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ON THE INFLUENCE OF FLUCTUATIONS OF THE DIRECTION OF AN EXTERNAL MAGNETIC FIELD ON PHASE AND AMPLITUDE CORRELATION FUNCTIONS OF SCATTERED RADIATION BY MAGNETIZED PLASMA SLAB

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Abstract—Statistical characteristics of scattered electromagnetic waves by turbulent magnetized plasma slab with electron density and magnetic field fluctuations are considered via the perturbation method and boundary conditions. Magnetic field fluctuates both in magnitude and direction. Analytical expressions for the component of scattered electric field, correlation functions of the amplitude and phase fluctuations, and also the phase structure function for arbitrary correlation functions of fluctuating plasma parameters are derived. The obtained results are valid for near and far zones. Under equal conditions, at strong magnetic fields, electron density fluctuations play the important role and in this case the imposed magnetic field decreases fluctuation intensity of the ordinary and extraordinary waves. Numerical calculations of statistical characteristics of scattered radiation were carried out for anisotropic Gaussian correlation function

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for electron density fluctuation and correlation tensor of the second order for the fluctuation of an external magnetic field. The phase portraits of correlation functions of the amplitude and phase fluctuations are constructed.

1. INTRODUCTION

At the present time the features of light propagation in random media have been rather well studied [1]. Many articles and reviews are related to statistical characteristics of scattered radiation and observations in the ionosphere [2–5]. The analysis of the statistical properties of small-amplitude electromagnetic waves that have passed through a plane turbulent plasma slab is very important in many practical applications associated with both natural and laboratory plasmas [6, 7]. In most papers statistically isotropic irregularities have been considered. However, in reality, irregularities in the ionosphere are anisotropic and mainly elongated along the geomagnetic field. The sizes of small-scale ionospheric irregularities have been obtained by several techniques, including topside sounding, radio star and radio-satellite scintillations and direct measurement by satellite probes. The irregularities have a variety of sizes and usually are elongated in the magnetic field direction. The dominant sizes observed will depend not only on the prevailing ionospheric conditions, but also on the sensitivity of the particular method to detect irregularities of different sizes. Investigation of statistical characteristics of scattered radiation in randomly inhomogeneous magnetized plasma is of a great practical importance. Statistical characteristics of the angular power spectrum (broadening and displacement of its maximum), scintillation effects and the angle-of-arrival of scattered electromagnetic waves by turbulent anisotropic collision magnetized ionospheric plasma layer for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations were investigated analytically in the complex geometrical optics approximation on the basis of stochastic eikonal equation and numerically by statistical simulation using the Monte Carlo method [8–10]. The geomagnetic field plays a key role in the dynamics of the plasma in the ionosphere. The dynamic processes are accompanied by the regular variations of the geomagnetic field. Intensive geomagnetic field perturbations are observed during strong earthquakes, launching of spacecraft and other phenomena. Therefore, geomagnetic field randomly varies both in magnitude and direction. Statistical characteristics of the scattered radiation in the ionospheric plasma at random variations of geomagnetic field magnitude in the geometrical optics approximation were considered in [11]. In the present paper special attention is also paid to the fluctuations of the external magnetic field directions.

In Section 2, the set of differential equations for electric field in homogeneous gyrotropic medium and corresponding dispersion equation are obtained if the angular frequency of an incident wave exceeds an ionic angular gyrofrequency. Two types of circularly polarized waves propagate along the direction of homogeneous external magnetic field. In Section 3 set of stochastic differential equations for perturbed electric field components has been obtained using the perturbation method if scattered electromagnetic waves propagate near the direction of an external magnetic field caused by electron density and magnetic field fluctuations. Analytical expressions for two-dimensional spectral components of scattered electric field by turbulent plasma slab are obtained using the boundary conditions. Attenuation coefficient for the mean field has been derived on the bases of energetic consideration. Statistical characteristics-correlation function of the amplitude and phase fluctuations of scattered radiation are obtained for arbitrary correlation function of electron density fluctuations and second-rank tensor of magnetic field fluctuations. We suppose that electron density and magnetic field fluctuations are statistically independent. The influence of direction fluctuations of an external magnetic field on the statistical characteristics of scattered electromagnetic field are considered for the first time in this paper. The obtained analytical expressions are valid for near and far zones. In Section 4, numerical calculations have been carried out for anisotropic Gaussian correlation function of electron density fluctuations and the second-rank tensor for magnetic field fluctuations. Conclusion is given in Section 5.

2. FORMULATION OF THE PROBLEM

Let's consider electromagnetic waves scattering by plasma slab having finite thickness L. If frequency of an incident wave satisfies the condition $\omega \gg \Omega_i = eH_0/Mc$, the ions can be considered immovable and only motion of electrons can be taken into account; Ω_i is the ion gyrofrequency, M is the mass of an ion, c is the speed of light in vacuum, and H_0 is the strength of the external magnetic field. If $\omega \gg \nu_{eff}$, ν_{eff} is the effective electron collision frequency with the ions and molecules, then conduction current can be neglected and the total current in the medium equals to the displacement current $\mathbf{j} = -eN\mathbf{w}$, $\mathbf{w}$ — velocity of electrons. If we assume that the fields are time-harmonic dependence, wave equation for electric field strength $\mathbf{E}$ can be written as:

$$rot\, rot\, \mathbf{E} - k_0^2 \mathbf{E} = -i \frac{4\pi k_0}{c} \, eN\mathbf{w}, \tag{1}$$

where: e — electron charge, N — electron density in magnetized plasma, $k_0 = \omega/c$, ω is the frequency. Taken into account that $i\omega\mathbf{w} = e\mathbf{E}/m + e[\mathbf{w} \cdot \mathbf{H_0}]/mc$, the Equation (1) can be written as:

$$\begin{aligned} & grad\, div\, \mathbf{E} - \Delta \mathbf{E} - k_0^2 \mathbf{E} \\ &= -\frac{\tilde{v} k_0^2}{1-\tilde{u}} \left\{ \mathbf{E} - \frac{ie}{mc\omega} [\mathbf{E} \cdot \mathbf{H_0}] - \left(\frac{e}{mc\omega}\right)^2 (\mathbf{E} \cdot \mathbf{H_0}) \mathbf{H_0} \right\}, \end{aligned} \tag{2}$$

where: $\tilde{u} = (eH_0/mc\omega)^2$, $\tilde{v} = \omega_p^2/\omega^2$ are the magneto-ionic parameters, $\omega_p = (4\pi N e^2/m)^{1/2}$ is the plasma frequency. Vector of electric induction $\mathbf{D} = \mathbf{E} - 4\pi i e N_0 \mathbf{w}/\omega$ is described by the expression:

$$\mathbf{D} = \left(1 - \frac{\tilde{v}}{1-\tilde{u}}\right) \mathbf{E} + i \, \frac{\tilde{v}}{1-\tilde{u}} \left[\mathbf{E} \cdot \frac{\mathbf{\Omega_H}}{\omega}\right] + \frac{\tilde{v}}{1-\tilde{u}} \left(\mathbf{E} \cdot \frac{\mathbf{\Omega_H}}{\omega}\right) \frac{\mathbf{\Omega_H}}{\omega}, \tag{3}$$

where $\Omega_H = eH_0/mc$ is the electron gyrofrequency.

For homogeneous gyrotropic medium without fluctuating plasma parameters the dispersion relation has the following form:

$$\begin{aligned} & \varepsilon_{zz} x^4 - \left[(\varepsilon_{xx} + \varepsilon_{yy})\varepsilon_{zz} + \varepsilon_{xz}^2 + \varepsilon_{yz}^2\right] x^2 \\ & + \left(\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{xz}^2 \varepsilon_{yy} + \varepsilon_{xx}\varepsilon_{yz}^2 + \varepsilon_{xy}^2 \varepsilon_{zz}\right) = 0, \end{aligned} \tag{4}$$

where $x = k_z/k_0$ is the non-dimensional parameter.

