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DESIGN OF TIME-MODULATED LINEAR ARRAYS WITH A MULTI-OBJECTIVE OPTIMIZATION APPROACH

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Abstract—This article proposes a Multi-objective Optimization (MO) framework for the design of time-modulated linear antenna arrays with ultra low maximum Side Lobe Level (SLL), maximum Side Band Level (SBL) and main lobe Beam Width between the First Nulls (BWFN). In contrast to the existing optimization-based methods that attempt to minimize a weighted sum of SLL, SBL, and BWFN, we treat these as three distinct objectives that are to be achieved simultaneously and use one of the best known Multi-Objective Evolutionary Algorithms (MOEAs) of current interest called MOEA/D-DE (Decomposition based MOEA with Differential Evolution operator) to determine the best compromise among these three objectives. Unlike the single-objective approaches, the MO approach provides greater flexibility in the design by yielding a set of equivalent final solutions from which the user can choose one that attains a suitable trade-off margin as per requirements. We compared time-modulated antenna structures with other methods for linear array synthesis such as the excitation method and the phase-position synthesis method on the basis of the approximated Pareto Fronts (PFs) yielded by MOEA/D-DE and the best compromise solutions determined from the Pareto optimal set with a fuzzy membership-function based method. The final results obtained with MOEA/D-DE were compared with the results achieved by two state-of-the-art single objective optimization algorithms and five other MO algorithms. Our simulation studies on three instantiations of the design problem reflect the superiority of the MOEA/D-DE based design of time-modulated linear arrays.

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1. INTRODUCTION

Time-modulated antenna arrays are recently receiving a good deal of attention from researchers due to their efficiency in realizing ultra-low sidelobe levels in the far-field pattern [1–5]. This feature is mainly attributed to the fact that time-modulated antenna arrays incorporate an additional degree of freedom in their design-the time. For an 8-element slotted linear array the time modulation method and the realization of a nearly ultra-low SLL (~ 39.8 dB) were first achieved by Kummer et al. [2]. The general principles for analysis of time-modulated antenna system were first put forward by Bickmore in [5].

Although antenna arrays of this kind have greater flexibility for design and offers significant reduction in the dynamic-range ratio of the excitation for ultra-low SLLs as compared to that required in ordinary SLLs, the design of time-modulated arrays is still complicated due to the presence of a multitude of sideband signals. Since these sideband signals are usually spaced at multiples of the modulation frequency, a significant portion of the radiated or received power is shifted to the sidebands. Certain applications demand complete removal of sideband signals and hence they should be suppressed as far as possible to improve the efficiency of array design. In [2], Yang et al. proposed a Differential Evolution (DE) based approach for the design of time-modulated linear arrays with effective suppression of sideband radiation patterns. Such designs offer severe challenges to the antenna researchers and as indicated by [6] metaheuristic algorithms can be the best ways to handle them. Yang et al.. presented the design of multiple radiation patterns from time-modulated linear antenna arrays in [7]. They illustrated that compared to the conventional linear arrays, in generating multiple patterns by switching among different phase distributions, the time-modulated linear arrays are capable of realizing more stringent requirements such as lower sidelobes for the multiple patterns. In [8], Yang and Nie presented the study of millimeter-wave low sidelobe linear arrays with time modulation and used single-objective DE and Genetic Algorithm (GA) to obtain the optimized time sequences. Li et al.. undertook an in-depth study of the Amplitude Modulation (AM) and Frequency Modulation (FM) signal transmission of time-modulated linear arrays in [9]. Yang et al.. undertook a single-objective design of uniform amplitude time-modulated linear arrays with both suppressed sidelobes and sidebands in [10]. The approach utilizes a direct optimization of the "switch-on" time sequence of each array element via the Simple Genetic Algorithm (SGA). Some other recently reported and significant research works on time-modulated antenna arrays include synthesis of shaped beam patterns [11], investigations on various time sequences [12, 13], full wave simulation of time-modulated antenna arrays in frequency domain [14], mutual coupling compensation [15], and time-domain analysis of time-modulated antenna arrays [16].

