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MICROSTRIP PATCH ANTENNA OPTIMIZATION USING MODIFIED CENTRAL FORCE OPTIMIZATION

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Abstract—Central force optimization (CFO) is a new simple deterministic multi-dimensional search evolutionary algorithm (EA) inspired by gravitational kinematics. This paper evaluates CFO's performance and provides further examples on its effectiveness. A new scheme, the acceleration clipping, is introduced, which enhances CFO's global search ability while maintaining its simplicity. The improved CFO algorithm is applied to the optimal design of two different wideband microstrip patch antennas. Specifically, a microstrip line fed E-shaped patch antenna and a coaxial line fed double-E-shaped patch antenna are designed and optimized using the CFO method. CFO's performance on these antennas is compared to that of the differential evolution (DE) optimization. Both the CFO and DE methods are interfaced with the full-wave IE3D software. It is found that the CFO results are very close to those obtained using the DE technique.

1. INTRODUCTION

Microstrip patch antennas are widely used in wireless and mobile communication systems because of their advantages, such as low profile, light weight, and ease of fabrication. Usually, the basic antenna topology can be chosen according to the desired antenna performance. The challenge is to determine the geometric parameters of the antenna, such as the patch dimensions and the feed position, to achieve the best design that satisfies certain criteria. Clearly, a trial-and-error process is time consuming and will not necessarily give the optimum patch parameters. Thus, a powerful optimization technique is needed, which will enable the antenna designer to design a specific antenna that meets specific requirements.

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To tackle this problem, in this paper a patch antenna design method is proposed using two evolutionary optimization techniques: the newly proposed central force optimization (CFO) [1–6] and the well-developed differential evolution (DE) [7–9]. Central Force Optimization (CFO) is an optimization algorithm analogizing gravitational kinematics [1–4]. Many nature inspired metaheuristics are based on biological metaphors, such as particle swarm optimization (PSO) [10], ant colony optimization (ACO) [11], and genetic algorithms (GA) [12]. These evolutionary algorithms are inherently stochastic, unlike CFO which is deterministic. In this paper, both the CFO and DE methods are interfaced with the standard software IE3D to accomplish the design of two patch antennas. Specifically, both the CFO and DE techniques are used to design and optimize a single band and wide band microstrip line fed E-shaped patch antennas, and a wide band coaxial line fed double-E-shaped patch antenna.

This paper is divided as follows: Section 2 briefly describes the CFO algorithm. The reader may consult [1–6] for more details. In Section 3, a new scheme, the acceleration clipping is introduced into the basic CFO algorithm. Section 4 describes the way the optimization techniques are connected to the IE3D simulator. Finally, Sections 5 and 6 present the optimal design of two different wide band microstrip patch antennas.

2. BASIC CFO ALGORITHM [1]

CFO finds the maxima of an objective function $f(x_i, \ldots, x_{N_d})$ by *flying* a set of probes through the decision space (DS) along trajectories computed using the gravitational analogy. In an N_d — dimensional real valued decision space (DS), each probe p with position vector $\vec{R}^p_{j-1} \in \mathbf{R}^{N_d}$ experiences an acceleration $\vec{A}^p_{j-1}$ at the discrete time step $(j-1)$ given by:

$$\vec{A}^p_{j-1} = G \sum_{\substack{k=1 \\ k \neq p}}^{N_p} U\left(M^k_{j-1} - M^p_{j-1}\right)\left(M^k_{j-1} - M^p_{j-1}\right)^\alpha \frac{\left(\vec{R}^k_{j-1} - \vec{R}^p_{j-1}\right)}{\left\|\vec{R}^k_{j-1} - \vec{R}^p_{j-1}\right\|^\beta} \quad (1)$$

where N_p is the total number of probes; $p = 1, \ldots, N_p$ is the probe number; $j = 0, \ldots, N_t$ is the time step; G is the gravitational constant; $\vec{R}^p_{j-1}$ is the position vector of probe p at step $j-1$; $M^p_{j-1} = f\left(\vec{R}^p_{j-1}\right)$ is the fitness value at probe p at time step $j-1$; $U()$ is the Unit Step function; and β, α are the CFO exponents [1–6].

CFO *mass* is defined as the difference of fitnesses raised to the power α multiplied by the Unit Step function. It should be emphasized that CFO *mass* is not the value of the objective function itself. Including the Unit Step $U()$ is essential because it creates positive mass, thus insuring that CFO's *gravity* is attractive. Each probe's position vector at step j is updated according to the following equation:

$$\vec{R}_j^p = \vec{R}_{j-1}^p + \frac{1}{2}\vec{A}_{j-1}^p \Delta t^2, \quad j \geq 1 \tag{2}$$

Δt in (2) is the time step increment (unity in this paper). CFO starts with a user-specified initial probes positions and accelerations distributions. The initial acceleration vectors are usually set to zero.

