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ON THE OPTIMAL SYNTHESIS OF RING SYMMETRIC SHAPED PATTERNS BY MEANS OF UNIFORMLY SPACED PLANAR ARRAYS

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Abstract—By taking inspiration from [1], a synthesis strategy is proposed for the case of planar arrays and ring shaped patterns which does not require the exploitation of global optimization procedures. In particular, the approach is able to determine a priori (that is, without solving the overall design problem) whether the given power pattern design constraints can be fulfilled or not, and, in the adfirmative case, to determine the needed excitation coefficients in a fast deterministic manner. Although the approach does not apply to generic planar arrays and generic constraints, it applies to a large number of problems of actual interest, and outperforms some recently published synthesis procedures. Moreover, it may serve both as a reference solution for more general synthesis procedures, and as an elementary brick for more cumbersome synthesis problems.

1. INTRODUCTION AND MOTIVATIONS

The optimal synthesis of shaped beams by means of array antennas is a classical problem in the literature [2], with applications ranging from radar and remote sensing to telecommunications [3]. As all antenna synthesis problems, it implies well defined constraints on the far field pattern, on the antenna geometry and structure, and on the feeding network.

By leaving aside the more cumbersome problem wherein the locations of the different elements are by themselves unknown, we focus herein on the simple canonical case where the different elements are uniformly spaced on a regular grid, and the degrees of freedom of the

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problem are the complex excitations of the different elements. As far as the far field constraints are concerned, the simple and widespread point of view of requiring that the far field power pattern lies in a given mask is adopted. In fact, unless a feed array is in order, one is not interested in the far field phase distribution, and a formulation in terms of a power pattern mask (rather than the fitting of a precise pattern) leaves an increased number of degrees of freedom to the designer, thus allowing to choose in a wider set of candidate solutions.

Such a problem deserves indeed interest for a number of reasons, the main one being that while a large number of contributions exists in the literature wherein this kind of problems is solved by means of global optimization techniques [4–8], this is not actually needed in a number of cases (as we are going to see below).

In fact, a straightforward solution strategy exists for the case of linear arrays which not only does not require the exploitation of global optimization techniques, but also allows to establish a priori the feasibility of given mask constrained power pattern synthesis problems [1]. Surprisingly, such a circumstance is ignored in a number of contributions (see f.i. [8]). Provided some additional hypothesis are fulfilled, it is shown below that the same kind of 'direct' and deterministic solution strategy can be applied to the case of planar arrays for shaped beams. In particular, an effective synthesis strategy can be devised as long as the pattern specifications imply 'factorable' or even 'ring-shaped' patterns. Notably, this class of patterns covers a large number of situations of actual interest, such as the synthesis of the elements of Very Large Arrays for Deep Space Applications [4, 7], the synthesis of the so-called 'iso-flux' patterns and many others. In all these cases, these approaches are capable to furnish different alternative solutions to the same problem, so that one can eventually 'pick' the more convenient one with respect to some additional performance parameter. Such a circumstance, together with the fact that these procedures are very fast, suggests that they can be eventually used as a reference source or a reference pattern for other more cumbersome problems, such as array locations synthesis via density taper techniques (along the guidelines of [9]).

In the following, both because of the fact they represent a basis for the subsequent planar array case, and in order to emphasize that a number of contributions in the literature are misleading, we briefly recall in Section 2 the results already available for the linear array case. Then, in Section 3, these results are extended to the case of planar arrays radiating ring symmetric shaped patterns, which are of interest in a number of applications ranging from 'iso-flux' patterns as generated from geostationary satellites [10] to some 'toroidal' patterns.

Suggestions are also given for a simple implementation of the suggested strategies. Section 4 is then devoted to a comparison with recent results in the literature (especially from [4] and [7]), as well as to the synthesis of a number of patterns of actual interest. Conclusions follow.

