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RESONANT MODES AND RESONANT TRANSMISSION IN MULTI-LAYER STRUCTURES

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Abstract—Resonant modes of multi-layer structures which contain the regions of negative epsilon material (such as a metal in the visible range) are analyzed. Existence of two separate classes of resonant modes is demonstrated. One is related to the excitation of the surface mode at the interface of the regions with opposite signs of the dielectric constant and involve energy transport by evanescent modes throughout the whole structure. The second class involves propagating modes (which form the resonant standing wave) in some regions and the evanescent waves in other layers with $\varepsilon < 0$. It is shown that the resonant transmission is related to the existence of quasi-stationary leaky modes having a finite life-time and characterized by large wave amplitude in the trapping region. It is shown that both types of

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resonances can coexist in multi-layer structures. It is also shown that the interaction of the symmetric and anti-symmetric surface eigenmodes widens the resonant transmission region.

1. INTRODUCTION

Over recent years there has been much work devoted to studies of metamaterials, which have both dielectric permittivity and magnetic permeability negative, $\varepsilon < 0$, $\mu < 0$. One of the central features of metamaterials is the amplification of evanescent waves [1, 2], and many of unique properties of these media are related to this phenomena [3–8]. Amplification of evanescent modes is responsible for a number of fascinating results, e.g., anomalously high transmission of electromagnetic waves through the metal films, which are normally opaque [9–11]. Such a high transmission occurs when the resonant conditions for the excitation of the evanescent waves are satisfied. In this paper, we are interested in situations when the transmission occurs in the tunneling regimes (i.e., when the waves are evanescent in the opaque region). In what follows, we will call this phenomena resonant transmission in tunneling structures, or resonant transmission, for brevity.

It has also been realized that amplification of evanescent waves and resonant transmission may also occur in composite structures with alternating layers of positive and negative ε without the requirement of negative μ. In this work we analyze the latter case, namely the wave propagation in composite structures consisting of layered structures of materials with negative and positive permittivity ε, while assuming that the magnetic permeability μ is always positive. Such structures, e.g., consisting of alternating dielectric and metal layers, are potential building blocks of various plasmonic devices [12]. Resonance transmission has been experimentally demonstrated in such structures [11, 13–16]. In particular, the effects of surface wave plasmons on reflection and guiding properties of multi-layer structures were analyzed in [17–20]. Resonant transmission has also been studied theoretically in earlier works [10, 21–23]. Tunneling phenomena in general remain to be of great interest and subject of controversies in quantum mechanics [24, 25] and the results of this paper are relevant to quantum mechanical resonant tunneling.

Resonant transmission has long been known in electronic devices [26, 27]. It also forms a basis for the operation of resonant Fabry-Perot optical cavities. In the latter case, the resonances occur at the frequencies corresponding to the propagating wave states inside the cavity. In this paper we show that there are two different mechanisms for the amplification of evanescent waves and resonant transmission. These mechanisms can coexist in multi-layer metal-dielectric structures thus creating the conditions for multiple resonances of different types.

Generically, resonant transmission occurs as a result of the excitation of the resonant eigen-modes of the system. It is important to emphasize here that these resonant modes are not truly stationary eigen-mode states but rather are quasi-stationary states experiencing weak amplitude decay in time due to the energy outflow into the surrounding region. Such modes are called leaky modes [28]. As it was noted above, there are two types of resonances. In one case, the resonant mode is related to the surface eigen-mode localized at the interfaces of two layers with opposite sign of the dielectric constant ε [29]. Such a mode will be called Surface-Plasmon-Polariton (SPP). The true eigen-mode exists at the boundary of two semi-infinite layers with opposite sign of the dielectric constant ε. Such a mode is evanescent in both layers and decays at infinity away from the interface. It becomes a leaky mode which amplitude slowly decays in time when both layers have finite width and there are propagating modes in vacuum regions outside of the layers. The mode amplitude decays in time due to the energy outflow by propagating modes. The second type of resonance occurs when the propagating modes is trapped inside the structure between two barriers (similar to Fabry-Perot resonances). For the infinite barriers, it is a truly stationary eigen-mode, but for a finite width barriers, the mode becomes quasi-stationary leaky mode due to energy tunneling through the finite width barrier. In both cases, the resonances lead to the amplification of evanescent modes and resonant transmission of electromagnetic waves via thick layers of opaque materials. In absence of dissipation, the transmission coefficient becomes unity, $T = 1$. Both regimes of resonant transmission can coexist for some configurations leading to multiple resonances in the transmission coefficient.

There are two specific configurations studied in this paper. The first one is a two-layer structure consisting of one layer of negative $\varepsilon < 0$ material and the other layer of the material with positive but small ε, $0 < \varepsilon < 1$. We call this configuration a single barrier structure. It is important to note, however that we consider the waves incident at the angle larger than critical, so in both regions waves are evanescent. The second layer with positive ε is required to create conditions for the resonant excitation of SPP which can be excited by the incident wave in vacuum. An interface of the $\varepsilon < 0$ material and vacuum also supports surface modes but such modes propagate along the interface boundary with subliminal phase velocity $\omega < k_y c$ [30].

Obviously, such a mode cannot be excited by the propagating wave which in vacuum has $\omega^2 = \left(k_y^2 + k_z^2\right) c^2$, where the y-direction is along the interfaces, and the z-direction is normal to it. Additional layer with $0 < \varepsilon < 1$ modifies the dispersion relation for SPP, so that the modes become superluminal, $\omega < k_y c$, and can be linearly excited by the incident wave. We show that in two-layer configuration, the leaky mode is a quasi-stationary state required for the resonant transmission. The two-layer configuration supports only the SPP assisted resonant transmission. Another configuration analyzed in our work is a three layer structure: two outer layers with negative ε and one layer with positive ε in between. We call this configuration a double-barrier structure. In the latter case, the resonant transmission may occur in two separate regimes. The first regime is related to the excitation of SPP modes at the interfaces. In this case, the modes are evanescent in all three regions. The second regime occurs when the waves are evanescent in the negative ε regions and propagating in the middle region with positive ε. This regime corresponds to the resonance with a standing wave in the central region.