Probes may fly outside the decision space and should be returned if they do. There are many possible probe retrieval methods. A useful one is the *reposition factor* F_{rep}, which plays an important role in CFO's convergence [1–6]. It is shown schematically in Figure 1.

for $i = 1$ *to* N_d

if $\vec{R}_j^p(i) < R_i^{\min}$ *Then*

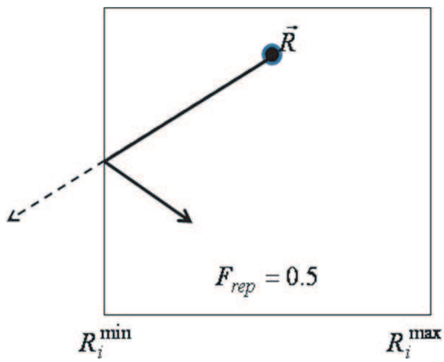
$\vec{R}_j^p(i) = R_i^{\min} + F_{rep}(\vec{R}_{j-1}^p(i) - R_i^{\min})$

if $\vec{R}_j^p(i) > R_i^{\max}$ *Then*

$\vec{R}_j^p(i) = R_i^{\max} - F_{rep}(R_i^{\max} - R_{j-1}^p(i))$

end

(a)



(b)

Figure 1. (a) Errant probe reposition factor retrieval. (b) Illustration of probe repositioning in 2-D DS.

F_{rep} is usually set to 0.5 or 0.9, or it may be variable [5]. $R_k^{\min}$ and $R_k^{\max}$ are the minimum and maximum values of the kth spatial dimension corresponding to the optimization problem constraints. The CFO algorithm flow chart appears in Figure 2 [5].

3. ACCELERATION CLIPPING (AC)

In fact, CFO requires optimizing six run parameters to reach the global optimum as fast as possible and to prevent probes from going into local optimum trapping [1–6]. In general, each objective function needs a different set of parameters. The deterministic nature of CFO helps on predicting the best choice quickly. But, this becomes more difficult

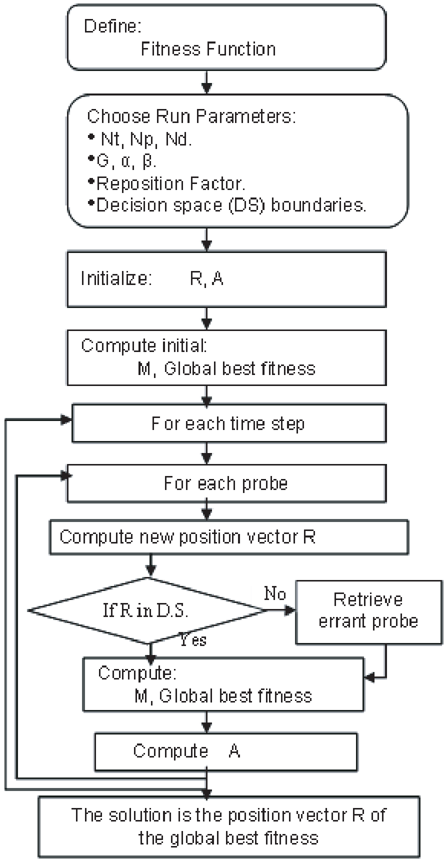


Figure 2. Flowchart of the main steps of the CFO algorithm [5].

and time consuming when the dimension of the problem increases where more probes are needed to cover the decision space effectively. Acceleration clipping is a new modification introduced here to release the CFO from its dependency on the run parameters, thus, making it more robust.

To damp the probes' motion and to prevent probes from flying out of the decision space too often, the acceleration clipping scheme is introduced to limit the maximum acceleration of the probes, as shown in Figure 3 [13].

When the length of the acceleration vector is greater than the diagonal length of the decision space multiplied by a predefined factor called $A_{\max}$, the acceleration vector will be clipped by multiplying it by the same factor $A_{\max}$. The decision space diagonal length is defined

$$
\begin{array}{l}
\textit{if } \left\| \vec{A}_j^{\,p} \right\| > A_{\max} \times \textit{Decision Space Diagonal Length} \\
\quad \vec{A}_j^{\,p} = A_{\max} \times \vec{A}_j^{\,p} \; ; \\
\textit{end}
\end{array}
$$

Figure 3. The acceleration clipping pseudocode.

as follows:

$$\text{Decision Space Diagonal Length} = \sqrt{\sum_{i=1}^{N_d} \left(R_i^{\max} - R_i^{\min}\right)^2} \tag{3}$$

In general, $A_{\max}$ is a constant between 0.001 to 0.5 and the default is 0.01, and $A_{\max} = 1$ refers to the original CFO.