In works like [2, 4, 10], separate objectives (which are often conflicting) are combined through a weighted linear sum into a single aggregated objective function. The weighted sum method is however, subjective and the solution obtained will depend on the values (more precisely, the relative values) of the weights specified. It is hard, if not impossible, to find a universal set of weights, that will click on different instantiations of the same problem. Motivated by the inherent multi-objective nature of the antenna array design problems and the overwhelming growth in the field of Multi-Objective Evolutionary Algorithms (MOEAs), we started to look for the most recently developed MOEAs that could solve the time-modulated linear array synthesis problem much more efficiently as compared to the conventional single-objective approaches. Our search converged to a decomposition-based MOEA, called MOEA/D-DE [17, 18], that ranked first among 13 state-of-the-art MOEAs in the unconstrained MOEA competition held under the IEEE Congress on Evolutionary Computation (CEC) 2009 [19]. MOEA/D-DE uses DE as its main search strategy and decomposes an MO problem into a number of scalar optimization sub-problems to optimize them simultaneously. Each sub-problem is optimized by only using information from its several neighboring sub-problems and this feature considerably reduces the computational complexity of the algorithm.

Although MOEAs have recently received some attention from the antenna array designers (e.g., see [43, 45]), to the best of our knowledge, no MOEA has so far been applied on the time-modulated linear array design problem till date. In this work we employ MOEA/D-DE to design linear time-modulated antenna arrays using the static excitation amplitude distribution and the switch on time intervals as the parameters to optimize. The MO framework attempts to achieve the best compromise among three design objectives: minimizing the Maximum Side Lobe Level (MSLL), Side Band Level (SBL) and the main lobe Beam Width between the First Nulls (BWFN) at the center frequency f_0. Since unlike single-objective optimization techniques (that finish with a single best solution) the MOEAs return a set of non-dominated solutions (the Pareto optimal set, to be briefly outlined in Section 2), we used a fuzzy membership function based approach [20] to identify the best compromise solutions over each case. In order to validate the MO design method of time-modulated linear arrays, we undertake a twofold comparative study over three significant instantiations of the design problem involving 16, 32, and 64 elements linear array. Firstly we compare the three kinds of design methodologies for linear arrays: the time modulation, the non-uniform excitation [21, 22] and the phase-position method [23] using MOEA/D-DE to achieve the design objectives in each case. This comparison reflects the superiority of the time-modulation method over the two others. Secondly the results of MOEA/D-DE over time-modulated array design are compared with the state-of-the-art variants of two popular single objective optimization algorithms of current interest, namely Differential Evolution (DE) [24] and Particle Swarm Optimization (PSO) [25, 26]. The comparison indicates that on the tested design instances MOEA/D-DE yields much better solutions as compared to the single-objective algorithms. In order to demonstrate the effectiveness of the decomposition-based approach taken in MOEA/D-DE, we compared its performance with five other MO algorithms: Normal-Boundary Intersection (NBI) [27], Non-dominated Sorting Genetic Algorithm (NSGA-II) [28], Pareto Archieved Evolution Strategy (PAES) [29, 30], Strength Pareto Evolutionary Algorithm (SPEA2) [31], and Multi-Objective Differential Evolution (MODE) [32]. MOEA/D-DE was found to outperform all these algorithms in terms of the hypervolume indicator and the R-indicator (I_{R_2}) [33] metrics.

2. MULTI-OBJECTIVE FORMULATION OF THE DESIGN PROBLEM

In this article, we consider the following three design methodologies for linear arrays and project each of them as an MO problem.

2.1. Time-modulated Antenna

We consider a time-modulated linear array of N isotropic elements which are equally spaced and each element is controlled by a high speed radio frequency (RF) switch and excited by complex amplitude. The array is used to transmit a rectangular pulse of width T, with a pulse repetition frequency $prf = 1/T_p$, and T_p is the pulse repetition period. Here the array factor is given by [4, 11, 16]:

$$F(\theta, t) = e^{j2\pi f_0 t} \sum_{k=0}^{N} A_k e^{j\alpha_k} U_k(t) e^{j\beta(k_1) d \sin\theta}, \tag{1}$$

where f_0 and β are the centre operating frequency and the wave number in free space, respectively; θ is the angle measured from the broadside direction; and d is the element spacing. A_k and α_k are the static excitation amplitude and phase of the kth element, respectively and U_k are the periodic "switch-on" time sequence functions in which each element is switched on for τ_k $(0 \leq \tau_k \leq T)$ in each period T_p. By decomposing (1) into a Fourier series, the radiation patterns at each harmonic frequency $m{\cdot}prf$ $(m = 0, \pm 1, \pm \ldots, \pm\alpha)$ are readily obtained and are given by:

$$F_m(\theta, t) = e^{j2\pi(f_0 + m \cdot prf)t} \cdot \sum_{k=1}^{t} a_{mk} \cdot e^{j[(k-1)\beta d \sin\theta + a_k]}, \tag{2}$$

where a_{mk} is the complex amplitude and is given by:

$$a_{mk} = A_k \tau_k \cdot prf \cdot \frac{\sin[\pi m \tau_k \cdot prf]}{\pi m \tau_k \cdot prf} \cdot e^{-j\pi m \tau_k \cdot prf}. \tag{3}$$