In this paper, we analytically derive the resonance conditions and emphasize the connection between the leaky mode conditions and the conditions for the resonant transmission through the multi-layer configuration in vacuum. We show that two conditions are equivalent. The leaky mode corresponds to a quasi-stationary state which is decaying in time due to energy radiation into the vacuum from the structure. The transmission regime corresponds to the scattering problem with the incident (and reflected) waves on one side and the outgoing wave on the other side of the structure. At the resonance (and in absence of dissipation), the reflection is absent, $R = 0$, and transmission is ideal, $T = 1$. The leaky mode regime corresponds to the same parameters of the structure. The role of the leaky modes in quantum mechanical systems with barriers were noted long time ago [31]. Here we also investigate the coupling of resonance modes in multi-layer configurations which leads to the broadening of the resonant transmission function.

In Sections 2 and 5, we consider the leaky modes, respectively in single and double layer configurations. In Sections 3 and 6, we analyze the resonant transmission in these two configurations. In Section 4, we analyze the energy transport by evanescent waves and emphasize that for a single barrier configuration, in both layers the energy is transported by two linearly independent evanescent modes. This approach offers an alternative derivation of resonant conditions as well as a simple derivation for the phase shift of the transmitted wave. The results are summarized and discussed in Section 7.

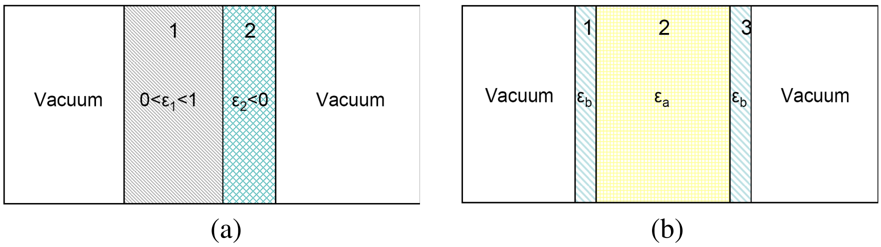

Figure 1. The schematics of the material layers. (a) The two layer structure; region 1 — positive epsilon material, region 2 — barrier region with negative epsilon. The incidence angle larger than critical are considered, so waves are evanescent in both regions. (b) The three layer structure; regions 1, 3 — barrier region with negative epsilon, region 2 — positive epsilon material. Depending on the angle of incidence, the waves can be evanescent or propagating in the region 2.

2. LEAKY MODES IN TWO-LAYER STRUCTURE

Here we consider a two-layer structure surrounded by the vacuum regions $V1$ and $V2$, as in Fig. 1. The layers are characterized by dielectric constants $\varepsilon = \varepsilon_1(\omega)$ and $\varepsilon = \varepsilon_2(\omega)$, respectively having the widths $2a_1$ and $2a_2$. To fix the notations, we note that we consider the electromagnetic wave with the electric field in the incidence (y, z) plane (p-polarization) $\mathbf{E} = (0, E_y, E_z)$ and the magnetic field is parallel to the interface plane $\mathbf{B} = (B, 0, 0)$. All fields do not depend on the x-direction and have the harmonic dependence in time and y-direction, $\sim \exp(-i\omega t + ik_y y)$. The dissipation is neglected in this paper and the parameter k_y is real. The exponential factor $\exp(-i\omega t + ik_y y)$ will be omitted below for brevity. We assume that $\varepsilon_1 > 0$ and $\varepsilon_2 < 0$ so that a second layer is an opaque barrier.

The Maxwell equations lead to the following equation for the p-polarized modes

$$\varepsilon \frac{d}{dz}\left(\frac{1}{\varepsilon}\frac{dB}{dz}\right) - \kappa^2 B = 0, \tag{1}$$

where

$$\kappa^2 = k_y^2 - \varepsilon\frac{\omega^2}{c^2}, \tag{2}$$

and k_y is the component of wave number in the y-direction parallel to the interfaces. This equation can be easily solved in each region. The corresponding solutions are matched at the interfaces using the continuity of B and $(dB/dz)/\varepsilon$.

In this configuration, the leaky mode is represented by the outgoing propagating modes in vacuum regions $V1$ and $V2$

$$B(z) = B_{v1} e^{-ik_v(z+2a_1)}, \qquad z < -2a_1, \tag{3}$$

$$B(z) = B_{v2} e^{ik_v(z-2a_2)}, \qquad z > 2a_2, \tag{4}$$

and evanescent modes in regions 1 and 2:

$$B(z) = A_c \cosh\left(\kappa_1(z+a_1)\right) + A_s \sinh\left(\kappa_1(z+a_1)\right), \quad -2a_1 < z < 0, \tag{5}$$

$$B(z) = B_c \cosh\left(\kappa_2(z-a_2)\right) + B_s \sinh\left(\kappa_2(z-a_2)\right), \quad 0 < z < 2a_2, \tag{6}$$

Here $\kappa_0, \kappa_v, \kappa_1$ and κ_2 are given by the expressions $k_v^2 = k_0^2 - k_y^2$, $k_0^2 = \omega^2/c^2$, $\kappa_1^2 = k_y^2 - \varepsilon_1 k_0^2$, and $\kappa_2^2 = k_y^2 - \varepsilon_2 k_0^2$.

The dispersion relation for the leaky mode is obtained by matching solutions across all three interfaces resulting in

$$\eta_2 \frac{\eta_2 \sinh(\varphi_2) - ik_v \cosh(\varphi_2)}{\eta_2 \cosh(\varphi_2) - ik_v \sinh(\varphi_2)} = -\eta_1 \frac{\eta_1 \sinh(\varphi_1) - ik_v \cosh(\varphi_1)}{\eta_1 \cosh(\varphi_1) - ik_v \sinh(\varphi_1)}, \tag{7}$$

where $\varphi_{1,2} = 4\kappa_{1,2} a_{1,2}$ and $\eta_{1,2} = \kappa_{1,2}/\varepsilon_{1,2}$.

For finite $\varphi_{1,2}$ this equation has no solutions with real ω but there are solutions with complex $\omega = \omega_0 + \omega^{(1)}$, where $\omega^{(1)}$ has the real and imaginary part, $\omega^{(1)} = \omega_r^{(1)} + i\gamma$. Such solutions corresponds to the quasi-stationary state with the amplitude decreasing in time due to outgoing energy flux into the vacuum regions, the so called leaky mode [28].