The difference between the velocity clamping in PSO [14] and the acceleration clipping in CFO is that the particle's velocity is clamped per dimension, while the entire acceleration vector is clipped. Therefore, the direction of the velocity vector changes after clamping, while the acceleration vector keeps its direction. Consequently, the probe still moves in the same resultant trajectory of the weighted difference position vectors. So, the acceleration clipping functionality is rather similar to the inertia weight functionality in PSO [14] rather than the velocity clamping. $A_{\max}$ is therefore an important parameter which refines the probes motions and decreases the number of outside flying probes. Therefore, it decreases the dependency of the CFO convergence on the reposition factor retrieval scheme and enhances the global search ability of CFO. A comprehensive comparison between the CFO and CFO-AC algorithms has been presented in [13]. Both algorithms were applied on finding the extrema (minima or maxima) of many mathematical functions, and it has been found that the CFO-AC algorithm outperforms the CFO algorithm. Moreover, both algorithms were applied on the optimal design of linear and circular antenna arrays, and again it was found that the CFO-AC outperforms the CFO basic algorithm [13].

4. INTEGRATION OF OPTIMIZATION ALGORITHMS WITH IE3D

IE3D is a full wave EM simulator [15] in which Maxwell's integral equations are solved using the frequency-domain method of moments. IE3D has several built-in optimization methods, including Powell optimizer, genetic optimizer, random optimizer, and an adaptive optimizer. The variables for optimization defined by IE3D are controlled by their directions and bounds. Optimization with complicated variations may cause an overlap problem in IE3D. CFO (with acceleration clipping) and DE algorithms, written by Matlab, are used to optimize the variables defined by IE3D and to compute the fitness functions according to the IE3D simulation results.

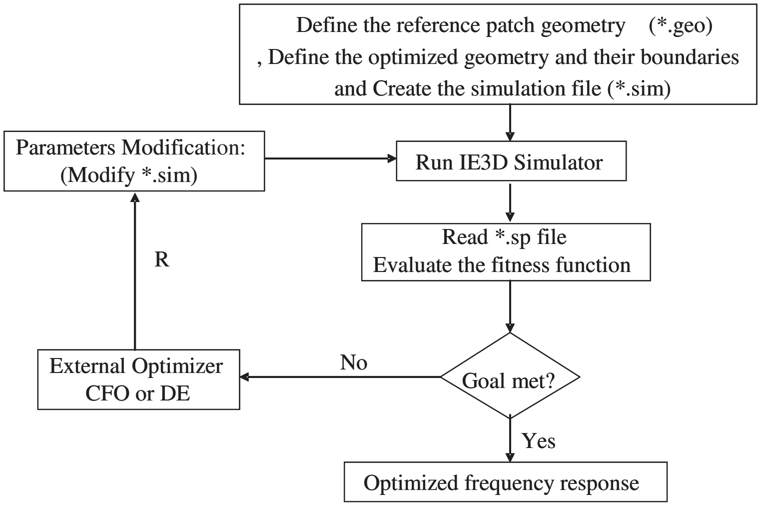


Figure 4. Flowchart of patch optimization.

Figure 4 shows the flowchart of the optimization procedure. Before the optimization, IE3D is used to model a reference patch antenna. The initial dimensions and the optimized geometry of the reference patch antenna are defined and saved in the file with extension *geo*. Then, the *sim* file is created which includes the simulation information, such as the optimized geometry with its offsets and the offsets' bounds. After that, CFO and DE are applied as the external optimizers. The optimization code is developed such that:

- It can replace the geometry's offsets in the *sim* file with the values of the elements of the position vector **R** in order to control the IE3D simulation.
- It can run the IE3D engine to simulate the new patch with new dimensions.
- It can read the S-parameters in the output file (with the extension *sp*) in order to evaluate the predefined fitness function.

5. E-SHAPED PATCH ANTENNA DESIGN

Recently, a coaxially fed E-shaped patch antennas with thick air substrate, reported in [16], has become prevalent in wireless communication applications [17]. The same antenna was optimized in [18] using PSO/FDTD optimizer to design a dual-frequency antenna and a broadband antenna. The dual-frequency antenna operated at 1.8 and 2.4 GHz, while the broadband antenna had a bandwidth from 1.79 to 2.43 GHz (30.5%). In [19], a low-profile microstrip line fed E-shaped patch antenna, shown in Figure 5, has been designed using the MPSO/IE3D method. The substrate has a thickness of $h = 2\,\text{mm}$, and dielectric constant ε_r of 2.55. The antenna is fed by an inserted microstrip line at $(W/2,\ L_f)$ with a fixed width ($W_\ell = 5.6\,\text{mm}$).

