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EFFECT OF STEERING ERROR VECTOR AND ANGULAR POWER DISTRIBUTIONS ON BEAMFORMING AND TRANSMIT DIVERSITY SYSTEMS IN CORRELATED FADING CHANNEL

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Abstract—A comparative analysis of transmit diversity and beamforming for linear and circular antenna arrays in a wireless communications system is presented. The objective is to examine the effect of random perturbations, angular power distributions on transmit diversity and beamforming system. The perturbations are modeled as additive random errors, following complex Gaussian multivariate distribution, to the antenna array steering vectors. Using outage probability, probability of error, and dynamic range of transmitter power as performance measures, we have shown significant effects of array perturbations on the two systems under spatially correlated Rayleigh fading channel. We also examine the effect of angular power distributions (uniform, truncated Gaussian, and truncated Laplacian), which corresponds to different propagation scenario, on the performance of the two systems. Results show that the central angle-of-arrival can

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have significant impact on system performance. And the transmit diversity system with truncated Laplacian distribution performs better as compared to other power distributions, and linear array is a preferable configuration for transmit diversity system. We conclude that array perturbations must not be neglected in the design of transmit diversity and beamforming systems.

1. INTRODUCTION

Given the demand for multimedia service provision to an increasingly large user base, space-time processing is likely to be an integral part of future system architectures as it provides a means of increasing data throughput and hence system capacity. This can be implemented by means of beamforming or spatial diversity [1]. In addition, the proliferation of MIMO technology as related to space-time coding has enabled the spectral efficient implementation of a transmit diversity schemes. Transmit diversity has been developed that allows the use of multiple antennas at the BS instead of at the mobile station (MS), while getting essentially the same diversity advantage. It is known that Transmit diversity system provides the benefit of diversity with no array gain, while the transmit beamforming provides array gain but no diversity [2, 3]. The beamforming system is normally designed so that the fading at the antennas is highly correlated for wide range of angular spreads, while transmit diversity is designed so that the fading will be decorrelated for small angle spreads.

The authors in [3], presented a comparative analysis and the tradeoffs between array gain and diversity for the two systems (i.e., transmit diversity and beamforming) for different number of antennas for the forward link cellular communication system with and without hand-off. In [4], we presented a comparative study of the two systems by taking into account the effect of angle-of-arrival (AOA) distributions. However, the analysis [3, 4], omitted array perturbations that are induced due to array element gain, phase, and element position imperfections in practical application of multiple antennas. In practice, antennas are subject to array perturbations which is due to mismatch between the actual array steering vectors of impinging waves and the ideal (presumed) ones [5, 6].

Motivated by this fact, we modify the system model in [3] to take into account the effect of array perturbations when calculating fading correlations. We present a comparative analysis/results of transmit diversity and beamforming system by taking into account the effects of random errors on steering vector and three angular power density function (pdf) in a correlated Rayleigh fading channel. We do so by comparing the performance of transmit diversity with a system that uses beamforming to point a relatively narrow beam at the mobile station. The objective is to examine the effect of random errors on steering vector and angular pdf [7–11] (i.e., Uniform, truncated Gaussian, and truncated Laplacian) on the performance of the two systems with angle spread, which is defined as standard deviation of the distribution under consideration. We consider two array configurations in our analysis, that is, uniform linear array (ULA) and uniform circular array (UCA). The principle parameter that determines the diversity performance is the fading correlation, which has been studied by several authors [12–17] among others, observed between the array elements. The level of correlation which is a function of angular pdf, angle spread, and array geometry determines the system performance. It is shown how this parameter impacts the system performance in terms of outage capacity and probability of error on the transmit diversity and beamforming system. Consequently, this has enabled recommendations relating to array configurations to be made suitable for different propagation environments.

Also note that in this paper, we only consider *open-loop* transmit diversity, where the base station has no knowledge of the downlink channel. Certainly, the close-loop transmit diversity has the potential to provide diversity as well as array gain advantage, but that is beyond the scope of this paper.

Rest of the paper is organized as follows. In Section 2, we present a detail description of the system model that includes a discussion on spatial correlation matrix. The system performance in terms of outage capacity, and probability of error, is presented in Section 3. Section 4 details the results and provides valuable discussions. Finally, conclusion is made in Section 5.

Notation: Lower case boldface letters are used to denote vectors and upper case boldface letters to denote matrices, $(.)^T$ denotes the transpose, and $(.)^H$ denotes the Hermitian transpose. In addition, $\Re(\mathbf{A})$ means the real part of $\mathbf{A}$, $\Im(\mathbf{A})$ means the imaginary part of $\mathbf{A}$.

2. SYSTEM MODEL

A wireless communication system with M array antennas (ULA or UCA) at the base station and a single antenna at the mobile station is considered. Specifically, the ULA consists of four elements located on the x direction while the UCA consists of four elements lying on a circle about the origin as shown in the Figure 1 below.