At the center frequency $(m = 0)$, (3) becomes:

$$a_{0k} = A_k \tau_k \cdot prf. \tag{4}$$

Thus we can use (3) and (4) to synthesize specific radiation patterns at f_0 and $f_0 + prf$, including ultra-low side lobe levels. The radiation pattern at central and first sideband frequency is given by Equation (4). The parameters to be optimized are the static excitation amplitudes A_k and "switch-on" times τ_k. The goal of design is to simultaneously minimize the MSLL, SBL_{max}, and BWFN. Let the array pattern at central frequency be $F_0(\theta, t)$ and at first sideband frequency be $F_1(\theta, t)$. $F_0(\theta, t)$ is a function of θ which is symmetric about 0°. Let $\theta_{\max}$ be the angle at which $F_0(\theta, t)$ attains global maxima. We calculate $F_0(\theta, t)$ and $F_1(\theta, t)$ for discrete values of θ. Let those discrete values be represented by set $\psi = [0, \pi/2]$. Let the discrete steps in which the array factor at central frequency is calculated be $\Delta\theta_0$. Similarly the discrete steps for the calculation of $F_1(\theta, t)$ be $\Delta\theta_1$.

The MSLL is taken as the decibel level of the maximum sidelobe. We first calculate where the array factor reaches its local maxima, and the maximum value of all the local maxima are then used for calculating MSLL. Let, $\zeta = [\theta \in \psi | \{F_0(\theta, t) > F_0(\theta - \Delta\theta_0, t)\} \Lambda \{F_0(\theta, t) > F_0(\theta + \Delta\theta_0, t)\} \Lambda \{\theta \neq \theta_{\max}\}]$ be the set of angles where local maxima of $F_0(\theta, t)$ occur.

Let $\Phi = \{\theta \in \psi | F_0(\theta, t) < F_0(\theta - \Delta\theta_0, t) \Lambda F_0(\theta, t) < F_0(\theta + \Delta\theta_0, t)\}$ be the set of angles where local minima of $F_0(\theta, t)$ is reached. Let the local minimum closest to 0° be α. Therefore $\alpha = \min(\Phi)$. Let $\theta'_{\max}$ be the angle at which $F_1(\theta, t)$ attains global maxima. Now we are at a position to define the three objective functions to be optimized by an MOEA as:

$$f_1 = \max\left[10 \log_{10}\left(\frac{F_0(\zeta, t)}{F_0(\theta_{\max}, t)}\right)\right] \text{dB} \tag{5a}$$

$$f_2 = 2\min(\Phi)\,\text{degrees} \tag{5b}$$

$$f_3 = 10\log\left(\frac{F_1\left(\theta'_{\max}, t\right)}{F_0\left(\theta_{\max}, t\right)}\right)\,\text{dB} \tag{5c}$$

The Dynamic Range Ratio (DRR) is usually given by:

$$DRR = I_{\max}/I_{\min}, \tag{6}$$

where I is the static current amplitude. In practical situations we need the dynamic-range to be low [4, 34]. Rather than minimizing dynamic range ratio as well we have imposed a constraint on dynamic range ratio as [4]:

$$I_{\max}/I_{\min} \leq 4. \tag{7}$$

The designer can impose a constraint according to his or her requirement. We have based our research on the assumption that a maximum dynamic range ratio of 4 can be allowed. However if the designer needs a still lower dynamic range ratio then the constraint needs to be changed. We again get a set of Pareto optimal solutions only that they will be slightly inferior to the ones who had lesser restriction on the DRR.

2.2. Non-uniform Excitation Method

In the non-uniform excitation method, only the excitations of the antenna elements are kept as optimization parameters. The array factor for an array of N isotropic radiators is given as follows:

$$AF(\varphi) = \sum_{n=1}^{N} I_n \cdot \cos[\beta \cdot x_n \cdot \cos(\varphi) + \phi_n], \tag{8}$$

where $\beta = \frac{2\pi}{\lambda}$ = wavenumber, I_n, φ_n, x_n are the excitation magnitude, phase and location of the n-th element. We vary only I_n and ϕ_n and the elements are assumed to be uniformly spaced at $\lambda/2$. This is similar to time-modulated antenna array synthesis only that here we don't have the extra degree of freedom pertaining to the switch-on times.