The microstrip line fed E-patch antenna is optimized here using the CFO/IE3D and DE/IE3D in order to assess the performance of CFO and DE optimizers in such real antenna problem. To avoid the overlap problem in IE3D simulations, the following conditions must hold as additional geometrical restrictions [19]:

$$P_s + 2W_s < W \quad L_s < L \quad L_f < L$$
$$P_s > W_\ell + 2W_f \ \text{ or } \ P_s + 2W_s < W_\ell, \text{ when } \ L_s + L_f >= L$$

Firstly, as a test of the CFO/IE3D and DE/IE3D methods, the optimizers are applied to achieve the simple objective of designing this E-shaped patch antenna to work at the resonance frequency (f_r) of 2.4 GHz. The fitness function to be maximized is formulated as:

$$\text{Fitness} = -S_{11}(2.4\ \text{GHz}) \tag{4}$$

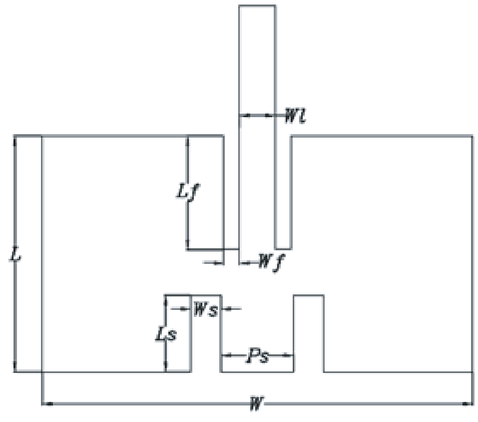


Figure 5. Geometry of low-profile E-shaped microstrip patch antenna [19].

The optimizers are applied for only one trial of 500 time steps and using 20 probes. The CFO parameters are: $G = 2$, $\alpha = 0.3$, $\beta = 1.5$, $F_{rep} = 0.5$, and $A_{\max} = 0.1$; while the DE parameters are: differentiation constant = 0.6, and crossover constant = 0.85. The obtained optimized dimensions for the antennas are listed in Table 1. Moreover, the low and high bounds for each parameter and the dimensions of the reference antenna are included in the same table. The comparison in Table 2 shows the good performance of both CFO and DE. A return loss of 73 dB ($f_r = 2.4$ GHz) is achieved by the CFO, but with very narrow bandwidth of 35.5 MHz. The reflection coefficient (in dB) for the optimized antennas and the reference antenna is shown in Figure 6.

Table 1. Optimized dimensions for the low-profile E-shaped patch antenna using the fitness function described by Equation (4) (all dimensions are in mm).

	W	L	W_s	L_s	P_s	W_f	L_f
Low Bounds	29	30	0	9	2	1	0
High Bounds	69	45	18	19	34	4	18
Reference Antenna	49	37.5	9	14	18	2.5	9
CFO Antenna I	53.7	39.58	9.59	15.46	22.16	2.15	12.79
DE Antenna I	64.2	39	10.95	14.05	14.86	2.25	9.44

Table 2. The results of optimizing the E-shaped patch antenna with the fitness function described by Equation (4).

	Fitness	S_{11} (2.4 GHz)	BW ($S_{11} < -10$ dB)
CFO	-2.24×10^{-4}	-73 dB	≈ 35.5 MHz
DE	-5.02×10^{-4}	-66 dB	≈ 44 MHz

Table 3. Optimized dimensions for the E-shaped patch antenna using the fitness function described by Equation (5) (all dimensions are in mm).

	W	L	W_s	L_s	P_s	W_f	L_f
CFO Antenna II	64.24	36.96	5.03	12.21	12.44	1.84	10.83
DE Antenna II	69	36.84	6.86	9.16	15.7	1.57	17.85
MPSO Antenna [19]	67	37.42	7.92	9.88	14.5	3.37	10.86

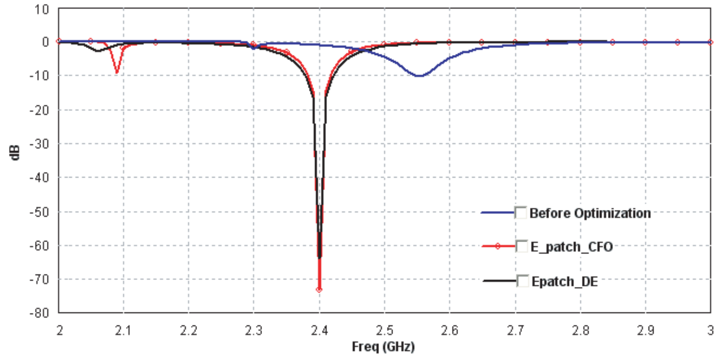


Figure 6. Reflection coefficient (in dB) of the CFO and DE optimized E-shaped antennas using the fitness function described by Equation (4) compared with the reference (un-optimized) E-shaped antenna.

