Progress In Electromagnetics Research, Vol. 105, 231–251, 2010

PARETO OPTIMAL YAGI-UDA ANTENNA DESIGN USING MULTI-OBJECTIVE DIFFERENTIAL EVOLUTION

S. K. Goudos, K. Siakavara, E. E. Vafiadis, and J. N. Sahalos

Radiocommunications Laboratory
Department of Physics
Aristotle University of Thessaloniki
GR-541 24 Thessaloniki, Greece

Abstract—Antenna design problems often require the optimization of several conflicting objectives such as gain maximization, sidelobe level (SLL) reduction and input impedance matching. Multi-objective Evolutionary Algorithms (MOEAs) are suitable optimization techniques for solving such problems. An efficient algorithm is Generalized Differential Evolution (GDE3), which is a multi-objective extension of Differential Evolution (DE). The GDE3 algorithm can be applied to global optimization of any engineering problem with an arbitrary number of objective and constraint functions. Another popular MOEA is Nondominated Sorting Genetic Algorithm-II (NSGA-II). Both GDE3 and NSGA-II are applied to Yagi-Uda antenna design under specified constraints. The numerical solver used for antenna parameters calculations is SuperNEC, an object-oriented version of the numerical electromagnetic code (NEC-2). Three different Yagi-Uda antenna designs are considered and optimized. Pareto fronts are produced for both algorithms. The results indicate the advantages of this approach and the applicability of this design method.

1. INTRODUCTION

Antenna design problems have general multi-objective. Common design objectives include gain maximization, sidelobe level reduction and input impedance matching. The above-mentioned objectives are often subject to constraints. A common problem that has

Corresponding author: S. K. Goudos (sgoudos@gmail.com).

been addressed in the literature is the design of Yagi-Uda antennas that satisfy such objectives [1–15]. The optimization goal is to find the optimum element lengths and spacings that fulfill the design specifications. Evolutionary algorithms (EAs) such as Genetic Algorithms (GAs) have been applied to a variety of antenna design problems [16–23]. Yagi-Uda antennas have been studied in the literature using different evolutionary algorithms. In [4] a binary coded GA is used for Yagi-Uda design by a single objective approach. This is produced from the aggregation of the objective functions using different weight values. A Computational Intelligence (CI) method is applied in [6]. Particle Swarm Optimization (PSO) [24] has also been used successfully in several constrained or unconstrained antenna design problems [25–44]. The authors in [7] optimize Yagi-Uda designs with the single objective function that combines all objectives using Comprehensive Learning PSO (CLPSO). In [8] Pareto GA, a multi-objective GA, is used for the generation of the Pareto front for the Yagi-Uda design problem.

Nondominated Sorting Genetic Algorithm-II (NSGA-II) [45] is a popular and efficient multi-objective genetic algorithm, which has been used in several engineering design problems [46, 47]. The major drawback of a GA approach is the difficulty in the implementation due to the algorithm's inherited complexity and long computational time.

Differential evolution (DE) is a population-based stochastic global optimization algorithm, proposed by Price and Storn [48, 49]. Several DE variants or strategies exist [50, 51]. The classical DE algorithm has been applied to several antenna and microwave design problems [12, 52–57]. Li and Guo presented a new Yagi-Uda design approach using different objective functions and DE as the optimization algorithm [12]. Their objective functions also use a combination of the objectives and different weight factors. One of the DE advantages is the fact that very few parameters have to be adjusted in order to produce results. Several DE extensions for multi-objective optimization have been proposed so far [58, 59]. Generalized Differential Evolution (GDE3) [60] is a multi-objective DE algorithm that has outperformed other multi-objective evolutionary algorithms for a given set of numerical problems [61, 62]. GDE3 has been applied to microwave filter design in [63].

In this paper, both GDE3 and NSGA-II are used for the multi-objective Yagi-Uda design problem. We apply both to three different Yagi-Uda antenna design cases with four, six and fifteen elements. We consider three objective functions subject to specific constraints. The 3D Pareto fronts produced by both algorithms are compared and discussed. Example design cases are given and compared with data from the literature. This is accomplished in conjunction with SuperNEC, a commercially available EM solver. This is an object-oriented version of the numerical electromagnetic code (NEC-2). SuperNEC and NEC-2 versions have also been used in the literature for Yagi-Uda antenna design.

The novelty in our work lies in the fact that we apply both GDE3 and NSGA-II to the Yagi-Uda antenna design problem. To the best of our knowledge this is the first time that GDE3 is applied to an antenna design problem. The advantages of the GDE3 and NSGA-II algorithm approachs for multi-objective antenna problems are clearly shown.

This paper is organized as follows: Section 2 describes the Yagi-Uda design problem. The definition of the general multi-objective optimization problem under constraints is given in Section 3. We also present the classical *DE/rand/1/bin* strategy and briefly outline the GDE3 algorithm details. Section 4 presents the numerical results for three distinct antenna design cases. Finally the conclusion is given in Section 5.

2. THE YAGI-UDA ANTENNA DESIGN PROBLEM

Figure 1 shows an N-element Yagi-Uda antenna. This antenna consists of a single driven element, one reflector element and $N-2$ director elements. Such a N-element Yagi-Uda antenna has $2N-1$ antenna parameters that determine the antenna characteristics, apart from the elements' radius. The design parameters are $\bar{x} = (L_1, L_2, \ldots, L_k, \ldots, L_N, S_1, S_2, \ldots, S_k, \ldots, S_{N-1})$ where $2L_k$ is the length of the kth element, and S_k is the spacing between the kth and $(k+1)$th elements.

