

## **PARETO OPTIMAL YAGI-UDA ANTENNA DESIGN USING MULTI-OBJECTIVE DIFFERENTIAL EVOLUTION**

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**Abstract**—Antenna design problems often require the optimization of several conflicting objectives such as gain maximization, sidelobe level (SLL) reduction and input impedance matching. Multi-objective Evolutionary Algorithms (MOEAs) are suitable optimization techniques for solving such problems. An efficient algorithm is Generalized Differential Evolution (GDE3), which is a multi-objective extension of Differential Evolution (DE). The GDE3 algorithm can be applied to global optimization of any engineering problem with an arbitrary number of objective and constraint functions. Another popular MOEA is Nondominated Sorting Genetic Algorithm-II (NSGA-II). Both GDE3 and NSGA-II are applied to Yagi-Uda antenna design under specified constraints. The numerical solver used for antenna parameters calculations is SuperNEC, an object-oriented version of the numerical electromagnetic code (NEC-2). Three different Yagi-Uda antenna designs are considered and optimized. Pareto fronts are produced for both algorithms. The results indicate the advantages of this approach and the applicability of this design method.

### **1. INTRODUCTION**

Antenna design problems have general multi-objective. Common design objectives include gain maximization, sidelobe level reduction and input impedance matching. The above-mentioned objectives are often subject to constraints. A common problem that has

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been addressed in the literature is the design of Yagi-Uda antennas that satisfy such objectives [1–15]. The optimization goal is to find the optimum element lengths and spacings that fulfill the design specifications. Evolutionary algorithms (EAs) such as Genetic Algorithms (GAs) have been applied to a variety of antenna design problems [16–23]. Yagi-Uda antennas have been studied in the literature using different evolutionary algorithms. In [4] a binary coded GA is used for Yagi-Uda design by a single objective approach. This is produced from the aggregation of the objective functions using different weight values. A Computational Intelligence (CI) method is applied in [6]. Particle Swarm Optimization (PSO) [24] has also been used successfully in several constrained or unconstrained antenna design problems [25–44]. The authors in [7] optimize Yagi-Uda designs with the single objective function that combines all objectives using Comprehensive Learning PSO (CLPSO). In [8] Pareto GA, a multi-objective GA, is used for the generation of the Pareto front for the Yagi-Uda design problem.

Nondominated Sorting Genetic Algorithm-II (NSGA-II) [45] is a popular and efficient multi-objective genetic algorithm, which has been used in several engineering design problems [46, 47]. The major drawback of a GA approach is the difficulty in the implementation due to the algorithm's inherited complexity and long computational time.

Differential evolution (DE) is a population-based stochastic global optimization algorithm, proposed by Price and Storn [48, 49]. Several DE variants or strategies exist [50, 51]. The classical DE algorithm has been applied to several antenna and microwave design problems [12, 52–57]. Li and Guo presented a new Yagi-Uda design approach using different objective functions and DE as the optimization algorithm [12]. Their objective functions also use a combination of the objectives and different weight factors. One of the DE advantages is the fact that very few parameters have to be adjusted in order to produce results. Several DE extensions for multi-objective optimization have been proposed so far [58, 59]. Generalized Differential Evolution (GDE3) [60] is a multi-objective DE algorithm that has outperformed other multi-objective evolutionary algorithms for a given set of numerical problems [61, 62]. GDE3 has been applied to microwave filter design in [63].

In this paper, both GDE3 and NSGA-II are used for the multi-objective Yagi-Uda design problem. We apply both to three different Yagi-Uda antenna design cases with four, six and fifteen elements. We consider three objective functions subject to specific constraints. The 3D Pareto fronts produced by both algorithms are compared and discussed. Example design cases are given and compared with

data from the literature. This is accomplished in conjunction with SuperNEC, a commercially available EM solver. This is an object-oriented version of the numerical electromagnetic code (NEC-2). SuperNEC and NEC-2 versions have also been used in the literature for Yagi-Uda antenna design.

The novelty in our work lies in the fact that we apply both GDE3 and NSGA-II to the Yagi-Uda antenna design problem. To the best of our knowledge this is the first time that GDE3 is applied to an antenna design problem. The advantages of the GDE3 and NSGA-II algorithm approaches for multi-objective antenna problems are clearly shown.