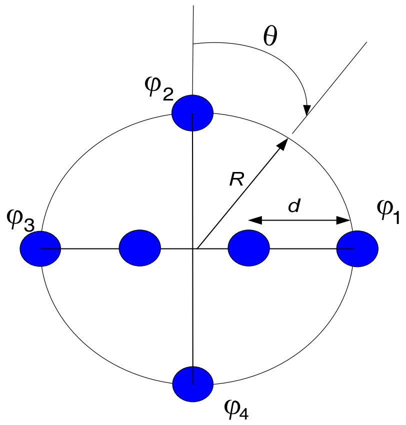

Figure 1. Four element ULA and UCA antenna array configuration at the base station. R is radius of circular array and d is the spacing between array elements, $d = 2R/3$.

We denote the channel response vector $\mathbf{h} = [h_1, h_2, \ldots, h_M]^T$, where h_m is the Rayleigh fading channel from mth transmit antenna at the BS to the MS antenna. In the case where waves are uniformly distributed over the channel angular spread, the covariance matrix of $\mathbf{h}$ is given by [3, Eq. (13)],

$$\mathbf{R}_h = 1/\theta_s \int_{-\theta_s/2}^{\theta_s/2} \alpha(\theta)\alpha(\theta)^H \, d\theta \tag{1}$$

where $\alpha(\theta)$ is the column steering vector at the BS array corresponding to the θ, where θ is the direction of the source relative to the array (that is perpendicular to the array). Assuming the ideal conditions and narrowband transmission, the mth entry of $\alpha(\theta)$ is given by

$$\alpha_m(\theta) = \exp\{-j2\pi(m-1)(d/\lambda)\} \tag{2}$$

where λ is the wavelength of carrier frequency and d is the spacing between array elements. Without loss of generality, we assume here that the phase of first element is zero. Note that adding the same fixed phase to all of the elements of the array does not change any of the results.

Similarly, for the case of UCA, it is given by,

$$\alpha_m(\theta) = \exp(j2\pi(m-1)(R/\lambda)\sin(\xi)\cos(\theta - \varphi_m)) \tag{3}$$

where $\alpha_m(\theta)$ is the mth entry $\alpha(\theta)$, R is the circular radius of the array, ξ is the elevation angle of arrival. For simplicity, only azimuth angles are considered in propagation geometry (i.e., $\xi = 90°$), and φ_m is the excitation angle of mth element that it makes with horizontal axis as shown in Figure 1.

2.1. Spatial Correlation

The spatial propagation parameters introduced here has a direct bearing on the spatial diversity performance of the array and consequently determines performance improvement as compared to a system using single antenna. In general, a receiver can receive signals that are line-of-sight (LOS) or non-line of sight (NLOS) of the transmitter. Assuming that the transmitter and receiver employ omni-directional antennas in a propagation environment, signals at the receiver will have undergone reflection and diffraction, therefore arriving at the receiver with a range of different powers due to the loss incurred over paths spread over the azimuth angles. These azimuth angles forms the azimuth angular power density function (pdf). The shape of this function is significant to this study. Given a LOS or NLOS scenario, the spatial distribution of scatters can be modeled as a uniform, ring or disc distribution [7] which yields the angular pdf. According to [7], the ring and disc distributions best suit a confined cluster of scatters observed in outdoor urban propagation environments. Given a NLOS scenario, spatial propagation can be modeled by a uniform distribution when signals arriving from both azimuth and elevation angles are considered over relatively short ranges in an indoor environment [8]. The LOS scenario can be modeled as Laplacian or Gaussian pdf as proposed in references [9], and [10] for indoor environments, such as rooms and offices etc. Some measurement results also show that the angular pdf in general has a shape which more closely resembles Gaussian or Laplacian pdf [11], while other experiments suggest the use of truncated Gaussian pdf, when the base station is near to the mobile station and truncated Laplacian pdf for micro-cellular radio environments. The effect of antenna directivity on channel characteristics (e.g., angular power distribution) for geometrical-based statistical models have been studied by [18], for macro-cellular and [25, 26], for micro-cellular radio environments. For the case when angular distribution of energy around the radio access point and the user equipment, the overall azimuthal response is obtained by multiplying the angle-resolved impulse response with beam pattern. For geometric channel models, the antenna effect is usually accounted for by deleting those scatterers not within the antenna beam-scanning range.

In this paper, we consider three angular pdfs, namely, uniform, truncated Gaussian and truncated Laplacian. Different angular pdf corresponds to different propagation scenario as detailed above. In [12], a detailed analysis on spatial fading correlation for uniform AOA on linear arrays was carefully studied. Further research on spatial fading correlation has been carried out for various angular pdf in [4, 13–19], which include Gaussian and Laplacian pdf etc. We denote truncated Gaussian pdf [13, 14] by,

$$f(\zeta_s) = C_g e^{-\frac{(\zeta_s-\theta)^2}{2\sigma_g^2}} \quad -\pi + \theta \leq \zeta_s \leq \pi + \theta \tag{4}$$

where

$$C_g = \left(erf\left(\frac{\pi}{\sqrt{2}\sigma_g}\right)\sqrt{2\pi}\sigma_g \right)^{-1} \tag{5}$$

Similarly truncated Laplacian AOA pdf is written as [16],

$$f(\zeta_s) = C_l e^{-s|\theta-\zeta_s|} \quad -\pi + \theta \leq \zeta_s \leq \pi + \theta \tag{6}$$

where

$$C_l = \frac{s}{2\left(1 - e^{-s\pi}\right)} \tag{7}$$

In (4)–(7), θ is the mean AOA and σ_g is the angle spread of truncated Gaussian distribution. Similarly, s is the decay factor which is related to the angle spread of truncated Laplacian distribution. Specifically, as s increases, the angle spread decreases.