2.3. Phase-position Synthesis Method

We will also compare the time-modulated antenna array design with Phase-position synthesis method which is a very well documented method of linear antenna array design. Let us assume that $2N$ isotropic radiators are placed symmetrically along the x-axis. The expression for the array factor can be written as

$$AF(\varphi) = 2 \cdot \sum_{n=1}^{N} I_n \cdot \cos[\beta \cdot x_n \cdot \cos(\varphi) + \phi_n]. \tag{9}$$

In this method of synthesis, we vary location x_n and phase φ_n of the nth element assuming $I_n = 1$. However we need to vary x_n in such a way such that mutual coupling effects is not introduced. Thus the element spacing needs to be constrained. In this problem the element spacing x_n is normalized with respect to $\lambda/2$. The constraints that need to be considered for normalized element spacing x'_n is given below.

$$0.5 \leq x'_{n+1} - x'_n \leq 1, \quad n \in [1, N-1] \tag{10a}$$

$$0.25 \leq x'_1 \leq 0.5 \tag{10b}$$

The second condition comes from the symmetry of the antenna array. Corresponding to the first element in the positive x-axis there is another element at the same distance from origin (i.e., x_1) on the negative x-axis. Condition (10b) ensures that the 1st element is neither too far nor too close to its mirror image.

For Non-uniform excitation method and Phase-position synthesis method the principal lobe beam-width and MSLL are taken as the two principal objectives.

3. THE MOEA/D-DE ALGORITHM-AN OUTLINE

Due to the multiple criteria nature of most real-world problems, Multi-objective Optimization (MO) problems are ubiquitous, particularly throughout engineering applications. As the name indicates, multi-objective optimization problems involve multiple objectives, which should be optimized simultaneously and that often are in conflict with each other. This results in a group of alternative solutions which must be considered equivalent in the absence of information concerning the relevance of the others. The concepts of *dominance* and *Pareto-optimality* may be presented more formally in the following way [35, 36]:

3.1. General MO Problems

Definition 1: Consider without loss of generality the following multi-objective optimization problem with D decision variables x (parameters) and n objectives y:

$$\text{Minimize}: \ \vec{Y} = f\left(\vec{X}\right) = \left(f_1(x_1, \ldots, x_D), \ldots, f_n(x_1, \ldots, x_D)\right), \tag{11}$$

where $\vec{X} = [x_1, \ldots, x_D]^T \in P$ and $\vec{Y} = [y_1, \ldots, y_n]^T \in O$ and $\vec{X}$ is called decision (parameter) vector, P is the parameter space, $\vec{Y}$ is the objective vector, and O is the objective space. A decision vector

$\vec{A} \in P$ is said to dominate another decision vector $\vec{B} \in P$ (also written as $\vec{A} \prec \vec{B}$ for minimization) if and only if:

$$\forall i \in \{1,\ldots,n\}: \; f_i\left(\vec{A}\right) \leq f_i\left(\vec{B}\right) \wedge \exists j \in \{1,\ldots,n\} : \; f_j\left(\vec{A}\right) < f_j\left(\vec{B}\right) \quad (12)$$

Based on this convention, we can define non-dominated, *Pareto-optimal* solutions as follows:

Definition 2: Let $\vec{A} \in P$ be an arbitrary decision vector.

(a) The decision vector $\vec{A}$ *is said to be* non-dominated regarding the set $P' \subseteq P$ if and only if there is no vector in P' which can dominate $\vec{A}$.

(b) The decision (parameter) vector $\vec{A}$ is called Pareto-optimal if and only if $\vec{A}$ is non-dominated regarding the whole parameter space P.

3.2. The MOEA/D-DE Algorithm

Multi-objective evolutionary algorithm based on decomposition was first introduced by Zhang and Li in 2007 [37] and extended with DE-based reproduction operators in [17, 18]. Instead of using non-domination sorting for different objectives, the MOEA/D algorithm decomposes a multi-objective optimization problem into a number of single objective optimization sub-problems by using weights vectors λ and optimizes them simultaneously. Each sub-problem is optimized by sharing information between its neighboring sub-problems with similar weight values. MOEA/D uses Tchebycheff decomposition approach [38] to convert the problem of approximating the PF into a number of scalar optimization problems. Let $\vec{\lambda}^1, \ldots, \vec{\lambda}^N$ be a set of evenly spread weight vectors and $\vec{Y}^* = (y_1^*, y_2^*, \ldots, y_M^*)$ be a reference point, i.e., for minimization problem, $y_i^* = \min\{f_i(\vec{X}) | \vec{X} \in \Omega\}$ for each $i = 1, 2 \ldots M$. Then the problem of approximation of the PF can be decomposed into N scalar optimization subproblems by Tchebycheff approach and the objective function of the j-th subproblem is:

$$g^{te}\left(\vec{X} | \vec{\lambda}^j, \vec{Y}^*\right) = \max_{1 \leq i \leq M} \left\{ \lambda_i^j \left| f_i(x) - y_i^* \right| \right\} \quad (13)$$

where $\lambda^j = (\lambda_1^j, \ldots, \lambda_M^j)^T$, $j = 1, \ldots, N$ is a weight vector, i.e., $\lambda_i^j \geq 0$ for all $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} \lambda_i^j = 1$. MOEA/D minimizes all these N objective functions simultaneously in a single run. Neighborhood relations among these single objective subproblems are defined based on the distances among their weight vectors. Each subproblem is then optimized by using information mainly from its neighboring subproblems. In MOEA/D, the concept of neighborhood, based on similarity between weight vectors with respect to Euclidean distances, is used to update the solution. The neighborhood of the i-th subproblem consists of all the subproblems with the weight vectors from the neighborhood of $\vec{\lambda}^i$. At each generation, the MOEA/D maintains following variables:

1. A population $(\vec{X}^1, \ldots, \vec{X}^N)$ with size N, where $\vec{X}_i$ is the current solution to the i-th subproblem.
2. The fitness values of each population corresponding to a specific subproblem.
3. The reference point $\vec{Y}^* = (y_1^*, y_2^*, \ldots, y_M^*)$, where y_i^* is the best value found so far for objective i.
4. An external population (EP), which is used to store non-dominated solutions found during the search.

The MOEA/D-DE algorithm is schematically presented in Table 1.

Table 1. The MOEA/D-DE algorithm.

1. *Initialization*	Initialize the External Population (EP) Compute the Euclidean distances between any two weight vectors and find out the T closest weight vectors to each weight vector where T is the neighborhood size. Randomly generate an initial population $X^1,\ldots,X^N$ and evaluate the fitness values. Initialize the reference points by a problem-specific method.
2. *Update*	Reproduction: reproduce the offspring $\vec{U}_i$ corresponding to parent $\vec{X}_i$ by DE/rand/1/bin scheme (Page 37 – 42, [24]). For j-th component of the i-th vector: $u_{i,j} = x_{r_1^i,j} + F.(x_{r_2^i,j} - x_{r_3^i,j})$, with probability Cr $= x_{j,i}$, with probability $1 - Cr$ Repair: Repair the solution if $\vec{U}$ is out of the boundary and the value is reset to be a randomly selected value inside the boundary. Update of reference points, if the fitness value of $\vec{U}$ is better than the reference point. Update the neighboring solutions, if the fitness value of $\vec{U}$ is better. Update of EP by removing all the vectors that are dominated by $\vec{U}$ and add $\vec{U}$ to EP if no vector in EP dominates it.
3. *Termination Criteria*	If stopping criteria is satisfied, then stop and output EP. Otherwise, go to Step 2

4. SIMULATION RESULTS

We consider three different antenna array designs as three problem instantiations. These are 16 element, 32 element, and 64 element time-modulated antenna array. The time-modulated arrays in each case are also compared with linear array synthesized by non-uniform Excitation method and Phase-Position Synthesis method. In non-uniform excitation the antenna array elements have non-uniform excitation. In phase-Position synthesis both phase and position of array elements are optimized. We have not considered just non-uniform phase and nonuniform spacing here because it is obvious that phase-position synthesis will yield better results than both of them due to more degrees of freedom.

For MOEA/D-DE, the best compromise solution was chosen from the PF using the method described in [20]. The ith objective function f_i is represented by a membership function μ_i defined as:

$$\mu_i = \begin{cases} 1 & f_i \leq f_i^{\min} \\ \frac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}} & f_i^{\min} < f_i < f_i^{\max} \\ 0 & f_i \geq f_i^{\max} \end{cases}, \tag{14}$$

where $f_i^{\min}$ and $f_i^{\max}$ are the minimum and maximum value of the ith objective solution among all nondominated solutions, respectively. For each nondominated solution q, the normalized membership function μ^q is calculated as:

$$\mu_q = \frac{\sum\limits_{i=1}^{N_{obj}} \mu_i^q}{\sum\limits_{k=1}^{N_s} \sum\limits_{i=1}^{N_{obj}} \mu_i^k} \tag{15}$$

where N_s is the number of non-dominated solution. The best compromise is the one having the maximum value of μ^q.