Now we consider statistical characteristics of scattered radiation by turbulent magnetized plasma slab with electron density and magnetic field fluctuations if wave propagates along the external magnetic field. As far as statistically isotropic scalar field not correlated with solenoidal vector field [12] we suppose that electron density and magnetic field fluctuations are statistically independent. When a wave passes through a region containing irregularities of refractive index both amplitude and phase fluctuations arise in the wave front. Each of the magnitudes in the Equation (2) can be presented as the sum of the mean value and small fluctuating terms. Therefore we will use the small perturbation method:

$$\mathbf{E} = <\mathbf{E}> + \mathbf{e}, \quad \mathbf{H_0} = <\mathbf{H_0}> + \mathbf{h_0}, \quad N = <N> + n. \tag{5}$$

The fluctuating values are random functions of the spatial coordinates. The angular brackets indicate the statistical average. We will assume that the mean values of electron density and magnetic field do not depend on coordinates. Substitution of (5) into (2) gives linearized set of equations for the mean and fluctuating fields. Hence fluctuating electric field satisfies the following differential equation:

$$\left(\frac{\partial^2}{\partial x_i \partial x_j} - \Delta\delta_{ij} - k_0^2 \varepsilon_{ij}\right) e_j = j_i. \tag{6}$$

Current density contains both electron density and magnetic field fluctuations:

$$\begin{aligned}
\mathbf{j} \;=\; & \frac{k_0^2 v}{1-u}\left\{i\sqrt{u}\,[<\mathbf{E}>\boldsymbol{\mu}] + u(<\mathbf{E}>\boldsymbol{\tau})\boldsymbol{\mu} + u(<\mathbf{E}>\boldsymbol{\mu})\boldsymbol{\tau}\right\}\\
& -\frac{k_0^2 v}{1-u}\left[n_1 + \frac{2u}{(1-u)^2}(\boldsymbol{\tau}\cdot\boldsymbol{\mu})\right]\\
& \cdot\left\{<\mathbf{E}> - i\sqrt{u}\,[<\mathbf{E}>\boldsymbol{\tau}] - u(<\mathbf{E}>\boldsymbol{\tau})\boldsymbol{\tau}\right\},\\
& v = \frac{\omega_{p0}^2}{\omega^2}, \quad \omega_{p0}^2 = \frac{4\pi e^2 <N>}{m\omega^2}, \quad u = \left(\frac{e<H_0>}{mc\,\omega}\right)^2,\\
& n_1 = \frac{n}{<N>}, \quad \boldsymbol{\mu} = \frac{\mathbf{h_0}}{|<\mathbf{H_0}>|}, \quad \boldsymbol{\tau} = \frac{<\mathbf{H_0}>}{|<\mathbf{H_0}>|}.
\end{aligned}$$

If electromagnetic wave propagates along oz axis and the vector $\boldsymbol{\tau}$ lies in the yz coordinate plane ($\mathbf{k} \parallel z$, $<\mathbf{H_0}> \in yz$), then components of the second-rank tensor ε_{ij} of collisionless magnetized plasma will have the following form:

$$\begin{aligned}
& \varepsilon_{xx} = 1 - \frac{v}{1-u}, \quad \varepsilon_{yy} = 1 - \frac{v\,(1-u\,\sin^2\theta)}{1-u}, \quad \varepsilon_{zz} = 1 - \frac{v(1-u\,\cos^2\theta)}{1-u},\\
& \varepsilon_{xy} = -\varepsilon_{yx} = i\frac{v\sqrt{u}\cos\theta}{1-u}, \quad \varepsilon_{yz} = \varepsilon_{zy} = \frac{u\,v\sin\theta\cos\theta}{1-u},\\
& \varepsilon_{xz} = -\varepsilon_{zx} = -\,i\,\frac{v\,\sqrt{u}\sin\theta}{1-u},
\end{aligned} \tag{7}$$

θ is the angle between the vectors $\mathbf{k}$ and $\mathbf{H}_0$. Below we will use the designation $\varepsilon_{xy} = i\,\tilde{\varepsilon}_{xy}$.

We seek the solution of the set of Equation (6) by expansion of the Fourier integral over x and y coordinates:

$$\mathbf{e}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \tilde{\mathbf{e}}(k_x, k_y, z)\exp\left[\,i\,(k_x x + k_y y)\right],$$

$$\mathbf{j}(\mathbf{r}) = \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y\, \tilde{\mathbf{g}}\,(k_x, k_y, z)\exp\left[\,i\,(k_x x + k_y y)\right].$$

As a result we obtain the set of differential equations for a perturbed electric field:

$$\begin{aligned}
&\frac{\partial^2 \tilde{e}_x}{\partial z^2} + A_1 \frac{\partial \tilde{e}_z}{\partial z} + B_1 \tilde{e}_x + C_1 \tilde{e}_y + D_1 \tilde{e}_z = G_1, \\
&\frac{\partial^2 \tilde{e}_y}{\partial z^2} + A_2 \frac{\partial \tilde{e}_z}{\partial z} + B_2 \tilde{e}_x + C_2 \tilde{e}_y + D_2 \tilde{e}_z = G_2, \\
&\frac{\partial \tilde{e}_x}{\partial z} + A_3 \frac{\partial \tilde{e}_y}{\partial z} + B_3 e_x + C_3 e_y + D_3 e_z = G_3,
\end{aligned} \tag{8}$$

where coefficients are:

$A_1 = -\,i\,k_x$, $\quad B_1 = k_0^2 \varepsilon_{xx} - k_y^2$, $\quad C_1 = k_0^2 \varepsilon_{xy} + k_x k_y$, $\quad D_1 = k_0^2 \varepsilon_{xz}$,
$G_1 = -\,k_0^2\, g_x$, $\quad A_2 = -\,i\,k_y$, $\quad B_2 = k_0^2 \varepsilon_{yx} + k_x k_y$, $\quad C_2 = k_0^2 \varepsilon_{yy} - k_x^2$,
$D_2 = k_0^2 \varepsilon_{yz}$, $\quad G_2 = -\,k_0^2\, g_y$, $\quad A_3 = k_y / k_x$, $\quad B_3 = i k_0^2\, \varepsilon_{zx} / k_x$,
$C_3 = i\, k_0^2 \varepsilon_{zy} / k_x$, $\quad D_3 = -i(k_x^2 + k_y^2 - k_0^2 \varepsilon_{zz}) / k_x$, $\quad G_3 = -\,i\,k_0^2\, g_z / k_x$.

The boundary conditions should be added to these equations. Let the plane xoy coincides with the lower boundary of slab, which is located in the half-space $z > 0$. Then the boundary conditions for the Equation (8) will be: at $z \geq L$ the waves propagating in the negative direction must be absent, and at $z \leq 0$ — in the positive direction. Free space is under and above the plasma slab. Since all functions are finite in a turbulent slab, $0 \leq z \leq L$, we solve the set of Equation (8) via the spectral method:

$$\tilde{e}_x(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\; A(\kappa) \exp[-\,i\,(L - z)\kappa],$$

$$\tilde{e}_y(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\; B(\kappa) \exp[-\,i\,(L - z)\kappa],$$

$$\tilde{e}_z(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\; C(\kappa) \exp[-\,i\,(L - z)\kappa],$$

$$G_1(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\; F_1(\kappa) \exp[-\,i\,(L - z)\kappa],$$

$$G_2(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\; F_2(\kappa) \exp[-\,i\,(L - z)\kappa],$$

$$G_3(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa\, F_3(\kappa) \exp[-i\,(L - z)\kappa]. \tag{9}$$

As a result the set of differential Equation (8) will be rewritten in the form of closed set of algebraic equations:

$$\begin{aligned}
\alpha_1(\kappa)A(\kappa) + C_1 B(\kappa) + \alpha_2(\kappa)C(\kappa) &= F_1(\kappa),\\
B_2 A(\kappa) + \beta_1(\kappa)B(\kappa) + \beta_2(\kappa)C(\kappa) &= F_2(\kappa),\\
\gamma_1(\kappa)A(\kappa) + \gamma_2(\kappa)B(\kappa) + D_3 C(\kappa) &= F_3(\kappa),
\end{aligned}$$

required coefficients $A(\kappa)$, $B(\kappa)$ and $C(\kappa)$ can be easily found via plasma parameters $\alpha_1(\kappa) = B_1 - \kappa^2$, $\alpha_2(\kappa) = D_1 + i\,A_1\,\kappa$, $\beta_1(\kappa) = C_2 - \kappa^2$, $\beta_2(\kappa) = D_2 + i\,A_2\,\kappa$, $\gamma_1(\kappa) = B_3 + i\,\kappa$, $\gamma_2(\kappa) = C_3 + i\,A_3\,\kappa$.