2.2. Spatial Correlation Matrix

Optimal diversity performance is achieved when the signals at the array elements are fully decorrelated. However, the degree of correlation between two signals, denoted by ρ, is dependent upon the angular pdf and array configuration. The ρ after normalization is in the range of $0 \leq |\rho| \leq 1$. Thus to achieve this assumption, it requires that the element spacing is such that the signals arriving at each element are decorrelated from one another for given propagation scenario. To determine the system performance, the first step is to calculate the spatial correlation function which will enable the values of ρ to be determined and subsequently the resulting spatial correlation matrix $\mathbf{R}_h$, to be formed, and finally the system performance can be computed.

The closed form expressions for the real and imaginary parts of spatial correlation matrix $\mathbf{R}_h$, for uniform linear array having uniform AOA, can easily be computed from [12]. For the case of truncated Gaussian AOA, the real and imaginary parts of spatial correlation matrix for ULA, and UCA are given by (8) and (9), respectively [13], but corrected in [14],

$$\begin{aligned}\Re\{\mathbf{R}(m,n)\} &= J_0(Z_c)+2\sqrt{2\pi}C_g\sigma_g\sum_{k=1}^{\infty}e^{-2k^2\sigma_g^2}J_{2k}(Z_c)\cos(2k(\theta))\\ \Im\{\mathbf{R}(m,n)\} &= 2\sqrt{2\pi}C_g\sigma_g\sum_{k=0}^{\infty}e^{-\frac{(2k+1)^2\sigma_g^2}{2}}J_{2k+1}(Z_c)\sin((2k+1)(\theta))\end{aligned} \tag{8}$$

$$\begin{aligned}\Re\{\mathbf{R}(m,n)\} &= J_0(Z_c)+2\sqrt{2\pi}C_g\sigma_g\sum_{k=1}^{\infty}e^{-2k^2\sigma_g^2}J_{2k}(Z_c)\cos(2k(\theta+\alpha))\\ \Im\{\mathbf{R}(m,n)\} &= 2\sqrt{2\pi}C_g\sigma_g\sum_{k=0}^{\infty}e^{-\frac{(2k+1)^2\sigma_g^2}{2}}J_{2k+1}(Z_c)\sin((2k+1)(\theta+\alpha))\end{aligned} \tag{9}$$

Similarly, for the case of truncated Laplacian AOA, the real and imaginary parts of spatial correlation matrix is given by (10) and (11), for ULA and UCA, respectively [16, 17],

$$\begin{aligned}\Re\{\mathbf{R}(m,n)\} &= J_0(Z_l)+2\sum_{k=1}^{\infty}\frac{s^2}{s^2+4k^2}J_{2k}(Z_l)\cos(2k\theta)\\ \Im\{\mathbf{R}(m,n)\} &= -2\sum_{k=0}^{\infty}\frac{s\left(1+e^{-s\pi}\right)}{\left[s^2+(2k+1)^2\right]\left(1-e^{-s\pi}\right)}J_{2k+1}(Z_l)\sin((2k+1)\theta)\end{aligned} \tag{10}$$

$$\begin{aligned}\Re\{\mathbf{R}(m,n)\} &= J_0(Z_c)+2\sum_{k=1}^{\infty}\frac{s^2}{s^2+4k^2}J_{2k}(Z_c)\cos(2k(\theta+\alpha))\\ \Im\{\mathbf{R}(m,n)\} &= -2\sum_{k=0}^{\infty}\frac{s\left(1+e^{-s\pi}\right)}{\left(s^2+(2k+1)^2\right)\left(1-e^{-s\pi}\right)}J_{2k+1}(Z_c)\sin((2k+1)(\theta+\alpha))\end{aligned} \tag{11}$$

where the parameters, Z_c, Z_l, and α are defined in [12, 16]. And $J_n(.)$ is the Bessel function of first kind and of order n. Eqs. (8) to (11) can be used to compute the spatial correlation matrix. Therefore, for a four-element antenna array the spatial correlation matrix can be given by,

$$\mathbf{R}_h=\begin{bmatrix}\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14}\\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24}\\ \rho_{31} & \rho_{31} & \rho_{33} & \rho_{34}\\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44}\end{bmatrix} \tag{12}$$

where $\rho_{mn} = \sqrt{\Re(\mathbf{R}_{mn})^2+\Im(\mathbf{R}_{mn})^2}$ is the correlation coefficient between element pairs m and n. For example, for a ULA $M = 4$, and $d = \lambda$, the spatial correlation matrix for three angular AOA pdf with angle spread of 3° is given below,