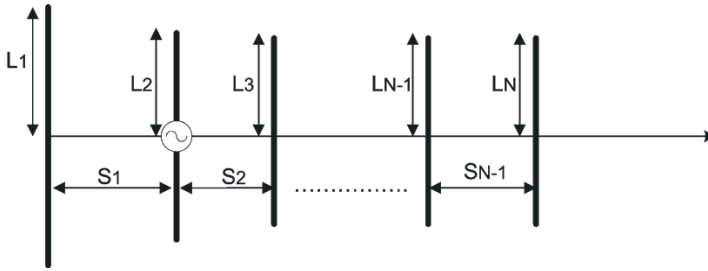
This paper is organized as follows: Section 2 describes the Yagi-Uda design problem. The definition of the general multi-objective optimization problem under constraints is given in Section 3. We also present the classical *DE/rand/1/bin* strategy and briefly outline the GDE3 algorithm details. Section 4 presents the numerical results for three distinct antenna design cases. Finally the conclusion is given in Section 5.

## 2. THE YAGI-UDA ANTENNA DESIGN PROBLEM

Figure 1 shows an  $N$ -element Yagi-Uda antenna. This antenna consists of a single driven element, one reflector element and  $N - 2$  director elements. Such a  $N$ -element Yagi-Uda antenna has  $2N - 1$  antenna parameters that determine the antenna characteristics, apart from the elements' radius. The design parameters are  $\bar{x} = (L_1, L_2, \dots, L_k, \dots, L_N, S_1, S_2, \dots, S_k, \dots, S_{N-1})$  where  $2L_k$  is the length of the  $k$ th element, and  $S_k$  is the spacing between the  $k$ th and  $(k + 1)$ th elements.

The Yagi-Uda antenna design goal is to find the optimum geometry that satisfies given performance specifications such as high gain,  $\text{Gain}(\bar{x})$ , low sidelobe level,  $\text{SLL}(\bar{x})$ , and input impedance close or equal to  $50 \Omega$ . The last objective can also be defined as having a Voltage Standing Wave Ratio (VSWR),  $\text{VSWR}(\bar{x})$ , close to one. It is obvious that such a problem is multi-objective. In the literature the above objectives have been combined in a single objective function using different weight factors [4, 7, 10, 12]. In this paper we express the Yagi-Uda antenna design problem as the minimization of the following objective functions:

$$\begin{aligned} F_1(\bar{x}) &= -\text{Gain}(\bar{x}) \\ F_2(\bar{x}) &= \text{SLL}(\bar{x}) \\ F_3(\bar{x}) &= \text{VSWR}(\bar{x}) \end{aligned} \quad (1)$$



**Figure 1.**  $N$ -element Yagi-Uda antenna.

Moreover, the design problem is subject to the following constraints:

$$\begin{aligned} g_1(\bar{x}) &= \text{Gain}(\bar{x}) \geq \text{Gain}_L \\ g_2(\bar{x}) &= \text{VSWR}(\bar{x}) \leq \text{VSWR}_L \end{aligned} \quad (2)$$

where  $\text{Gain}_L$ ,  $\text{VSWR}_L$  are the minimum allowable gain and maximum allowable VSWR respectively. In this paper, the methods of moments-based numerical electromagnetics code (NEC-2) has been used in the analysis. In particular, SuperNEC, a commercially available NEC-2 version with MATLAB interface, is used.

### 3. MULTI-OBJECTIVE OPTIMIZATION WITH CONSTRAINTS

The general constrained multi-objective optimization problem (MOOP) definition is [64]:

$$\text{Minimize } F(\bar{x}) = [F_1(\bar{x}), F_2(\bar{x}), \dots, F_n(\bar{x})] \quad (3)$$

$$\text{Subject to } g_i(\bar{x}) \leq 0 \quad i = 1, 2, \dots, k \quad (4)$$

$F(\bar{x})$  is the vector of the objective functions, and  $g_i$  are the constraint functions.  $n$  is the number of objective functions, and  $k$  is the number of constraint functions.