The Yagi-Uda antenna design goal is to find the optimum geometry that satisfies given performance specifications such as high gain, $\mathrm{Gain}(\bar{x})$, low sidelobe level, $\mathrm{SLL}(\bar{x})$, and input impedance close or equal to $50\,\Omega$. The last objective can also be defined as having a Voltage Standing Wave Ratio (VSWR), $\mathrm{VSWR}(\bar{x})$, close to one. It is obvious that such a problem is multi-objective. In the literature the above objectives have been combined in a single objective function using different weight factors [4, 7, 10, 12]. In this paper we express the Yagi-Uda antenna design problem as the minimization of the following objective functions:

$$
\begin{aligned}
F_1(\bar{x}) &= -\mathrm{Gain}(\bar{x}) \\
F_2(\bar{x}) &= \mathrm{SLL}(\bar{x}) \\
F_3(\bar{x}) &= \mathrm{VSWR}(\bar{x})
\end{aligned}
\tag{1}
$$

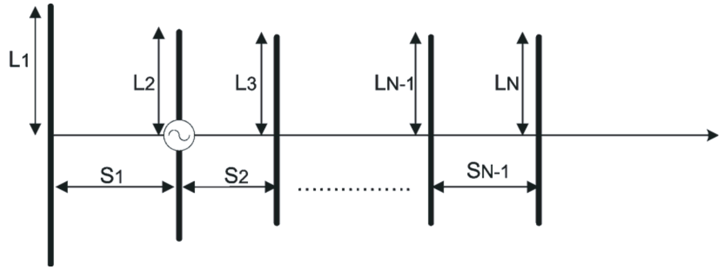

Figure 1. N-element Yagi-Uda antenna.

Moreover, the design problem is subject to the following constraints:

$$\begin{aligned} g_1(\bar{x}) &= \mathrm{Gain}(\bar{x}) \geq \mathrm{Gain}_L \\ g_2(\bar{x}) &= \mathrm{VSWR}(\bar{x}) \leq \mathrm{VSWR}_L \end{aligned} \tag{2}$$

where Gain_L, VSWR_L are the minimum allowable gain and maximum allowable VSWR respectively. In this paper, the methods of moments-based numerical electromagnetics code (NEC-2) has been used in the analysis. In particular, SuperNEC, a commercially available NEC-2 version with MATLAB interface, is used.

3. MULTI-OBJECTIVE OPTIMIZATION WITH CONSTRAINTS

The general constrained multi-objective optimization problem (MOOP) definition is [64]:

$$\text{Minimize} \quad F(\bar{x}) = [F_1(\bar{x}), F_2(\bar{x}), \ldots, F_n(\bar{x})] \tag{3}$$

$$\text{Subject to} \quad g_i(\bar{x}) \leq 0 \;\; i = 1, 2, \ldots, k \tag{4}$$

$F(\bar{x})$ is the vector of the objective functions, and g_i are the constraint functions. n is the number of objective functions, and k is the number of constraint functions.

In principle, multi-objective optimization is different from single-objective optimization. In single-objective optimization one attempts to obtain the best solution, which is usually the global minimum or global maximum depending on the optimization problem. In case of multiple objectives, there may not exist one best solution (global minimum or maximum) with respect to all objectives. In a typical MOOP, it is often necessary to determine a set of points that all fit a predetermined definition for an optimum. The predominant concept in defining an optimal point is that of Pareto optimality. Pareto-optimal solutions are those solutions (from the set of feasible solutions) that cannot be improved in any objective without causing degradation in at least one other objective.

Therefore, the above problem can be solved in two ways. The first way is to convert it to a single-objective optimization problem. This can be accomplished by using weights for different objective functions and penalty terms for the constraint functions. This method leads to a single solution. The second way is to use Pareto optimization, which means to optimize all the objectives simultaneously giving them equal importance. If none of the objective function values can be further improved without impairing the value of at least one objective for a given solution then this solution is Pareto-optimal and belongs to the set of non-dominated solutions which is called Pareto front. The main goal is to find some points (solutions) that belong to the Pareto front. From this set of non-dominated solutions, optimal antenna designs that provide a suitable compromise between the objectives for the desired constraints can be realized. More details about constraint handling in MOEAs can be found in a recent paper [65].

A multi-objective evolutionary algorithm can be used to solve this problem. Multi-objective evolutionary algorithms have gained popularity and have been used extensively over the last years in several design problems in electromagnetics. EAs use vectors to model the possible solutions. In order to distinguish the members of the non-dominated set from the population members we refer to the first as solutions and the second ones as vectors. The definitions of dominance relations between two vectors (or individuals of the population) are given below. The weak dominance $\preceq$ relation between two vectors $\bar{x}_1$, $\bar{x}_2$ in the search space is defined as [60]:

$$\bar{x}_1 \text{ weakly dominates } \bar{x}_2 \qquad \bar{x}_1 \preceq \bar{x}_2 \text{ iff } \forall i : F_i(\bar{x}_1) \leq F_i(\bar{x}_2) \tag{5}$$

while the dominance $\prec$ relation is defined as:

$$\bar{x}_1 \text{ dominates } \bar{x}_2 \qquad \bar{x}_1 \prec \bar{x}_2 \text{ iff } \bar{x}_1 \preceq \bar{x}_2 \wedge \exists i : F_i(\bar{x}_1) < F_i(\bar{x}_2) \tag{6}$$

The above relations can be extended to include constraint dominance $\prec_c$ [60]:

$\bar{x}_1$ constraint-dominates $\bar{x}_2$ $\bar{x}_1 \prec_c \bar{x}_2$ when any of the following conditions are true:

1) $\bar{x}_1$ belongs to the feasible design space, and $\bar{x}_2$ is infeasible.
2) $\bar{x}_1$, $\bar{x}_2$ are both infeasible, but $\bar{x}_1$ dominates $\bar{x}_2$ in constraint function space.
3) $\bar{x}_1$, $\bar{x}_2$ both belong the feasible design space, but $\bar{x}_1$ dominates $\bar{x}_2$ in objective function space.