When the dielectric layers are thick, $\varphi_1, \varphi_2 \gg 1$, the expression can be reduced to

$$\eta_1 + \eta_2 = 2\eta_2 \frac{\eta_2 + ik_v}{\eta_2 - ik_v} \exp(-2\varphi_2) + 2\eta_1 \frac{\eta_1 + ik_v}{\eta_1 - ik_v} \exp(-2\varphi_1) \tag{8}$$

Since $\varphi_1, \varphi_2 \gg 1$, we can consider the right hand side in Eq. (8) as a small parameter and, in zero order, neglect it. Then, in the lowest order we find the following dispersion equation

$$\eta_1 + \eta_2 \equiv \frac{\kappa_1}{\varepsilon_1} + \frac{\kappa_2}{\varepsilon_2} = 0. \tag{9}$$

This a familiar dispersion relation for the SPP mode at the interface of two semi-infinite regions, $(a_{1,} a_2) \to \infty$, with dielectric permittivities ε_1 and ε_2 [29]. It can be written in the form

$$k_0^2 = k_y^2 \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 \varepsilon_2}. \tag{10}$$

We assume evanescent modes in regions 1 and 2, which means $\kappa_1^2 > 0$ and $\kappa_2^2 > 0$. The latter requires sufficiently large incidence angles,

$k_y^2 > \varepsilon\omega^2/c^2$. [The positive roots are chosen for the localization of the surface mode so that $\kappa_1 > 0$ and $\kappa_2 > 0$.] It follows then, that two regions should have the dielectric constants of the opposite signs, $\varepsilon_1\varepsilon_2 < 0$. It is also required that $\varepsilon_1 + \varepsilon_2 \leq 0$. It is easy to see that for $\varepsilon_2 < 0$, the dielectric constant of the second layer is positive $0 < \varepsilon_1 < 1$.

To find the time rate of the decay of the quasi-stationary mode one can expand the left hand side in the vicinity of ω_0, which corresponds to the solution of (10), $\omega = \omega_0 + \omega^{(1)}$, $\omega^{(1)} = \omega_r^{(1)} + i\gamma$. After some algebra we find

$$\frac{\omega^{(1)}}{\omega_0} = \frac{2}{\eta_0^2 + k_v^2} \frac{\begin{matrix}\left(\eta_0^2 - k_v^2\right)\left[\exp\left(-2\varphi_1\right) - \exp\left(-2\varphi_2\right)\right] \\ +2ik_v\eta_0\left[\exp\left(-2\varphi_1\right) + \exp\left(-2\varphi_2\right)\right]\end{matrix}}{\Delta}, \tag{11}$$

where

$$\Delta = k_0^2\left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1}\right) + \frac{1}{\varepsilon_1}\frac{\partial \ell n(\varepsilon_1)}{\partial \ell n\omega}\left(\frac{k_0^2}{2} - \frac{k_y^2}{\varepsilon_1}\right) - \frac{1}{\varepsilon_2}\frac{\partial \ell n(\varepsilon_2)}{\partial \ell n\omega}\left(\frac{k_0^2}{2} - \frac{k_y^2}{\varepsilon_2}\right). \tag{12}$$

The imaginary part of $\omega^{(1)}$ defines the characteristic mode life-time, γ^{-1}. For Drude model of $\varepsilon_{1,2}(\omega)$ we find that for $\varepsilon_1 > 0$ one has $\partial \ell n(\varepsilon_1)/\partial \ell n\omega > 0$ while for $\varepsilon_2 < 0$ we have $\partial \ell n(\varepsilon_2)/\partial \ell n\omega < 0$. As a result the signs of the decrement γ and the wave vector k_v are opposite. It simply shows that the mode will decay or grow, depending on the direction of the energy flux in vacuum, to or away from the slab, which are determined by the sign of k_v.

3. RESONANT TRANSMISSION FOR THE SINGLE BARRIER STRUCTURE

To analyze the transmission problem for the single barrier structure we should assume the incident wave in the vacuum region $V1$ (on the left) and take into account the reflected wave in the same region. Then the solution in this region takes the form

$$B(z) = B_{v1}^{+} e^{ik_v(z+2a_1)} + B_{v1}^{-} e^{-ik_v(z+2a_1)}. \tag{13}$$

Here B_{v1}^{+} and B_{v1}^{-} are amplitudes of the incident and reflected waves. In the region $V2$, the solution remains in the form (4) and B_{v2} is the amplitude of the transmitted wave. After matching of the solutions in various region one finds

$$B_{v1}^{-} = \frac{r}{r_0 - r}, \tag{14}$$

where r is the reflection parameter

$$r = \eta_1 \frac{\eta_1 \sinh(\phi_1) + ik_v \cosh(\phi_1)}{\eta_1 \cosh(\phi_1) + ik_v \sinh(\phi_1)} - \eta_2 \frac{\eta_2 \sinh(\phi_2) + ik_v \cosh(\phi_2)}{\eta_2 \cosh(\phi_2) + ik_v \sinh(\phi_2)}, \quad (15)$$

and

$$r_0 = 2ik_v \eta_1 \frac{\sinh(\phi_2) + \cosh(\phi_2)}{\eta_1 \cosh(\phi_2) + ik_v \sinh(\phi_2)}, \quad (16)$$

where $\phi_{1,2}$ are define in Section 2. Zero reflection, $r = 0$, requires the condition $\eta_1 + \eta_2 = 0$ which is fully equivalent to the surface mode dispersion relation (9). Additional condition $\phi_1 = \phi_2$ is also required. This condition means [21]

$$\kappa_1 a_1 = \kappa_2 a_2. \quad (17)$$

The real parameters $\kappa_{1,2} a_{1,2}$ characterize the amplitude increase/decrease in regions 1 and 2. Respectively, the amplification in the region 1 is equal to the amplitude decrease in region 2. We will call this condition as a matched amplification condition for evanescent waves, c.f. this with the standard interference condition for the propagating waves $k_1 a_1 = k_2 a_2$. The condition (17), together with the surface wave resonance condition (9), actually means that the average dielectric permittivity of the total structure at the resonance is zero:

$$\bar{\varepsilon} \equiv \varepsilon_1 a_1 + \varepsilon_2 a_2 = 0. \quad (18)$$

The matched amplification condition (17) was earlier obtained for multilayer structures containing negative ε, negative μ materials [5, 6]. Somewhat analogous condition for evanescent waves also exists in the problem of superlense [1, 30]. It is worth noting that the resonant transmission in the evanescent mode regime is a manifestation of strongly nonlocal response of near-zero-epsilon materials that have attracted much attention recently [32].