In principle, multi-objective optimization is different from single-objective optimization. In single-objective optimization one attempts to obtain the best solution, which is usually the global minimum or global maximum depending on the optimization problem. In case of multiple objectives, there may not exist one best solution (global minimum or maximum) with respect to all objectives. In a typical MOOP, it is often necessary to determine a set of points that all fit a predetermined definition for an optimum. The predominant concept in defining an optimal point is that of Pareto optimality. Pareto-optimal solutions are those solutions (from the set of feasible solutions) that

cannot be improved in any objective without causing degradation in at least one other objective.

Therefore, the above problem can be solved in two ways. The first way is to convert it to a single-objective optimization problem. This can be accomplished by using weights for different objective functions and penalty terms for the constraint functions. This method leads to a single solution. The second way is to use Pareto optimization, which means to optimize all the objectives simultaneously giving them equal importance. If none of the objective function values can be further improved without impairing the value of at least one objective for a given solution then this solution is Pareto-optimal and belongs to the set of non-dominated solutions which is called Pareto front. The main goal is to find some points (solutions) that belong to the Pareto front. From this set of non-dominated solutions, optimal antenna designs that provide a suitable compromise between the objectives for the desired constraints can be realized. More details about constraint handling in MOEAs can be found in a recent paper [65].

A multi-objective evolutionary algorithm can be used to solve this problem. Multi-objective evolutionary algorithms have gained popularity and have been used extensively over the last years in several design problems in electromagnetics. EAs use vectors to model the possible solutions. In order to distinguish the members of the non-dominated set from the population members we refer to the first as solutions and the second ones as vectors. The definitions of dominance relations between two vectors (or individuals of the population) are given below. The weak dominance  $\preceq$  relation between two vectors  $\bar{x}_1$ ,  $\bar{x}_2$  in the search space is defined as [60]:

$$\bar{x}_1 \text{ weakly dominates } \bar{x}_2 \quad \bar{x}_1 \preceq \bar{x}_2 \text{ iff } \forall i : F_i(\bar{x}_1) \leq F_i(\bar{x}_2) \quad (5)$$

while the dominance  $\prec$  relation is defined as:

$$\bar{x}_1 \text{ dominates } \bar{x}_2 \quad \bar{x}_1 \prec \bar{x}_2 \text{ iff } \bar{x}_1 \preceq \bar{x}_2 \wedge \exists i : F_i(\bar{x}_1) < F_i(\bar{x}_2) \quad (6)$$

The above relations can be extended to include constraint dominance  $\prec_c$  [60]:

$\bar{x}_1$  constraint-dominates  $\bar{x}_2$   $\bar{x}_1 \prec_c \bar{x}_2$  when any of the following conditions are true:

- 1)  $\bar{x}_1$  belongs to the feasible design space, and  $\bar{x}_2$  is infeasible.
- 2)  $\bar{x}_1$ ,  $\bar{x}_2$  are both infeasible, but  $\bar{x}_1$  dominates  $\bar{x}_2$  in constraint function space.
- 3)  $\bar{x}_1$ ,  $\bar{x}_2$  both belong the feasible design space, but  $\bar{x}_1$  dominates  $\bar{x}_2$  in objective function space.

### 3.1. Differential Evolution

A population in DE consists of  $NP$  vectors  $\bar{x}_{iG}$ ,  $i = 1, 2, \dots, NP$ , where  $G$  is the generation number. The population is initialized randomly from a uniform distribution. Each  $D$ -dimensional vector represents a possible solution. The initial population evolves in each generation with the use of three operators: Mutation, crossover and selection. Depending on the form of these operators several DE variants or strategies exist in the literature [24, 25]. The most popular is the one known as *DE/rand/1/bin* strategy. In this strategy a mutant vector  $\bar{v}$  for every target vector  $\bar{x}_{iG}$  is computed by:

$$\bar{v}_{i,G+1} = \bar{x}_{r_1,G} + F(\bar{x}_{r_2,G} - \bar{x}_{r_3,G}), \quad r_1 \neq r_2 \neq r_3 \quad (7)$$