2. A BRIEF SUMMARY OF SOME RESULTS IN THE OPTIMAL SYNTHESIS OF SHAPED BEAMS VIA LINEAR ARRAYS

In the case of a linear (equispaced) array of N antennas lying along the z axis, one may easily get a representation for the squared amplitude of the array factor as:

$$\mathcal{P}(u) = \sum_{p=-N+1}^{N-1} D_p e^{jpu} \tag{1}$$

where, because of the fact that $\mathcal{P}(u)$ is a real function, it is

$$D_p = D_{-p}^* \tag{2}$$

and $u = \beta d \cos(\theta)$, d and θ being respectively the uniform spacing amongst neighboring elements and the angle with respect to the array axis. As (1), by construction, is able to represent all possible power patterns radiated from the given array, a necessary condition for the existence of a field fulfilling given constraints on the power pattern is that the following system of functional linear inequalities in the variables D_p is satisfied:

$$\mathcal{P}(u) = \sum_{p=-N+1}^{N-1} D_p e^{jpu} \leq UB(u) \tag{3a}$$

$$\mathcal{P}(u) = \sum_{p=-N+1}^{N-1} D_p e^{jpu} \geq LB(u) \tag{3b}$$

$$D_p = D_{-p}^* \tag{3c}$$

$$\mathcal{P}(u) = \sum_{p=-N+1}^{N-1} D_p e^{jpu} \geq 0 \quad \forall u \tag{3d}$$

wherein functions $UB(u)$ and $LB(u)$ denote respectively the upper and lower bounds for the power pattern, and derive from the initial conditions on the pattern as a function of θ. It is worth noting that also the element factor can be eventually taken into account. Then, the condition (3d) is implied from the condition (3b) as long as the spacing is not smaller than half a wavelength. Note that, whenever the spacing is smaller than half a wavelength [so that the initial conditions on the pattern do not cover the periodicity interval $(-\pi, \pi)$ in terms of u], it turns out useful to enforce anyway some (upper bound) constraints on (1) in the overall periodicity interval in order to avoid 'superdirective' [11] arrays.

If we take into account the bandlimitedness of $\mathcal{P}(u)$ [12], Equation (3) can be substituted with a sufficiently fine discretization, so that (3) becomes a system of ordinary linear inequalities in the D_p variables [1]. The solvability of a system of linear inequalities is a well known problem, and it is equivalent to assess the existence of a 'feasible point' for a 'Linear Programming' problem [13]. As a consequence, consideration of the system derived from (3) allows to establish a priori (i.e., without solving the overall synthesis problem) whether the given design problem admits a solution or not. More precisely, the existence of a solution to (3) represents a necessary condition in order the overall problem to admit a solution. As such, it represents a *feasibility criterion* for the given design problem.

More interestingly, in the case of linear arrays, the satisfaction of the system deriving from (3) also constitutes a sufficient condition in order the overall problem to admit a solution. Also, a straightforward way exists to find the array excitations. In fact, by defining the auxiliary variable

$$z = e^{ju} \tag{4}$$

expression (1) can be expressed in terms of a polynomial of order $2N-2$. Also, by virtue of the fact that (1) gives rise, because of (2), to a real polynomial, one can also show that if z_i is a root of such a polynomial, then $1/z_i^*$ (where * means conjugation) is also a root of this polynomial. Then, as [see (3d)] $\mathcal{P}(u)$ is a real and positive semidefinite function, the roots of the kind $z_i = e^{ju_i}$ have an even multiplicity. As a consequence, using (3c) and (4), if $z_i = e^{ju_i}$, (1) can be expressed as

$$\mathcal{P}(u) = |D_{N-1}|^2 e^{j(1-N)u} \prod_{i=1}^{N-1} (z - z_i) \prod_{i=1}^{N-1} \left(z - \frac{1}{z_i^*} \right) \tag{5}$$

Then, by straightforward algebraic manipulations, (5) becomes

$$\mathcal{P}(u) = K^2 \prod_{i=1}^{N-1} (e^{ju} - z_i) \prod_{i=1}^{N-1} (e^{ju} - z_i)^* \tag{6}$$

where

$$K^2 = \frac{|D_{n-1}|^2 (-1)^{N-1}}{\prod_{i=1}^{N-1} z_i^*} \tag{7}$$

and K^2 is necessarily a real and positive quantity. By developing the two products and embedding a constant value K in each of them, (7) can be finally expressed as:

$$\mathcal{P}(u) = \mathcal{F}(u)\mathcal{F}^*(u) \tag{8}$$

wherein

$$\mathcal{F}(u) = \sum_{p=0}^{N-1} \mathcal{F}_p e^{jpu} \tag{9}$$

which can be regarded as the array factor of an N element array, and solves the overall synthesis problem. Note that, because the factorization (5) is not unique (as one can always exchange the position of two zeroes of the kind z_i and $1/z_i^*$), there exist 2^{N_0} distinct sets of coefficients able to do it, wherein $2N_0$ is the number of zeros of $\mathcal{P}(z)$ not belonging to the unitary circle (flipping of the other ones being unessential). This multiplicity of solutions may be useful to extract some advantageous characteristic, such as the solution presenting the minimum phase variation, or the minimum excitation dynamics, and so on.