Table 4. The results of optimizing the E-shaped patch antenna with the fitness function described by Equation (5).

	Fitness	$S_{11}(f)$	SWR
CFO	-0.20654	< -13 dB	< 1.58
DE	-0.16013	< -15.5 dB	< 1.4
MPSO [19]	–	< -13.9 dB	< 1.5

Now, the same microstrip line fed E-shaped patch antenna is optimized again to be suitable for the WLAN system with the operating frequency band from 2.4 GHz to 2.484 GHz. The fitness function is changed to insure the minimization of the return loss in the entire frequency band and is described as follows:

$$\text{Fitness} = -\max\left(S_{11}(f)\right), 2.4\,\text{GHz} \leq f \leq 2.484\,\text{GHz} \qquad (5)$$

The new optimized antennas geometries are shown in Table 3. In addition to the DE and CFO results, MPSO results [19] are included in the table. According to the fitness convergence plots in Figure 7 and the comparison results in Table 4, the DE performance is somewhat better than the performance of the CFO and MPSO. The reflection coefficient achieved by the DE is -15.5 dB which is around 2 dB less than that obtained by CFO and MPSO. Figure 8 shows the reflection coefficient (in dB) for the optimized antennas. The DE-optimized antenna has a slightly larger bandwidth than the CFO-optimized patch.

The gain of the designed antennas versus frequency is shown in Figure 9. Within the frequency band of interest, the minimum

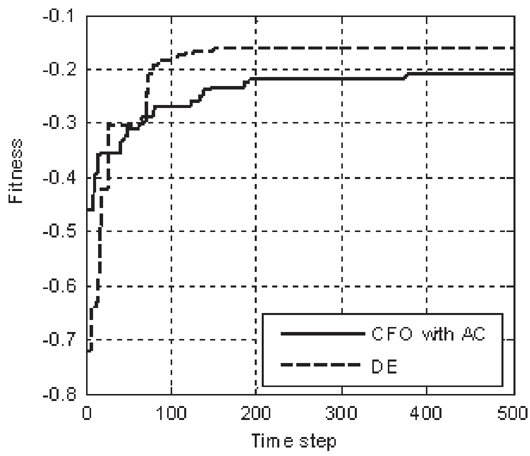


Figure 7. The fitness convergence of the CFO and DE optimizers applied on the design of the E-shaped antenna using the fitness function described by Equation (5).

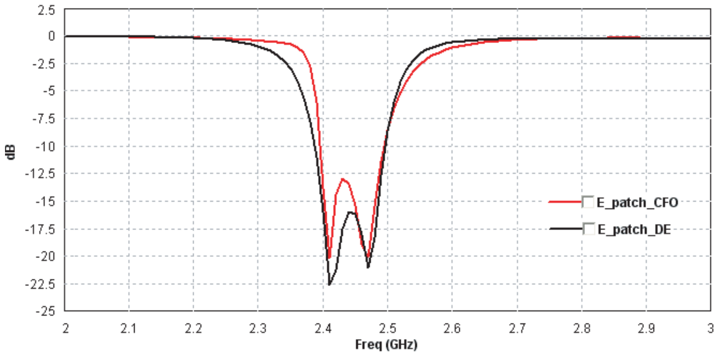


Figure 8. Reflection coefficient (in dB) of the CFO and DE optimized E-shaped antennas using the fitness function described by Equation (5).

gain achieved by the DE antenna is 5.6 dBi, while the CFO antenna minimum gain is 4.6 dBi. The antennas radiation patterns at two different frequencies are shown in Figure 10, which are typical patterns for patch antennas.

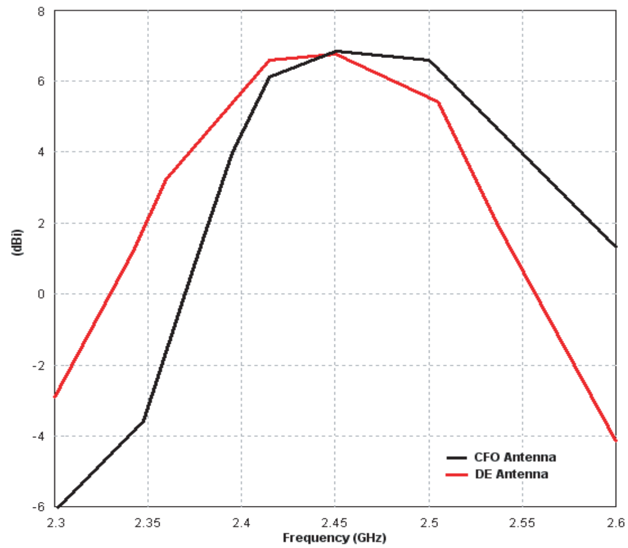


Figure 9. The gain versus frequency plots of the CFO and DE optimized E-shaped patch antennas.

