

FOCAL REGION FIELDS OF GREGORIAN SYSTEM PLACED IN HOMOGENEOUS CHIRAL MEDIUM

M. Q. Mehmood, M. J. Mughal, and T. Rahim

Faculty of Electronic Engineering
GIK Institute of Engineering Sciences and Technology
Topi, Swabi, 23640, N.W.F.P., Pakistan

Abstract—This work presents the derivation of high frequency electromagnetic field expressions for two dimensional Gregorian system embedded in a chiral medium. Two cases have been analyzed. Firstly, the chirality parameter is adjusted to support positive phase velocity (PPV) for both left circularly polarized (LCP) and right circularly polarized (RCP) modes traveling in the medium. Secondly, the chirality is adjusted in such a way that one mode travels with PPV and other with negative phase velocity (NPV). Method proposed by Maslov is used, for finding the field expressions, to overcome the problem of Geometrical Optics (GO) because GO fails at caustics. The results for both the cases are given in the paper.

1. INTRODUCTION

A chiral medium is macroscopically continuous medium composed of equivalent chiral objects, uniformly distributed and randomly oriented. A chiral object is a three dimensional body that cannot be brought into congruence with its mirror image through any translation and rotation. Chiral medium supports both LCP and RCP modes [1]. It may supports NPV propagation for both modes, or NPV for one mode and PPV for the other modes [2]. NPV mediums are those, in which direction of power flow is opposite to the direction of phase velocity [3]. Conditions for chiral medium to support NPV propagation have been derived in [4]. In the present work we have embedded the Gregorian reflector in chiral medium. Placing a Gregorian Reflector in chiral medium have many advantages over ordinary medium due to unique characteristics of chiral medium like polarization control, impedance matching and cross coupling of electric and magnetic fields.

Corresponding author: M. Q. Mehmood (qasim145ps@gmail.com).

By changing the chiral media parameters ω , μ , β the desired value of the wave impedance and propagation constant can be achieved. In particular, reflections can be adjusted (decreased or increased) desirably. In this respect, the chiral medium can be controlled by variations of three parameters ω , μ , β , whereas an achiral medium has only two variable parameters, ω , μ . Moreover, in case of negative reflection caused by NPV, it also gives the advantage of invisibility. We have considered the two cases, in the first case chiral medium supports PPV for both LCP and RCP modes. In the second case chiral medium supporting PPV for one mode and NPV for other mode is taken into account. As GO fails in the focal regions, so Maslov's method is used to study the fields at the focal regions [5, 6]. Maslov's method combines the simplicity of asymptotic ray theory and the generality of the Fourier transform method. This is achieved by representing the geometrical optics fields in hybrid coordinates consisting of space coordinates, and wave vector coordinates, that is by representing the field in terms of six coordinates. It may be noted that information of ray trajectories is included in both space coordinates and wave vector coordinates. Solving the Hamiltonian equations under the prescribed initial conditions, one can construct the geometrical optics field in space R , which is valid except in the vicinity of focal point. Near the focal point, the expression for the geometrical optics field in spatial coordinates is rewritten in hybrid domain. The expression in hybrid domain is related to the original domain R through the asymptotic Fourier transform. The reason for considering the hybrid domain is that, in general the singularities in different domain do not coincides. This means that a domain always exist in which the solution is bounded [11]. Analysis of focusing systems has been worked out by various authors using Maslov's method [7–16]. In this paper our goal is to find the focal region fields of two dimensional Gregorian reflector placed in chiral medium. Section 2 is about the receiving characteristics of two dimensional Gregorian reflector placed in chiral medium for both $k\beta < 1$ and $k\beta > 1$. In Section 3 results and discussions are given and Section 4 is about conclusions in the light of results given in Section 3.

2. GEOMETRICAL OPTICS FIELDS OF TWO DIMENSIONAL GREGORIAN REFLECTOR PLACED IN CHIRAL MEDIUM

The reflection of plane waves, from Perfect Electric Conductor (PEC), traveling in chiral has been considered in [17]. When both LCP and RCP hit PEC plane boundary there are four reflected waves. The

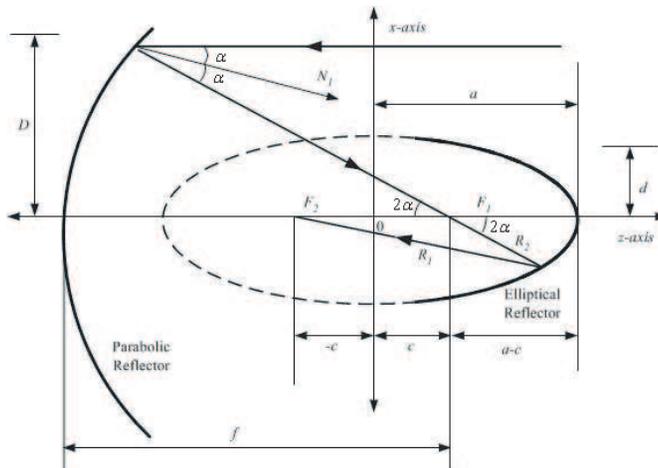


Figure 1. Gregorian system.