where  $r_1, r_2, r_3$  are randomly chosen indices from the population, and  $F$  is a mutation control parameter. After mutation the crossover operator is applied to generate a trial vector  $\bar{u}_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \dots, u_{Di,G+1})$  whose coordinates are given by:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1}, & \text{if } \text{rand}(j) \leq CR \text{ or } j = rn(i) \\ x_{ji,G}, & \text{if } \text{rand}(j) > CR \text{ and } j \neq rn(i) \end{cases} \quad (8)$$

where  $j = 1, 2, \dots, D$ ,  $\text{rand}(j)$  is a number from a uniform random distribution from the space  $[0, 1]$ ,  $rn(i)$  a randomly chosen index from  $(1, 2, \dots, D)$  and  $CR$  the crossover constant from the space  $[0, 1]$ . DE uses a greedy selection operator. According to this selection scheme for minimization problems:

$$\bar{x}_{i,G+1} = \begin{cases} \bar{u}_{i,G+1}, & \text{if } f(\bar{u}_{i,G+1}) < f(\bar{x}_{i,G}) \\ \bar{x}_{i,G}, & \text{otherwise} \end{cases} \quad (9)$$

where  $f(\bar{u}_{i,G+1})$ ,  $f(\bar{x}_{i,G})$  are the fitness values of the trial and the old vector respectively. Therefore, the newly found trial vector  $\bar{u}_{i,G+1}$  replaces the old vector  $\bar{x}_{i,G}$  only when it produces a lower objective function value than the old one. Otherwise, the old vector remains in the next generation. The stopping criterion for the DE is usually the generation number or the number of objective function evaluations.

### 3.2. Generalized Differential Evolution (GDE3)

Multi-objective DE algorithms extend the classical DE algorithm for solving MOOP. Generalized Differential Evolution (GDE3) introduced in [60] can solve problems that have  $n$  objectives and  $k$  constraint functions. Recently, GDE3 has outperformed other evolutionary algorithms in numerical benchmark problems [61, 62]. It has been successfully applied to the molecular sequence alignment problem [66] and to microwave filters design [63]. To the best of the authors'

knowledge this is the first time that the GDE3 algorithm is applied to an antenna design problem. The algorithm uses the concept of Crowding Distance (CD), which approximates the crowdedness of a vector in its non-dominated set as NSGA-II [45]. The vectors are sorted based on non-dominance and crowdedness. A basic difference exists between NSGA-II and GDE3 regarding the population size after a generation. In NSGA-II the population size after a generation is increased to  $2NP$ . Then non-dominated ranking is applied, and  $NP$  non-dominated vectors are selected. In GDE3 after a generation the population size is  $NP + m$ , where  $m \in [0, NP]$ , because the population size is increased only when the trial  $u_{i,G+1}$  and old vector  $x_{i,G}$  are feasible and do not dominate each other. Therefore, non-dominated ranking is applied to  $NP + m$  population size, which can be less in general than  $2NP$ , thus resulting in less computational time than NSGA-II [60].

The GDE3 algorithm is outlined below:

- 1) Initialize random population of  $NP$  individuals. Set  $m = 0$ .
- 2) Evaluate objective function and constraint function values for every vector of the population.
- 3) Apply the mutation and crossover operators according to (7) and (8) and create a trial vector  $u_{i,G+1}$ .
- 4) Evaluate objective function and constraint function values for the trial vector.
- 5) Apply the selection operator according to the following criterion:

$$x_{i,G+1} = \begin{cases} u_{i,G+1}, & \text{if } u_{i,G+1} \preceq_c x_{i,G} \\ x_{i,G}, & \text{otherwise} \end{cases} \quad (10)$$

- 6) Set  $m = m + 1$ ,  $x_{NP+m,G+1} = u_{i,G+1}$   
 if  $\forall j : g_j(u_{i,G+1}) \leq 0 \wedge x_{i,G+1} == x_{i,G} \wedge x_{i,G} \not\prec u_{i,G+1}$  (11)
- 7) Apply non-dominated ranking to  $NP + m$  vectors. Select  $NP$  non-dominated vectors and set  $m = 0$ .
- 8) Repeat step 3 until the maximum number of generations  $G_{\max}$  is reached.