Summarizing, the following synthesis strategy can be devised:

- By using the *feasibility criterion*, establish the minimum number of equispaced antennas which are needed to fulfill design constraints. To this end, amongst the many possibilities, the subroutine *fmincon* of MATLAB [14] may be used. As a byproduct, a polynomial representation [of the kind (1, 2)] will be obtained for the power pattern;
- Extract the zeroes of the polynomial entering the power mask. To this end, amongst the many possibilities, the subroutine *roots* of MATLAB [14] may be used;
- Use (5) in such a way that if z_i belongs to the first product, then $1/z_i^*$ belongs to the second product;
- Compute the polynomial expressions. Such a step could be performed by using a numerical library routine (such as *poly* of MATLAB [14]) or a point matching technique (i.e., equating the expression in terms of roots to the one in terms of excitations in a sufficiently dense grid of points).

  Flipping of the zeroes may be used to extract solutions which are particularly convenient in terms of excitation coefficients characteristics.

3. OPTIMAL SYNTHESIS OF RING SYMMETRIC SHAPED BEAMS BY MEANS OF PLANAR ARRAYS

In order to possibly extend, at least in a partial fashion, the above results to the planar case, let us consider an array on a rectangular grid in the xy plane, and let d_x, d_y be the regular spacing along the x and y direction respectively. Then, the usual spectral variables u and v can be defined as:

$$u = \beta d_x \sin\theta\cos\phi \quad v = \beta d_y \sin\theta\sin\phi \tag{10}$$

where β is the wavenumber, and θ, ϕ are the usual (angular) spherical coordinates.

A first obvious extension of the above theory is possible whenever the pattern one is looking for can be considered an u-v factorable pattern. In fact, the problem can be decomposed in two 1-D auxiliary problems along the main axes. In so doing, some care has to be anyway taken in defining the 1-D masks. More interestingly, an extension of the theory is also possible whenever ring symmetric shaped beams are looked for. In order to discuss such a case, let us consider the case wherein an equal (odd) number of elements, say $2N + 1$, are located along the x and y axes. Also, let us suppose the excitations are centrosymmetric, i.e.,

$$I_{n,m} = I_{n,-m} = I_{-n,m} = I_{-n,-m} \tag{11}$$

Then, the array factor of such an array can be conveniently written as:

$$\begin{aligned}\mathcal{F}(u,v) \;=\; & I_{0,0} + 2\sum_{n=1}^{N} I_{n,0}\cos(nu) + 2\sum_{m=1}^{N} I_{0,m}\cos(mv) \\ & +4\sum_{m=1}^{N}\sum_{n=1}^{N} I_{n,m}\cos(nu)\cos(mv)\end{aligned} \tag{12}$$

and, provided a certain number of excitations are assumed to be zero, (12) is still valid if the array has not a rectangular contour. The key step in the overall optimal synthesis strategy developed for the linear array case was that of using the properties of one dimensional polynomials. In order to exploit similar arguments, by taking inspiration from [15] one can consider the auxiliary variable w defined as

$$w = w_0 \cos\left(\frac{u'}{\sqrt{2}}\right)\cos\left(\frac{v'}{\sqrt{2}}\right) \tag{13}$$

where u' and v' are defined as:

$$u' = (u+v)\frac{\sqrt{2}}{2} \quad v' = (u-v)\frac{\sqrt{2}}{2} \tag{14}$$

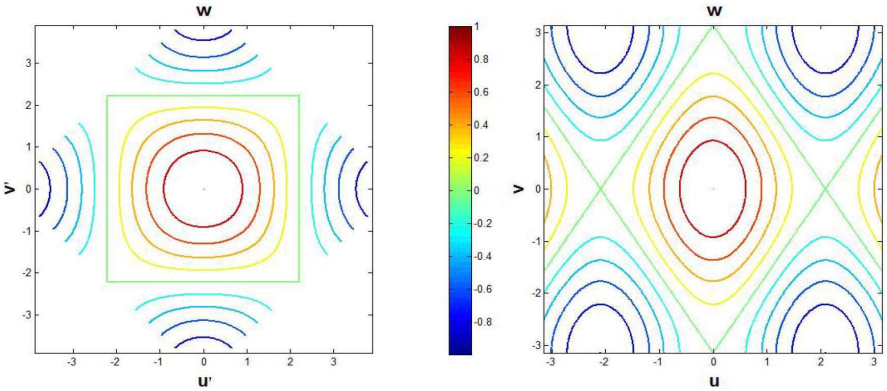

Figure 1. Behaviour of w: (left side) as a function of u' and v' for $w_0 = 1$ and $d_x = d_y = \lambda/2$; (right side) as a function of u and v for $w_0 = 1$, $d_y = \lambda/2$ and $d_x = 0.75\lambda$.

and correspond to a 45° rotation of the usual spectral variables u and v. Notably, in the spectral plane u, v (or, which is the same, in the plane u', v'), the equation

$$w = constant \tag{15}$$

defines a circle for sufficiently small values of u and v, and gently degenerates into a square of side w_0 for u' and v' approaching $\pi/\sqrt{2}$ (see Fig. 1 in the left side). Also, note that the circles correspond to ellipses in the more usual spherical coordinates u and v whenever $d_x \neq d_y$ (see Fig. 1 in the right side). If we consider a polynomial of degree N in the auxiliary variable w, i.e.,

$$\sum_{h=0}^{N} a_h w^h \tag{16}$$

one can prove that (16) can be interpreted as the array factor of a planar array. More precisely, it can be proved that (16) is representative of the array factor of a planar array (on a rectangular grid) having a rhombic shape. In fact, by virtue of (13) and (14), such a polynomial can be written as:

$$\sum_{h=0}^{N} a_h w_0^h \cos^h\left(\frac{u+v}{2}\right) \cos^h\left(\frac{u-v}{2}\right) \tag{17}$$

As (see [16])

$$\cos^h(\alpha) = \frac{1}{2^h} \sum_{k=0}^{h} \binom{h}{k} \cos[(h-2k)\alpha] \tag{18}$$

polynomial (7) is also equal to:

$$\sum_{h=0}^{N} a_h \frac{w_0^h}{2^{2h}} \left\{ \sum_{k=0}^{h} \binom{h}{k} \cos\left[(h-2k)\left(\frac{u+v}{2}\right)\right] \right. \\ \left. \sum_{p=0}^{h} \binom{h}{p} \cos\left[(h-2p)\left(\frac{u-v}{2}\right)\right] \right\} \tag{19}$$

Then, by using well known trigonometric identities, (19) can be finally written as:

$$\sum_{h=0}^{N} a_h \frac{w_0^h}{2^{2h+1}} \left\{ \sum_{k=0}^{h} \sum_{p=0}^{h} \left\{ \binom{h}{p} \binom{h}{k} \right. \right. \\ \left. \left. \cos[(h-k-p)u] \cos[(p-k)v] + \cos[(h-k-p)v] \cos[(p-k)u] \right\} \right\} \tag{20}$$

Then, by comparing expressions (12) and (20), one can conclude that (20) can be interpreted as the array factor of a planar array. In fact, as for any fixed value of h the sum of the indices respectively multiplying u and v never exceeds h (so that the sum of the indices never exceeds N), it can be concluded that (10) can be interpreted as the array factor of a planar array (on a rectangular grid) having a rhombic shape and $2N + 1$ elements along the two diagonals.

As a consequence of all the above, the same strategy already described for the linear array case can be adopted, i.e.,

- By using a polynomial of order $2N$, which is meant to represent the square amplitude of (13), establish the minimum number of equispaced antennas such that the (ring symmetric) power pattern design constraints can be fulfilled. As a byproduct, a polynomial representation will be obtained for the square amplitude of (16). Note that, opposite to the case of Section 2 where the fulfillment of such a condition is automatic for spacings of the order of half a wavelength (or anyway simple), some care has to be taken herein in order to enforce non negativity of this polynomial;
- Factorize the above polynomial representation. Because of the fact the polynomial is real and does not change sign, the roots will appear either as complex quantities (in complex conjugate pairs) or as real roots with even multiplicity;

- In analogy with (5) and (6), organize the power pattern representation in two different products of factors in such a way that if the root w_i belongs to the first product, then the root w_i^* belongs to the second product. The two terms will be complex conjugate each to the other;
- Find the coefficients of (16) by computing the polynomial representation of one of the two factors above. Then, the array excitation coefficients can be computed by means, e.g., of (20). These two steps can be more easily executed in a contemporary fashion by means of a point matching procedure (i.e., equating the expression in terms of roots to the one in terms of array excitations).