system in our problem is Gregorian which consists of two reflectors, one is parabolic main reflector and other is elliptical sub reflector as shown in Fig. 1. Here we will consider the receiving characteristics of this system. Both RCP and LCP waves are incident on main parabolic reflector, it will cause four reflected waves designated as LL, RR, LR and RL [18]. These four waves are then reflected from the secondary elliptical subreflector and will cause eight reflected waves designated as LLL, RRR, LLR, RRL, RLR, RLL, LRR and LRL. Only four of these waves (LLL, RRR, LLR, RRL) will converge in the focal region while other four waves (RLR, RLL, LRR, LRL) will diverge. In this paper we are considering only four converging rays after reflection from elliptical subreflector as shown in Fig. 2. Quantities designated as RRR and RRL are RCP and LCP reflected wave components, respectively, when RCP is incident. Quantities designated as LLL and LLR are LCP and RCP reflected waves, respectively, when LCP is incident wave.

In the second case when chirality parameter $k\beta > 1$. It causes $n_1 = \frac{1}{1-k\beta} < 0$ and $n_2 = \frac{1}{1+k\beta} > 0$, so LCP wave travels with NPV and RCP wave with PPV. For $k\beta < -1$ RCP wave travels with NPV and LCP wave with PPV. We have depicted here the case of $k\beta > 1$ only because for $k\beta < -1$, we can get the solutions from $k\beta > 1$ by interchanging the role of LCP and RCP modes [19]. Gregorian system for $k\beta > 1$ is shown in Fig. 3. LLL and RRR are reflected at the same angle while RRL and LLR have different response. It can be seen that only three rays are contributing to the focus in this case. while LLR is divergent.

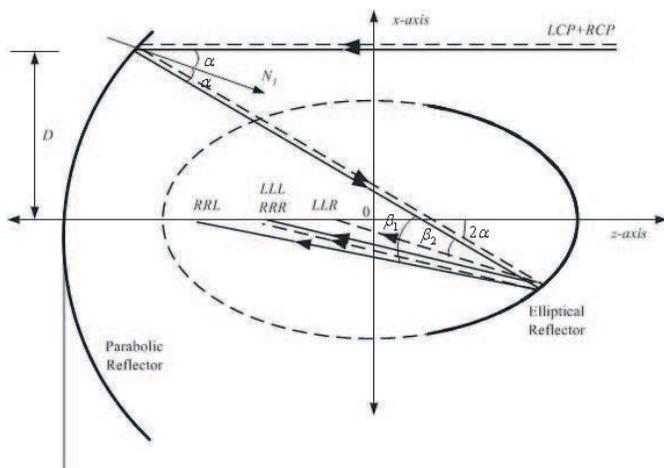


Figure 2. Gregorian system in chiral medium.

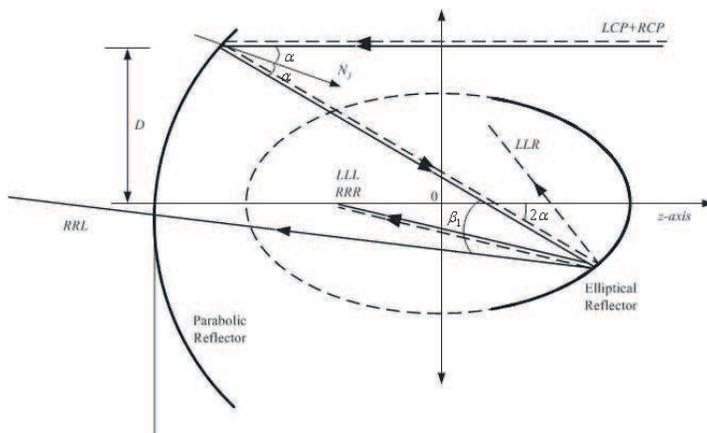


Figure 3. Gregorian system in chiral medium, $k\beta > 1$.