At quasi-longitudinal propagation determinant of set of Equation (8) reduces to the biquadratic equation:

$$\begin{aligned}
&x^4 - \left\{(\varepsilon_{xx} + \varepsilon_{yy}) - \frac{1}{\varepsilon_{zz}}\left[(\varepsilon_{xx} + \varepsilon_{zz})\gamma_x^2 + (\varepsilon_{yy} + \varepsilon_{zz})\gamma_y^2\right]\right\} x^2\\
&+ \left[(\varepsilon_{xx}\varepsilon_{yy} - \tilde{\varepsilon}_{xy}^2) - \frac{1}{\varepsilon_{zz}}\left(\varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{xx}\varepsilon_{zz} - \tilde{\varepsilon}_{xy}^2\right)\cdot\left(\gamma_x^2 + \gamma_y^2\right)\right.\\
&\left.+\frac{1}{\varepsilon_{zz}}(\varepsilon_{xx} + \varepsilon_{yy})\,\gamma_x^2\gamma_y^2 + \frac{1}{\varepsilon_{zz}}\left(\varepsilon_{xx}\,\gamma_x^4 + \varepsilon_{yy}\,\gamma_y^4\right)\right] = 0,
\end{aligned} \tag{10}$$

the roots of which are: $x_1 = \zeta_1 - \zeta_2\,(\gamma_x^2 + \gamma_y^2)$, $x_2 = \zeta_3 - \zeta_4\,(\gamma_x^2 + \gamma_y^2)$, $x_3 = -x_1$, $x_4 = -x_2$; $\gamma_x = k_x/k_0$, $\gamma_y = k_y/k_0$, $\gamma^2 = \gamma_x^2 + \gamma_y^2 (\gamma^2 \ll 1)$, $\zeta_1 = (\varepsilon_{xx} + \tilde{\varepsilon}_{xy})^{1/2}$, $\zeta_2 = \frac{1}{4\varepsilon_{zz}}\frac{\varepsilon_{xx}+\varepsilon_{zz}+\tilde{\varepsilon}_{xy}}{\sqrt{\varepsilon_{xx}+\tilde{\varepsilon}_{xy}}}$, $\zeta_3 = (\varepsilon_{xx} - \tilde{\varepsilon}_{xy})^{1/2}$, $\zeta_4 = \frac{1}{4\varepsilon_{zz}}\frac{\varepsilon_{xx}+\varepsilon_{zz}-\tilde{\varepsilon}_{xy}}{\sqrt{\varepsilon_{xx}-\tilde{\varepsilon}_{xy}}}$.

3. FLUCTUATION OF THE PHASE AND THE AMPLITUDE OF SCATTERED RADIATION BY MAGNETIZED PLASMA SLAB

Further, we will investigate phase and amplitude fluctuations of electromagnetic fields, passed through plasma slab with random inhomogeneities of electron density and magnetic field fluctuations. In the perturbation theory fluctuations of the phase φ_1 and relative amplitude A_1 are determined by the expressions:

$$\varphi_1 = \mathrm{Im}\left(\frac{e}{< E >}\right) \quad , \quad A_1 = \mathrm{Re}\left(\frac{e}{< E >}\right) \quad . \tag{11}$$

These formulae are valid for the components of the electric field vector, the mean values of which are nonzero. In this formula, the indices indicating the direction of the components are omitted as these fluctuating values are the same for all components. Correlation functions at the points $\mathbf{r_1}$ and $\mathbf{r_2}$ are determined as:

$$W_\varphi(\mathbf{r_1},\mathbf{r_2}) = <\varphi_1(\mathbf{r_1})\,\varphi_1(\mathbf{r_2})>,$$
$$W_A(\mathbf{r_1},\mathbf{r_2}) = <A_1(\mathbf{r_1})\,A_1(\mathbf{r_2})> . \qquad (12)$$

It is also necessary to make an assumption concerning to the mean field. Further we will consider scattering of only plane normal waves (ordinary and extraordinary) on electron density and magnetic field fluctuations. Let's each component of the mean field $<E_j> = |<E_j>|\exp(i\,q_{0j}\,z)$ is a slowly varying function of the coordinate z due to the attenuation of the mean field caused by transition of the mean field energy to the fluctuating one. We guess that the thickness of plasma slab is much smaller than the attenuation length (see below) and $<E_j>$ is a constant. Square of the refractive index [6] corresponding to the two normal waves propagating in a homogeneous magnetized plasma (without taking into account thermal motion) is

$$N_j^2 = \frac{q_{oj}}{k_0} = 1 - \frac{2v(1-v)}{2(1-v) - u\sin^2\theta \mp \sqrt{u^2\sin^4\theta + 4u(1-v)^2\cos^2\theta}} \qquad (13)$$

minus sign and index $j = 1$ correspond to the extraordinary wave, plus sign and index $j = 2$ — to the ordinary wave. Generally speaking ordinary and extraordinary waves in collisionless magnetized plasma are elliptically polarized, moreover the relations of the mean electric field components are determined with the well-known formulae [6]:

$$\frac{<E_y>_{1,2}}{<E_x>_{1,2}} = i\,Z_{1,2}, \qquad \frac{<E_z>_{1,2}}{<E_x>_{1,2}} = i\,P_{1,2}, \qquad (14)$$

were the polarization coefficients are

$$Z_{1,2} = \frac{2\sqrt{u}(1-v)\cos\theta}{u\sin^2\theta \pm \sqrt{u^2\sin^4\theta + 4u\,(1-v)^2\cos^2\theta}},$$
$$P_{1,2} = -\frac{v\sqrt{u}\sin\alpha + Z_j\,u\,v\sin\alpha\,\cos\alpha}{1 - u - v + u\,v\cos^2\alpha},$$

In the first expression the upper sign is related to the extraordinary while the lower one — to the ordinary waves. The magnitude $Z_{1,2}$ characterises relations of ellipse axes, describing by the electric field in the plane, perpendicular to the direction of the wave vector $\mathbf{k}$, i.e., ellipticity of the ordinary waves in plasma; The value $P_{1,2}$ determines relative value of the longitudinal component (with respect to the vector $\mathbf{k}$) of an electric field of normal wave. From (14) follows that at $\theta = 0°$ waves become transversal and polarization is circular, having clockwise rotation in the extraordinary wave and counterclockwise — in the ordinary wave (if looking in the direction of wave propagation). The above mentioned directions of rotation correspond to waves propagation along $\mathbf{H_0}$ (or at an acute angle between the wave vector $\mathbf{k}$ and vector $\mathbf{H_0}$). From the formula (14) follows that at $\theta = 0°$, for the ordinary wave we have $< E_y > = i < E_x >$, $< E_z > = 0$ or $< E_x > = -\, i < E_y >$, $< E_z > = 0$; for the extraordinary wave $< E_y > = -\, i < E_x >$, $< E_z > = 0$ or $< E_x > = \; i < E_y >$, $< E_z > = 0$. Below we will consider only this case.

Unknown functions $A(x)$, $B(x)$ and $C(x)$ are determined by the expressions:

$$\begin{aligned}
A(x) = \frac{1}{k_0^2\,\varepsilon_{zz}} &\left\{\left[\varepsilon_{xx}\varepsilon_{zz} - (\varepsilon_{xx}+\varepsilon_{zz})\,\gamma_x^2 - \varepsilon_{xx}\gamma_y^2 - (\varepsilon_{zz}-\gamma_x^2)\,x^2\right]F_1(x)\right.\\
&+\left[-i\tilde{\varepsilon}_{xy}\varepsilon_{zz} + i\tilde{\varepsilon}_{xy}(\gamma_x^2+\gamma_y^2) - \varepsilon_{zz}\gamma_x\gamma_y + \gamma_x\gamma_y x^2\right]F_2(x)\\
&\left.+\left[(i\,\tilde{\varepsilon}_{xy}\,\gamma_y - \varepsilon_{xx}\,\gamma_x)x + \gamma_x\,x^3\right]F_3(x)\right\},\\
B(x) = \frac{1}{k_0^2\varepsilon_{zz}} &\left\{\left[i\,\tilde{\varepsilon}_{xy}\varepsilon_{zz} - i\,\tilde{\varepsilon}_{xy}(\gamma_x^2+\gamma_y^2) - \varepsilon_{zz}\,\gamma_x\gamma_y + \gamma_x\gamma_y\,x^2\right]F_1(x)\right.\\
&+\left[\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xx}(\gamma_x^2+\gamma_y^2) - \varepsilon_{zz}\gamma_y^2 + (\gamma_y^2-\varepsilon_{zz})x^2\right]F_2(x)\\
&\left.+\left[(\varepsilon_{xx}\,\gamma_y - i\,\tilde{\varepsilon}_{xy}\,\gamma_x)x - \gamma_y\,x^3\right]F_3(x)\;\right\},\\
C(x) = \frac{1}{k_0^2\,\varepsilon_{zz}} &\left\{\left[-(\,i\,\tilde{\varepsilon}_{xy}\,\gamma_y + \varepsilon_{xx}\,\gamma_x)\,x + \gamma_x\,x^3\right]F_1(x)\right.\\
&+\left[(i\,\tilde{\varepsilon}_{xy}\,\gamma_x - \varepsilon_{xx}\,\gamma_y)\,x + \gamma_y\,x^3\right]F_2(x)\\
&+\left[(\varepsilon_{xx}^2 - \tilde{\varepsilon}_{xy}^2 - -\varepsilon_{xx}(\gamma_x^2+\gamma_y^2)\right.\\
&\left.\left.+(\gamma_x^2+\gamma_y^2-2\varepsilon_{xx})x^2 + x^4\right]F_3(x)\right\},
\end{aligned} \tag{15}$$

These formulae have been obtained at the assumption that $k_0\,l_{D,M} \gg 1$, where l_D and l_M are characteristic linear scales of random inhomogeneities of electron density and magnetic field fluctuations, respectively. This inequality is equivalent to the condition $\gamma_x, \gamma_y \ll 1$ and therefore we kept the small terms of the second-order.