6. DOUBLE-E-SHAPED PATCH ANTENNA DESIGN

In order to design a wide band antenna, a patch antenna with four slots, shown in Figure 11, is proposed. Two extra slots parallel to those in the E-shaped patch antenna are appended to shape what could be called a *double-E-shaped* patch antenna. For simplicity, a coaxial probe is used as a feeder. A *double-E-shaped* patch antenna with substrate thickness of ($h = 15\,\text{mm}$) and filled with air ($\varepsilon_r = 1$) is designed using the CFO/IE3D and DE/IE3D methods. The antenna is fed by a coaxial probe at a distance F from the origin along the x-axis in the middle branch to achieve the best return loss. To avoid the overlap problem in IE3D simulations, the following conditions must also hold as additional geometrical restrictions:

$$L_s < L \quad P_1 > \frac{1}{2}(W_1 + W_2) \quad P_2 > P_1 + \frac{1}{2}(W_2 + W_3) \quad |F| < L/2$$

The antenna is designed to work in the frequency band from 1.7 to 2.5 GHz to be suitable for the following wireless systems: DCS-1800 (1.71 ∼ 1.88 GHz), PCS-1900 (1.85 ∼ 1.99 GHz), IMT-2000/ UMTS (1.885 ∼ 2.2 GHz), WLAN (2.4 ∼ 2.483 GHz), and Bluetooth (2.4 ∼ 2.5 GHz). The fitness function is similar to Equation (5) by taking the frequency samples from 1.7 to 2.5 GHz. The obtained

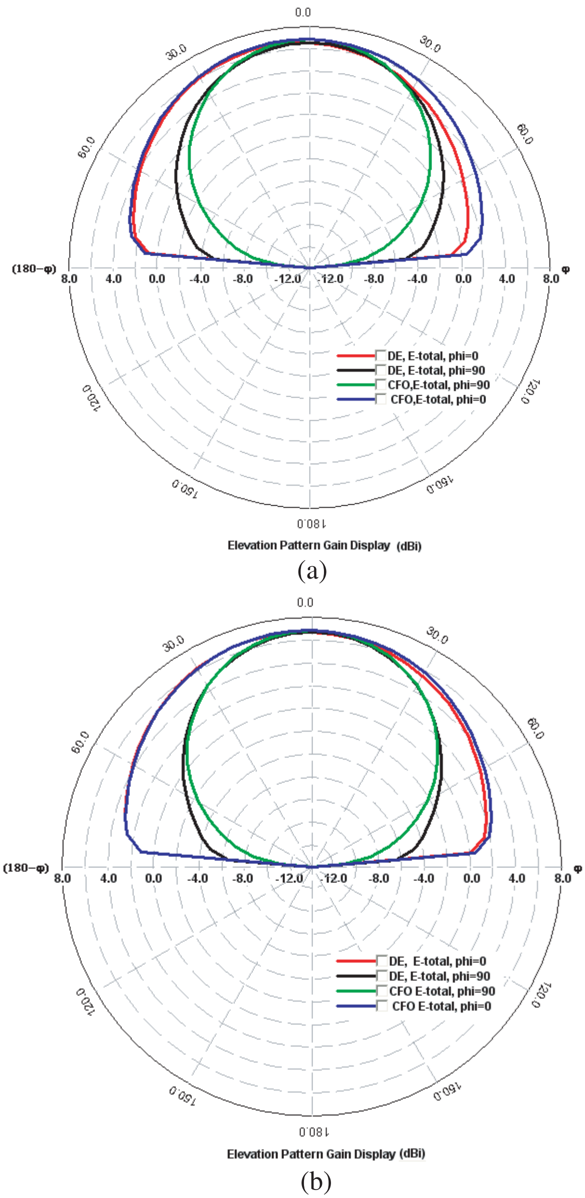
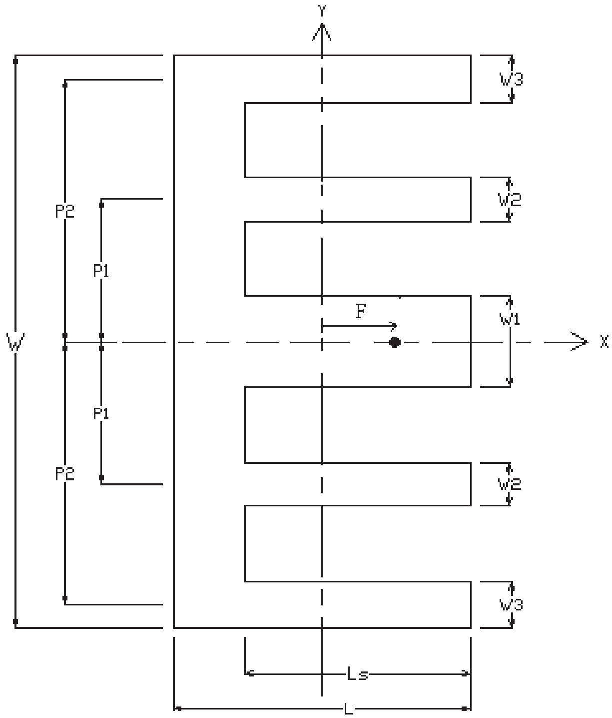
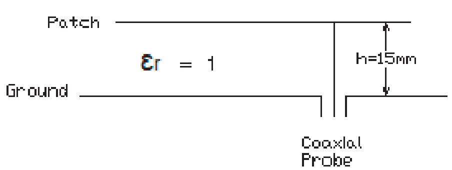