Equations for parabolic and elliptical reflector of Gregorian system are given by

$$\zeta_1 = \frac{\xi_1^2}{4f} - f + c \tag{1}$$

$$\zeta_2 = a \left[1 - \frac{\xi_2^2}{b^2} \right]^{\frac{1}{2}} \tag{2}$$

$$c^2 = a^2 - b^2 \tag{3}$$

where (ξ_1, ζ_1) and (ξ_2, ζ_2) are the Cartesian coordinates of the points on the parabolic and elliptical reflectors, respectively. Incident waves on main parabolic reflector having unit amplitude are given by.

$$Q_L = \exp(jkn_1z), \quad Q_R = \exp(jkn_2z) \quad (4)$$

Consider the case of normal incidence such that these waves are incident at angle α with surface normal \vec{N}_1 as shown in Figs. 1–3. The wave vectors of the waves reflected by the parabolic cylinder are given by [18].

$$\vec{P}_{LL} = -n_1 \sin 2\alpha \hat{i}_x + n_1 \cos 2\alpha \hat{i}_z \quad (5)$$

$$\vec{P}_{RR} = -n_2 \sin 2\alpha \hat{i}_x + n_2 \cos 2\alpha \hat{i}_z \quad (6)$$

$$\begin{aligned} \vec{P}_{RL} = & -n_1 \sin \left[\alpha + \sin^{-1} \left(\frac{n_2}{n_1} \sin \alpha \right) \right] \hat{i}_x \\ & + n_1 \cos \left[\alpha + \sin^{-1} \left(\frac{n_2}{n_1} \sin \alpha \right) \right] \hat{i}_z \end{aligned} \quad (7)$$

$$\begin{aligned} \vec{P}_{LR} = & -n_2 \sin \left[\alpha + \sin^{-1} \left(\frac{n_1}{n_2} \sin \alpha \right) \right] \hat{i}_x \\ & + n_2 \cos \left[\alpha + \sin^{-1} \left(\frac{n_1}{n_2} \sin \alpha \right) \right] \hat{i}_z \end{aligned} \quad (8)$$

Now we will take two, RR and LL, waves that will incident on elliptical subreflector and converge as well after reflection. Initial amplitudes of these two waves are as following [18].

$$A_{0LL} = \frac{\cos \alpha - \cos \alpha_2}{\cos \alpha + \cos \alpha_2} \quad (9)$$

$$A_{0RR} = \frac{\cos \alpha - \cos \alpha_1}{\cos \alpha + \cos \alpha_1} \quad (10)$$

where

$$\sin \alpha = \frac{\xi_1}{\sqrt{\xi_1^2 + 4f^2}} \quad (11)$$

$$\cos \alpha = \frac{2f}{\sqrt{\xi_1^2 + 4f^2}} \quad (12)$$

$$\vec{N}_1 = -\sin \alpha \hat{i}_x + \cos \alpha \hat{i}_z \quad (13)$$

The wave vectors of the waves reflected by the elliptical subreflector

are as following.

$$\overrightarrow{P_{LLL}} = -n_1 \sin(2\alpha - 2\psi) \hat{i}_x - n_1 \cos(2\alpha - 2\psi) \hat{i}_z \quad (14)$$

$$\overrightarrow{P_{RRR}} = -n_2 \sin(2\alpha - 2\psi) \hat{i}_x - n_2 \cos(2\alpha - 2\psi) \hat{i}_z \quad (15)$$

$$\overrightarrow{P_{RRL}} = -n_1 \sin(\beta_1 - \psi) \hat{i}_x - n_1 \cos(\beta_1 - \psi) \hat{i}_z \quad (16)$$

$$\overrightarrow{P_{LLR}} = -n_2 \sin(\beta_2 - \psi) \hat{i}_x - n_2 \cos(\beta_2 - \psi) \hat{i}_z \quad (17)$$

Corresponding initial amplitudes for these four rays are

$$A_{0LLL} = \left[\frac{\cos \alpha - \cos \alpha_2}{\cos \alpha + \cos \alpha_2} \right] \left[\frac{\cos \beta - \cos \beta_2}{\cos \beta + \cos \beta_2} \right] \quad (18)$$

$$A_{0RRR} = \left[\frac{\cos \alpha - \cos \alpha_1}{\cos \alpha + \cos \alpha_1} \right] \left[\frac{\cos \beta - \cos \beta_1}{\cos \beta + \cos \beta_1} \right] \quad (19)$$

$$A_{0RRL} = \left[\frac{\cos \alpha - \cos \alpha_1}{\cos \alpha + \cos \alpha_1} \right] \left[\frac{2 \cos \beta}{\cos \beta + \cos \beta_1} \right] \quad (20)$$

$$A_{0LLR} = \left[\frac{\cos \alpha - \cos \alpha_2}{\cos \alpha + \cos \alpha_2} \right] \left[\frac{2 \cos \beta}{\cos \beta + \cos \beta_2} \right] \quad (21)$$

and the corresponding initial phases are

$$S_{0LLL} = -n_1 \zeta_1 = n_1 \left[2f \frac{\cos 2\alpha}{1 + \cos 2\alpha} - c \right] \quad (22)$$