GDE3 variations with different DE strategies can be easily created simply by using different equations for crossover and mutation other than (7) and (8). More details about the GDE3 algorithm can be found in [60].

#### 4. NUMERICAL RESULTS

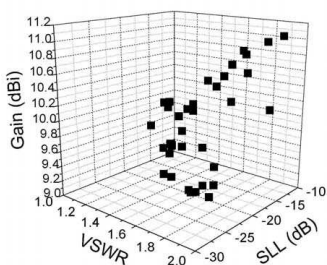
All algorithms are executed 20 times. The best results are compared. All algorithms are compiled using the same compiler (Borland C++ Builder 5.0) in a PC with Intel Core 2 Duo E8500 at 3.16 GHz with 4GB RAM running Windows XP. The population size is set to 40 for both algorithms in all cases. The iteration number is set to 1000 for the cases of four and six elements, while for the 15-element Yagi-Uda is set to 2000. The control parameters chosen for GDE3 are according to [61, 67] for solving problems with three objectives  $F = 0.2$ ,  $CR = 0.2$ . These values have been verified with several trials. In NSGA-II the crossover and mutation probabilities are set equal to 0.9 and 0.1, respectively.

The design cases selected are those that appear in the literature in order to compare results [4, 6–8, 12]. The first design case is that of a four-element Yagi-Uda antenna. For this case it is  $\text{Gain}_L = 9$  dBi,  $\text{VSWR}_L = 2$ . The dipole radius is set to  $0.00225\lambda$  as in the literature [4, 6–8, 12]. The 3D Pareto fronts found by GDE3 and NSGA-II are given in Figures 2 and 3, respectively. Each point of the Pareto front denotes a feasible design solution with three coordinates. We notice that both algorithms produce similar results. Table 1 has designs taken from the literature [4, 6, 7]. In Table 2 we present three different example cases from the Pareto front found by GDE3. The radiation patterns for these cases are plotted in Figure 4. As expected, no single optimum solution exists. Each of the example cases is better than the others in one objective. Design 1 presents a lower gain than the others but its VSWR is closer to one than others. One may notice that it has a higher gain value than the case of [7]. Design 2 has a high gain of 9.96 dBi and a relative low SLL of  $-15.00$  dB. It outperforms the design from [6] in terms of gain value. The third example case obtained by GDE3 presents almost perfect impedance matching to  $50\ \Omega$  and has the lowest SLL of all. The tradeoff for this case is the lowest gain value. The average execution time for GDE3 and NSGA-II is 1546.34 and 1618.73 seconds respectively. For this case the total number of objective function evaluations is 40,000 compared with 200,000 from [8] using the Pareto GA.

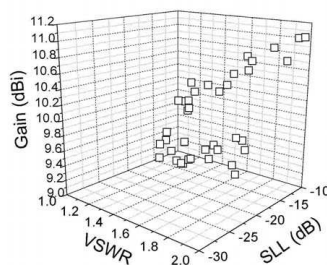
The next example is also common in literature [4, 7, 8]. It is that of a six-element Yagi-Uda antenna. For this case we set  $\text{Gain}_L = 11$  dBi,  $\text{VSWR}_L = 2$  and the dipole radius to  $0.003369\lambda$ . The Pareto fronts for this case are shown in Figures 5 and 6 for GDE3 and NSGA-II, respectively. Table 3 holds results obtained from the literature using GA, Pareto GA and CLSPO. Three example cases that were found by GDE3 are reported in Table 4. Figure 7 depicts the corresponding radiation patterns. The results obtained by GDE3 have in average



lower SLL values than the those found by NSGA-II. In terms of gain and VSWR both algorithms produce similar results. The three designs found by GDE3 have high gain values. Design 1 outperforms the designs from [4] and [7] in terms of gain and sidelobe level values. Design 2 presents the highest gain (13 dBi). For this case there is a tradeoff between gain and VSWR value. Design 3 provides the lowest SLL (-15.50 dB) of all but the VSWR deteriorates at the value of 1.72. The average execution time for GDE3 and NSGA-II is 1790.63 and 1804.62 seconds respectively. The total number of objective function evaluations is again 40,000. In [8] for the six-element case a total number of 300,000 objective is required.