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.832 & 0.427 & 0.051 \\ 0.832 & 1 & 0.832 & 0.427 \\ 0.427 & 0.832 & 1 & 0.832 \\ 0.051 & 0.427 & 0.832 & 1 \end{bmatrix}; \qquad \text{(a)}$$

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.947 & 0.805 & 0.615 \\ 0.947 & 1 & 0.947 & 0.805 \\ 0.805 & 0.947 & 1 & 0.947 \\ 0.615 & 0.805 & 0.947 & 1 \end{bmatrix}; \qquad \text{(b)}$$

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.661 & 0.327 & 0.176 \\ 0.661 & 1 & 0.661 & 0.327 \\ 0.327 & 0.661 & 1 & 0.661 \\ 0.176 & 0.327 & 0.661 & 1 \end{bmatrix} \qquad \text{(c)}$$

Similarly for higher angle spread of 12° we have:

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.271 & 0.149 & 0.074 \\ 0.271 & 1 & 0.271 & 0.149 \\ 0.149 & 0.271 & 1 & 0.271 \\ 0.074 & 0.149 & 0.271 & 1 \end{bmatrix}; \qquad \text{(a)}$$

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.427 & 0.025 & 0.005 \\ 0.427 & 1 & 0.427 & 0.025 \\ 0.025 & 0.427 & 1 & 0.427 \\ 0.005 & 0.025 & 0.427 & 1 \end{bmatrix}; \qquad \text{(b)}$$

$$\mathbf{R}_h = \begin{bmatrix} 1 & 0.317 & 0.107 & 0.051 \\ 0.317 & 1 & 0.317 & 0.107 \\ 0.107 & 0.317 & 1 & 0.317 \\ 0.051 & 0.107 & 0.317 & 1 \end{bmatrix} \qquad \text{(c)}$$

where (a) uniform, (b) truncated Gaussian, and (c) truncated Laplacian.

Also note that the correlation matrix is symmetric along its diagonal. It is obvious that the diagonal elements of the correlation matrix is 1, however, the less obvious is that the off-diagonal elements of the correlation matrix have different values for three angular pdf for small and large angle spread. It is this distribution of the values upon which the performance of spatial diversity system depends. This process is easily repeated for the geometry associated with each array configuration, taking into account the variations of mean AOA and angle spread. Thus $\mathbf{R}_h$, can be constructed for any given array configuration and propagation scenarios, assuming that the angular pdf is known.

2.3. Antenna Array Perturbations

In practice, antennas are subjected to array perturbations which is due to mismatch between the actual array steering vectors of impinging waves and the ideal (presumed) ones. For small perturbations the first order approximation applies, where the actual steering vector is modeled with [5, P. 501–510],

$$\alpha'(\theta) \approx \alpha(\theta) + \alpha_\varepsilon \tag{13}$$

where α_ε is a steering error vector. The probability density function (pdf) of α_ε follows complex Gaussian multivariate distribution with zero-mean and covariance matrix $\mathbf{R}_\varepsilon$ and it is given by [6],

$$f(\alpha_\varepsilon) = \left[\pi^M \det(\mathbf{R}_\varepsilon)\right]^{-1} \exp\left\{-\alpha_\varepsilon^H \mathbf{R}_\varepsilon \alpha_\varepsilon\right\} \tag{14}$$

It is assumed that α_ε is independent of $\alpha(\theta)$ and spatially uncorrelated, i.e., $\mathbf{R}_\varepsilon = \sigma_\varepsilon^2 \mathbf{I}_M$ where σ_ε^2 is the variance of perturbations and $\mathbf{I}_M$ denotes an identity matrix of size M. The model in (13) and (14) has also been used by [23], and [24]. It is not our intention to determine the ways to reduce the array perturbations, but rather incorporate the above model in our analysis to see its effects on the performance of transmit diversity and beamforming systems.

The normalized covariance matrix $\mathbf{R}'$ of the channel response vector in the presence of array perturbations becomes,

$$\mathbf{R}' = \left(1 + \sigma_\varepsilon^2\right)^{-1} \left(\mathbf{R}_h + \sigma_\varepsilon^2 \mathbf{I}_M\right) \tag{15}$$

We use Eq. (15), for providing results in the presence of array perturbations.

3. PERFORMANCE ANALYSIS

Let S denotes the average signal-to-noise ratio (SNR) at the MS, the instantaneous SNR observed by transmit diversity system, which employs widely spaced antennas, is given by $\gamma = (S/M)\chi$ where,

$$\chi = \sum\nolimits_{m=1}^{M} |h_m|^2 = \mathbf{h}^H \mathbf{h} = \mathbf{u}^H \mathbf{R}' \mathbf{u} \tag{16}$$

And $\mathbf{u}$ is vector of M zero-mean unit variance independent Gaussian random variables [2, 3]. The pdf of χ is given by [20],

$$f^{TD}(\chi) = \sum\nolimits_{m=1}^{M} \left(\prod\nolimits_{i=1, i\neq m}^{M} \frac{\lambda_m}{\lambda_m - \lambda_i}\right) \frac{1}{\lambda_m} \exp\left(-\frac{\chi}{\lambda_m}\right) \tag{17}$$

where λ_m are singular values (eigenvalues) of $\mathbf{R}'$ and $\mathbf{R}_h$, with and without array perturbations, respectively.