$$S_{0RRR} = -n_2 \zeta_1 = n_2 \left[2f \frac{\cos 2\alpha}{1 + \cos 2\alpha} - c \right] \quad (23)$$

$$S_{0RRL} = -n_2 \zeta_1 = n_2 \left[2f \frac{\cos 2\alpha}{1 + \cos 2\alpha} - c \right] \quad (24)$$

$$S_{0LLR} = -n_1 \zeta_1 = n_1 \left[2f \frac{\cos 2\alpha}{1 + \cos 2\alpha} - c \right] \quad (25)$$

where

$$\beta_1 = \sin^{-1} \left(\frac{n_2}{n_1} \sin \beta \right) \quad (26)$$

$$\beta_2 = \sin^{-1} \left(\frac{n_1}{n_2} \sin \beta \right) \quad (27)$$

$$\beta = (2\alpha - \psi) \quad (28)$$

$$\sin \psi = \frac{-1}{\sqrt{R_1 R_2}} \frac{a}{b} \xi_2 \quad (29)$$

$$\cos \psi = \frac{1}{\sqrt{R_1 R_2}} \frac{b}{a} \zeta_2 \quad (30)$$

$$\vec{N}_2 = -\sin \psi \hat{i}_x + \cos \psi \hat{i}_z \quad (31)$$

In the above equation R_1 and R_2 are the distances from the point (ξ_2, ζ_2) to the focal points $z = -c$ and $z = c$ respectively with $c^2 = a^2 - b^2$. The cartesian coordinates of the ray reflected by the elliptical cylinder is given by.

$$x_{LLL} = \xi_2 + P_{xLLL}t = \xi_1 + P_{xLL}t_1 + P_{xLLL}t \tag{32}$$

$$x_{RRR} = \xi_2 + P_{xRRR}t = \xi_1 + P_{xRR}t_1 + P_{xRRR}t \tag{33}$$

$$x_{RRL} = \xi_2 + P_{xRRL}t = \xi_1 + P_{xRR}t_1 + P_{xRRL}t \tag{34}$$

$$x_{LLR} = \xi_2 + P_{xLLR}t = \xi_1 + P_{xLL}t_1 + P_{xLLR}t \tag{35}$$

$$z_{LLL} = \zeta_2 + P_{zLLL}t = \zeta_1 + P_{zLL}t_1 + P_{zLLL}t \tag{36}$$

$$z_{RRR} = \zeta_2 + P_{zRRR}t = \zeta_1 + P_{zRR}t_1 + P_{zRRR}t \tag{37}$$

$$z_{RRL} = \zeta_2 + P_{zRRL}t = \zeta_1 + P_{zRR}t_1 + P_{zRRL}t \tag{38}$$

$$z_{LLR} = \zeta_2 + P_{zLLR}t = \zeta_1 + P_{zLL}t_1 + P_{zLLR}t \tag{39}$$

where

$$t_1 = \sqrt{(\xi_2 - \xi_1)^2 + (\zeta_2 - \zeta_1)^2} \tag{40}$$

$$t = \sqrt{(x - \xi_2)^2 + (z - \zeta_2)^2} \tag{41}$$

After transforming cartesian coordinates (x, z) to the ray fixed coordinates (ξ_1, t) and then finding the jacobian transform we will get following expressions.

$$J_{LLL} = 1 - \frac{n_1 t}{R_1} \tag{42}$$

$$J_{RRR} = 1 - \frac{n_2 t}{R_1} \tag{43}$$

$$J_{RRL} = 1 - \frac{n_1 t \cos \beta}{R_2 \cos \beta_1} \times \left[\frac{n_2 R_1 \cos \beta - a \left\{ \sqrt{n_1^2 - n_2^2 \sin^2 \beta} + n_2 \cos \beta \right\}}{R_1 \sqrt{n_1^2 - n_2^2 \sin^2 \beta}} \right] \tag{44}$$

$$J_{LLR} = 1 - \frac{n_2 t \cos \beta}{R_2 \cos \beta_2} \times \left[\frac{n_1 R_1 \cos \beta - a \left\{ \sqrt{n_2^2 - n_1^2 \sin^2 \beta} + n_1 \cos \beta \right\}}{R_1 \sqrt{n_2^2 - n_1^2 \sin^2 \beta}} \right] \tag{45}$$

The GO field for each ray can now be written as

$$U(r)_{LLL} = A_{0LLL}(\xi) [J_{LLL}]^{-1/2} \times \exp[-jk(S_{0LLL} + n_1^2 t + n_1 t_1)] \quad (46)$$

$$U(r)_{RRR} = A_{0RRR}(\xi) [J_{RRR}]^{-1/2} \times \exp[-jk(S_{0RRR} + n_2^2 t + n_2 t_1)] \quad (47)$$

$$U(r)_{RRL} = A_{0RRL}(\xi) [J_{RRL}]^{-1/2} \times \exp[-jk(S_{0RRL} + n_1^2 t + n_1 t_1)] \quad (48)$$

$$U(r)_{LLR} = A_{0LLR}(\xi) [J_{LLR}]^{-1/2} \times \exp[-jk(S_{0LLR} + n_2^2 t + n_2 t_1)] \quad (49)$$

where $A_0(\xi)$ and $S_0(\xi)$ are the initial phases and amplitudes. Their expressions are given in Eqs. (18)–(25).