Two-dimensional spectral densities of current density fluctuation causing electric field strength fluctuations for the ordinary wave at $\theta = 0°$ are determined by the expressions:

$$\begin{aligned}
g_x(\text{æ}, z) = &-\frac{v}{1-u} < E_x > \left[\left(1+\sqrt{u}\right)n_1(\text{æ}, z)\right.\\
&\left.+\left(\sqrt{u} + 2\,u\frac{1+\sqrt{u}}{1-u}\right)\;\mu_z(\text{æ}, z)\right]\\
&\equiv < E_x > \Upsilon_0\left[\,\Upsilon_1\,n_1(\text{æ}, z) + \Upsilon_2\,\mu_z(\text{æ}, z)\,\right],\\
g_y(\text{æ}, z) = &\;i\;g_x(\text{æ}, z),
\end{aligned}$$

$$\begin{aligned} g_z(\text{æ}, z) &= v \; \frac{u+\sqrt{u}}{1-u} \; < E_x > [\mu_x(\text{æ}, z) + i\,\mu_y(\text{æ}, z)] \\ &\equiv \; < E_x > \Upsilon_3 \, [\, \mu_x(\text{æ}, z) + i\,\mu_y(\text{æ}, z)] \,, \end{aligned} \tag{16}$$

where $\text{æ} = \{k_x, k_y\}$. As a result electric current density fluctuations in the plane xy are caused by both electron density and magnetic field fluctuations, while z component of the Fourier transform of current density contains only magnetic field fluctuations in perpendicular plane with respect to the external magnetic field, directed along the axis z in this case.

Substituting coefficients (15) and (16) in (9), applying the residue theory, two-dimensional spectral densities of electric field scattered by inhomogeneous plasma slab (with electron density and magnetic field fluctuations) satisfying the boundary conditions, at $z = L$ have the following form:

$$\begin{aligned} \tilde{e}_x(\text{æ}, L) = \frac{2k_0}{\delta_1\,\varepsilon_{zz}} < E_x > \Big\{ -\Upsilon_0 \, (a_1 - b_1\,\gamma_x^2 \, + c_1\,\gamma_y^2 + i d_1\,\gamma_x\,\gamma_y) \\ \int_0^L dz' \; \big[\Upsilon_1\, n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\big] \cdot \sin\big[(L-z')\,k_0\,x_1\big] \\ + \Upsilon_3(\, i\, f_1\,\gamma_x - g_1\,\gamma_y) \int_0^L dz' \; \big[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\big] \\ \cos\big[(L-z')\,k_0\,x_1\big] \Big\} + \frac{2\,k_0}{\delta_2\,\varepsilon_{zz}} < E_x > \Big\{ -\Upsilon_0 \, (a_2 - b_2\,\gamma_x^2 \\ + c_2\,\gamma_y^2 + i\,d_2\,\gamma_x\,\gamma_y) \int_0^L d\,z' \, \big[\Upsilon_1\, n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\big] \\ \cdot \sin\big[(L-z')\,k_0\,x_2\big] + \Upsilon_3(\, i\, f_2\,\gamma_x - g_2\,\gamma_y) \\ \int_0^L d\,z' \, \big[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\big] \cos\big[(L-z')\,k_0\,x_2\big] \Big\}. \end{aligned} \tag{17}$$

where: $a_1 = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{zz}\zeta_1^2 + \tilde{\varepsilon}_{xy}\varepsilon_{zz}$, $a_2 = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{zz}\zeta_3^2 + \tilde{\varepsilon}_{xy}\varepsilon_{zz}$, $b_1 = \zeta_1^2 + 2\,\zeta_1\,\zeta_2\,\varepsilon_{zz} + \varepsilon_{xx} + \varepsilon_{zz} + \tilde{\varepsilon}_{xy}$, $b_2 = \zeta_3^2 + 2\,\zeta_3\,\zeta_4\,\varepsilon_{zz} + \varepsilon_{xx} + \varepsilon_{zz} + \tilde{\varepsilon}_{xy}$, $c_1 = \varepsilon_{xx} + 2\,\zeta_1\,\zeta_2\,\varepsilon_{zz} - \tilde{\varepsilon}_{xy}$, $c_2 = \varepsilon_{xx} + 2\,\zeta_3\,\zeta_4\,\varepsilon_{zz} - \tilde{\varepsilon}_{xy}$, $d_1 = \zeta_1^2 - \varepsilon_{zz}$, $d_2 = \zeta_3^2 - \varepsilon_{zz}$, $f_1 = \zeta_1^3 - \varepsilon_{xx}\,\zeta_1$, $f_2 = \zeta_3^3 - \varepsilon_{xx}\,\zeta_3$, $g_1 = \tilde{\varepsilon}_{xy}\zeta_1$, $g_2 = \tilde{\varepsilon}_{xy}\,\zeta_3$,

$\delta_1 = 2x_1(x_1^2 - x_2^2)$, $\delta_2 = -\,2x_2(x_1^2 - x_2^2)$;

$$\begin{aligned}\tilde{e}_y(\text{æ}, L) = \frac{2\,k_0}{\delta_1\,\varepsilon_{zz}} < E_x > \Big\{ & -\Upsilon_0\,(a_3\,\gamma_x\,\gamma_y + i\,b_3 + i\,c_3\,\gamma_x^2 + i\,d_3\,\gamma_y^2) \\ & \int_0^L dz'\,\left[\Upsilon_1\,n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\right] \cdot \sin\left[(L - z')\,k_0\,x_1\right] \\ & + \Upsilon_3(\,i\,g_3\,\gamma_y + f_3\,\gamma_x)\int_0^L dz'\,\left[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\right] \\ & \cos\left[(L - z')\,k_0\,x_1\right]\Big\} + \frac{2\,k_0}{\delta_2\,\varepsilon_{zz}} < E_x > \{-\Upsilon_0(a_4\,\gamma_x\,\gamma_y + i\,b_4 \\ & + i\,c_4\,\gamma_x^2 + i\,d_4\,\gamma_y^2)\int_0^L dz'\left[\Upsilon_1\,n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\right] \\ & \cdot \sin\left[(L - z')\,k_0\,x_2\right] + \Upsilon_3(\,i\,g_4\,\gamma_y + f_4\,\gamma_x) \\ & \left.\int_0^L dz'\left[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\right]\cos\left[(L - z')\,k_0\,x_2\right]\right\}, \end{aligned} \tag{18}$$

where: $a_3 = \zeta_1^2 - \varepsilon_{zz}$, $a_4 = \zeta_3^2 - \varepsilon_{zz}$, $b_3 = \varepsilon_{zz}(\tilde{\varepsilon}_{xy} + \varepsilon_{xx} - \zeta_1^2)$, $b_4 = \varepsilon_{zz}(\tilde{\varepsilon}_{xy} + \varepsilon_{xx} - \zeta_3^2)$, $c_3 = 2\,\zeta_1\,\zeta_2\,\varepsilon_{zz} - \tilde{\varepsilon}_{xy} - \varepsilon_{xx}$, $c_4 = 2\,\zeta_3\,\zeta_4\,\varepsilon_{zz} - \tilde{\varepsilon}_{xy} - \varepsilon_{xx}$, $d_3 = 2\,\zeta_1\,\zeta_2\,\varepsilon_{zz} + \zeta_1^2 - \tilde{\varepsilon}_{xy} - \varepsilon_{xx} - \varepsilon_{zz}$, $d_4 = 2\,\zeta_3\,\zeta_4\,\varepsilon_{zz} + \zeta_3^2 - \tilde{\varepsilon}_{xy} - \varepsilon_{xx} - \varepsilon_{zz}$, $g_3 = \varepsilon_{xx} - \zeta_1^3$, $g_4 = \varepsilon_{xx} - \zeta_3^3$, $f_3 = \varepsilon_{xy}\,\zeta_1$, $f_4 = \varepsilon_{xy}\,\zeta_3$;

$$\begin{aligned}\tilde{e}_z(\text{æ}, L) = \frac{2\,k_0}{\delta_1\,\varepsilon_{zz}} < E_x > \{ & \Upsilon_0\,a_5\,(\,i\,\gamma_x - \,\gamma_y) \\ & \int_0^L dz'\,\left[\Upsilon_1\,n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\right]\cos\left[(L - z')\,k_0\,x_1\right] \\ & -\Upsilon_3(\,c_5 + d_5\,\gamma_x^2 + d_5\,\gamma_y^2)\int_0^L dz'\left[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\right] \\ & \sin\left[(L - z')\,k_0\,x_1\right]\big\} + \frac{2\,k_0}{\delta_2\,\varepsilon_{zz}} < E_x > \{\Upsilon_0\,a_6(\,i\,\gamma_x - \,\gamma_y) \\ & \int_0^L dz'\left[\Upsilon_1\,n_1(\text{æ}, z') + \Upsilon_2\,\mu_z(\text{æ}, z')\right]\cos\left[(L - z')\,k_0\,x_2\right] \end{aligned}$$

$$-\Upsilon_3(c_6 + d_6\,\gamma_x^2 + d_6\,\gamma_y^2)\int\limits_0^L dz'\left[\mu_x(\text{æ}, z') + i\,\mu_y(\text{æ}, z')\right]$$

$$\sin\left[(L - z')\,k_0\,x_1\right]\Big\}\,; \tag{19}$$

where: $a_5 = \zeta_1(\zeta_1^2 - \varepsilon_{xx} - \tilde{\varepsilon}_{xy})$, $a_6 = \zeta_3(\zeta_3^2 - \varepsilon_{xx} - \tilde{\varepsilon}_{xy})$, $c_5 = (\zeta_1^2 - \varepsilon_{xx})^2 - \tilde{\varepsilon}_{xy}^2$, $c_6 = (\zeta_3^2 - \varepsilon_{xx})^2 - \tilde{\varepsilon}_{xy}^2$, $d_5 = (4\,\zeta_1\,\zeta_2 - 1)(\varepsilon_{xx} - \zeta_1^2)$, $d_6 = (4\,\zeta_3\,\zeta_4 - 1)(\varepsilon_{xx} - \zeta_3^2)$.