The beamforming system employs closely spaced antennas with half-wavelength, pointing a relatively narrow beam toward MS. It provides an effective array gain of $g = \mathbf{w}^H\mathbf{R}'\mathbf{w}$, where $\mathbf{w}$ is the weight vector. The instantaneous *SNR* observed by the beamforming system can be written as, $\gamma = gS\chi$, where χ follows an exponential distribution and is given by $f^{BF}(\chi) = \exp(-\chi)$.

3.1. Outage Capacity

A convenient approach to compare the two systems is to calculate their respective channel capacities. This provides a comparison that is independent of the specific system details (such as modulation, coding, frame/block size, etc.). The authors in [3] have used outage capacity to provide a more realistic characterization of performance of wireless communication systems than is provided by Shannon capacity. The capacity is treated as a random variable, being a function of randomly varying *SNR* in these works. We define the outage capacity $C_{out}(p)$ with probability$p$, which has an intuitive interpretation as the highest transmission rate that can be sustained with probability $(1-p)$.

The "random capacity" of wireless communication system is given by $C = \log_2(1+\gamma)$, which is a function of instantaneous *SNR*. For transmit diversity systems, it can be written as $C = \log_2(1 + S/M\chi)$. The pdf $f(C)$ can be computed from the pdf of χ following the same approach as in [3], using standard random variable transformation and can be given by,

$$f^{TD}(C) = \frac{2^C M \log 2}{S} f^{TD}\left(\left(2^C - 1\right)\frac{M}{S}\right) \tag{18}$$

where $f^{TD}(x)$ is given by (17). Similarly, for beamforming system it is, $C = \log_2(1 + gS\chi)$. Using random variable transformation we obtain,

$$f^{BF}(C) = \frac{2^C M \log 2}{gS} f^{BF}\left(\left(2^C - 1\right)\frac{1}{gS}\right) \tag{19}$$

The outage capacity $C_{out}(p)$ for probability p can be computed by,

$$p = \int_{-\infty}^{C_{out}(p)} f(C)dC \tag{20}$$

3.2. Power Control

It is known that the average transmitter power provides a reasonable measure of performance for code-division multiple access (CDMA) systems, where power is transmitted simultaneously to a large number of users. In that case, the instantaneous total BS power is approximately equal to the average transmit power per user times the total number of users. In contrast, in time division multiple access (TDMA) systems, BS transmit only to one user at a time; in that case, the average transmitter power may not be an adequate performance measure. Instead, we need to look at the probability distribution of the transmitter power. In [3], a cumulative probability density function (cdf) for the transmitter power of transmit diversity and beamforming systems is plotted where no correlation was assumed. In this paper, following the same approach, we shall show the effect of random perturbation errors, where there exists some amount of correlation, on the transmit diversity and beamforming systems. When the multipath fading is slow, power control attempts to adjust the transmitter power by keeping the SNR fixed at nominal value $S_0 = \gamma P$. With γ defined above, the transmitter powers of the transmit diversity and beamforming systems are $P = MS_0/(\chi S)$ and $P = S_0/(g\chi S)$, respectively. Without loss of generality we assume $2S_0/S = 1$, for both systems [3]. Using standard variable transformation, P can easily be expressed as a random variable dependent on χ whose pdf is already given above. Therefore, for transmit diversity system we have,

$$f(P) = \frac{M}{2P^2} f^{TD}\left(\frac{M}{2P}\right) \tag{21}$$

Similarly for beamforming system we have that,

$$f(P) = \frac{1}{2gP^2} f^{BF}\left(\frac{1}{2gP}\right) \tag{22}$$

3.3. Bit Error Rate

It is evident from the analysis in section II that the level of correlation is dependent upon angular pdf of the channel and array geometry. How these parameters impact the probability of error is analyzed in this section. Assuming a degree of correlation exists between the diversity branches, the probability of error for BPSK modulation operating with maximal-ratio combining (MRC) is given by [21, Chapter 14], [22],

$$P_e = \frac{1}{2} \sum_{\substack{m=1 \\ m \neq k}}^{M} \frac{\lambda_m^{M-1}}{\prod_{m \neq k} (\lambda_m - \lambda_k)} \left(1 - \sqrt{\frac{\lambda_m S}{(1+\lambda_m) S}}\right) \tag{23}$$

where all the parameters are already defined above. In (23), the eigenvalues of the correlation matrix is used here replacing the variance of the received signal as in [21]. We have already described the methodology to compute the spatial correlation matrix, $\mathbf{R}'$ and $\mathbf{R}_h$, in Section 2.3.