Since GO becomes infinite at caustics, so we find approximate field at the caustics by Maslov's method. To calculate the field at caustic we need expression $J(t) \frac{\partial P_z}{\partial z}$ for all four rays, reflected from elliptical subreflector, which are found below.

$$\left[J(t)_{LLL} \frac{\partial P_{zLLL}}{\partial z} \right] = \frac{n_1 \sin^2(2\alpha - 2\psi)}{R_1} \quad (50)$$

$$\left[J(t)_{RRR} \frac{\partial P_{zRRR}}{\partial z} \right] = \frac{n_2 \sin^2(2\alpha - 2\psi)}{R_1} \quad (51)$$

$$\left[J(t)_{RRL} \frac{\partial P_{zRRL}}{\partial z} \right] = \left[\frac{n_1^2 \sin^2(\beta_1 - \psi) \cos^2 \beta}{\cos \beta_1 \sqrt{n_1^2 - n_2^2 \sin^2 \beta}} \right] \left[\frac{n_2 b R_1 - a \sqrt{(R_1 R_2)(n_1^2 - n_2^2 \sin^2 \beta)} - n_2 a b}{b R_1 R_2} \right] \quad (52)$$

$$\left[J(t)_{LLR} \frac{\partial P_{zLLR}}{\partial z} \right] = \left[\frac{n_2^2 \sin^2(\beta_2 - \psi) \cos^2 \beta}{\cos \beta_2 \sqrt{n_2^2 - n_1^2 \sin^2 \beta}} \right] \left[\frac{n_1 b R_1 - a \sqrt{(R_1 R_2)(n_2^2 - n_1^2 \sin^2 \beta)} - n_1 a b}{b R_1 R_2} \right] \quad (53)$$

The phase functions are given by

$$S_{LLL} = S_{0LLL} + n_1 t_1 + n_1^2 t - z(x, P_{zLLL})P_{zLLL} + zP_{zLLL} \quad (54)$$

$$S_{RRR} = S_{0RRR} + n_2 t_1 + n_2^2 t - z(x, P_{zRRR})P_{zRRR} + zP_{zRRR} \quad (55)$$

$$S_{RRL} = S_{0RRL} + n_1 t_1 + n_1^2 t - z(x, P_{zRRL})P_{zRRL} + zP_{zRRL} \quad (56)$$

$$S_{LLR} = S_{0LLR} + n_2 t_1 + n_2^2 t - z(x, P_{zLLR})P_{zLLR} + zP_{zLLR} \quad (57)$$

In these phase functions S_0 and t_1 are given above. While the extra terms are given by

$$\begin{aligned} S_{exLLL} &= n_1^2 t - z(x, P_{zLLL})P_{zLLL} + zP_{zLLL} \\ &= n_1^2 t - (\zeta_2 + P_{zLLL}t)P_{zLLL} + zP_{zLLL} \\ &= (z - \zeta_2)P_{zLLL} + P_{xLLL}^2 t \\ &= (z - \zeta_2)P_{zLLL} + (x - \xi_2)P_{xLLL} \\ &= n_1 [-x \sin(2\alpha - 2\psi) - z \cos(2\alpha - 2\psi)] \\ &\quad + n_1 [\xi_2 \sin(2\alpha - 2\psi) + \zeta_2 \cos(2\alpha - 2\psi)] \end{aligned} \quad (58)$$

Similarly

$$\begin{aligned} S_{exRRR} &= n_2 [-x \sin(2\alpha - 2\psi) - z \cos(2\alpha - 2\psi)] \\ &\quad + n_2 [\xi_2 \sin(2\alpha - 2\psi) + \zeta_2 \cos(2\alpha - 2\psi)] \end{aligned} \quad (59)$$

$$\begin{aligned} S_{exRRL} &= n_1 [-x \sin(\beta_1 - \psi) - z \cos(\beta_1 - \psi)] \\ &\quad + n_1 [\xi_2 \sin(\beta_1 - \psi) + \zeta_2 \cos(\beta_1 - \psi)] \end{aligned} \quad (60)$$

$$\begin{aligned} S_{exLLR} &= n_2 [-x \sin(\beta_2 - \psi) - z \cos(\beta_2 - \psi)] \\ &\quad + n_2 [\xi_2 \sin(\beta_2 - \psi) + \zeta_2 \cos(\beta_2 - \psi)] \end{aligned} \quad (61)$$