4. EVANESCENT WAVES AND ENERGY TRANSPORT

It is instructive to consider energy transport via the combination of evanescent waves as in Eqs. (5) and (6). The energy flux is easily calculated from the Poynting flux

$$S_z = -\frac{c}{4\pi} E_y B_y, \quad (19)$$

Consider the magnetic field in the form of the sum of two evanescent waves

$$B_x = C \exp(-\kappa z) + D \exp(\kappa z). \quad (20)$$

Here κ is a real (positive) number, while coefficients C and D can be complex. The electric field is

$$E_y = -i\frac{c}{\omega}\frac{1}{\varepsilon}\frac{\partial B_x}{\partial z}, \tag{21}$$

then the time average of Poynting vector takes the form

$$S_z = \frac{ic}{16\pi\omega}\frac{1}{\varepsilon}\left(-B_x^*\frac{\partial B_x}{\partial z} + B_x\frac{\partial B_x^*}{\partial z}\right) = \frac{ic\kappa}{8\pi\omega}\frac{1}{\varepsilon}\left(CD^* - C^*D\right). \tag{22}$$

It is evident from this expression that a combination of two evanescent waves with a finite phase shift between the amplitudes of the waves is required for a finite energy flux, or in other words, the ratio C/D should have a finite imaginary part for S_z to be finite. To illustrate the energy transport evanescent waves we consider the two-layer structure as in Section 2 for large angles $k_y^2 > \varepsilon\omega^2/c^2$.

It is convenient to define the solutions in the vacuum region on the right (incident wave) and the solution in the vacuum region on the left (transmitted wave) such that they have a common reference point $z = 0$. We represent the incident wave as $\Psi_v^< = A_i \exp(ik_0 z)$ in the vacuum region on the left, and the transmitted wave one the right is represented by $\Psi_v^> = A_t \exp(ik_0 z)$. In both regions 1 and 2, the waves are evanescent. The explicit form of the wave fields in these regions is given in Appendix. The solution is constructed from the pair of symmetric and antisymmetric solutions Ψ_{cc} and Ψ_{ss} given by Eqs. (A1) and (A2). These solutions exist when both equations for the SPP resonance (9) and the matched amplification condition (17) are satisfied. When these conditions are satisfied, from Eqs. (A8), (A10), and (A12), it follows that the amplitude of the transmitted wave is related to the amplitude of the transmitted wave by the relation

$$A_t = A_i \exp\left[-2ik_v(a_1 + a_2)\right], \tag{23}$$

which demonstrates the 100% transmission. It is also worth noting that the relation between A_t and A_i means that at the resonance, the transmitted wave at $z = 2a_2$ has a zero phase shift with respect to the incident wave at $z = -2a_1$. The latter is consistent with the effective dielectric parameter for this structure $\bar{\varepsilon} = 0$.

5. LEAKY MODES IN TWO-BARRIER STRUCTURE

In this section, we consider a three layer configuration with two symmetrical barriers 1 and 3, bounded by two semi-infinite vacuum regions on both sides, as depicted in Fig. 1(b). The barriers with dielectric constant $\varepsilon_b < 0$ and width $2b$ are separated by the region 2 with a dielectric permittivity $\varepsilon_a > 0$ and the width $2a$. We are interested in a leaky eigen -mode so that the solutions in the vacuum regions $V1$ and $V2$ have the same form as given by Eqs. (3) and (4). The solutions in regions 1, 2 and 3 are respectively given by the expressions

$$B(z)=A_1 \cosh\left(\kappa(z-z_1)+A_2 \sinh\left(\kappa(z-z_1)\right.\right., \quad z_1-b<z<z_1+b, \tag{24}$$

$$B(z)=B_1 \cos\left(k_a z\right)+B_2\left(k_a z\right), \quad -a<z<a, \tag{25}$$

$$B(z)=C_1 \cosh\left(\kappa(z-z_3)+C_2 \sinh\left(\kappa(z-z_3)\right.\right., \quad z_3-b<z<z_3+b. \tag{26}$$

Here $\kappa^2 = k_y^2 - k_0^2\varepsilon_b$ and $k_a^2 = k_0^2\varepsilon_a - k_y^2$, $-z_1 = z_3 = (a+b)$.

Matching these solutions at interfaces, after some algebra, one gets the following system of equations

$$(B_{v1} + B_{v2})\, D_1 = 0, \tag{27}$$

$$(B_{v1} - B_{v2})\, D_2 = 0. \tag{28}$$

Here

$$D_1 \equiv \sin\left(k_a a\right)\cosh\left(2\kappa b\right) - \zeta\cos\left(k_a a\right)\sinh\left(2\kappa b\right) - \frac{ik_v\varepsilon_b}{\kappa}\left(\sin\left(k_a a\right)\sinh\left(2\kappa b\right) - \zeta\cos\left(k_a a\right)\cosh\left(2\kappa b\right)\right), \tag{29}$$

and

$$D_2 \equiv \cos\left(k_a a\right)\cosh\left(2\kappa b\right) + \zeta\sin\left(k_a a\right)\sinh\left(2\kappa b\right) - \frac{ik_v\varepsilon_b}{\kappa}\left(\cos\left(k_a a\right)\ \sinh\left(2\kappa b\right) + \zeta\sin\left(k_a a\right)\cosh\left(2\kappa b\right)\right). \tag{30}$$

It is easy to see that there are two independent eigen-mode dispersion equations corresponding to symmetric and anti-symmetric solutions. The dispersion equation $D_1 = 0$, describes the odd modes with $B_{v1} = -B_{v2}$, and the dispersion equation $D_2 = 0$ describes the even modes with $B_{v1} = B_{v2}$. These dispersion equations do not have the solutions with real ω except in the limit of thick barriers, $\kappa b \to \infty$. In the latter case, $\sinh(2\kappa b) \to \exp(2\kappa b)$, $\cosh(2\kappa b) \to \exp(2\kappa b)$, and one gets the standard equations for a finite depth well problem

$$\tan\left(k_a a\right) = \zeta, \tag{31}$$

$$\tan\left(k_a a\right) = -\frac{1}{\zeta}. \tag{32}$$

respectively for odd and even eigen-functions, where $\zeta = \kappa\varepsilon_a/\left(\varepsilon_b k_a\right)$. These dispersion equations describe the standard symmetric and antisymmetric modes of a finite depth well bounded by infinite barriers. For the finite width barriers, these solutions become leaky modes that decay in time due to energy tunneling through the finite barriers. The expressions for the decay rates of such quasistationary modes are given in the Appendix B.