**Figure 2.** Pareto front for four-element Yagi-Uda antenna found by GDE3.



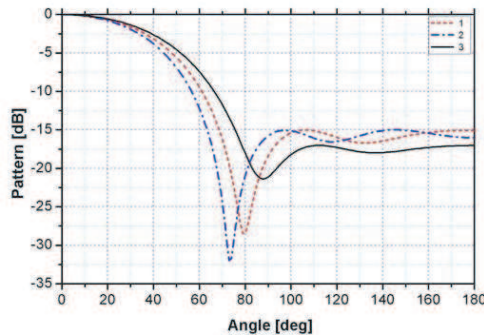
**Figure 3.** Pareto front for four-element Yagi-Uda antenna found by GDE3.

**Table 1.** Design parameters and results obtained from the literature for four-element Yagi-Uda antenna.

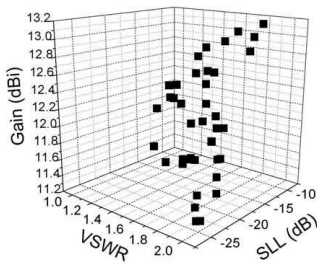
	GA [4]		CI [6]		CLPSO [7]	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.245	-	0.238	-	0.238	-
2	0.236	0.283	0.237	0.288	0.233	0.311
3	0.221	0.179	0.220	0.200	0.217	0.205
4	0.212	0.279	0.212	0.265	0.206	0.279
Gain (dBi)	9.84		9.83		9.44	
Z ( $\Omega$ )	$38.5 - j2.3$		$46.19 + j8.12$		$49.56 + j0.11$	
VSWR	1.31		1.20		1.01	
SLL (dB)	-14.50		-15.1		-15.02	

**Table 2.** Design parameters and results found by GDE3 for four-element Yagi-Uda antenna.

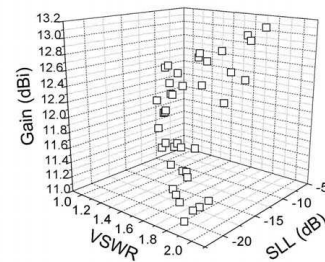
	1		2		3	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.243	-	0.236	-	0.248	-
2	0.235	0.266	0.232	0.294	0.237	0.251
3	0.221	0.175	0.220	0.220	0.219	0.156
4	0.211	0.248	0.211	0.268	0.205	0.272
Gain (dBi)	9.56		9.96		9.17	
Z ( $\Omega$ )	49.59 - j0.11		41.80 + j0.54		48.56 - j0.06	
VSWR	1.01		1.20		1.03	
SLL (dB)	-15.02		-15.00		-17.02	



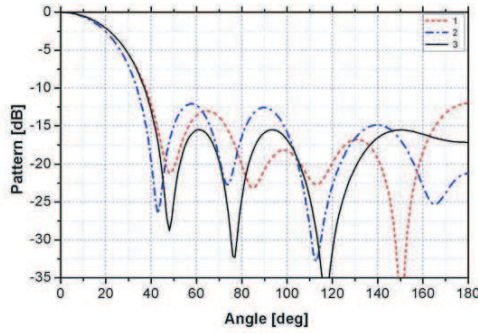
**Figure 4.** Radiation patterns of four-element Yagi-Uda design cases found by GDE3.



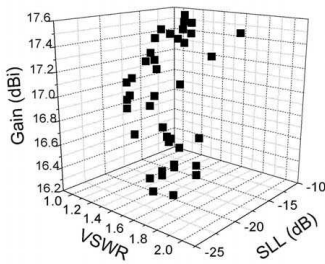
**Figure 5.** Pareto front for six-element Yagi-Uda antenna found by GDE3.



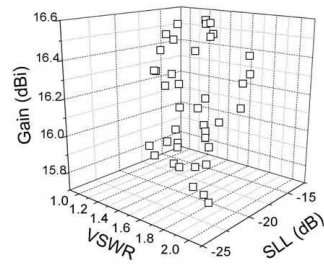
**Figure 6.** Pareto front for six-element Yagi-Uda antenna found by NSGA-II.