Figure 10. The radiation pattern of the the CFO and DE optimized E-shaped patch antennas. (a) Frequency = 2.415 GHz. (b) Frequency = 2.45 GHz.



(a)



(b)

Figure 11. Geometry of the double-E-shaped microstrip patch antenna.

optimized parameters for the antenna are listed in Table 5. The CFO and DE run parameters used here are the same as those used in the design of the E-shaped patch antenna.

According to the comparison in Table 6, the CFO gives results that are very close to those obtained by DE. The reflection coefficient (in dB) of the optimized double-E-shaped patch antennas is shown in Figure 12. Both the CFO and DE were able to give the desired frequency response.

Table 5. Optimized dimensions for the coaxial probe fed double-E-shaped patch antenna (all dimensions are in mm).

	W	L	L_s	W_1	W_2	W_3	P_1	P_2	F
Low Bound	—	34	27	4	0	0	10	26	−20
High Bound	—	74	67	24	12	12	40	66	20
Reference Antenna	98	54	47	14	6	6	25	46	0
CFO Antenna	80.48	55.26	50.36	12.44	9.08	7.78	25.42	36.35	17.52
DE Antenna	81	55.24	50.32	12.66	6.76	11.78	23.77	34.61	17.92

Table 6. The results of optimizing the coaxial probe fed double-E-shaped patch antenna.

	Fitness	$S_{11}(n)$	SWR
CFO	−0.3208	< -9.87 dB	< 1.944
DE	−0.3202	< -9.89 dB	< 1.942

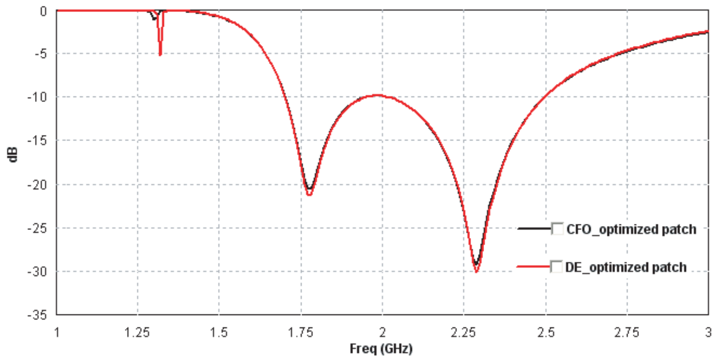


Figure 12. Reflection coefficient (in dB) of the CFO and DE optimized coaxial fed double-E-shaped antenna.

The fractional bandwidth of both antennas is about 38%, and the minimum gain achieved within the frequency band of interest is about 6.5 dBi. Figure 13 shows the gain versus frequency for the optimized antennas. The antennas' radiation patterns, at three different frequencies, are shown in Figure 14.