Knowledge of the expressions of scattered fields allows us to calculate the attenuation coefficient of the mean field on the basis of energetic point of view. It is known that in the absence of Ohmic dissipation and external currents we have [6] $\operatorname{div}\operatorname{Re}\left[\mathbf{E}\cdot\mathbf{H}\right] = 0$. Components of magnetic-field strength can be found from the Maxwell equations: $H_x = -(\kappa/k_0)E_y$, $H_y = (\kappa/k_0)E_x$, $H_z = 0$, where κ are the roots of the dispersion equation corresponding to the homogeneous set of Equation (8). Applying (5) it can be written:

$$\frac{d}{dz}\left(< E_x >^2 + < E_y >^2\right) + \frac{d}{dz}\left(< e_x^2 > + < e_y^2 >\right) = 0 \quad . \tag{20}$$

This expression represents energy conservation law. Integrating this expression we find $< E_x >^2 + < E_y >^2 + < e_x^2 > + < e_y^2 > = const$, where the right-hand part corresponds to the intensity of an incident radiation on inhomogeneous plasma slab with electron density and magnetic field fluctuations $\left[< E_x >^2 + < E_y >^2\right]_{L=0} = I_0 = const$. Now let's integrate (20) by volume enclosed between the z and $z + L$ planes and then statistically average the obtained expression. Introducing the notation $I(z) \equiv < E_x(z) >^2 + < E_y(z) >^2$ integration result can be rewritten as:

$$I(z+L) - I(z) = -\left(< e_x^2(z+L) > + < e_y^2(z+L) >\right). \tag{21}$$

Since we are considering a case of large-scale irregularities, we neglect back scattering of electromagnetic radiation. From the expressions (17) and (18) taking into account (16) we have $e_x(z) = (\sigma L)^{1/2} < E_x >$, here σ is a dumping coefficient much exceeding L). The expression (21) has a meaning only if $(\sigma L) \ll 1$. Substituting $< e_x^2 >$ and $< e_y^2 >$ in (21) and expanding left-hand part into a series with respect to L, we obtain $I(z) = I_0\exp(-\sigma\,z)$. For the arbitrary distance integrating (20) we get $< e_x^2 > + < e_y^2 >= I_0\left[1 - \exp(-\,\sigma z)\right]$. It is obvious that the boundary condition is satisfied.

From the expressions (17)–(19) it is not difficult to separate real and imaginary parts and obtain second-order statistical characteristics of scattered radiation. Let's introduce new variables: $z' - z'' = \rho_z$ and $z' + z'' = 2\eta$; $< n_1(\text{æ}, z')\,n_1(\text{æ}', z'') > = W_D(\text{æ}, \rho_z)\delta(\text{æ} + \text{æ}')$.

After integration with respect to variable η for the arbitrary correlation function of electron density fluctuation $W_D(æ, \rho_z)$, correlation functions of the amplitude and phase fluctuations of scattered electromagnetic waves in the direction of the y axis (only this case will be considered in this paper due to the balkiness of the formulae) have the following form:

$$< A_1(x+\rho_x, y+\rho_y, L) A_1(x, y, L) >_{yD} = -2\frac{\Upsilon_0^2 \Upsilon_1^2 L}{\varepsilon_{zz}^2 k_0^2}$$

$$\int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y\, k_x^2 k_y^2 \ \exp(i k_x \rho_x + i k_y \rho_y) \cdot \int\limits_{-\infty}^{\infty} d\rho_z W_D(k_x, k_y, \rho_z)$$

$$\left\{ \frac{a_3^2}{\delta_1^2} \left[\frac{\sin(2x_1 k_0 L)}{2x_1 k_0 L} - \cos(x_1 k_0 \rho_z) \right] + \frac{a_4^2}{\delta_2^2} \left[\frac{\sin(2x_2 k_0 L)}{2x_2 k_0 L} - \cos(x_2 k_0 \rho_z) \right] \right.$$

$$\left. +2\frac{a_3 a_4}{\delta_1 \delta_2} \left[\frac{\sin(q k_0 L)}{q k_0 L} \cos\left(\frac{t}{2} k_0 \rho_z\right) - \frac{\sin(t k_0 L)}{t k_0 L} \cos\left(\frac{q}{2} k_0 \rho_z\right) \right] \right\}, \quad (22)$$

$$< \varphi_1(x+\rho_x, y+\rho_y, L)\ \varphi_1(x, y, L) >_{yD} = -\frac{2\ k_0^2 L}{\varepsilon_{zz}^2} \Upsilon_0^2 \Upsilon_1^2$$

$$\int\limits_{-\infty}^{\infty} dk_x \int\limits_{-\infty}^{\infty} dk_y \exp(i k_x \rho_x + i k_y \rho_y) \cdot \int\limits_{-\infty}^{\infty} d\rho_z\, W_D(k_x, k_y, \rho_z)$$

$$\left\{ \frac{d_3}{\delta_1^2} \left[d_3 + \frac{2}{k_0^2}(c_3 k_x^2 + a_3 k_y^2) \right] \left[\frac{\sin(2x_1 k_0 L)}{2x_1 k_0 L} - \cos(x_1 k_0 \rho_z) \right] \right.$$

$$+\frac{d_4}{\delta_2^2} \left[d_4 + \frac{2}{k_0^2}(c_4 k_x^2 + a_4 k_y^2) \right] \cdot \left[\frac{\sin(2x_2 k_0 L)}{2x_2 k_0 L} - \cos(x_2 k_0 \rho_z) \right]$$

$$+\frac{2}{\delta_1 \delta_2} \left[d_3 d_4 + (d_3 c_4 + d_4 c_3)\ \frac{k_x^2}{k_0^2} + (d_3 a_4 + d_4 a_3)\ \frac{k_y^2}{k_0^2} \right]$$

$$\left. \cdot \left[\frac{\sin(q\, k_0 L)}{q\, k_0 L} \cos\left(\frac{t}{2} k_0\, \rho_z\right) - \frac{\sin(t\, k_0 L)}{t\, k_0 L} \cos\left(\frac{q}{2} k_0\, \rho_z\right) \right] \right\}, \quad (23)$$

here $t = x_1 - x_2 = t_1 - t_2\,\gamma^2$, $q = x_1 + x_2 = q_1 - q_2\,\gamma^2$.

In the absence of an external magnetic field, taking into account $\gamma_x^2 \ll 1$ and $\gamma_y^2 \ll 1$, determinant (10) has two doubly degenerate roots (zero approximation): $x_1 = x_3 = \sqrt{\varepsilon}$, $x_2 = x_4 = -\sqrt{\varepsilon}$. Using the residue theory and Cauchy theorem it can be shown that at $k_0 L \ll 1$, $\rho_x = \rho_y = 0$, in case of isotropic Gaussian correlation function for variance of the phase we obtain the well-known result $\sigma_\varphi^2 = (\sqrt{\pi}/4)\,\sigma_N^2\, k_0^2\, l_D\, L\, v^2$ [7]. Taking into account that electron density and magnetic field fluctuations are statistically independent, statistical characteristics of scattered electromagnetic waves for the arbitrary correlation tensor of an external magnetic field fluctuation $V_{ij}(æ,\ \rho_z)$ have the following form:

$$< A_1(x+\rho_x, y+\rho_y, L) A_1(x,y,L) >_{y\,M} = 2k_0^2\, L \frac{\Upsilon_3^2}{\varepsilon_{zz}^2}$$

$$\int\limits_{-\infty}^{\infty} dæ \int\limits_{-\infty}^{\infty} d\rho_z \exp(i\,k_x\rho_x + i\,k_y\rho_y)\cdot\left(\frac{1}{\delta_1^2}\left[-f_1^2\,\gamma_x^2\,V_{xx}(æ,\rho_z)\right.\right.$$