3.1. Differential Evolution

A population in DE consists of NP vectors $\bar{x}_{iG}$, $i = 1, 2, \ldots, NP$, where G is the generation number. The population is initialized randomly from a uniform distribution. Each D-dimensional vector represents a possible solution. The initial population evolves in each generation with the use of three operators: Mutation, crossover and selection. Depending on the form of these operators several DE variants or strategies exist in the literature [24, 25]. The most popular is the one known as *DE/rand/1/bin* strategy. In this strategy a mutant vector $\bar{v}$ for every target vector $\bar{x}_{iG}$ is computed by:

$$\bar{v}_{i,G+1} = \bar{x}_{r_1,G} + F\left(\bar{x}_{r_2,G} - \bar{x}_{r_3,G}\right), \quad r_1 \neq r_2 \neq r_3 \tag{7}$$

where r_1, r_2, r_3 are randomly chosen indices from the population, and F is a mutation control parameter. After mutation the crossover operator is applied to generate a trial vector $\bar{u}_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \ldots, u_{Di,G+1})$ whose coordinates are given by:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1}, & \text{if } \mathrm{rand}(j) \leq CR \text{ or } j = rn(i) \\ x_{ji,G}, & \text{if } \mathrm{rand}(j) > CR \text{ and } j \neq rn(i) \end{cases} \tag{8}$$

where $j = 1, 2, \ldots, D$, $\mathrm{rand}(j)$ is a number from a uniform random distribution from the space [0, 1], $rn(i)$ a randomly chosen index from $(1, 2, \ldots, D)$ and CR the crossover constant from the space [0, 1]. DE uses a greedy selection operator. According to this selection scheme for minimization problems:

$$\bar{x}_{i,G+1} = \begin{cases} \bar{u}_{i,G+1}, & \text{if } f(\bar{u}_{i,G+1}) < f(\bar{x}_{i,G}) \\ \bar{x}_{i,G}, & \text{otherwise} \end{cases} \tag{9}$$

where $f(\bar{u}_{i,G+1})$, $f(\bar{x}_{i,G})$ are the fitness values of the trial and the old vector respectively. Therefore, the newly found trial vector $\bar{u}_{i,G+1}$ replaces the old vector $\bar{x}_{i,G}$ only when it produces a lower objective function value than the old one. Otherwise, the old vector remains in the next generation. The stopping criterion for the DE is usually the generation number or the number of objective function evaluations.

3.2. Generalized Differential Evolution (GDE3)

Multi-objective DE algorithms extend the classical DE algorithm for solving MOOP. Generalized Differential Evolution (GDE3) introduced in [60] can solve problems that have n objectives and k constraint functions. Recently, GDE3 has outperformed other evolutionary algorithms in numerical benchmark problems [61, 62]. It has been successfully applied to the molecular sequence alignment problem [66] and to microwave filters design [63]. To the best of the authors' knowledge this is the first time that the GDE3 algorithm is applied to an antenna design problem. The algorithm uses the concept of Crowding Distance (CD), which approximates the crowdedness of a vector in its non-dominated set as NSGA-II [45]. The vectors are sorted based on non-dominance and crowdedness. A basic difference exists between NSGA-II and GDE3 regarding the population size after a generation. In NSGA-II the population size after a generation is increased to $2NP$. Then non-dominated ranking is applied, and NP non-dominated vectors are selected. In GDE3 after a generation the population size is $NP+m$, where $m \in [0, NP]$, because the population size is increased only when the trial $u_{i,G+1}$ and old vector $x_{i,G}$ are feasible and do not dominate each other. Therefore, non-dominated ranking is applied to $NP+m$ population size, which can be less in general than $2NP$, thus resulting in less computational time than NSGA-II [60].

The GDE3 algorithm is outlined below:

1) Initialize random population of NP individuals. Set $m = 0$.
2) Evaluate objective function and constraint function values for every vector of the population.
3) Apply the mutation and crossover operators according to (7) and (8) and create a trial vector $u_{i,G+1}$.
4) Evaluate objective function and constraint function values for the trial vector.
5) Apply the selection operator according to the following criterion:

$$x_{i,G+1} = \begin{cases} u_{i,G+1}, & \text{if } u_{i,G+1} \preceq_c x_{i,G} \\ x_{i,G}, & \text{otherwise} \end{cases} \tag{10}$$

6) Set $m = m + 1$, $x_{NP+m,G+1} = u_{i,G+1}$

$$\text{if } \forall j : g_j(u_{i,G+1}) \leq 0 \wedge x_{i,G+1} == x_{i,G} \wedge x_{i,G} \not\preceq u_{i,G+1} \tag{11}$$

7) Apply non-dominated ranking to $NP + m$ vectors. Select NP non-dominated vectors and set $m = 0$.
8) Repeat step 3 until the maximum number of generations $G_{\max}$ is reached.

GDE3 variations with different DE strategies can be easily created simply by using different equations for crossover and mutation other than (7) and (8). More details about the GDE3 algorithm can be found in [60].

4. NUMERICAL RESULTS

All algorithms are executed 20 times. The best results are compared. All algorithms are compiled using the same compiler (Borland C++ Builder 5.0) in a PC with Intel Core 2 Duo E8500 at 3.16 GHz with 4GB RAM running Windows XP. The population size is set to 40 for both algorithms in all cases. The iteration number is set to 1000 for the cases of four and six elements, while for the 15-element Yagi-Uda is set to 2000. The control parameters chosen for GDE3 are according to [61, 67] for solving problems with three objectives $F = 0.2$, $CR = 0.2$. These values have been verified with several trials. In NSGA-II the crossover and mutation probabilities are set equal to 0.9 and 0.1, respectively.

The design cases selected are those that appear in the literature in order to compare results [4, 6–8, 12]. The first design case is that of a four-element Yagi-Uda antenna. For this case it is $\mathrm{Gain}_L = 9\,\mathrm{dBi}$, $\mathrm{VSWR}_L = 2$. The dipole radius is set to 0.00225λ as in the literature [4, 6–8, 12]. The 3D Pareto fronts found by GDE3 and NSGA-II are given in Figures 2 and 3, respectively. Each point of the Pareto front denotes a feasible design solution with three coordinates. We notice that both algorithms produce similar results. Table 1 has designs taken from the literature [4, 6, 7]. In Table 2 we present three different example cases from the Pareto front found by GDE3. The radiation patterns for these cases are plotted in Figure 4. As expected, no single optimum solution exists. Each of the example cases is better than the others in one objective. Design 1 presents a lower gain than the others but its VSWR is closer to one than others. One may notice that it has a higher gain value than the case of [7]. Design 2 has a high gain of 9.96 dBi and a relative low SLL of -15.00 dB. It outperforms the design from [6] in terms of gain value. The third example case obtained by GDE3 presents almost perfect impedance matching to $50\,\Omega$ and has the lowest SLL of all. The tradeoff for this case is the lowest gain value. The average execution time for GDE3 and NSGA-II is 1546.34 and 1618.73 seconds respectively. For this case the total number of objective function evaluations is 40,000 compared with 200,000 from [8] using the Pareto GA.