Over the time-modulated linear array design instances we also compare the performance of MOEA/D-DE with that of two single-objective optimization techniques, namely DEGL (DE with Global and Local Neighborhood) [39] and CLPSO (Comprehensive Learning PSO) [40] that are the state-of-the-art variants of DE and PSO, two metaheuristic algorithms widely used in past for various electromagnetic optimization [2, 4, 26, 41–44]. For single-objective optimization techniques, we use a weighted linear sum of the objective functions given in (5a)–(5c). We also compared MOEA/D-DE results with five other MO algorithms: NBI [27], NSGA-II [28], PAES [29, 30], SPEA2 [31], and MODE [32]. Parameters for all the algorithms are selected from their respective literatures and the detailed parametric setup for MOEA/D-DE and the two single-objective optimization algorithms have been shown in Table 2. For comparing the performance of the MO algorithms, we used the following performance indices:

(1) R **indicator (**I_{R2}**) [33]**: It can be expressed as

$$I_{R2} = \frac{\sum_{\lambda \in \Lambda} u^*(\lambda, A) - u^*(\lambda, R)}{|\Lambda|}, \tag{16}$$

where R is a reference set, u^* is the maximum value reached by the utility function u with weight vector λ on an approximation set A, i.e., $u^* = \max_{y \in A} u_\lambda(y)$. We choose the augmented Tchebycheff function as the utility function.

(2) Hypervolume difference to a reference set ($I_{\bar{H}}$) **[33]**: The hypervolume indicator I_H measures the hypervolume of the objective space that is weakly dominated by an approximation set A, and is to be maximized. Here we consider the hypervolume difference to a reference set R, and we will refer to this indicator as $I_{\bar{H}}$, which is defined as $I_{\bar{H}} = I_H(R) - I_H(A)$ where smaller values correspond to higher quality as opposed to the original hypervolume I_H.

In what follows, we report the best results obtained from a set of 25 independent runs of MOEA/D-DE and its competitors, where

Table 2. Parametric set-up for single-objective optimization algorithms.

MOEA/D-DE		CLPSO		DEGL	
Param.	Val.	Param.	Val.	Param.	Val.
Pop_size	150	Swarm size	150	Pop_size	150
Crossover Probability CR	0.9	C_1	1.494	Crossover Probability CR	0.9
F	0.8	C_2	1.494	F	0.8
distribution index η	20	Inertial Weight w	linearly decreased from 0.9 to 0.2	Neighborhood size	15% of Pop_ size
mutation rate p_m	$1/D$	$v_{d,\max}$	0.9*r_d	weight factor	fixed, 0.5

each run for each algorithm is continued up to 3×10^5 Function Evaluations (FEs). Note that for MOEA/D-DE, after each run we extract the best compromise solution obtained with the fuzzy membership function based method outlined above. In the study that follows the time-modulated antenna arrays are assumed to have the following parameters: Time period $T = 1\,\mu\text{s}$, modulating frequency $prf = 1\,\text{MHz}$ and central frequency $f_0 = 3.0\,\text{GHz}$.

4.1. Design Results for 16 Element Array

A 16-element linear array of isotropic radiating elements, with $\lambda/2$ spacing, is considered for the time-modulated antenna array. In Table 3 we provide the R-indicator and hypervolume indicator-values calculated over the best run of the following MO algorithms-MOEA/D-DE, NSGA2, MODE, PAES, SPEA2, and NBI. Best values of these performance metrics were obtained by MOEA/D-DE. Table 4 presents

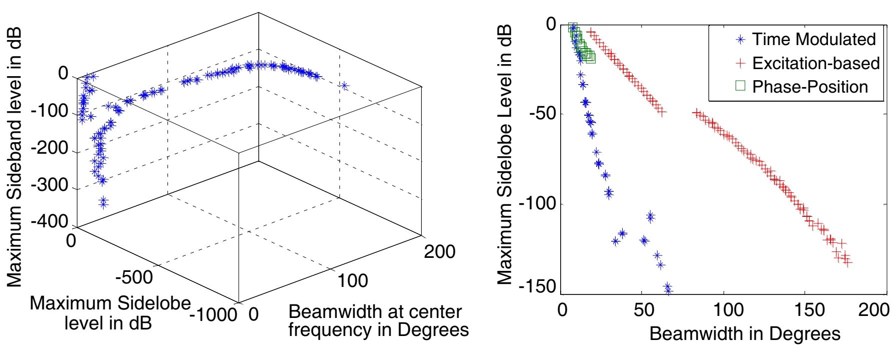
Table 3. Best, worst, mean, and standard deviations of the performance metrics for comparing the MO algorithms (16 element array).