4. RESULTS AND DISCUSSION

A wireless communication system with four ULA or UCA antennas at the BS and a single antenna at the MS is considered. The TD-U, TD-G, and TD-L in the figures stand for transmit diversity system with uniform pdf, transmit diversity with truncated Gaussian pdf, and transmit diversity with truncated Laplacian, respectively. BF or Bf stands for beamforming, and angle spread is denoted by σ in the figures. All the results shown consider mean angle-of-arrival, $\theta = 0°$, except only in Figure 9.

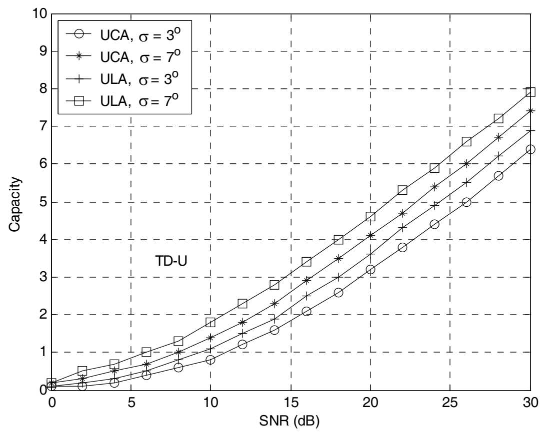

Figure 2. Outage capacity for transmit diversity for ULA/UCA with uniform AOA distribution, $M = 4$, $p = 2\%$.

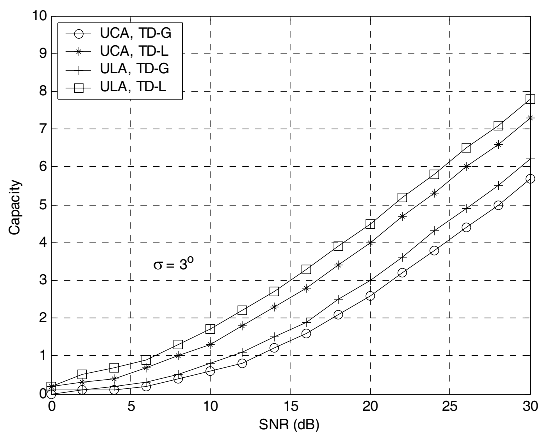

Figure 3. Outage capacity for transmit diversity for ULA and UCA; angle spread $= 3°$, $M = 4$, $p = 2\%$.

Figure 2 and Figure 3, show the outage capacity curve as a function of SNR of transmit diversity system at $p = 2\%$. In Figure 2, a comparison for TD-U is given for ULA and UCA for two values of angle spread ($\sigma = 3^\circ$ and $\sigma = 7^\circ$). We can see that, base station with ULA configuration provides higher capacity than that of UCA. Specifically, at $\sigma = 7^\circ$ and $C = 3$ bits/s/Hz, UCA is 2 dB inferior to ULA. Similarly for TD-L and TD-G a comparison is also plotted in Figure 3, for ULA and UCA at an angle spread of 3°. If we compare Figure 2 and Figure 3, it can be noticed that the TD-L provides higher capacity than the other two pdfs.

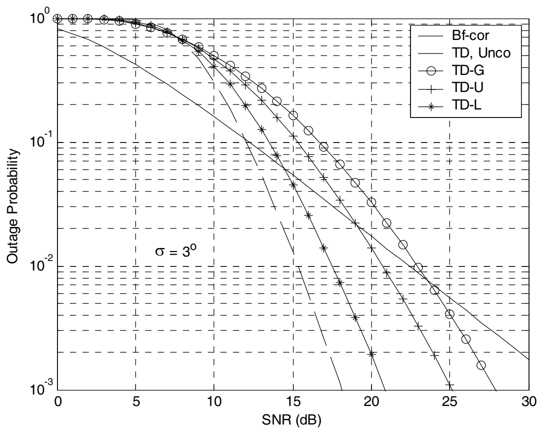

Figure 4. Outage probability for transmit diversity and beamforming systems for UCA; angle spread = 3°, $C_{out}(p) = 3$, $M = 4$.

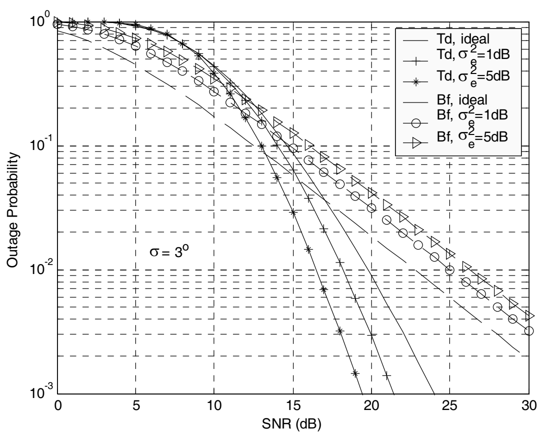

Figure 5. Outage probability for transmit diversity and beamforming system; $C_{out} = 3$ bits/s/Hz, $M = 4$, angular spread of 3°.