As in the case of linear arrays, factorization in two complex conjugate factors is not unique. As a consequence, some advantage could be again taken from a 'zero flipping' procedure in order to extract the most convenient set of excitations amongst the different possibilities (all producing the same power pattern).

4. NUMERICAL EXAMPLES

The procedure above only can be applied to the case wherein a ring pattern is in order [wherein ring shapes are determined from condition (10)]. Also, the procedure is limited to the case where an equal number of antennas is located along the x and y axes. This notwithstanding, a number of cases of practical interest do exist where the above procedure is of interest.

Both as a proof of concept and as a proof of usefulness, three different numerical experiments are considered in the following.

First, a comparison with recent contributions in the literature dealing with the synthesis of circularly symmetric shaped patterns is furnished. In particular, let us consider the problem, analyzed in both [4] and [7], of synthesizing a pattern having a flat-top shape in an angular region extending from $\theta = -10°$ up to $\theta = 10°$.

As underlined in [4], the synthesis of array radiating such a kind of pattern is of interest for the design of the single elements of Very Large Arrays for Deep Space Detection. Also, because of the fact that the Earth, as seen from a geostationary satellites, roughly corresponds to an angular region extending from $\theta = -8.6°$ up to $\theta = 8.6°$, the same kind of pattern (but for the compensation of geometrical attenuation, see below) is of interest for the synthesis of arrays radiating from geostationary satellites [10, 15].

By considering the same kind of pattern as in [4], and isotropic element patterns, the first step of the proposed synthesis procedure

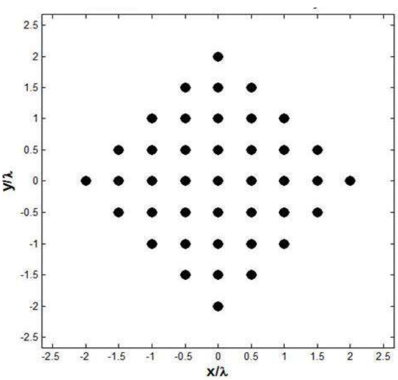

Figure 2. The 41-element array synthesized in the first test case.

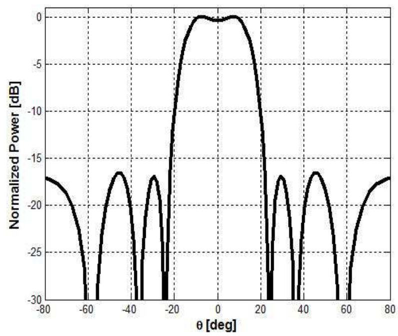

Figure 3. Power pattern synthesized in the first numerical experiment.

Table 1. Amplitude (in the top) and phase (in the bottom) of the excitations $I_{n,m}$ of the array shown in Fig. 2, corresponding to the radiation performances reported in Fig. 3.

n \ m	0	1	2	3	4
0	2.11	1.76	1.04	0.36	0.07
1		1.50	0.83	0.28	
2			0.43		
0	−3.11	0.03	−3.11	0.02	−3.11
1		−3.11	0.02	−3.11	
2			−3.11		

suggests that the minimum allowed value of N in order to fulfill the power mask is $N = 4$, which corresponds to a rhombic array composed by 41 antennas half a wavelength spaced (see Fig. 2). Then, the second step of the procedure (without any 'zero flipping' optimization) gives rise to the excitations of Table 1, corresponding to the circularly symmetric pattern shown in Fig. 3. In particular, only a part of the synthesized currents is shown, and the remaining values can be easily determined by exploiting (11) and the fact that $I_{n,m} = I_{m,n}$. Notably, the Peak Sidelobe Level (-16.7 dB), the maximal ripple (0.4 dB, in the shaped-beam region), and also the number of radiating elements have been significantly reduced with respect to the final result reported in [4] (wherein, however, an actual element pattern is used). With respect to the results shown in [7], one can notice that the present method provides very similar radiation performances by exploiting roughly the 50% of the antennas. Finally, the fact that isotropic elements are used herein suggests that some improvements will be possible with respect to [4] even when actual element patterns will be considered (see also Conclusions).