Performance Metric	Value type	MOEA/D-DE	NSGA-2	MODE	PEAS	SPEA2	NBI
R-indicator	Best	**5.67e-07**	1.30e-04	3.41e-05	6.52e-04	5.63e-04	5.42e-03
	Worst	**6.93e-05**	5.61e-02	9.88e-02	0.1312	8.71e-02	0.3412
	Mean	**1.13e-05**	4.56e-03	6.71e-03	4.23e-02	2.11e-02	7.89e-02
	Std. Dev.	**6.04e-06**	1.21e-03	3.92e-03	2.31e-02	4.52e-03	1.19e-02
Hypervolume-indicator	Best	**4.52e-03**	8.86e-03	7.81e-03	2.83e-02	9.85e-03	4.87e-02
	Worst	**1.15e-02**	6.52e-02	8.12e-02	0.2419	0.1219	0.2613
	Mean	**5.81e-03**	1.42e-02	4.19e-02	0.1219	7.98e-02	0.1532
	Std. Dev.	**1.34e-03**	2.31e-03	8.49e-03	6.32e-02	5.76e-02	6.321e-02

Table 4. Optimal compromise table for 16 element antenna array design.

Method of Design	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)
Time-modulated	**13.015**	**−33.46**	**−58.88**
Non-uniform Excitation	28.045	−14.09	NA
Phase-Position	16.2855	−16.06	NA

three design objectives of the best compromise solution achieved by MOEA/D-DE for the time-modulated array. BWFN and MSLL for the best compromise solution achieved by MOEA/D-DE corresponding to non-uniform excitation and phase-position based design methods have also been shown in the same table. Figure 1(a) presents the 3-dimensional approximated PF or trade-off curve obtained with MOEA/D-DE for the linear time-modulated array. In Figure 1(b), we show the 2-dimensional PF for all the three methods of linear array design. Figure 1(b) indicates that it is possible to achieve much better trade-off between MSLL and BW for time-modulated linear arrays with MOEA/D-DE. The same fact is supported by Table 4 that shows for time-modulated arrays much smaller values of BW and MSLL were obtained. Finally in Table 5, we provide values of the three design objectives finally achieved with MOEA/D-DE and the two single-objective algorithms. The static amplitude excitations



(a) (b)

Figure 1. Best approximated PFs obtained with MOEA/D-DE over 16-element array design instance. (a) 3-dimensional PF for 16 element time-modulated array design. (b) 2-dimensional PF for three design methods over 16 element array.

Table 5. Best values of the three design-objectives achieved by best MO algorithm (MOEA/D-DE) and the two single-objective optimization algorithms over 16-element array design.

Algorithm	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)	Dynamic Range
MOEA/D	**13.015**	**−33.46**	**−58.88**	**1.852**
DEGL	15.08	−22.71	−56.79	3.823
CLPSO	15.08	−22.63	−48.21	3.742

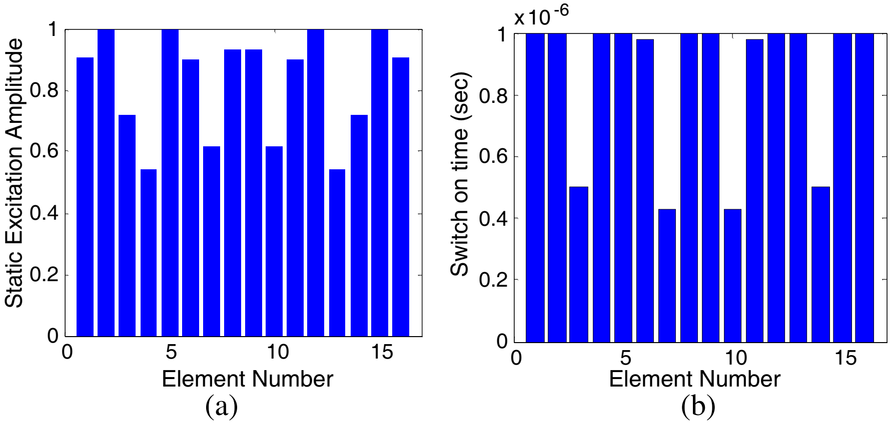


Figure 2. (a) Static excitations obtained by MOEA/D-DE. (b) Switch on time sequence obtained by MOEA/D-DE.