$$\left.-g_1^2\gamma_y^2 V_{yy}(æ,\rho_z) + 2f_1 g_1 \gamma_x\,\gamma_y V_{xx}(æ,\rho_z)\right]\left[\frac{\sin(2x_1\,k_0\,L)}{2x_1\,k_0\,L} + \cos(k_0\,x_1\,\rho_z)\right]$$

$$+\frac{1}{\delta_2^2}\left[-f_2^2\,\gamma_x^2\,V_{xx}(æ,\ \rho_z) - g_2^2\,\gamma_y^2\,V_{yy}(æ,\ \rho_z) + 2f_2\,g_2\,\gamma_x\,\gamma_y\,V_{xx}(æ,\ \rho_z)\right]$$

$$\left[\frac{\sin(2x_2\,k_0\,L)}{2x_2\,k_0\,L} + \cos(k_0\,x_2\,\rho_z)\right] + \frac{1}{\delta_1\delta_2}\left[-f_1\,f_2\,\gamma_x^2\,V_{xx}(æ,\ \rho_z)\right.$$

$$\left.-g_1\,g_2\,\gamma_y^2\,V_{yy}(æ,\ \rho_z) + (f_1\,g_2 + f_2\,g_1)\,\gamma_x\,\gamma_y\,V_{xy}(æ,\ \rho_z)\right]$$

$$\cdot\left\{\frac{1}{q\,k_0 L}\left[\sin\left(\frac{t}{2}k_0\,\rho_z + q\,k_0\,L\right) - \sin\left(\frac{t}{2}k_0\,\rho_z - q\,k_0\,L\right)\right]\right.$$

$$\left.\left.+\frac{1}{t\,k_0 L}\left[\sin\left(\frac{q}{2}k_0\,\rho_z + t\,k_0\,L\right)\ -\ \sin\left(\frac{q}{2}k_0\,\rho_z - t\,k_0\,L\right)\right]\right\}\right), \tag{24}$$

$$< \varphi_1(x+\rho_x, y+\rho_y, L)\ \varphi_1(x,y,L) >_{y\,M} = -2\frac{k_0^2\,L}{\varepsilon_{zz}^2}$$

$$\int\limits_{-\infty}^{\infty} dæ \int\limits_{-\infty}^{\infty} d\rho_z \exp(i\,k_x\rho_x + i\,k_y\rho_y)\cdot\left\{\frac{1}{\delta_1^2}\left[\Upsilon_0^2\Upsilon_2^2(b_1^2 + 2\,b_1\,c_1\,\gamma_x^2\right.\right.$$

$$\left.+2\,b_1\,d_1\,\gamma_y^2)\,V_{zz}(æ,\ \rho_z)\right]\left[\frac{\sin(2x_1\,k_0\,L)}{2x_1\,k_0\,L} - \cos(k_0\,x_1\,\rho_z)\right]$$

$$+\Upsilon_3^2\ \left[g_1^2\,\gamma_y^2\,V_{xx}(æ,\ \rho_z) + f_1^2\,\gamma_x^2\,V_{yy}(æ,\ \rho_z) + 2\,g_1\,f_1\,\gamma_x\,\gamma_y\,V_{xy}(æ,\ \rho_z)\right]$$

$$\left[\frac{\sin(2x_1\,k_0\,L)}{2x_1\,k_0\,L} + \cos(k_0\,x_1\,\rho_z)\right] + 2\,\Upsilon_0\Upsilon_2\Upsilon_3\left[b_1\,g_1\,\gamma_y\,V_{xz}(æ,\ \rho_z)\right.$$

$$\left.\left.+b_1\,f_1\,\gamma_x\,V_{yz}(æ,\ \rho_z)\right]\sin(k_0 x_1 \rho_z)\right\} + \cdot\left\{\frac{1}{\delta_2^2}\left[\Upsilon_0^2\,\Upsilon_2^2\,(b_2^2 + 2\,b_2\,c_2\,\gamma_x^2\right.\right.$$

$$\left.+2\,b_2\,d_2\,\gamma_y^2)\,V_{zz}(æ,\ \rho_z)\right]\left[\frac{\sin(2x_2\,k_0\,L)}{2x_2\,k_0\,L} - \cos(k_0\,x_2\,\rho_z)\right]$$

$$+\Upsilon_3^2\ \left[g_2^2\,\gamma_y^2\,V_{xx}(\text{æ},\ \rho_z)+f_2^2\,\gamma_x^2\,V_{yy}(\text{æ},\ \rho_z)+2\,g_2\,f_2\,\gamma_x\,\gamma_y\,V_{xy}(\text{æ},\ \rho_z)\right]$$
$$\left[\frac{\sin(2x_2\,k_0\,L)}{2x_2\,k_0\,L}+\cos(k_0\,x_2\,\rho_z)\right]+2\,\Upsilon_0\Upsilon_2\Upsilon_3\left[b_2\,g_2\,\gamma_y\,V_{xz}(\text{æ},\ \rho_z)\right.$$
$$\left.+b_2 f_2\gamma_x V_{yz}(\text{æ},\rho_z)\right]\sin(k_0x_2\rho_z)\}-\frac{1}{\delta_1\delta_2}\left\{\Upsilon_0^2\Upsilon_2^2\left[b_1\,b_2+(b_1\,c_2+b_2\,c_1)\,\gamma_x^2\right.\right.$$
$$\left.+(b_1\,d_2+b_2\,d_1)\,\gamma_y^2\right]\ V_{zz}(\text{æ},\ \rho_z)\,\frac{1}{q\,k_0\,L}\left[\sin\left(\frac{t}{2}k_0\rho_z-q\,k_0\,L\right)\right.$$
$$\left.-\sin\left(\frac{t}{2}k_0\,\rho_z+q\,k_0\,L\right)\right]+\Upsilon_0\Upsilon_2\Upsilon_3\left[(g_1\,b_2-g_2\,b_1)\,\gamma_y\,V_{xz}(\text{æ},\ \rho_z)\right.$$
$$\left.+\,(f_1\,b_2-f_2\,b_1)\,\gamma_x\,V_{yz}(\text{æ},\ \rho_z)\right]\frac{1}{q\,k_0\,L}\ \cdot\left[\cos\left(\frac{t}{2}k_0\,\rho_z-q\,k_0\,L\right)\right.$$
$$\left.-\cos\left(\frac{t}{2}k_0\,\rho_z+q\,k_0\,L\right)\right]+\Upsilon_3^2\left[g_1\,g_2\,\gamma_y^2\,V_{xx}(\text{æ},\rho_z)+f_1\,f_2\,\gamma_x^2\,V_{yy}(\text{æ},\rho_z)\right.$$
$$\left.+\,(g_1\,f_2+g_2\,f_1)\,\gamma_x\,\gamma_y\,V_{xy}(\text{æ},\ \rho_z)\right]\frac{1}{q\,k_0\,L}\left[\sin\left(\frac{t}{2}k_0\,\rho_z-q\,k_0\,L\right)\right.$$
$$\left.-\sin\left(\frac{t}{2}k_0\,\rho_z+q\,k_0\,L\right)\right]+\Upsilon_0^2\Upsilon_2^2\ \left[b_1\,b_2+(b_1\,c_2+b_2\,c_1)\,\gamma_x^2\right.$$
$$\left.+(b_1\,d_2+b_2\,d_1)\,\gamma_y^2\right]\ V_{zz}(\text{æ},\ \rho_z)\ \ \frac{1}{t\,k_0\,L}\ \left[\sin\left(\frac{q}{2}k_0\,\rho_z-t\,k_0\,L\right)\right.$$
$$\left.-\sin\left(\frac{q}{2}k_0\,\rho_z+t\,k_0\,L\right)\right]+\Upsilon_0\Upsilon_2\Upsilon_3\left[(g_1\,b_2-g_2\,b_1)\,\gamma_y\,V_{xz}(\text{æ},\ \rho_z)\right.$$
$$\left.+(f_1\,b_2-f_2\,b_1)\,\gamma_x\,V_{yz}(\text{æ},\ \rho_z)\right]\frac{1}{t\,k_0\,L}\ \cdot\left[\cos\left(\frac{q}{2}k_0\,\rho_z-t\,k_0\,L\right)\right.$$
$$\left.-\cos\left(\frac{q}{2}k_0\rho_z+tk_0\,L\right)\right]+\Upsilon_3^2\left[g_1\,g_2\,\gamma_y^2\,V_{xx}(\text{æ},\ \rho_z)+f_1\,f_2\,\gamma_x^2\,V_{yy}(\text{æ},\ \rho_z)\right.$$
$$\left.+\,(g_1\,f_2+g_2\,f_1)\,\gamma_x\,\gamma_y\,V_{xy}(\text{æ},\ \rho_z)\right]\ \ \frac{1}{t\,k_0\,L}$$
$$\left.\left[\sin\left(\frac{q}{2}k_0\,\rho_z-t\,k_0\,L\right)-\sin\left(\frac{q}{2}k_0\,\rho_z+t\,k_0\,L\right)\right]\right\}. \tag{25}$$

It is evident that at $L=0$ boundary conditions are satisfied. Taking into account the inequalities $\gamma_x,\gamma_y\sim 2\pi/l\ll k_0$ in the exponent of the integrand (17)–(19) three terms should be retained, since the second and third terms can essential contribution at great distance z. From (22)–(25) follow that statistical characteristics for the ordinary wave tend to zero with increasing of the external magnetic field. Under other equal conditions fluctuations of plasma density plays the important role at strong magnetic fields ($u>1$) and the imposed magnetic field in this case reduces fluctuation level of parameters of both waves. If electron density in the layer varies so that $0 < v < 1$, then reflection area does not exist for the ordinary wave. If applying the same requirement also for the extraordinary wave, then it should be $0 < v < 1 - \sqrt{u}$.