The next example is also common in literature [4, 7, 8]. It is that of a six-element Yagi-Uda antenna. For this case we set $\mathrm{Gain}_L = 11\,\mathrm{dBi}$, $\mathrm{VSWR}_L = 2$ and the dipole radius to 0.003369λ. The Pareto fronts for this case are shown in Figures 5 and 6 for GDE3 and NSGA-II, respectively. Table 3 holds results obtained from the literature using GA, Pareto GA and CLSPO. Three example cases that were found by GDE3 are reported in Table 4. Figure 7 depicts the corresponding radiation patterns. The results obtained by GDE3 have in average lower SLL values than the those found by NSGA-II. In terms of gain and VSWR both algorithms produce similar results. The three designs found by GDE3 have high gain values. Design 1 outperforms the designs from [4] and [7] in terms of gain and sidelobe level values. Design 2 presents the highest gain (13 dBi). For this case there is a tradeoff between gain and VSWR value. Design 3 provides the lowest SLL (−15.50 dB) of all but the VSWR deteriorates at the value of 1.72. The average execution time for GDE3 and NSGA-II is 1790.63 and 1804.62 seconds respectively. The total number of objective function evaluations is again 40,000. In [8] for the six-element case a total number of 300,000 objective is required.

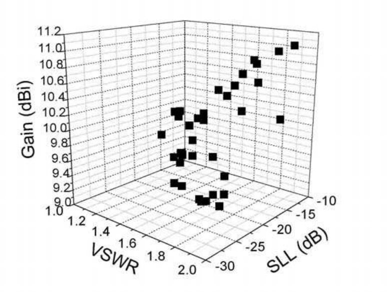

Figure 2. Pareto front for four-element Yagi-Uda antenna found by GDE3.

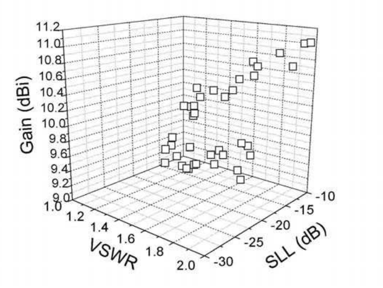

Figure 3. Pareto front for four-element Yagi-Uda antenna found by GDE3.

Table 1. Design parameters and results obtained from the literature for four-element Yagi-Uda antenna.

	GA [4]		CI [6]		CLPSO [7]	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.245	-	0.238	-	0.238	-
2	0.236	0.283	0.237	0.288	0.233	0.311
3	0.221	0.179	0.220	0.200	0.217	0.205
4	0.212	0.279	0.212	0.265	0.206	0.279
Gain (dBi)	9.84		9.83		9.44	
Z (Ω)	$38.5 - j2.3$		$46.19 + j8.12$		$49.56 + j0.11$	
VSWR	1.31		1.20		1.01	
SLL (dB)	-14.50		-15.1		-15.02	

Table 2. Design parameters and results found by GDE3 for four-element Yagi-Uda antenna.

	1		2		3	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.243	-	0.236	-	0.248	-
2	0.235	0.266	0.232	0.294	0.237	0.251
3	0.221	0.175	0.220	0.220	0.219	0.156
4	0.211	0.248	0.211	0.268	0.205	0.272
Gain (dBi)	9.56		9.96		9.17	
Z (Ω)	$49.59 - j0.11$		$41.80 + j0.54$		$48.56 - j0.06$	
VSWR	1.01		1.20		1.03	
SLL (dB)	-15.02		-15.00		-17.02	

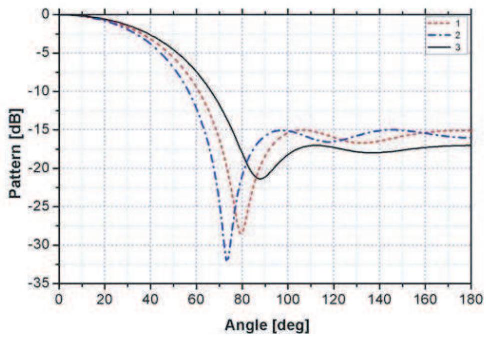

Figure 4. Radiation patterns of four-element Yagi-Uda design cases found by GDE3.

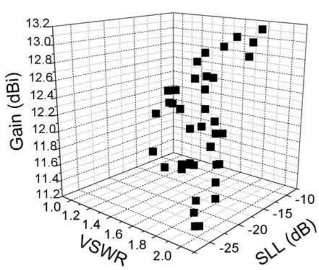

Figure 5. Pareto front for six-element Yagi-Uda antenna found by GDE3.

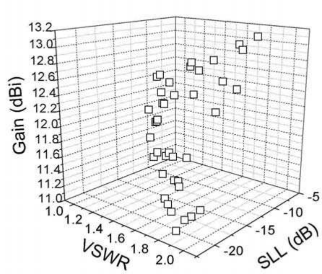

Figure 6. Pareto front for six-element Yagi-Uda antenna found by NSGA-II.

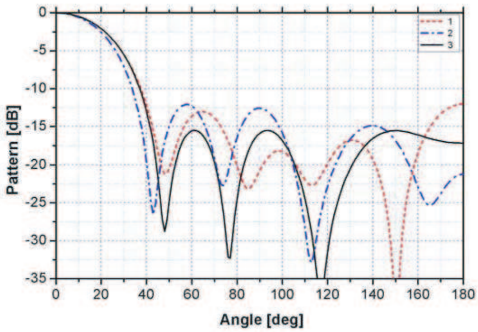

Figure 7. Radiation patterns of six-element Yagi-Uda design cases found by GDE3.