6. RESONANT TRANSMISSION IN A TWO-BARRIER STRUCTURE

Now we consider the resonant transmission in the two barrier structure as in Fig. 1(b). It is straightforward to derive general equations for transmission and reflection coefficients in this geometry. Here we only investigate the solution for the resonant case when $T = 1$.

For the analysis of the resonant transmission problem for $T = 1$ we assume that there is no reflected wave in region $V1$. Then the solutions in vacuum regions $V1$ and $V2$ are

$$B(z) = B_{v1} e^{ik_v(z-z_1+a)}, \qquad z < -a-2b, \tag{33}$$

$$B(z) = B_{v2} e^{ik_v(z-z_3-a)}, \qquad z > a+2b. \tag{34}$$

The solutions in the regions 1, 2 and 3 are given by the expressions (24)–(26). Matching all these solutions at interfaces, after some algebra, one gets the following system of equations

$$B_{v1} h_{11} + B_{v2} h_{12} = 0, \tag{35}$$

$$B_{v1} h_{21} + B_{v2} h_{22} = 0. \tag{36}$$

Here, the coefficients are defined by the expressions

$$\begin{aligned} h_{11} &= \cos(k_a a)\cosh(2\kappa b) + \zeta \sin(k_a a)\sinh(2\kappa b) \\ &\quad + i\frac{k_v \varepsilon_b}{\kappa}\left[\cos(k_a a)\ \sinh(2\kappa b) + \zeta \sin(k_a a)\cosh(2\kappa b)\right], \quad (37) \\ h_{12} &= -\cos(k_a a)\cosh(2\kappa b) - \zeta \sin(k_a a)\sinh(2\kappa b) \quad (38) \\ &\quad + i\frac{k_v \varepsilon_b}{\kappa}\left[\cos(k_a a)\ \sinh(2\kappa b) + \zeta \sin(k_a a)\cosh(2\kappa b)\right], \quad (39) \\ h_{21} &= \sin(k_a a)\cosh(2\kappa b) - \zeta \cos(k_a a)\sinh(2\kappa b) \\ &\quad + \frac{i k_v \varepsilon_b}{\kappa}\left(\sin(k_a a)\sinh(2\kappa b) - \zeta \cos(k_a a)\cosh(2\kappa b)\right), \quad (40) \\ h_{22} &= \sin(k_a a)\cosh(2\kappa b) - \zeta \cos(k_a a)\sinh(2\kappa b) \\ &\quad - \frac{i k_v \varepsilon_b}{\kappa}\left(\sin(k_a a)\sinh(2\kappa b) - \zeta \cos(k_a a)\cosh(2\kappa b)\right). \quad (41) \end{aligned}$$

Solubility condition for the system (35), (36) is $h_{11}h_{22} - h_{12}h_{21} = 0$. This equation is equivalent to the resonant transmission condition $T = 1$. This equation can be transformed to the form

$$\sin(2k_a a)\cosh(4\kappa b)\left[1-\zeta^2\right]\left[1+\frac{k_v^2\varepsilon_b^2}{\kappa^2}\right] - 2\zeta\cos(2k_a a)\sinh(4\kappa b) \left[1+\frac{k_v^2\varepsilon_b^2}{\kappa^2}\right] + \sin(2k_a a)\left[1+\zeta^2\right]\left[1-\frac{k_v^2\varepsilon_b^2}{\kappa^2}\right] = 0. \tag{42}$$

This equation describes the resonant transmission of two different types. One is related to the standing waves resonantly excited in the region 2. For this case, one can get a simple resonance condition in the limit of thick barriers, $\kappa b \gg 1$. In this limit, the last term in (42) can be neglected and one gets from (42)

$$\tan(2k_a a) = \frac{2\zeta}{1-\zeta^2}. \tag{43}$$

It is easy to see that this equation has two solutions: $\tan(k_a a) = \zeta$ and $\tan(k_a a) = -1/\zeta$ which exactly correspond to the leaky eigen-modes solutions (31) and (32). It is interesting to note that the existence of the leaky modes in a similar two barrier configurations (with inner parabolic well) was noted in Ref. [31].

Another type of resonance is related to the SPP excitation. In this case, the waves in the region (2) are also evanescent and $k_a = i\kappa_a$ where κ_a is real. There are two roots of (42) corresponding to the symmetric and antisymmetric solutions. In general, both roots are close to the solution defined by (9). There is a special situation when the average permittivity of the structure is zero, $\bar{\varepsilon} = 0$. In the latter case, one of the roots coincides exactly with (9) and there opens a wide resonant

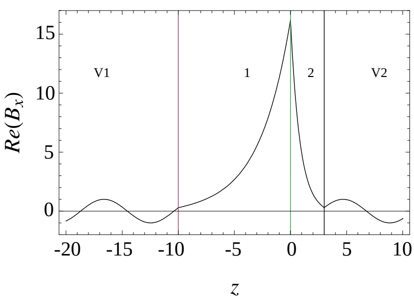

Figure 2. The resonant eigenmode in a two-layer structure. The width of the first layer $2a_1 = 10c/\omega$, and the width of the second layer $2a_2 = 3c/\omega$, $\varepsilon_1 = 0.3$, $\varepsilon_2 = -1$.