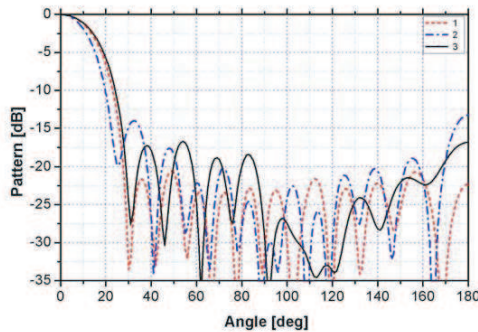
**Figure 7.** Radiation patterns of six-element Yagi-Uda design cases found by GDE3.



**Figure 8.** Pareto front for fifteen-element Yagi-Uda antenna found by GDE3.



**Figure 9.** Pareto front for fifteen-element Yagi-Uda antenna found by NSGA-II.



**Figure 10.** Radiation patterns of fifteen-element Yagi-Uda design cases found by GDE3.

**Table 3.** Design parameters and results obtained from the literature for six-element Yagi-Uda antenna.

Element	GA [4]		Pareto GA [8]		CLPSO [7]	
	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.239	-	0.2394	-	0.236	-
2	0.225	0.182	0.2339	0.2853	0.231	0.257
3	0.224	0.152	0.2214	0.2386	0.221	0.192
4	0.217	0.229	0.2167	0.2908	0.215	0.296
5	0.211	0.435	0.2065	0.3759	0.211	0.334
6	0.220	0.272	0.2049	0.3965	0.214	0.345
Gain (dBi)	12.58		12.9		12.65	
Z ( $\Omega$ )	49.64 - $j$ 5.08		N/A		50.013 - $j$ 0.013	
VSWR	1.11		1.47		1.0004	
SLL (dB)	-10.4		-15.0		-9.6	

**Table 4.** Design parameters and results found by GDE3 for six-element Yagi-Uda antenna.

Element	1		2		3	
	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.241	-	0.240	-	0.238	-
2	0.222	0.128	0.225	0.329	0.230	0.295
3	0.221	0.104	0.219	0.292	0.221	0.238
4	0.217	0.268	0.215	0.303	0.214	0.307
5	0.213	0.361	0.208	0.383	0.206	0.383
6	0.215	0.342	0.213	0.386	0.208	0.393
Gain (dBi)	12.72		13.00		12.88	
Z ( $\Omega$ )	49.67 - $j$ 5.49		30.99 - $j$ 3.22		29.16 + $j$ 1.97	
VSWR	1.12		1.62		1.72	
SLL (dB)	-12.02		-12.09		-15.50	

The final example is that of a fifteen-element Yagi-Uda antenna. We have selected for this case  $\text{Gain}_L = 15$  dBi,  $\text{VSWR}_L = 2$ . We set the dipole radius to  $0.003369\lambda$ . Figure 8 and Figure 9 have the Pareto fronts produced by GDE3 and NGSa-II respectively. The GDE3 has found solutions with higher gain values. The design parameters for solutions reported in the literature are given in Table 5 [4, 7, 12].

Table 6 holds the design parameters for three example design cases found by GDE3. The radiation patterns for these are depicted in Figure 10.

Design 1 presents a better performance than all designs cited in the literature in terms of sidelobe level. Also it presents a higher gain value than those from [4, 7]. Design 2 provides the highest gain value among all designs from both tables. It presents also a low VSWR value of 1.10, and the SLL is lower than those reported in the literature. For the parameters of design 3 a lower VSWR is achieved by the tradeoff in gain. The SLL is lower than the designs found in the literature. The average execution time for GDE3 and NSGA-II is 6451.45 and 6466.50 seconds respectively. The total number of objective function evaluations is 80,000. For this case of fifteen-