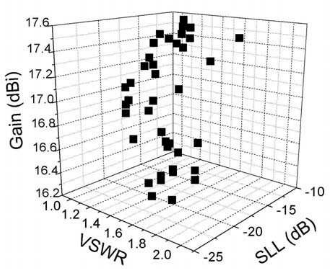

Figure 8. Pareto front for fifteen-element Yagi-Uda antenna found by GDE3.

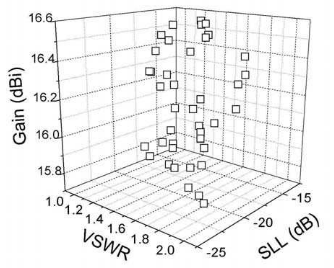

Figure 9. Pareto front for fifteen-element Yagi-Uda antenna found by NSGA-II.

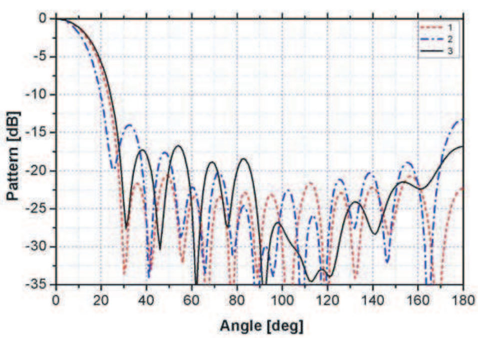

Figure 10. Radiation patterns of fifteen-element Yagi-Uda design cases found by GDE3.

Table 3. Design parameters and results obtained from the literature for six-element Yagi-Uda antenna.

	GA [4]		Pareto GA [8]		CLPSO [7]	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.239	-	0.2394	-	0.236	-
2	0.225	0.182	0.2339	0.2853	0.231	0.257
3	0.224	0.152	0.2214	0.2386	0.221	0.192
4	0.217	0.229	0.2167	0.2908	0.215	0.296
5	0.211	0.435	0.2065	0.3759	0.211	0.334
6	0.220	0.272	0.2049	0.3965	0.214	0.345
Gain (dBi)	12.58		12.9		12.65	
Z (Ω)	$49.64 - j5.08$		N/A		$50.013 - j0.013$	
VSWR	1.11		1.47		1.0004	
SLL (dB)	-10.4		-15.0		-9.6	

Table 4. Design parameters and results found by GDE3 for six-element Yagi-Uda antenna.

	1		2		3	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.241	-	0.240	-	0.238	-
2	0.222	0.128	0.225	0.329	0.230	0.295
3	0.221	0.104	0.219	0.292	0.221	0.238
4	0.217	0.268	0.215	0.303	0.214	0.307
5	0.213	0.361	0.208	0.383	0.206	0.383
6	0.215	0.342	0.213	0.386	0.208	0.393
Gain (dBi)	12.72		13.00		12.88	
Z (Ω)	$49.67 - j5.49$		$30.99 - j3.22$		$29.16 + j1.97$	
VSWR	1.12		1.62		1.72	
SLL (dB)	-12.02		-12.09		-15.50	

The final example is that of a fifteen-element Yagi-Uda antenna. We have selected for this case $\text{Gain}_L = 15\,\text{dBi}$, $\text{VSWR}_L = 2$. We set the dipole radius to 0.003369λ. Figure 8 and Figure 9 have the Pareto fronts produced by GDE3 and NGSA-II respectively. The GDE3 has found solutions with higher gain values. The design parameters for solutions reported in the literature are given in Table 5 [4, 7, 12].

Table 6 holds the design parameters for three example design cases found by GDE3. The radiation patterns for these are depicted in Figure 10.

Design 1 presents a better performance than all designs cited in the literature in terms of sidelobe level. Also it presents a higher gain value than those from [4, 7]. Design 2 provides the highest gain value among all designs from both tables. It presents also a low VSWR value of 1.10, and the SLL is lower than those reported in the literature. For the parameters of design 3 a lower VSWR is achieved by the tradeoff in gain. The SLL is lower than the designs found in the literature. The average execution time for GDE3 and NSGA-II is 6451.45 and 6466.50 seconds respectively. The total number of objective function evaluations is 80,000. For this case of fifteen-

Table 5. Design parameters and results obtained from the literature for fifteen-element Yagi-Uda antenna.

	GA [4]		CLPSO [7]		DE [12]	
Element	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.236	-	0.239	-	0.236	-
2	0.230	0.249	0.226	0.168	0.229	0.293
3	0.221	0.155	0.222	0.171	0.222	0.195
4	0.205	0.185	0.216	0.260	0.217	0.307
5	0.216	0.191	0.210	0.311	0.214	0.346
6	0.210	0.252	0.201	0.216	0.206	0.450
7	0.210	0.442	0.210	0.262	0.207	0.395
8	0.189	0.431	0.205	0.378	0.203	0.430
9	0.191	0.362	0.197	0.336	0.204	0.520
10	0.200	0.205	0.204	0.376	0.204	0.386
11	0.204	0.268	0.199	0.324	0.189	0.439
12	0.215	0.414	0.190	0.406	0.199	0.556
13	0.174	0.197	0.197	0.210	0.209	0.439
14	0.199	0.130	0.203	0.328	0.202	0.433
15	0.204	0.362	0.202	0.369	0.205	0.399
Gain (dBi)	15.41		16.40		17.24	
Z (Ω)	$50.01 - j0.5$		$50.09 + j0.15$		$53.2 + j4.0$	
VSWR	1.01		1.0035		1.10	
SLL (dB)	-10.31		-13.03		-10.64	

Table 6. Design parameters and results found by GDE3 for fifteen-element Yagi-Uda antenna.