In Figure 4, the outage probability curve as a function of *SNR* of transmit diversity and beamforming system for UCA is plotted with $C_{out}(p) = 3$, and for angle spread of 3°. It can be seen that the performance of TD-U and BF (fully correlated case) is roughly the same at $p = 2\%$, but inferior to the case when ULA used at the base station [3]. However, that is not the case for TD-G and TD-L, in contrast to the conclusion made in [3], please see [4, Figure 2]. It is clear from the Figure 4 that the transmit diversity systems do not perform well at small angle spread, especially for TD-G.

Figure 5 shows the outage probability of the two systems for an angular spread of 3°, and $M = 4$. The curves of ideal arrays for both transmit diversity and beamforming systems are obtained based on ideal conditions as reported in [3, see Figure 8]. It is interesting to note from Figure 5 that the performance of beamforming system deteriorates while transmit diversity exhibits improved performance over that is predicted in [3]. This is because when σ_ε^2 increases, $\mathbf{R}'$ approaches an identity matrix, where an effective array gain for the beamforming system reduces and the fading correlation of transmit diversity system decreases. Hence, in the case of independent fading $\mathbf{R}' = \mathbf{I}_M$, and $g = 1$, meaning that the beamforming system provides no gain over a single antenna, whereas the transmit diversity system is very effective, providing M degrees of freedom [2]. In contrast to the conclusion of [3], the transmit diversity system performs better than beamforming system in situations where array perturbation exists.

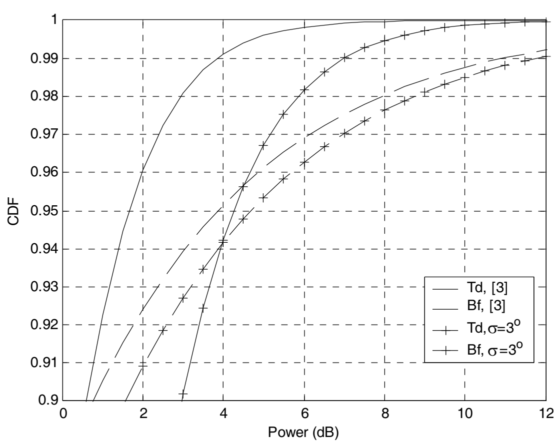

Figure 6. Cumulative density function of transmitter power for transmit diversity and Beamforming system for $M = 4$.

In situations, where BS only transmits to one MS, for example in TDMA systems, dynamic range of transmitter power is a better measure than the average transmitter power [3]. In Figure 6, the cumulative distribution function (CDF) of transmitter power for both systems is shown. If the peak power is defined as the power level that is exceeded 1% of the time, then we can see from Figure 6, that the beamformer will require roughly 6.5 dB higher peak power than that of transmit diversity [3], however, with an angle spread of 3° it will require only 4.5 dB higher peak power than that of transmit diversity system. This is due to the fact the transmit diversity system do not perform well in correlated signal environment, while beamforming system are robust in that case.

In Figure 7, we plot the CDF of transmitter power for both systems in the presence of array perturbation. Again, if the peak power is defined as the power level that is exceeded 1% of the time, then we can see from Figure 7 that effects of array perturbations on the peak powers of transmit diversity and beamforming systems are opposite. It can be seen that the curve of ideal array conditions overvalues the peak power of transmit diversity, whereas it undervalues the peak power of beamforming system.

The results shown in Figures 2 to 7 consider the information theoretic aspect of the system performance in terms of capacity and outage probability with results considering central angle-of-arrival at $\theta = 0°$. Figures 8 and 9 show results in terms of probability of error as function of central angle-of-arrival θ. The objective is to examine the impact of central angle-of-arrival, angle spread for three angular pdf, (i.e., uniform, truncated Gaussian, and truncated Laplacian).

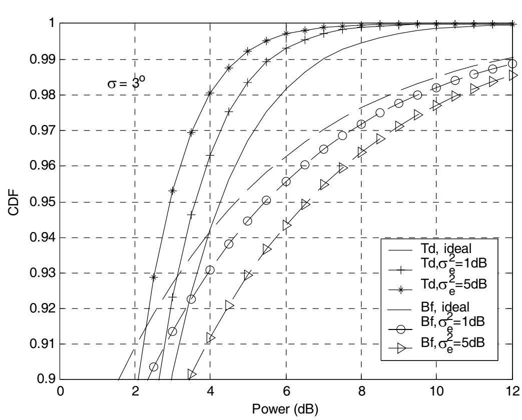

Figure 7. Cumulative density function of transmitter power for transmit diversity and beamforming system for $M = 4$, angular spread of 3°.