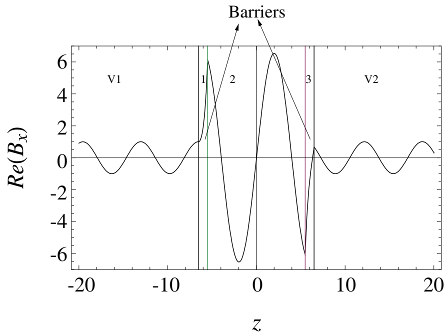

Figure 3. The resonant eigenmode in a two-barrier structure. The barrier layers 1 and 3 have the width $2b = c/\omega$, with $\varepsilon_b = -6$; the width of the middle layer 2 is $2a = 11c/\omega$, with $\varepsilon_a = 0.7$. There is an antisymmetric propagating mode between the barriers. The resonant conditions are satisfied for $k_y c/\omega = 0.2668$.

region. Indeed, the surface wave dispersion Eq. (9) can be put in the form $\zeta = i$. The last term in (42) then disappears and (42) takes the form

$$\sinh(2\kappa_a a - 4\kappa b) = 0. \tag{44}$$

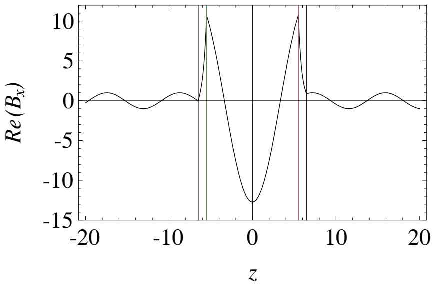

Figure 4. The resonant eigenmode in a two-barrier structure. There is a symmetric propagating mode between the barriers. The resonant conditions are satisfied for $k_y c/\omega = 0.69363$, $\varepsilon_b = -6$, $\varepsilon_a = 0.7$, $2a = 11c/\omega$, $2b = c/\omega$.

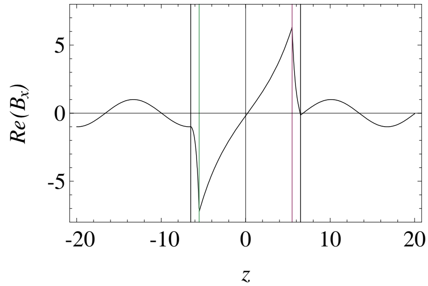

Figure 5. The resonant eigenmode in a two-barrier structure in the evanescent regime. There is an antisymmetric evanescent mode between the barriers. The resonant conditions are satisfied for $k_y c/\omega = 0.88115$.

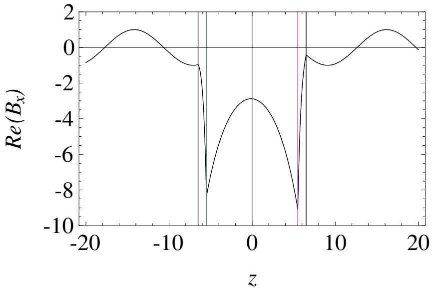

Figure 6. The resonant eigenmode in a two-barrier structure in the evanescent regime. There is a symmetric evanescent mode between the barriers. The resonant conditions are satisfied for $k_y c/\omega = 0.89655$.

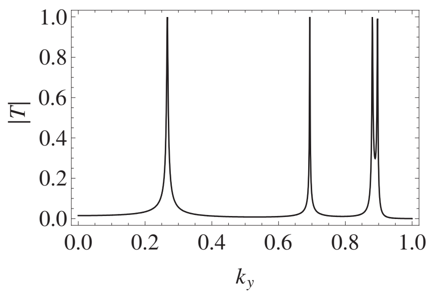

Figure 7. The resonant eigenmodes of a two-barrier structure as a function of the incidence angle $\theta = \arcsin(k_y/k_0)$. Two resonances for lower k_y corresponds to the propagating modes and two resonances for larger k_y are related to the surface wave modes.

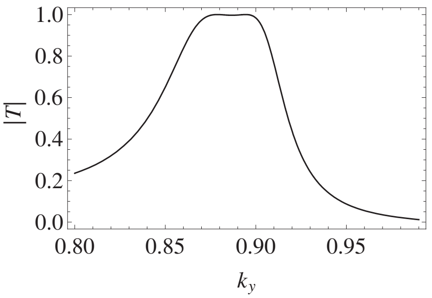

Figure 8. The coupling of the surface wave resonances for $\bar{\varepsilon} \neq 0$: $\varepsilon_b = -5.6$, $\varepsilon_a = 0.7$, $2a = 8c/\omega$, $2b = 0.5c/\omega$.

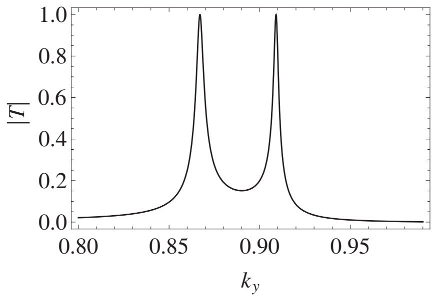

Figure 9. The coupling of the surface wave resonances for $\bar{\varepsilon} = 0$: $\varepsilon_b = -5.6$, $\varepsilon_a = 0.7$, $2a = 8c/\omega$, $2b = c/\omega$.

The surface wave resonant amplification condition for this configuration then

$$2\kappa_a a = 4\kappa b. \tag{45}$$

This condition together with the surface wave resonant condition (9) means that the effective dielectric parameter of the total structure is zero, $\bar{\varepsilon} \equiv 4\varepsilon_b b + 2\varepsilon_a a = 0$, similar to the situation for a two-layer structure of Section 4. There are multiple degenerated roots in the latter case and the resonant transmission region becomes wide as in Fig. 9.

7. DISCUSSION AND CONCLUSION

We have analyzed the resonant transmission in multi-layer structures which include layers of opaque material with negative dielectric permittivity, $\varepsilon < 0$. It is shown that the resonant transmission ($T = 1$) (in absence of dissipation) occurs near the resonant eigen-modes of the structures. In general, there are two basic types of resonances. The first one is based on the excitation of the SPP modes localized at the interface of two regions with opposite signs of the dielectric constant. In this case, waves are evanescent in all layers. There are two required conditions for the resonant transmission for $T = 1$: one is the SPP resonance condition given by the equation and the other one, is the matched amplification condition for the evanescent waves given by (17). Since the surface wave modes require finite k_y, this type of resonance tunneling may occur only for a finite incidence angle $\theta \neq 0$, $\sin\theta = k_y/k_0$. The profile of the magnetic field at the resonance for a single barrier configuration is shown in Fig. 2. The relation of the resonant transmission to the SPP guided modes has also been noted in [17].