**Table 5.** Design parameters and results obtained from the literature for fifteen-element Yagi-Uda antenna.

Element	GA [4]		CLPSO [7]		DE [12]	
	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.236	-	0.239	-	0.236	-
2	0.230	0.249	0.226	0.168	0.229	0.293
3	0.221	0.155	0.222	0.171	0.222	0.195
4	0.205	0.185	0.216	0.260	0.217	0.307
5	0.216	0.191	0.210	0.311	0.214	0.346
6	0.210	0.252	0.201	0.216	0.206	0.450
7	0.210	0.442	0.210	0.262	0.207	0.395
8	0.189	0.431	0.205	0.378	0.203	0.430
9	0.191	0.362	0.197	0.336	0.204	0.520
10	0.200	0.205	0.204	0.376	0.204	0.386
11	0.204	0.268	0.199	0.324	0.189	0.439
12	0.215	0.414	0.190	0.406	0.199	0.556
13	0.174	0.197	0.197	0.210	0.209	0.439
14	0.199	0.130	0.203	0.328	0.202	0.433
15	0.204	0.362	0.202	0.369	0.205	0.399
Gain (dBi)	15.41		16.40		17.24	
Z ( $\Omega$ )	50.01 - j0.5		50.09 + j0.15		53.2 + j4.0	
VSWR	1.01		1.0035		1.10	
SLL (dB)	-10.31		-13.03		-10.64	

**Table 6.** Design parameters and results found by GDE3 for fifteen-element Yagi-Uda antenna.

Element	1		2		3	
	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.237	-	0.239	-	0.237	-
2	0.227	0.204	0.219	0.210	0.223	0.208
3	0.223	0.149	0.226	0.124	0.220	0.132
4	0.218	0.285	0.218	0.256	0.218	0.279
5	0.213	0.327	0.213	0.337	0.215	0.303
6	0.210	0.433	0.208	0.435	0.172	0.271
7	0.206	0.393	0.207	0.438	0.206	0.160
8	0.205	0.401	0.207	0.407	0.203	0.386
9	0.194	0.449	0.205	0.402	0.195	0.285
10	0.192	0.335	0.174	0.328	0.195	0.319
11	0.191	0.307	0.201	0.320	0.199	0.429
12	0.192	0.448	0.202	0.377	0.197	0.447
13	0.189	0.432	0.200	0.437	0.200	0.449
14	0.183	0.446	0.204	0.438	0.212	0.356
15	0.202	0.448	0.208	0.434	0.168	0.303
Gain (dBi)	16.88		17.58		16.46	
Z ( $\Omega$ )	45.85 - $j$ 7.20		46.11 - $j$ 0.71		47.84 - $j$ 0.70	
VSWR	1.19		1.10		1.05	
SLI (dB)	-20.29		-13.62		-15.93	

element Yagi-Uda design we have not found in the literature a multi-objective approach that optimizes all three objectives. In [8] 400,000 objective function evaluations are reported for an eight and twelve-element antenna.

## 5. CONCLUSION

Multi-objective evolutionary algorithms can be used successfully for Yagi-Uda antenna design. A novel antenna design method using the GDE3 algorithm has been presented. GDE3 is a new multi-objective DE algorithm, which has been compared against NSGA-II. Both algorithms are quite efficient in producing the Pareto front for different antenna design cases. For the Yagi-Uda design case GDE3

can produce similar or slightly better results than NSGA-II for the same population size and the same number of generations. One of the GDE3 advantages is the fact that it requires less computational load than NSGA-II. This is due to the fact that the population size which is ranked after a generation is usually less than the one required by NSGA-II. The antenna design cases found by GDE3 outperform those reported in the literature. The biggest advantage of using GDE3 for Yagi-Uda design is probably the fact that it can efficiently handle this tri-objective problem subject to any number of design constraints using a multi-objective approach. It must also be pointed out that GDE3 and NSGA-II require less number of objective function evaluations for the same Yagi-Uda design than the Pareto GA [8]. Both algorithms can be easily applied to other microwave and antenna design problems, and they can also be used in conjunction with an EM solver software. In our future work we plan to explore the applicability of other state-of-the-art MOEAs to antenna and microwave design problems such as the MOEA/D [59], multi-objective evolutionary algorithms based on summation of normalized objective values and diversified selection (SNOV-DS) [68] and the recently proposed new MOPSO [69]. The application of multi-objective evolutionary algorithms to antenna design problems provides researchers with a set of solutions. Then the most suitable design case for given antenna specifications can be selected.

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