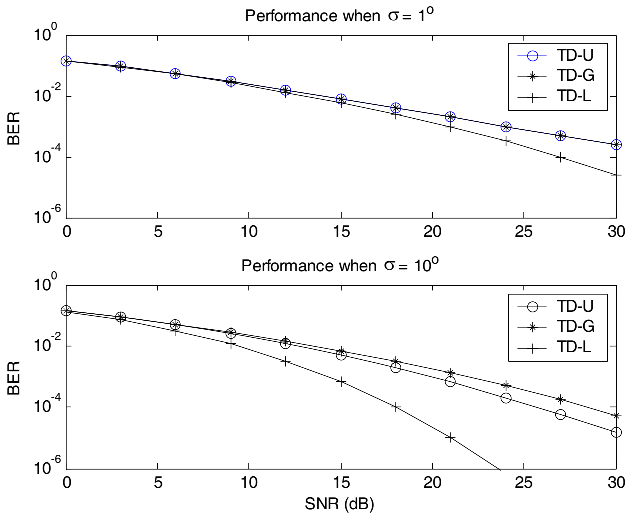

Figure 8. Bit error rate of transmit diversity system for ULA; mean AOA $\theta = 0^\circ$, average SNR $= 15\,\mathrm{dB}$, $M = 4$.

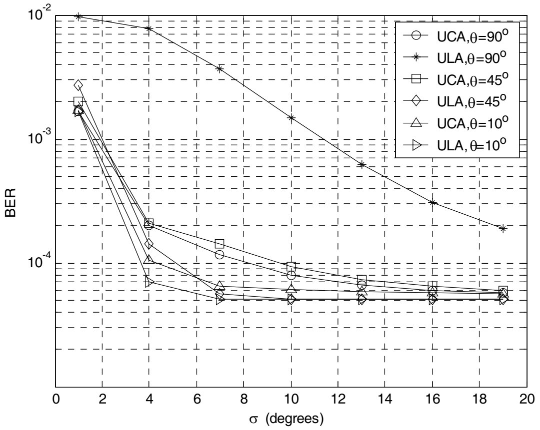

Figure 9. Bit error rate of transmit diversity system for ULA/UCA; truncated Gaussian AOA, average SNR $= 15\,\mathrm{dB}$, $M = 4$, and ($R = 5\lambda$, $d = 2R/3$).

Figure 8 shows bit error rate curve as a function of average *SNR* for two values of angle spread (1° and 10°), which corresponds to small and large angle spreads, respectively. For small angle spread (upper part of Figure 8), the performance is roughly the same for three angular pdfs, except only when the *SNR* is higher, the TD-U shows better performance. Similarly, when angle spread is larger, a significant difference in the performance of transmit diversity, which can be observed from Figure 8 (lower part), especially with truncated Laplacian pdf. This is due to distribution of off-diagonal values of correlation matrix (please refer to the section of spatial correlation matrix for more insight on it). Moreover, as discussed earlier that truncated Laplacian pdf is best suited for LOS propagation scenarios in indoor environments with high angle spread, and it is obvious that the performance is always better in LOS propagation. In a similar manner, TD-U shows a better performance as compared to TD-G for high angle spread, though not much difference, only 1 dB SNR improvement at 1e-03. The results in Figure 8, for transmit diversity system having uniform angular pdf are consistent with those reported in [22] for MRC case.

Figure 9 shows the performance of the UCA and the ULA as function of angle spread. The array geometry is given in Figure 1. As shown, the UCA significantly outperforms the ULA at $\theta = 90°$. However, at angles lower than 60°, linear array performs similar to or even better than the UCA. The variability in the performance of both arrays is more pronounced at moderate angle spreads of say 5° (This is typical angle spread observed in outdoor sub-urban propagation environments), where the UCA outperforms the ULA by over an order of magnitude at $\theta = 90°$. This variability in the ULA performance is due to the fact that the correlation between elements is high for the ULA when $\theta = 90°$, unless angle spread is very large. Similarly, ULA provides an order of magnitude improvement over the UCA at $\theta = 45°$. This is the worst case for the UCA, since in that case elements three and four are directly behind the elements one and two, and thus strongly correlated, as shown in Figure 1. We also note that all curves approach the performance of four branch diversity as angle spread increases.

5. CONCLUSIONS

In this paper, we presented a comparative analysis of transmit diversity system and transmit beamforming for the downlink of wireless communication system, using outage capacity and probability of error as performance measure for uniform linear and circular antenna arrays. We examined the effect of angular azimuth power density functions (i.e., Uniform, truncated Gaussian, and truncated Laplacian), on the performance of the two systems under Rayleigh fading channel for several values of angle spreads. These angular pdf corresponds to different propagation environments. We also examined the effect of array perturbations, which is modeled as additive random errors, following complex Gaussian multivariate distribution, to the antenna array steering vectors. We have shown significant effects of array perturbations on the performance of the two systems. We conclude based on the results that array perturbations must not be neglected in the design of transmit diversity and beamforming systems. Results also show that the transmit diversity system using ULA or UCA at base station with truncated Laplacian angular pdf, always perform better even at smaller angle spreads as compared to other energy distributions. Further it is observed that central angle-of-arrival can have significant impact on the system performance of spatial diversity system. It is also shown that in general ULA provides higher capacity or better performance than that of UCA, however, the worst cases for ULA and UCA is at the central angles of 90, and 45 degrees, respectively.

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