After substituting all the required parameters and simplifying them we will get the following final expressions at caustics.

$$\begin{aligned} U(r)_{LLL} &= \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0LLL}(\xi) \sqrt{R_1} \\ &\quad \times \exp[-jk\{S_{0LLL} + n_1 t_1 + S_{exLLL}\}] d(2\alpha) \end{aligned} \quad (62)$$

$$\begin{aligned} U(r)_{RRR} &= \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0RRR}(\xi) \sqrt{R_1} \\ &\quad \times \exp[-jk\{S_{0RRR} + n_2 t_1 + S_{exRRR}\}] d(2\alpha) \end{aligned} \quad (63)$$

$$\begin{aligned}
U(r)_{RRL} = & \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0RRL}(\xi) \frac{1}{\sqrt{n_1^2 - n_2^2 \sin^2 \beta}} \\
& \times \left[\frac{R_1 R_2 b n_2 \cos \beta_1}{abn_2 + a\sqrt{(R_1 R_2)(n_1^2 - n_2^2 \sin^2 \beta)} - bn_2 R_1} \right]^{-1/2} \\
& \times \exp[-jk\{S_{0RRL} + n_1 t_1 + S_{exRRL}\}] d(2\alpha) \quad (64)
\end{aligned}$$

$$\begin{aligned}
U(r)_{LLR} = & \sqrt{\frac{k}{2j\pi}} \left[\int_{A_1}^{A_2} + \int_{-A_1}^{-A_2} \right] A_{0LLR}(\xi) \frac{1}{\sqrt{n_2^2 - n_1^2 \sin^2 \beta}} \\
& \times \left[\frac{R_1 R_2 b n_1 \cos \beta_2}{abn_1 + a\sqrt{(R_1 R_2)(n_2^2 - n_1^2 \sin^2 \beta)} - bn_1 R_1} \right]^{-1/2} \\
& \times \exp[-jk\{S_{0LLR} + n_2 t_1 + S_{exLLR}\}] d(2\alpha) \quad (65)
\end{aligned}$$

Eqs. (62)–(65) are the field expressions at caustics and found using Maslov's method.

3. RESULTS AND DISCUSSIONS

Field pattern around the caustic of a Gregorian system are determined using Eqs. (62)–(65) by performing the integration numerically. Values of the different parameters are: $kf = 125$, $ka = 80$, $kb = 70$, $kd = 70$, $kD = 110$. Limits of integration are selected using the following relations [20].

$$A_1 = 2 \tan^{-1} \left(\frac{D}{2f} \right) \quad (66)$$

$$A_2 = \tan^{-1} \left(\frac{d}{2c} \right) \quad (67)$$

Variations in magnitude of the fields are shown versus kz in Fig. 4 to Fig. 10. Equations of caustics for U_{LLL} and U_{RRR} are given by Eq. (62) and Eq. (63). These are similar as in the case ordinary medium [20]. LLL and RRR coincide for all values of $k\beta$. As the value of $k\beta$ increases, magnitude of the field around caustic increases. This behavior is depicted in Fig. 4 and Fig. 5. For $k\beta = 0$, $n_1 = n_2 = 1$ and

$$U_{LLL} = U_{RRR} = 0 \quad (68)$$

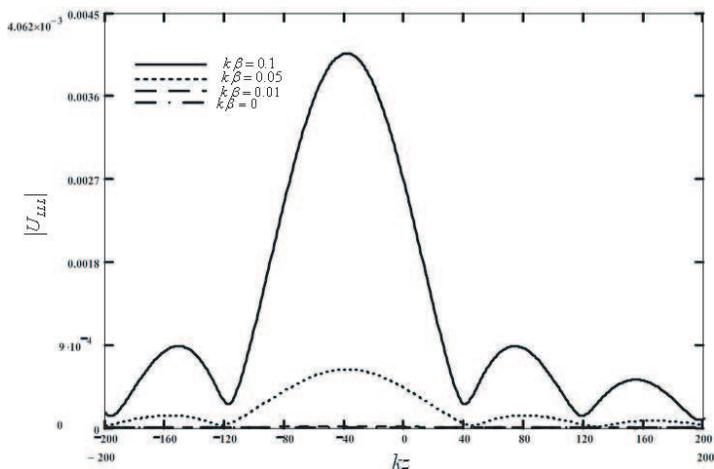


Figure 4. $|U_{LLL}|$ of Gregorian system at $kx = 0$, $k\beta = 0, 0.01, 0.05, 0.1$.