As a second example, let us consider the problem of synthesizing a pattern for an array antenna to be mounted on a Geostationary Earth Orbit (GEO) satellite such that the same amount of power density has to be realized on each (visible) portion of the Earth. This kind of pattern is known as 'iso-flux' pattern. In such a design problem, when stating the power mask to adopt in the synthesis procedure, one has to take into account the Earth curvature as seen from the satellite. In fact, in order to compensate the power attenuation on the planet border [which is due to a (5900 km) longer path], a depression of 1.3 dB is required at center of the coverage zone. Similar reasonings have to be exploited for the case of Medium Earth Orbit (MEO) satellites. Notably, as long as the satellite height is sufficiently large (so that the maximum angle of the Earth cone as seen from satellite is sufficiently small) transformation (13) guarantees circular symmetry in the shaped zone (see Fig. 1), so that the proposed synthesis approach can be safely adopted. In particular, by considering the GEO satellites case and adopting the power pattern mask shown in Fig. 4, the feasibility criterion states that a value $N = 5$ is required in order

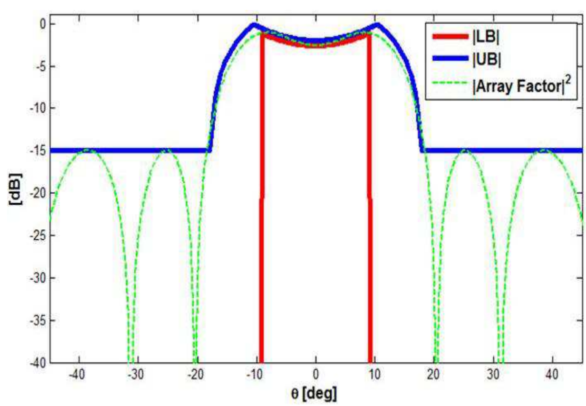

Figure 4. Iso-flux pattern (synthesized by means of a 61-element array) covering the earth from GEO satellites. The adopted power pattern mask is also reported.

Table 2. Amplitude (in the top) and phase (in the bottom) of the excitations $I_{n,m}$ corresponding to the radiation performances reported in Fig. 4.

$n \backslash m$	0	1	2	3	4	5
0	2.0468	1.6225	0.8375	0.3256	0.0930	0.0155
1		1.2836	0.6694	0.2625	0.0781	
2			0.3667	0.1573		
0	1.6493	−1.5835	1.2944	−2.3064	0.4629	−2.8352
1		1.4724	−1.9987	0.6898	−2.8361	
2			0.7542	−2.8430		

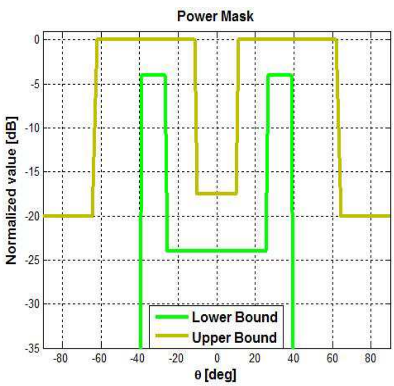

Figure 5. Power pattern mask adopted for the third test case.

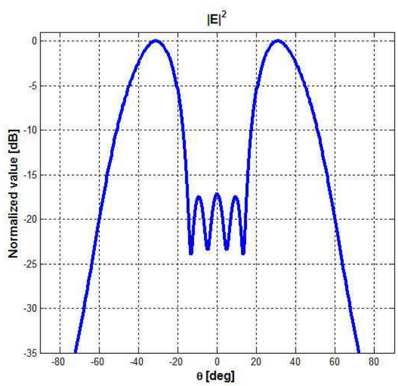

Figure 6. Power pattern corresponding to the excitations of Table 3 (u-cut through main beam power pattern).

to fulfill constraints. Then, the subsequent synthesis step furnishes the pattern of Fig. 4 and the excitations of Table 2 (wherein the condition $I_{n,m} = I_{m,n} = I_{n,-m} = I_{-n,m} = I_{-n-m}$ has been exploited in order to show the coefficients in a suitable fashion). The synthesized rhombic array is composed by 61 isotropic elements half a wavelength spaced. Notably, for all the reasonings above, no array with the same structure and spacings can fulfill the given constraints with a lower number of radiators.