	1		2		3	
Element	$L(\lambda)$	S(λ)	$L(\lambda)$	$S(\lambda)$	$L(\lambda)$	$S(\lambda)$
1	0.237	-	0.239	-	0.237	-
2	0.227	0.204	0.219	0.210	0.223	0.208
3	0.223	0.149	0.226	0.124	0.220	0.132
4	0.218	0.285	0.218	0.256	0.218	0.279
5	0.213	0.327	0.213	0.337	0.215	0.303
6	0.210	0.433	0.208	0.435	0.172	0.271
7	0.206	0.393	0.207	0.438	0.206	0.160
8	0.205	0.401	0.207	0.407	0.203	0.386
9	0.194	0.449	0.205	0.402	0.195	0.285
10	0.192	0.335	0.174	0.328	0.195	0.319
11	0.191	0.307	0.201	0.320	0.199	0.429
12	0.192	0.448	0.202	0.377	0.197	0.447
13	0.189	0.432	0.200	0.437	0.200	0.449
14	0.183	0.446	0.204	0.438	0.212	0.356
15	0.202	0.448	0.208	0.434	0.168	0.303
Gain (dBi)	16.88		17.58		16.46	
Z (Ω)	$45.85 - j7.20$		$46.11 - j0.71$		$47.84 - j0.70$	
VSWR	1.19		1.10		1.05	
SLL (dB)	-20.29		-13.62		-15.93	

element Yagi-Uda design we have not found in the literature a multi-objective approach that optimizes all three objectives. In [8] 400,000 objective function evaluations are reported for an eight and twelve-element antenna.

5. CONCLUSION

Multi-objective evolutionary algorithms can be used successfully for Yagi-Uda antenna design. A novel antenna design method using the GDE3 algorithm has been presented. GDE3 is a new multi-objective DE algorithm, which has been compared against NSGA-II. Both algorithms are quite efficient in producing the Pareto front for different antenna design cases. For the Yagi-Uda design case GDE3 can produce similar or slightly better results than NGSA-II for the same population size and the same number of generations. One of the GDE3 advantages is the fact that it requires less computational load than NSGA-II. This is due to the fact that the population size which is ranked after a generation is usually less than the one required by NSGA-II. The antenna design cases found by GDE3 outperform those reported in the literature. The biggest advantage of using GDE3 for Yagi-Uda design is probably the fact that it can efficiently handle this tri-objective problem subject to any number of design constraints using a multi-objective approach. It must also be pointed out that GDE3 and NSGA-II require less number of objective function evaluations for the same Yagi-Uda design than the Pareto GA [8]. Both algorithms can be easily applied to other microwave and antenna design problems, and they can also be used in conjunction with an EM solver software. In our future work we plan to explore the applicability of other state-of-the-art MOEAs to antenna and microwave design problems such as the MOEA/D [59], multi-objective evolutionary algorithms based on summation of normalized objective values and diversified selection (SNOV-DS) [68] and the recently proposed new MOPSO [69]. The application of multi-objective evolutionary algorithms to antenna design problems provides researchers with a set of solutions. Then the most suitable design case for given antenna specifications can be selected.

ACKNOWLEDGMENT

The authors thank Poynting Innovations for proving the SuperNEC software. We would like also to thank Professor P. N. Suganthan for proving the source code of his paper with Yagi-Uda antenna design.

REFERENCES

1. Cheng, D. K. and C. A. Chen, "Optimum element spacings for Yagi-Uda arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 21, 615–623, 1973.
2. Chen, C. A. and D. K. Cheng, "Optimum element lengths for Yagi-Uda arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 23, 8–15, 1975.
3. Cheng, D. K., "Gain optimization for Yagi-Uda arrays," *IEEE Antennas and Propagation Magazine*, Vol. 33, 42–45, 1991.
4. Jones, E. A. and W. T. Joines, "Design of Yagi-Uda antennas

using genetic algorithms," *IEEE Transactions on Antennas and Propagation*, Vol. 45, 1386–1392, 1997.
5. Venkatarayalu, N. V. and T. Ray, "Single and multi-objective design of Yagi-Uda antennas using computational intelligence," *Congr. Evol. Comput.*, Vol. 2, 1237–1242, 2003.
6. Venkatarayalu, N. V. and T. Ray, "Optimum design of Yagi-Uda antennas using computational intelligence," *IEEE Transactions on Antennas and Propagation*, Vol. 52, 1811–1818, 2004.
7. Baskar, S., A. Alphones, P. M. Suganthan, and J. J. Liang, "Design of Yagi-Uda antennas using comprehensive learning particle swarm optimisation," *IEE Proceedings: Microwaves, Antennas and Propagation*, Vol. 152, 340–346, 2005.
8. Kuwahara, Y., "Multiobjective optimization design of Yagi-Uda antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 53, 1984–1992, 2005.
9. Misra, I. S., R. S. Chakrabarty, and B. B. Mangaraj, "Design, analysis and optimization of V-dipole and its three-element Yagi-Uda array," *Progress In Electromagnetics Research*, Vol. 66, 137–156, 2006.
10. Rattan, M., M. S. Patterh, and B. S. Sohi, "Optimization of Yagi-Uda antenna using simulated annealing," *Journal of Electromagnetic Waves and Applications*, Vol. 22, Nos. 2–3, 291–299, 2008.
11. Bemani, M. and S. Nikmehr, "A novel wide-band microstrip Yagi-Uda array antenna for WLAN applications," *Progress In Electromagnetics Research B*, Vol. 16, 389–406, 2009.
12. Li, J. Y. and J. L. Guo, "Optimization technique using differential evolution for Yagi-Uda antennas," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 4, 449–461, 2009.
13. Teisbaek, H. B. and K. B. Jakobsen, "Koch-fractal Yagi-Uda antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 2–3, 149–160, 2009.
14. Chou, H. T., K. L. Hung, and C. Y. Chen, "Utilization of a Yagi antenna director array to synthesize a shaped radiation pattern for optimum coverage in wireless communications," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 7, 851–861, 2009.
15. Sun, B. H., S. G. Zhou, Y. F. Wei, and Q. Z. Liu, "Modified two-element Yagi-Uda antenna with tunable beams," *Progress In Electromagnetics Research*, Vol. 100, 175–187, 2010.
16. Mahanti, G. K., N. Pathak, and P. Mahanti, "Synthesis of

thinned linear antenna arrays with fixed sidelobe level using real-coded genetic algorithm," *Progress In Electromagnetics Research*, Vol. 75, 319–328, 2007.