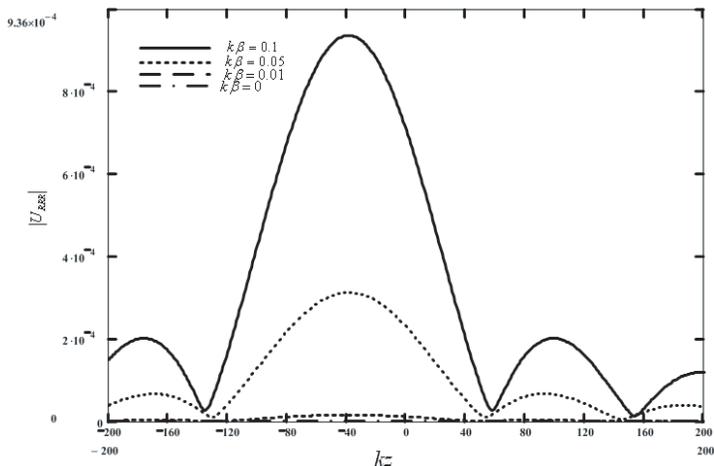


Figure 5. $|U_{RRR}|$ of Gregorian system at $kx = 0$, $k\beta = 0, 0.01, 0.05, 0.1$.

Equations of caustics of RRL and LLR rays are given by Eq. (64) and Eq. (65). With the increase in value of chirality parameter $k\beta$, the gap between the focal points of RRL and LLR rays increases as shown in Fig. 6 and Fig. 7. For $k\beta = 0$, $n_1 = n_2 = 1$ and

$$U_{RRL} = U_{LLR} = 0 \tag{69}$$

Eq. (68) and Eq. (69) justifies the fact that for $k\beta = 0$, LL and RR rays

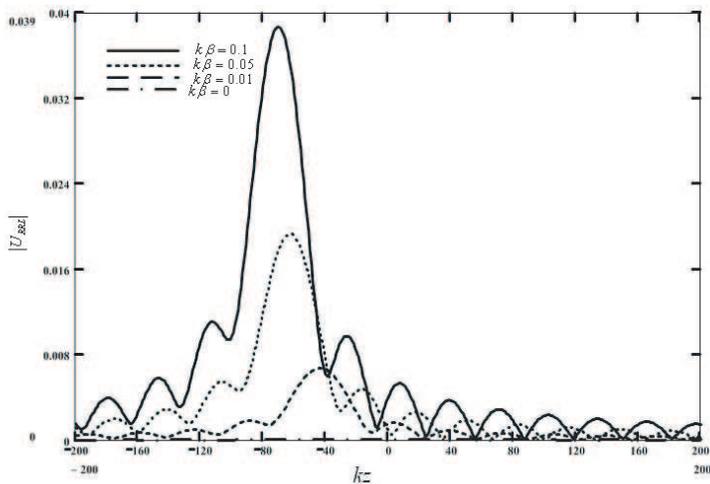


Figure 6. $|U_{RRL}|$ of Gregorian system at $kx = 0$, $k\beta = 0, 0.01, 0.05, 0.1$.

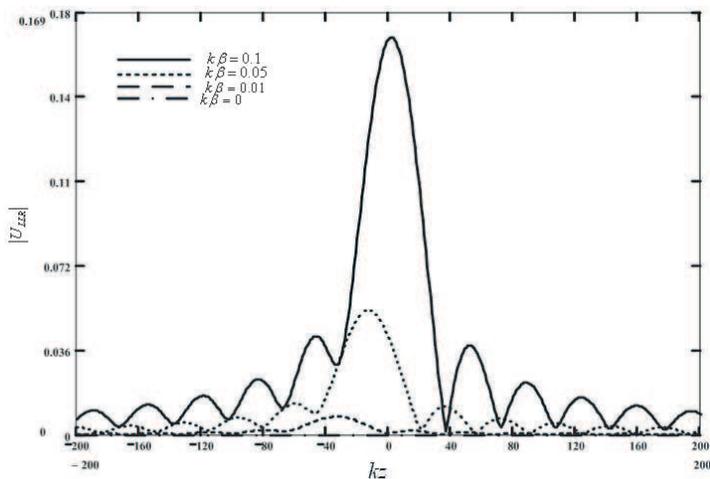


Figure 7. $|U_{LLR}|$ of Gregorian system at $kx = 0$, $k\beta = 0, 0.01, 0.05, 0.1$.

goes to zero. So its quite obvious that LLL, RRR, LLR and RRL will also be zero for zero chirality. While other four rays LRL, LRR, RLR, RRL, caused due the incidence of RL and LR will be like ordinary medium waves [20] for this zero chirality case.