As a third and last example, which is of interest in the ground segment of some satellite communication links, let us consider the

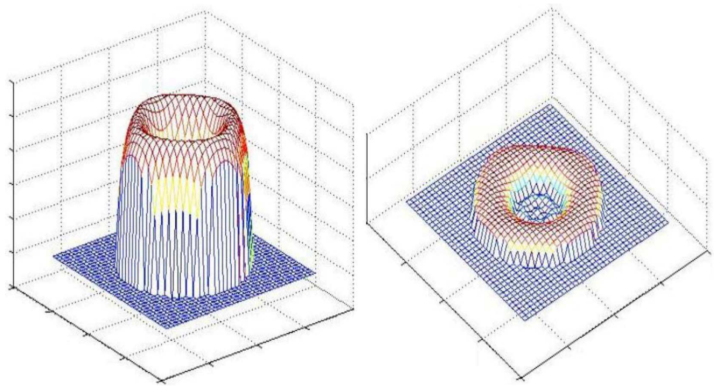

Figure 7. Pictorial main-beam-view of the power pattern shown in Fig. 6.

Table 3. Amplitude (in the top) and phase (in the bottom) of the excitations $I_{n,m}$ corresponding to the radiation performances reported in Fig. 6 and in Fig. 7.

n \ m	0	1	2	3	4	5	6
0	3.2639	4.5642	3.8692	2.6774	1.3519	0.5222	0.1255
1		3.9545	2.7745	1.6696	0.7587	0.2770	
2			2.7258	2.3025	1.2976		
3				1.9553			
0	2.7636	−1.4644	0.8281	2.8558	−1.4874	0.3137	1.9445
1		1.1676	−2.6558	−0.3562	1.7703	−2.1912	
2			0.2751	2.4690	−1.8190		
3				−1.7751			

case wherein a toroidal beam pattern is required. Amongst the many cases, such a kind of pattern is of interest whenever one requires that an antenna on (the roof of) a vehicle has to connect with a GEO satellite in a way independent from the vehicle orientation, and avoiding the need of phasing the excitations. In fact, the toroidal shaped pattern, adjusted according to the latitude of the vehicle with respect to the one of the satellite, will provide better directivity performances with respect to an almost isotropic pattern. Obviously, a toroidal pattern is also of interest whenever a relatively large pattern is required while a particular direction (or region) is not of interest (or has to be avoided). As an example for this class of problems, let us consider the case wherein the power pattern mask is given as in Fig. 5. Then, by supposing the antennas are half a wavelength spaced (i.e., $d_x = d_y = \lambda/2$), the application of the feasibility criterion suggested that the minimum value of N such to fulfill constraints is given by $N = 6$. Then, by applying the synthesis procedure as developed in Section 3, an 85-elements array has been obtained. The power pattern ($\cos\theta$ element factor embedded) reported in Fig. 6 and in Fig. 7 has been finally achieved.

The amplitude and the phase of the excitations of such an array are reported in Table 3. Again, only a part of the synthesized currents is shown due to their symmetrical behaviour. Also, note that no zero flipping procedure has been exploited in order to optimize excitations.

5. CONCLUSIONS

Opposite to a large body of literature wherein a number of different global optimization procedures are used to synthesize shaped patterns by means of uniformly spaced arrays [4–8], deterministic and straightforward synthesis procedures have been discussed herein. In particular, in case of linear array a not very well known synthesis strategy, already available since [1], has been briefly reviewed and discussed. Then, the strategy and the corresponding procedures have been extended to the case of ring shaped patterns generated by planar arrays with a rhombic shape. Although the strategy does not apply to generic planar arrays, it has been evidenced in the numerical analysis that it applies indeed to a number of problems of actual interest, to which it furnishes in a fast and effective fashion solutions which cannot be furtherly ameliorated. In the authors' view, the solutions provided by such an approach can be of interest in many other ways.

First, they can act as a benchmark for other more general synthesis procedures.

Second, they can act as an effective starting guess for problems wherein one or more of the assumptions (rhombic shape, quadrantal simmetry, equal number of antennas along the x and y axes) are removed. In this case, advantage could be taken from the fact that through zero flipping procedures the proposed strategy makes all the different solutions to the canonical case at hand readily available, so that an exploitation of multiplicity can be of help. Obviously, the availability of a whole set of possible solutions also allows a better control of possible mutual coupling effects arising when considering actual (rather than ideal) radiating elements.

Last but not least, in analogy with the Woodward Lawson technique [17], the approach could be used to synthesize more complex footprints by a composition of simpler (this time, shaped) patterns.

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