17. Mahanti, G. K., A. Chakrabarty, and S. Das, "Phase-only and amplitude-phase synthesis of dual-pattern linear antenna arrays using floating-point genetic algorithms," *Progress In Electromagnetics Research*, Vol. 68, 247–259, 2007.
18. Panduro, M. A., C. A. Brizuela, L. I. Balderas, and D. A. Acosta, "A comparison of genetic algorithms, particle swarm optimization and the differential evolution method for the design of scannable circular antenna arrays," *Progress In Electromagnetics Research B*, Vol. 13, 171–186, 2009.
19. Zhang, Y. J., S. X. Gong, and Y. X. Xu, "Radiation pattern synthesis for arrays of conformal antennas mounted on an irregular curved surface using modified genetic algorithms," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 10, 1255–1264, 2009.
20. Zhang, S., S. X. Gong, Y. Guan, P. F. Zhang, and Q. Gong, "A novel IGA-edsPSO hybrid algorithm for the synthesis of sparse arrays," *Progress In Electromagnetics Research*, Vol. 89, 121–134, 2009.
21. Lim, S. and H. Ling, "Comparing electrically small folded conical and spherical helix antennas based on a genetic algorithm optimization," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 11–12, 1585–1593, 2009.
22. Siakavara, K., "Novel fractal antenna arrays for satellite networks: Circular ring sierpinski carpet arrays optimized by genetic algorithms," *Progress In Electromagnetics Research*, Vol. 103, 115–138, 2010.
23. Luo, Z., X. Chen, and K. Huang, "A novel electrically-small microstrip genetic antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 4, 513–520, 2010.
24. Kennedy, J. and R. Eberhart, "Particle swarm optimization," *IEEE International Conference on Neural Networks*, Piscataway, NJ, 1995.
25. Ababneh, J., M. Khodier, and N. Dib, "Synthesis of interdigital capacitors based on particle swarm optimization and artificial neural networks," *International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 16, 322–330, 2006.
26. Mahmoud, K. R., M. El-Adawy, S. M. M. Ibrahem, R. Bansal, K. R. Mahmoud, and S. H. Zainud-Deen, "A comparison between circular and hexagonal array geometries for smart antenna

systems using particle swarm optimization algorithm," *Progress In Electromagnetics Research*, Vol. 72, 75–90, 2007.
27. Mikki, S. M. and A. A. Kishk, "Physical theory for particle swarm optimization," *Progress In Electromagnetics Research*, Vol. 75, 171–207, 2007.
28. Semnani, A. and M. Kamyab, "An enhanced method for inverse scattering problems using fourier series expansion in conjunction with FDTD and PSO," *Progress In Electromagnetics Research*, Vol. 76, 45–64, 2007.
29. Najjar, Y., Y. Moneer, and N. Dib, "Design of optimum gain pyramidal horn with improved formulas using particle swarm optimization," *International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 17, 505–511, 2007.
30. Dib, N. and M. Khodier, "Design and optimization of multi-band Wilkinson power divider," *International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 18, 14–20, 2008.
31. Shihab, M., Y. Najjar, N. Dib, and M. Khodier, "Design of non-uniform circular antenna arrays using particle swarm optimization," *Journal of Electrical Engineering*, Vol. 59, 216–220, 2008.
32. Chamaani, S., S. A. Mirtaheri, M. Teshnehlab, M. A. Shoorehdeli, and V. Seydi, "Modified multi-objective particle swarm optimization for electromagnetic absorber design," *Progress In Electromagnetics Research*, Vol. 79, 353–366, 2008.
33. Huang, C. H., C. C. Chiu, C. L. Li, and K. C. Chen, "Time domain inverse scattering of a two-dimensional homogenous dielectric object with arbitrary shape by particle swarm optimization," *Progress In Electromagnetics Research*, Vol. 82, 381–400, 2008.
34. Li, W. T., X. W. Shi, and Y. Q. Hei, "An improved particle swarm optimization algorithm for pattern synthesis of phased arrays," *Progress In Electromagnetics Research*, Vol. 82, 319–332, 2008.
35. Li, W. T., X. W. Shi, L. Xu, and Y. Q. Hei, "Improved GA and PSO culled hybrid algorithm for antenna array pattern synthesis," *Progress In Electromagnetics Research*, Vol. 80, 461–476, 2008.
36. Lu, Z. B., A. Zhang, and X. Y. Hou, "Pattern synthesis of cylindrical conformal array by the modified particle swarm optimization algorithm," *Progress In Electromagnetics Research*, Vol. 79, 415–426, 2008.
37. Benedetti, M., G. Oliveri, P. Rocca, and A. Massa, "A fully-adaptive smart antenna prototype: Ideal model and experimental validation in complex interference scenarios," *Progress In*

Electromagnetics Research, Vol. 96, 173–191, 2009.