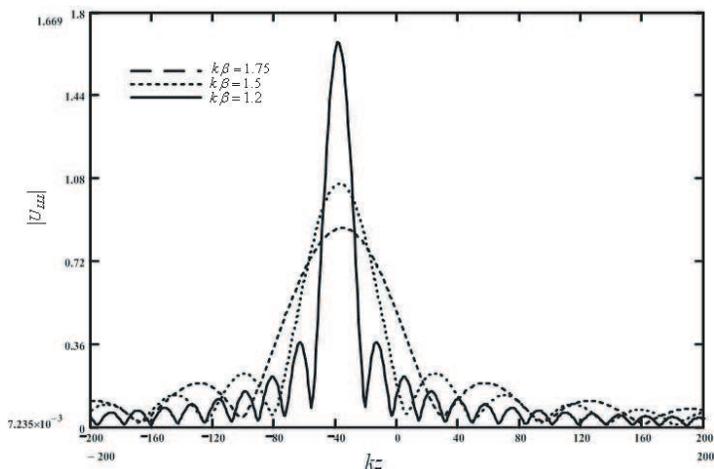


Figure 8. $|U_{LLL}|$ of Gregorian system at $kx = 0$, $k\beta = 1.2, 1.5, 1.75$.

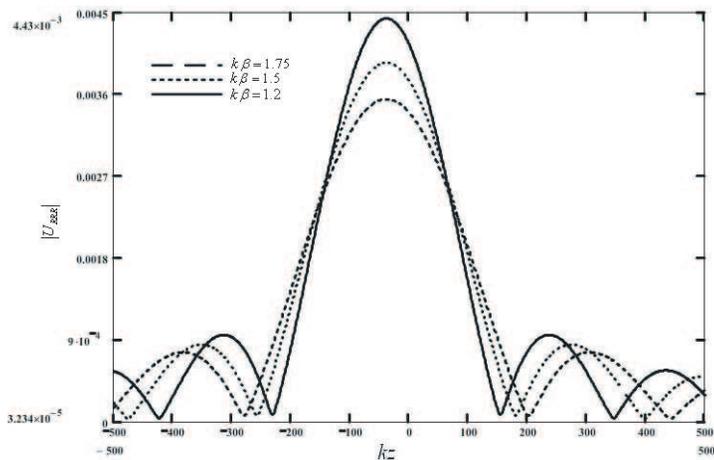


Figure 9. $|U_{RRR}|$ of Gregorian system at $kx = 0$, $k\beta = 1.2, 1.5, 1.75$.

Plots of LLL, RRR, RRL and LLR rays for $k\beta > 1$ are given in Fig. 8 to Fig. 10. We have the same values of kf , ka , kb , kd , and $kD = 110$ as for PPV case. It is seen clearly that for $k\beta > 1$ LCP wave is traveling with NPV and RCP with PPV. LLR wave diverges out and do not form a real focus while RRL wave forms a focus with much shifted towards left.

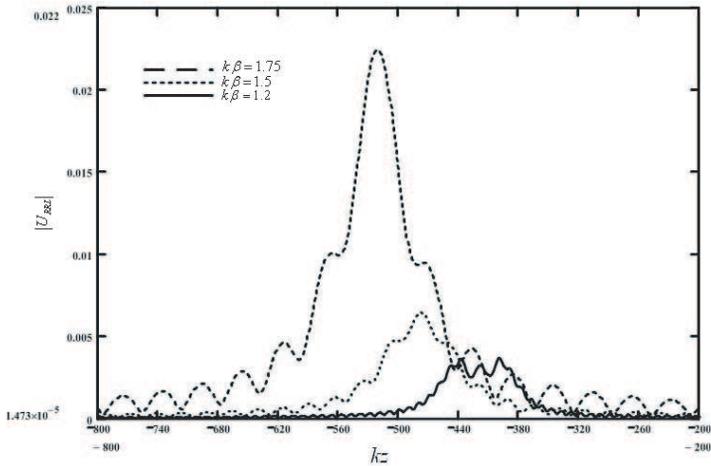


Figure 10. $|U_{RRL}|$ of Gregorian system at $kx = 0$, $k\beta = 1.2, 1.5, 1.75$.

4. CONCLUSION

It is found that excitation of a Gregorian reflector, placed in reciprocal and homogenous chiral medium, by plane wave may yield eight rays four of which converge and their field expressions are determined in this paper. Two of them, LLL and RRR, are located at the same location as if the reflector is placed in ordinary medium [20]. It is seen that for PPV case other two focal points, LLR and RRL, are on the opposite sides of caustic located at ordinary medium location [20]. As the chirality parameter increases, the gap between LLR and RRL increases. For NPV case, the LLR ray diverges and does not form a focus, while focal point of RRL ray shifts to left with increase in chirality parameter.

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