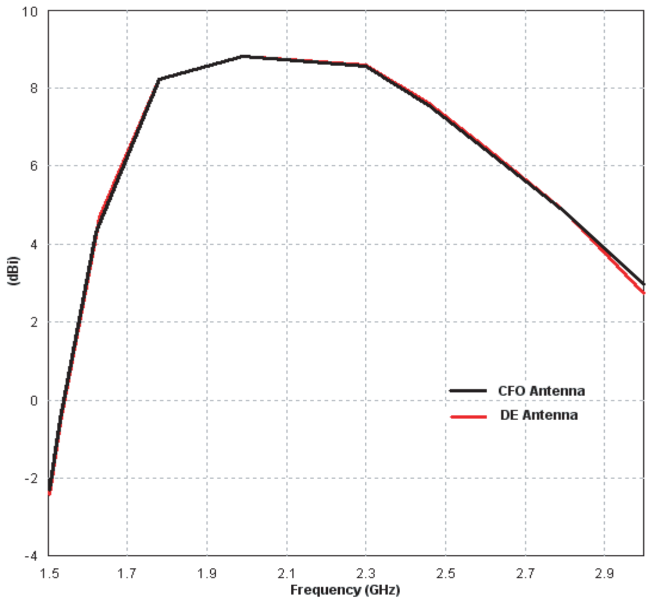
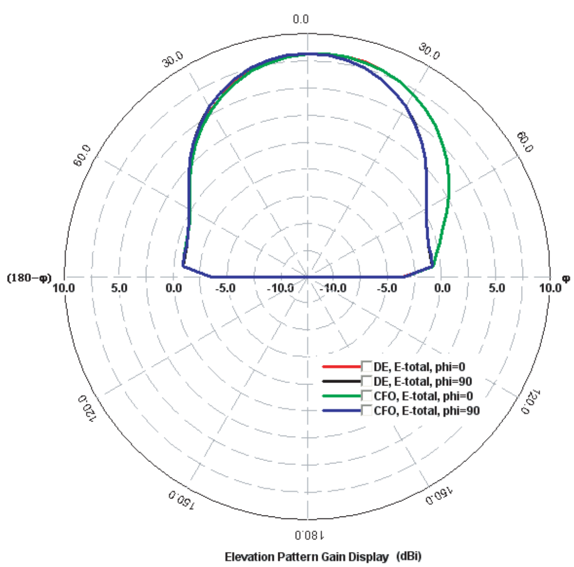
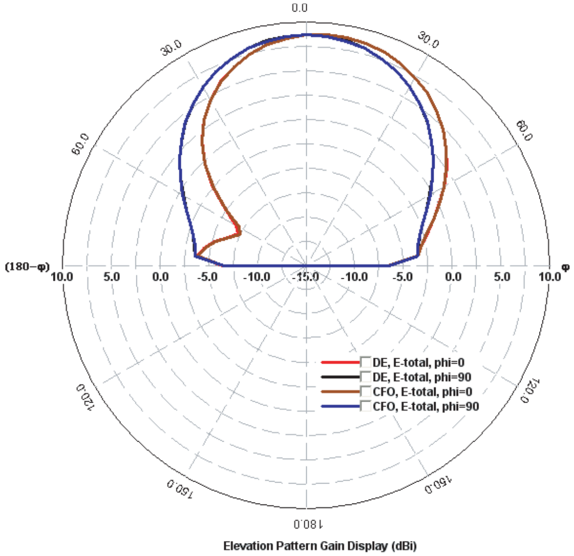


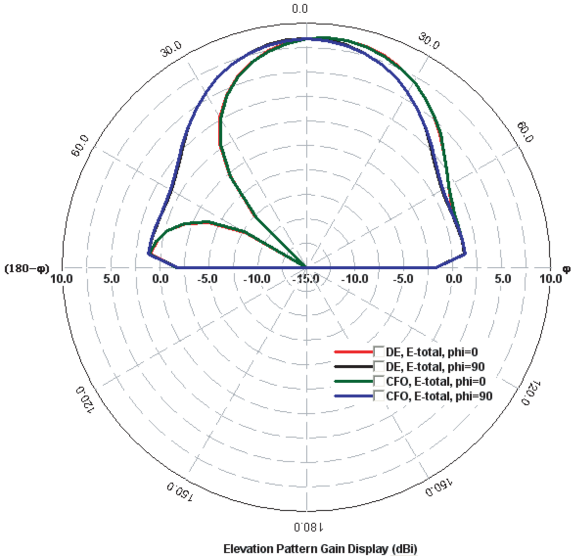
Figure 13. The gain versus frequency for the CFO and DE optimized double-E-shaped patch antennas.



(a)



(b)



(c)

Figure 14. The radiation pattern of the the CFO and DE optimized double-E-shaped patch antennas. (a) Frequency = 1.78 GHz. (b) Frequency = 1.99 GHz. (c) Frequency = 2.3 GHz.

7. CONCLUSIONS

In this paper, the CFO and DE techniques have been applied to the design of microstrip patch antennas. To make it more robust, the acceleration clipping has been introduced into the basic CFO algorithm. It has been shown that the CFO-AC algorithm exhibits very good performance and holds what appears to be considerable promise. Two different wide band patch antennas were optimized using the CFO-AC and DE algorithms. Specifically, an E-shaped and a double-E-shaped patch antennas were designed and optimized using the CFO and DE methods, which were interfaced with the IE3D software. The E-shaped patch antenna covered the WLAN band (2.4 GHz to 2.484), while the double-E-shaped patch covered the frequency range of 1.7–2.5 GHz. The obtained CFO results were in very good agreement with those obtained using the differential evolution (DE) optimization method. The accuracy of the CFO and DE methods validates their potential application in antenna design problems.

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