In the previous section statistical characteristics of scattered radiation by inhomogeneous plasma slab were obtained for the arbitrary correlation function of electron density and magnetic field fluctuations. These formulae are valid for near $(L/k_0 l^2) \ll 1$ and far $(L/k_0 l^2) \gg 1$ zones from a plasma slab boundaries. It is necessary to specify these correlation functions for performing numerical calculations.

3.1. Correlation Functions of the Fluctuating Parameters of Turbulent Plasma Slab

The spectral density function which describes the irregularities in random medium depends on the particular case and may differ from medium to medium. An irregularity model is conveniently described by a three-dimensional correlation function of electron density. The most widely used spectral density function is the Gaussian, which has certain mathematical advantages. In the theoretical study forward scattering assumption is valid when $< \varepsilon_1^2 > k_0 L \ll 1 \ll k_0 l_D$, where $< \varepsilon_1^2 >$ is the variance of the medium fluctuations. If the single scattering condition is also fulfilled $< \varepsilon_1^2 > k_0^2 l_D L \ll 1$ a medium is characterized by the Gaussian irregularity spectrum. On the lower boundary of inhomogeneous slab having thickness 100 km, locating at the altitudes from 300 up to 500 km it is easy to show that the these conditions are satisfied for the electromagnetic waves with a frequency of few tens MHz and higher. Therefore in the analytical calculations we use anisotropic Gaussian correlation function of electron density fluctuation [13].

$$W_D(k_x, k_y, \rho_z) = \frac{< N_1^2 >}{4\pi} \frac{l_\parallel^2}{\chi \Gamma_0} \exp\left(-\frac{m^2}{l_\parallel^2} \rho_z^2 + i\, n\, k_x\, \rho_z \right) \exp\left(-\frac{k_x^2 l_\parallel^2}{4\Gamma_0^2} - \frac{k_y^2 l_\parallel^2}{4\chi^2} \right), \tag{26}$$

where: $m^2 = \frac{\chi^2}{\Gamma_0^2}$, $\Gamma_0^2 = \sin^2\gamma_0 + \chi^2 \cos^2\gamma_0$, $n = \frac{\chi^2 - 1}{\Gamma_0^2} \sin\gamma_0 \cos\gamma_0$. The average shape of electron density irregularities has the form of elongate ellipsoid of rotation. The rotation axis is located in the plane of geomagnetic meridian. The ellipsoid is characterized with two parameters: the anisotropy factor for the irregularities equaling to the ratio of ellipsoid axes $\chi = l_\parallel / l_\perp$ (ratio of longitudinal and transverse linear scales of plasma irregularities with respect to the external magnetic field) and orientation equaling to the inclination angle of the rotation axis with respect to the magnetic field (sometimes with respect to horizon) γ_0. Anisotropy of the shape of irregularities is connected with the difference of diffusion coefficients in the field align and field perpendicular directions.

For the external magnetic field fluctuation we suggest correlation tensor of the second rank

$$< \mu_i'(\mathbf{r_1})\mu_j'(\mathbf{r_2}) >= V_{ij}(\boldsymbol{\rho}) = \frac{1}{12} < \mu'^2 > \left(\frac{\partial^2}{\partial\rho_i\partial\rho_j} - \delta_{ij}\frac{\partial^2}{\partial\rho_s\partial\rho_s}\right)\Phi(\rho). \tag{27}$$

Fourier transform (27) if $\Phi(\rho) = l_M^2 \exp\left(-\rho^2/l_M^2\right)$ (l_M is the characteristic linear scale of an external magnetic field fluctuation, $< \mu'^2 >$ the variance of an external magnetic field fluctuation) has the following form

$$V_{ij}(\mathbf{k}) = < \mu'^2 > \ \frac{\pi^{3/2}}{12} l_M^5 \left(k^2\delta_{ij} - k_i\, k_j\right) \exp\left(-\frac{k^2 l_M^2}{4}\right), \tag{28}$$

Numerical calculations for the electron density fluctuation are carried out in Cartesian coordinate system, and polar coordinate system is applied for the magnetic field fluctuation with application of so-called Markov assumption, $V_{ij}(\text{æ},\, \rho_z) \sim V_{ij}(\text{æ})\,\delta(\rho_z)$. This amounts to assuming that the random medium is "delta-function" correlated along the direction of propagation. The error in computing the mean-square fluctuation of the log amplitude of the wave due to the Markov assumption is proportional to $((\lambda L)^{1/2}/L)^{1/3}$, where L is the distance the wave traveled in the turbulent medium. In the ionosphere, typical value for L range is from 50 km to a few hundred kilometers. Therefore, for wave frequency at a few hundred MHz and higher, the Markov assumption is valid [14].

4. NUMERICAL CALCULATIONS

In our numerical calculations frequency of radiated electromagnetic wave is 40 MHz. Plasma slab 100 km thick is situated at a height 300 km over the Earth. The small thickness of the irregularity layer gives rise to pronounced Fresnel oscillations in the amplitude and phase power spectra. The mean values of turbulent plasma slab in the F region of the Earth's ionosphere are $u = 0.0012$, $v = 0.0133$.

Figure 1 illustrates the dependence of the phase structure function $D_\varphi(\rho_x, \rho_y, L)$ on the normalized distance $Y = \rho_y/l_\parallel$ at $\xi = 1000$ and different values of anisotropy parameter χ of electron density fluctuations (irregularities are stretched along the geomagnetic field lines ($\gamma_0 = 0°$)) in the principal plane. Fig. 2 shows dependence of the function $D_\varphi(\rho_x, \rho_y, L)$ on the parameter Y at $\chi = 2$ and different values of the inclination angle γ_0 of elongated irregularities with respect to

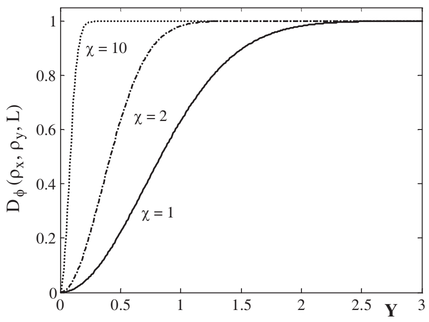


Figure 1. The dependence of the normalized wave structure function $D_\varphi(\rho_x, \rho_y, L)$ on normalized distance Y between observation points at different values of anisotropy factor of irregularities χ; $\xi = 1000$, $\gamma_0 = 0°$ and $X = 0$.

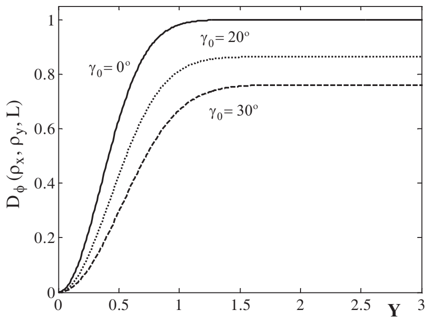


Figure 2. The dependence of the normalized wave structure function $D_\varphi(\rho_x, \rho_y, L)$ on normalized distance Y at $\chi = 2$, $\xi = 1000$ and $X = 0$.