Another type of resonance occurs when the waves are evanescent in the region of negative ε and the wave is propagating in other regions. The resonant transmission with $T = 1$ occurs when the standing wave is excited in the propagating region. This regime is similar to the standard Fabry-Perot resonances. These resonances are also possible for normal incidence, $k_y = 0$. Therefore, the double-layer structure in Fig. 1(a) allows only the surface wave resonance, while the three layer structure, as in Fig. 1(b), have both SPP and Fabry-Perot resonances. The profiles of the magnetic field at the resonance for a two barrier configuration are shown in Figs. 3–6. The regime when the standing wave is excited in the propagating layer 2 is shown in Figs. 3 and 4, respectively for antisymmetric and symmetric cases. The regime when the waves are evanescent in all three regions is shown in Figs. 5 and 6, respectively for antisymmetric and symmetric cases.

As it was noted above, for the double layer structure, the resonant transmission requires two conditions: (9) and (17). The frequency of the incident wave should match with the frequency of the surface wave (9). This condition therefore determines the value of ω/k_y, or the incidence angle. An additional constraint (17), a condition of matched amplification of the evanescent waves, is also required to achieve $T = 1$ transmission. It is interesting to note that three layer structure is less constrained. The resonant transmission condition $T = 1$ is given by a single Eq. (42). The surface mode resonance at the $\zeta = i$ corresponds to the symmetric solution (Fig. 6). There is also an antisymmetric solution (Fig. 5) which occurs near the point $\zeta = i$.

In general, the three-layer structure can have several resonances as shown in Fig. 7, where there are two resonances associated with the propagating modes in the region 3 (for lower k_y), and two resonances associated with SPP modes. Additional resonances associated with propagating modes will appear as the width of the propagating region 3 increases in accordance with general properties of the eigen-states of the quantum mechanical potential well of a finite depth, which are described by Eqs. (31) and (32). Therefore there can be multiple resonances for the regimes when the mode in the region 2 is propagating. Additional resonances occur due to existence of multiple eigen-modes in a finite depth well when the width of the well is increasing. When additional barriers are added to the system, new resonances will appear due to the mode splitting cause by coupling across the tunneling regions. Such multiple resonances in a system with left handed martials were considered in [33].

The SPP resonance described by the condition (9) has an interesting property in a system with finite width layer. In fact, the relation (9) is a condition for the SPP at the interface of two half-infinite media. For a finite width layer, as in our situation, the surface waves are excited on both sides. The interaction of the surface wave localized on opposite sides results of the splitting of the resonance into two [34]. Two resonances correspond to symmetric and antisymmetric solutions that are formed due to coupling of the surface waves localized at each boundary. The two resonances near (9) are shown in Fig. 8 for parameters when $\bar{\varepsilon} \neq 0$. The surface waves coupling takes a peculiar form when $\bar{\varepsilon} = 0$. As it was discussed in Section 6, when the condition $\bar{\varepsilon} = 0$ is satisfied, the total composite structure has an exact resonance at (9), despite of the finite width of the barrier. Then, the $\bar{\varepsilon} = 0$ condition result in a relatively wide region of high transparency near the resonant value given by (9). This situation is shown in Fig. 9.

The resonant properties of the multi-layer structures studied in this paper can be of interest for various plasmonic applications. It is important to note that in the considered configurations neither SPP modes defined by the dispersion relation (9) nor standing wave resonances are true stationary eigen-modes. Rather, these modes are quasi-stationary leaky modes with a finite life time defined as an inverse of the leaky mode decay rate, $\tau^{-1} \sim Im(\omega)$. The leaky mode regimes (or equivalently, the resonant transmission regimes) are characterized by the strong enhancement of the wave amplitude inside the structure [4, 13]. The strong enhancement of the wave amplitude is evident in Figs. 2, 5 and 6. Such regimes, which can be realized at the dimensions below the half wavelength, possibly can be used as a laser cavity resonator. The control of the dielectric permittivity, in particular, regimes with $1 > \varepsilon > 0$, can be achieved with the external electric field applied to a semiconductor materials.

The properties of the resonances studied in this paper are similar to general resonant properties of multi-barrier structures in quantum mechanic and therefore are of interest for the problem of superliminality and Hartman effect. We conjecture that the characteristic mode life-time (characteristic time of the leaky mode decay, γ^{-1}) is an effective measure of the signal propagation in the tunneling problem. It is expected that the effective propagation (tunneling) time will further increase in the resonant regimes due to the increased time of the energy accumulation for the evanescent waves [35]. As it is well known, the dielectric permittivity ε cannot be negative unless it depends on ω. It is worth noting here that in calculations of the leaky mode life-time (effective propagation time) is it critical to take into account the dispersion of the media, $\varepsilon = \varepsilon(\omega)$. Our calculations shows that the effects of dispersion significantly affect the leaky mode decay time and the corresponding propagation time. Such modifications are important for Hartman effect.

ACKNOWLEDGMENT

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APPENDIX A. RESONANT EVANESCENT MODE SOLUTION WITH $T = 1$ FOR A DOUBLE LAYER STRUCTURE

One can easily construct a solution across the two layers that illustrates the nature of the resonant transmission. It is convenient to build such a solution from the odd and even functions which are symmetric with respect to the middle point in each region, respectively points $z = a_2$ and $z = -a_1$. There are two such solutions in each region $A_c \cosh(\kappa_1(z+a_1))$ and $A_s \sinh(\kappa_1(z+a_1))$ $-2a_1 < z < 0$, and $C_c \cosh(\kappa_2(z-a_2))$ and $C_s \sinh(\kappa_2(z-a_2))$ for $0 < z < 2a_2$, respectively. The combination of both even solutions gives for the whole region

$$\Psi_{cc} = \begin{cases} A_c \cosh(\kappa_1(z+a_1)), & -2a_1 < z < 0 \\ C_c \cosh(\kappa_2(z-a_2)), & 0 < z < 2a_2 \end{cases}, \tag{A1}$$

$$\Psi_{ss} = \begin{cases} A_s \sinh(\kappa_1(z+a_1)), & -2a_1 < z < 0 \\ C_s \sinh(\kappa_2(z-a_2)), & 0 < z < 2a_2 \end{cases}. \tag{A2}$$

