. Guney, K. and S. Basbug, "Interference suppression of linear antenna arrays by amplitude-only control using a bacterial foraging algorithm," *Progress In Electromagnetics Research*, PIER 79, 475–497, 2008.
5. Mahanti, G. K., A. Chakrabarty, and S. Das, "Phase-only and amplitude-phase only synthesis of dual-beam pattern linear antenna arrays using floating-point genetic algorithms," *Progress In Electromagnetics Research*, PIER 68, 247–259, 2007.
6. Rahmat-Samii, Y. and E. Michielssen, *Electromagnetic Optimization by Genetic Algorithms*, John Wiley and Sons, 1999.
7. Kirkpatrick, S., C. D. Gellatt, and M. P. Vecchi, "Optimization by simulated annealing," *Science*, Vol. 220, 671–680, 1983.
8. Ciuprina, G., D. Ioan, and I. Munteanu, "Use of intelligent-particle swarm optimization in electromagnetics," *IEEE Trans. on Magnetics*, Vol. 38, 1037–1040, 2002.
9. Robinson, J. and Y. Rahmat-Samii, "Particle swarm optimization in electromagnetics," *IEEE Trans. on Antennas and Propagation*, Vol. 52, 397–407, 2004.
10. Feoktistov, V., *Differential Evolution, in Search of Solutions*, Springer Science, 2006.
11. Dorigo, M., M. Birattari, and T. Stützle, "Ant colony optimization," *IEEE Comp. Intelligent Magazine*, Vol. 1, 28–39, 2006.
12. Formato, R. A., "Central force optimization: A new metaheuristic with applications in applied electromagnetics," *Progress In Electromagnetics Research*, PIER 77, 425–491, 2007.
13. Formato, R. A., "Central force optimization: A new nature inspired computational framework for multidimensional search and optimization," *Studies in Computational Intelligence (SCI)*, Vol. 129, 221–238, Springer-Verlag, Heidelberg, 2008.

14. Storn, R. and K. Price, "Differential evolution — A simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Vol. 11, No. 4, 341–359, Dec. 1997.
15. Kurup, D. G., M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm," *IEEE Trans. on Antennas and Propagation*, Vol. 51, No. 9, 2210–2217, 2003.
16. Panduro, M. A. and C. del Rio Bocio, "Design of beam-forming networks for scannable multi-beam antenna arrays using corps," *Progress In Electromagnetics Research*, PIER 84, 173–188, 2008.
17. Brest, J., S. Greiner, B. Boskovic, M. Mernik, and V. Zumer, "Self-adapting control parameters in differential evolution: A comparative study on numerical benchmark problems," *IEEE Transactions on Evolutionary Computation*, Vol. 10, No. 6, 646–657, Dec. 2006.
18. Goudos, S. K., "Design of microwave broadband absorbers using a self-adaptive differential evolution algorithm," *International Journal of RF and Microwave Computer-aided Engineering*, Vol. 19, No. 3, 364–372, 2009.
19. Taguchi, G., S. Chowdhury, and Y. Wu, *Taguchi's Quality Engineering Handbook*, John Wiley and Sons Inc., NJ, 2005.
20. Weng, W. C., F. Yang, and A. Elsherbini, *Electromagnetics and Antenna Optimization Using Taguchi's Method*, Morgan & Claypool, San Rafael, CA, 2007.
21. Weng, W., F. Yang, and A. Elsherbeni, "Linear antenna array synthesis using Taguchi's method: A novel optimization technique in electromagnetics," *IEEE Trans. on Antennas and Propagation*, Vol. 55, 723–730, 2007.
22. Weng, W. C. and C. Choi, "Optimal design of CPW slot antennas using Taguchi's method," *IEEE Trans. on Magnetics*, Vol. 45, No. 3, 1542–1545, Mar. 2009.
23. Vesterstrom, J. and R. Thomsen, "A comparative study of differential evolution, particle swarm optimization, and evolutionary algorithms on numerical benchmark problems," *Proceedings of Congress on Evolutionary Computation*, 1980–1987, Portland, USA, 2004.
24. Isernia, T. and G. Panariello, "Optimal focusing of scalar fields subject to arbitrary upper bounds," *Electronics Letters*, Vol. 34, 162–164, 1998.
25. Isernia, T., P. D. Iorio, and F. Soldovieri, "An effective approach for the optimal focusing of array fields subject to arbitrary upper

bounds," *IEEE Trans. on Antennas and Propagation*, Vol. 48, 1837–1847, 2000.

26. Bucci, O. M., L. Caccavale, and T. Isernia, "Optimal far-field focusing of uniformly spaced arrays subject to arbitrary upper bounds in nontarget directions," *IEEE Trans. on Antennas and Propagation*, Vol. 50, 1539–1554, 2002.
27. Isernia, T., F. J. Ares Pena, O. M. Bucci, M. D'Urso, J. F. Gomez, and J. A. Rodriguez, "A hybrid approach for the optimal synthesis of pencil beams through array antennas," *IEEE Trans. on Antennas and Propagation*, Vol. 52, 2912–2918, 2004.
28. Lanza Diego, M., J. R. Perez Lopez, and J. Basterrechea, "Synthesis of planar arrays using a modified particle swarm optimization algorithm by introducing a selection operator and elitism," *Progress In Electromagnetics Research*, PIER 93, 145–160, 2009.
29. Al-Aqil, M., "Synthesis of antenna arrays using the particle swarm optimization method," Master Thesis, Jordan University of Science and Technology, Jan. 2009.
30. Lee, K.-C. and J.-Y. Jhang, "Application of electromagnetism-like algorithm to phase-only synthesis of antenna arrays," *Progress In Electromagnetics Research*, PIER 83, 279–291, 2008.