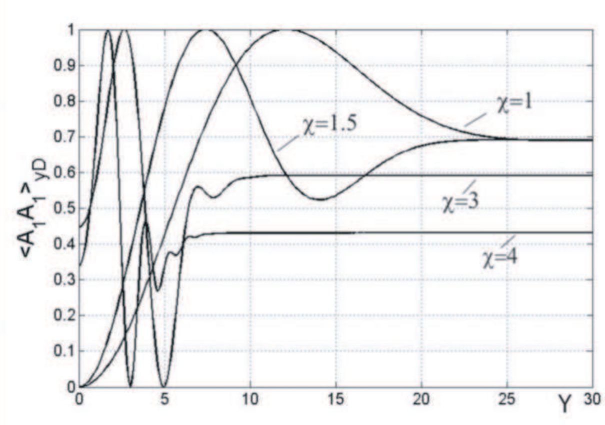


Figure 3. The dependence of the wave structure function of the amplitude fluctuations of scattered electromagnetic waves caused by electron density fluctuations on normalized distance Y in the principle plane for different values of the parameters χ.

the external magnetic field. In both cases the phase structure function tends to saturation [1] with increasing distance between observation points. Fig. 3 illustrates the dependence of the wave structure functions of the amplitude of scattered radiation caused by electron density fluctuations for different values χ of scattered irregularities. Numerical calculations show that at $\xi = 600$ wave structure function of the amplitude caused by electron density fluctuations are more sensitive at small distances between the receiving antennas in the principal plane $Y < 5$ than for phase structure function. They tend to saturation with removal of receiving antennas. In particular, at $\chi = 4$ saturation of the wave structure function of the amplitude starts from $Y = 8$, at $\chi = 1.5$ — from $Y = 25$; its magnitude decreases inversely proportional to the anisotropy coefficient. Wave structure function of the phase fluctuations fast increases and then tends to saturation: at $\chi = 4$ starting from $Y = 7$, and at $\chi = 1.5$ — from $Y = 21$. Figs. 4–7 illustrate phase portraits (the dependence of an integrand (30) on the polar angle of the cylindrical coordinate system) of scattered radiation caused by fluctuations of the direction of an external magnetic field at different relations of characteristic linear scales of considered task: thickness of plasma layer, characteristic linear scale of an external magnetic field fluctuation and distance between observation points, at $u = 0.0012$, $v = 0.0133$ (Fig. 4 and 6) and $u = 0.22$, $v = 0.28$ (Figs. 5 and 7).

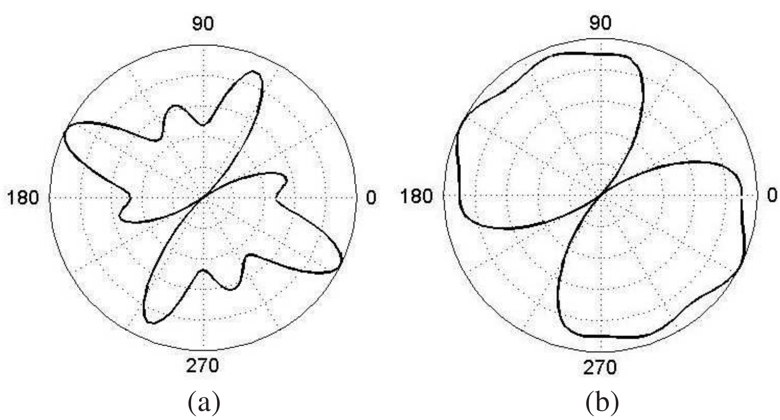
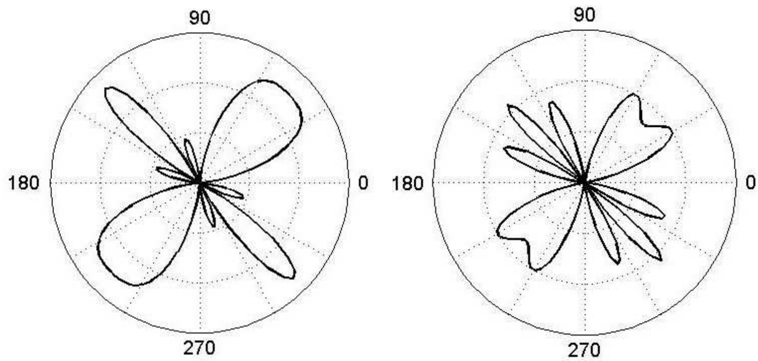
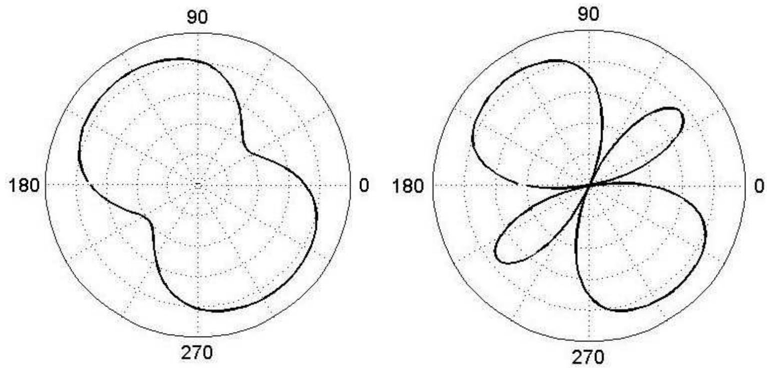


Figure 4. Phase portraits of correlation function of the amplitude fluctuations of scattered radiation caused by fluctuations of the direction of an external magnetic field at $v = 0.0133$, $u = 0.0012$, $L/l_M = 10$; $\rho_x/l_M = 0.8$ (a), $\rho_x/l_M = 1$ (b).



(a) (b)

Figure 5. Phase portraits of correlation function of the amplitude fluctuations of scattered radiation caused by fluctuations of the direction of an external magnetic field at $v = 0.28$, $u = 0.22$; $L/l_M = 0.8$, $\rho_x/l_M = 0.08$ (a) and $L/l_M = 1.2$, $\rho_x/l_M = 0.06$ (b).



(a) (b)

Figure 6. Phase portraits of correlation function of the phase fluctuations of scattered radiation caused by fluctuations of the direction of an external magnetic field at $v = 0.28$, $u = 0.22$, $L/l_M = 10$; $\rho_x/l_M = 0.8$ (a), $\rho_x/l_M = 1$ (b).

From these figures follows that fluctuations of the direction of an external magnetic field substantially influences the behavior of phase portraits of statistical characteristics of scattered radiation by magnetized plasma slab in both principle and perpendicular planes. The dispersion of the external magnetic field fluctuations is very small (particularly, in Galaxy it is of the order of $\sim 10^{-8} - 10^{-10}$). Different

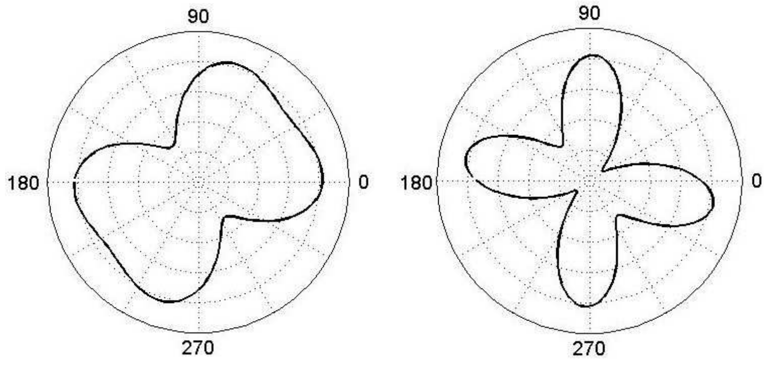


Figure 7. Phase portraits of correlation function of the phase fluctuations of scattered radiation caused by fluctuations of the direction of an external magnetic field at $v = 0.28$, $u = 0.22$, $L/l_M = 10$; $\rho_x/l_M = 0.8$ (a), $\rho_x/l_M = 1$ (b).

external factors such as: earthquake, magnetic storms, etc. can substantially influence the value of $< \mu'^2 >$. Therefore, in this case besides the dispersion of electron density fluctuation, both value and direction of an external magnetic field fluctuation should be taken into account.

5. CONCLUSIONS

In this paper, statistical characteristics of the scattered electromagnetic waves by irregularities plasma slab with electron density and magnetic field fluctuations were investigated via the perturbation method if wave propagates along the external magnetic field. Analytical expressions for the component of fluctuating electric field in principal and perpendicular planes have been obtained. Second statistical moments and wave structure functions of the phase and amplitude of scattered radiation for the arbitrary correlation functions of fluctuating magnetized plasma parameters have been calculated. Phase portraits of these statistical characteristics caused by fluctuations of the direction of an external magnetic field for anisotropic Gaussian correlation function have been constructed. If a wave propagates at big angle with respect to the external magnetic field $\theta \neq 0$, unlike the case discussed in this paper, linear and cubic terms appear additionally in the determinant, which considerably complicates finding of biquadratic equation and statistical characteristics of the scattered field. The transition from the ordinary to the extraordinary wave is already nontrivial. In particular, at $\theta = \pi/2$: for the ordinary wave $< E_x >= 0$, $< E_z > = 0$, and for the extraordinary wave $< E_y > = 0$, $< E_z > = -(\varepsilon_{zx}/\varepsilon_{zz}) < E_x >$; besides, unlike the case discussed above, scattering of the electric field in the principal plane is caused by both magnetic field and electron density fluctuations, while component x of current density contains only magnetic field fluctuations. In separate paper we will investigate statistical characteristics of scattered radiation at $\theta = \pi/2$.

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