Matching conditions at $z = 0$ give the following equations for Ψ_{cc}:

$$A_c \cosh(\kappa_1 a) = C_c \cosh(\kappa_2 a_2) \quad \frac{k_1}{\varepsilon_1} A_s \sinh(\kappa_1 a_1) = -\frac{k_2}{\varepsilon_2} C_s \sinh(\kappa_2 a_2), \tag{A3}$$

and respectively for Ψ_{ss}:

$$A_s \sinh(\kappa_1 a_1) = -C_s \sinh(\kappa_2 a_2) \quad \frac{k_1}{\varepsilon_1} A_c \cosh(k_1 a_1) = \frac{k_2}{\varepsilon_2} C_c \cosh(\kappa_2 a_2). \tag{A4}$$

The matching conditions (A3) and (A4) impose the following constraints:

$$\frac{\kappa_1}{\varepsilon_1} \tanh(k_1 a_1) = -\frac{\kappa_2}{\varepsilon_2} \tanh(\kappa_2 a_2), \tag{A5}$$

and

$$\frac{\kappa_1}{\varepsilon_1} \coth(k_1 a_1) = -\frac{\kappa_2}{\varepsilon_2} \coth(\kappa_2 a_2) \tag{A6}$$

Since our solutions (A1) and (A6) all real, we will need both of them to maintain the finite energy flux. For the solutions to exist, the constraints (A5) and (A6) have to be consistent with each other which requires the following conditions

$$\frac{\kappa_1}{\varepsilon_1} + \frac{\kappa_2}{\varepsilon_2} = 0, \quad \kappa_2 a_2 = \kappa_1 a_1. \tag{A7}$$

The first equation is easily recognized as the dispersion relation for the surface mode at the interface between two layers with ε_1 and ε_2. The second equation is a matched amplification condition obtained earlier (17).

To illustrate the $T = 1$ tunneling regime and investigate the phase delay across the two layers we use the solutions Ψ_{ss} and Ψ_{cc}.

Matching at $z = -2a_1$ gives

$$A_i \exp\left(-2ik_0 a_1\right) = A_c \cosh\left(k_1 a_1\right) - A_s \sinh(k_1 a_1), \tag{A8}$$

$$ik_0 A_i \exp\left(-2ik_0 a_1\right) = \frac{k_1}{\varepsilon_1}\left(-A_c \sinh\left(k_1 a_1\right) + A_s \cosh(k_1 a_1)\right), \tag{A9}$$

while the matching at $z = 2a_2$ gives

$$A_t \exp\left(2ik_0 a_2\right) = B_c \cosh\left(k_2 a_2\right) + B_s \sinh(k_2 a_2), \tag{A10}$$

$$ik_0 A_t \exp\left(2ik_0 a_2\right) = \frac{k_2}{\varepsilon_2}\left(B_c \sinh\left(k_2 a_2\right) + B_s \cosh(k_2 a_2)\right). \tag{A11}$$

Matching conditions at $z = 0$ give

$$A_c \cosh\left(k_1 a_1\right) + A_s \sinh(k_1 a_1) = B_c \cosh\left(k_2 a_2\right) - B_s \sinh(k_2 a_2), \tag{A12}$$

$$\frac{\kappa_1}{\varepsilon_1}\left[A_c \cosh\left(k_1 a_1\right) + A_s \sinh(k_1 a_1)\right]$$
$$= \frac{\kappa_2}{\varepsilon_2}\left[-B_c \sinh\left(k_2 a_2\right) + B_s \cosh(k_2 a_2)\right], \tag{A13}$$

Eqs. (A8), (A10), and (A12), result in the relation (23) that demonstrates $T = 1$ transmission.

APPENDIX B. LEAKY MODES DECAY RATES FOR THE THREE LAYER STRUCTURE

In the limit of thick barriers $\kappa b \gg 1$, one can develop a perturbative solution for decaying leaky modes with decreasing in time amplitudes. Assuming that $\phi = 2\kappa b$ is large, $\phi \gg 1$, one gets from Eq. (29)

$$D_{10}\left(1 - \frac{ik_v \varepsilon_b}{\kappa}\right) + 2\zeta exp\left(-2i\phi\right)\cos\left(k_a a\right)\left(1 + \frac{ik_v \varepsilon_b}{\kappa}\right) = 0, \tag{B1}$$

for odd modes, and from Eq. (30) for even modes

$$D_{20}\left(1-\frac{ik_v\varepsilon_b}{\kappa}\right)+2\zeta exp\left(-2i\phi\right)\cos\left(k_a a\right)\left(1+\frac{ik_v\varepsilon_b}{\kappa}\right)=0. \quad \text{(B2)}$$

In the lowest order, the eigen mode frequency are defined from the equations $D_{10}=0$ and $D_{20}=0$, respectively for the odd and even modes. Here

$$D_{10}\equiv\tan\left(k_a a\right)-\zeta, \quad \text{(B3)}$$

and

$$D_{20}\equiv\tan\left(k_a a\right)+\frac{1}{\zeta}. \quad \text{(B4)}$$

The next order defines the small correction to the real frequency and small decrement $\omega=\omega_0+\omega^{(1)}$, $\omega^{(1)}=\omega_r^{(1)}-i\gamma$.

$$\begin{aligned}\omega_{odd}^{(1)} &= 2\left(\zeta exp\left(-2i\phi\right)\cos\left(k_a a\right)\left(1+\frac{ik_v\varepsilon_b}{\kappa}\right)2\zeta exp\left(-2i\phi\right)\right.\\ &\qquad\left.\cos\left(k_a a\right)\left(1-\frac{ik_v\varepsilon_b}{\kappa}\right)^{-1}\frac{\partial D_{10}}{\partial\omega}\right)_{\omega=\omega_0}, \quad \text{(B5)}\\ \omega_{even}^{(1)} &= 2\left(\zeta exp\left(-2i\phi\right)\cos\left(k_a a\right)\left(1+\frac{ik_v\varepsilon_b}{\kappa}\right)2\zeta \exp\left(-2i\phi\right)\right.\\ &\qquad\left.\cos\left(k_a a\right)\left(1-\frac{ik_v\varepsilon_b}{\kappa}\right)^{-1}\frac{\partial D_{20}}{\partial\omega}\right)_{\omega=\omega_0}. \quad \text{(B6)}\end{aligned}$$

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