38. Khodier, M. and M. Al-Aqeel, "Linear and circular array optimization: A study using particle swarm intelligence," *Progress In Electromagnetics Research B*, Vol. 15, 347–373, 2009.
39. Lanza, M., J. R. Perez Lopez, and J. Basterrechea, "Synthesis of planar arrays using a modified particle swarm optimization algorithm by introducing a selection operator and elitism," *Progress In Electromagnetics Research*, Vol. 93, 145–160, 2009.
40. Li, G., S. Yang, Y. Chen, and Z. Nie, "A novel electronic beam steering technique in time modulated antenna arrays," *Progress In Electromagnetics Research*, Vol. 97, 391–405, 2009.
41. Perez, J. R. and J. Basterrechea, "Hybrid particle swarm-based algorithms and their application to linear array synthesis," *Progress In Electromagnetics Research*, Vol. 90, 63–74, 2009.
42. Zhang, S., S. X. Gong, and P. F. Zhang, "A modified PSO for low sidelobe concentric ring arrays synthesis with multiple constraints," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 11–12, 1535–1544, 2009.
43. Islam, M. T., M. Moniruzzaman, N. Misran, and M. N. Shakib, "Curve fitting based particle swarm optimization for UWB patch antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 23, 2421–2432, Nos. 17–18, 2009.
44. Zhang, L., F. Yang, and A. Z. Elsherbeni, "On the use of random variables in particle swarm optimizations: A comparative study of gaussian and uniform distributions," *Journal of Electromagnetic Waves and Applications*, Vol. 23, Nos. 5–6, 711–721, 2009.
45. Deb, K., A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Transactions on Evolutionary Computation*, Vol. 6, 182–197, 2002.
46. OuYang, J., F. Yang, S. W. Yang, and Z. P. Nie, "The improved NSGA-II approach," *Journal of Electromagnetic Waves and Applications*, Vol. 22, Nos. 2–3, 163–172, 2008.
47. Jiang, L., J. Cui, L. Shi, and X. Li, "Pareto optimal design of multilayer microwave absorbers for wide-angle incidence using genetic algorithms," *IET Microwaves Antennas & Propagation*, Vol. 3, 572–579, 2009.
48. Storn, R. and K. Price, "Differential evolution — A simple and efficient adaptive scheme for global optimization over continuous spaces," Tech. Rep. TR-95-012, 1995, http://citeseer.ist.psu.edu/article/storn95differential.html.
49. Storn, R. and K. Price, "Differential evolution — A simple and

efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, Vol. 11, 341–359, 1997.
50. Das, S., A. Konar, and U. K. Chakraborty, "Two improved differential evolution schemes for faster global search," *GECCO 2005 — Genetic and Evolutionary Computation Conference*, Washington, D.C., 2005.
51. Das, S., A. Abraham, U. K. Chakraborty, and A. Konar, "Differential evolution using a neighborhood-based mutation operator," *IEEE Transactions on Evolutionary Computation*, Vol. 13, 526–553, 2009.
52. Panduro, M. A. and C. Del Rio Bocio, "Design of beam-forming networks for scannable multi-beam antenna arrays using CORPS," *Progress In Electromagnetics Research*, Vol. 84, 173–188, 2008.
53. Shahoei, H., H. Ghafoori-Fard, and A. Rostami, "A novel design methodology of multiclad single mode optical fiber for broadband optical networks," *Progress In Electromagnetics Research*, Vol. 80, 253–275, 2008.
54. Dib, N. I., S. K. Goudos, and H. Muhsen, "Application of taguchi's optimization method and self-adaptive differential evolution to the synthesis of linear antenna arrays," *Progress In Electromagnetics Research*, Vol. 102, 159–180, 2010.
55. Li, G., S. Yang, M. Huang, and Z. Nie, "Sidelobe suppression in time modulated linear arrays with unequal element spacing," *Journal of Electromagnetic Waves and Applications*, Vol. 24, Nos. 5–6, 775–783, 2010.
56. Pal, S., B. Qu, S. Das, and P. N. Suganthan, "Linear antenna array synthesis with constrained multi-objective differential evolution," *Progress In Electromagnetics Research B*, Vol. 21, 87–111, 2010.
57. Pal, S., S. Das, A. Basak, and P. N. Suganthan, "Synthesis of difference patterns for monopulse antennas with optimal combination of array-size and number of subarrays — A multi-objective optimization approach," *Progress In Electromagnetics Research B*, Vol. 21, 257–280, 2010.
58. Zhang, Q., W. Liu, and H. Li, "The performance of a new version of MOEA/D on CEC09 unconstrained MOP test instances," *IEEE Congress on Evolutionary Computation*, Trondheim, 2009.
59. Li, H. and Q. Zhang, "Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *IEEE Transactions on Evolutionary Computation*, Vol. 13, 284–302, 2009.

60. Kukkonen, S. and J. Lampinen, "GDE3: The third evolution step of generalized differential evolution," *Proceedings of the 2005 IEEE Congress on Evolutionary Computation*, 2005.
61. Kukkonen, S. and J. Lampinen, "Performance assessment of generalized differential evolution 3 (GDE3) with a given set of problems," *Proceedings of the IEEE Congress on Evolutionary Computation*, 2007.
62. Tan, K. C., "CEC 2007 conference report," *IEEE Computational Intelligence Magazine*, Vol. 3, 72–73, 2008.
63. Goudos, S. K. and J. N. Sahalos, "Pareto optimal microwave filter design using multiobjective differential evolution," *IEEE Transactions on Antennas and Propagation*, Vol. 58, 132–144, 2010.
64. Marler, R. T. and J. S. Arora, "Survey of multi-objective optimization methods for engineering," *Structural and Multidisciplinary Optimization*, Vol. 26, 369–395, 2004.
65. Qu, B. Y. and P. N. Suganthan, "Constrained multi-objective optimization algorithm with ensemble of constraint handling methods," *Engineering Optimization*, In press, 2010.
66. Kukkonen, S., S. R. Jangam, and N. Chakraborti, "Solving the molecular sequence alignment problem with generalized differential evolution 3 (GDE3)," *Proceedings of IEEE Symposium on Computational Intelligence in Multicriteria Decision Making*, 2007.
67. Kukkonen, S. and J. Lampinen, "An empirical study of control parameters for the third version of generalized differential evolution (GDE3)," *Proceedings of the IEEE Congress on Evolutionary Computation*, 2006.
68. Qu, B. Y. and P. N. Suganthan, "Multi-objective evolutionary algorithms based on the summation of normalized objectives and diversified selection," *Information Sciences*, In press, Uncorrected Proof, doi:10.1016/j.ins.2010.05.013, 2010.
69. Zhao, S. Z. and P. N. Suganthan, "Two-lbests based multi-objective particle swarm optimizer," *Engineering Optimization*, In press, 2010.