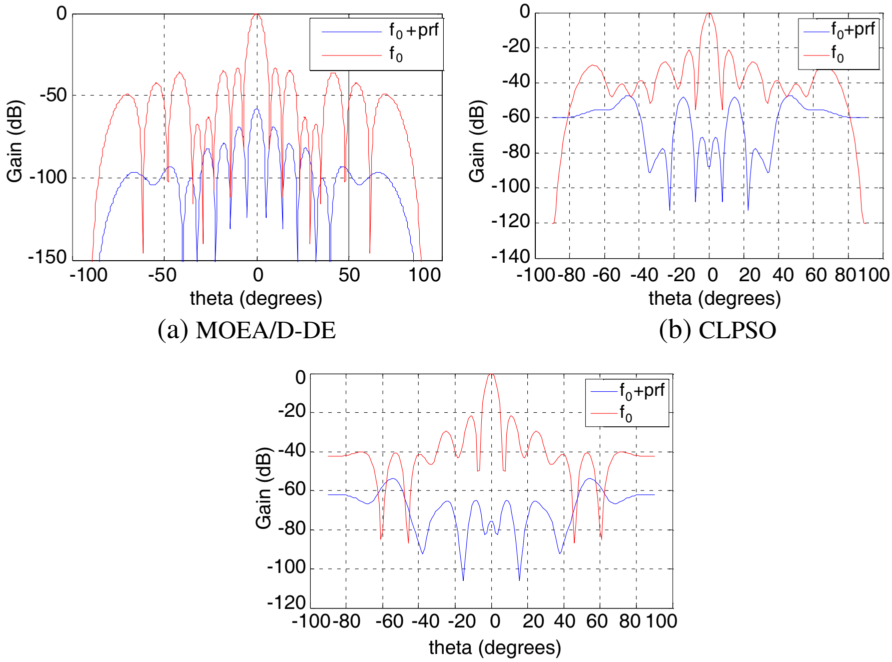
Table 6. Best, worst, mean, and standard deviations of the performance metrics for comparing the MO algorithms (32 element array).

Performance Metric	Value type	MOEA/D-DE	NSGA-2	MODE	PEAS	SPEA2	NBI
R-indicator	Best	**8.23e-08**	7.81e-05	8.12e-05	3.12e-03	6.80e-04	8.41e-03
	Worst	**4.53e-06**	3.41e-03	5.61e-03	2.31e-02	7.94e-03	0.3192
	Mean	**1.90e-06**	4.79e-04	8.47e-04	9.63e-03	1.62e-03	4.60e-02
	Std. Dev.	**7.54e-07**	8.91e-05	9.72e-05	2.18e-03	7.45e-04	7.13e-03
Hypervolume-indicator	Best	**9.45e-05**	3.19e-04	5.37e-04	3.02e-03	9.98e-04	7.43e-03
	Worst	**4.53e-04**	2.95e-03	5.43e-03	8.74e-02	6.01e-03	0.1029
	Mean	**1.67e-04**	7.52e-04	8.95e-04	6.59e-02	2.94e-03	9.60e-02
	Std. Dev.	**6.76e-05**	2.96e-04	4.57e-04	8.43e-03	6.93e-04	1.19e-02

and switch on time intervals are shown in Figure 2. Corresponding array patterns have been shown in Figure 3. Table 5 indicates that the best compromise solution of MOEA/D-DE is considerably superior in comparison to the best results obtained with DEGL and CLPSO. The Pareto front shown in Figure 1(b) for the time-modulated antenna arrays might not seem optimal. Actually there are some points that seem dominated by other points but in actuality they are non-dominated because they have better 3rd objective (lower SBL) compared to the other solutions.

4.2. Design Results for 32 Element Array

Next the algorithms have been applied to a 32 element time-modulated linear array with an equal spacing of $\lambda/2$. Table 6 shows that the best R-indicator and hypervolume indicator values are obtained for MOEA/D-DE. Table 7 shows three design objectives of the best



(a) MOEA/D-DE (b) CLPSO (c) DEGL

Figure 3. Normalized power patterns of the time-modulated linear array with optimized static excitations and switch-on time intervals: f_0 and $f_0 + prf$ for 16 element array.

Table 7. Optimal compromise table for 32 element antenna array design.

Method of Design	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)
Time-modulated	**9.684**	**−51.76**	**−83.647**
Non-uniform Excitation	24.42	−37.16	NA
Phase-Position	9.95	−20.43	NA

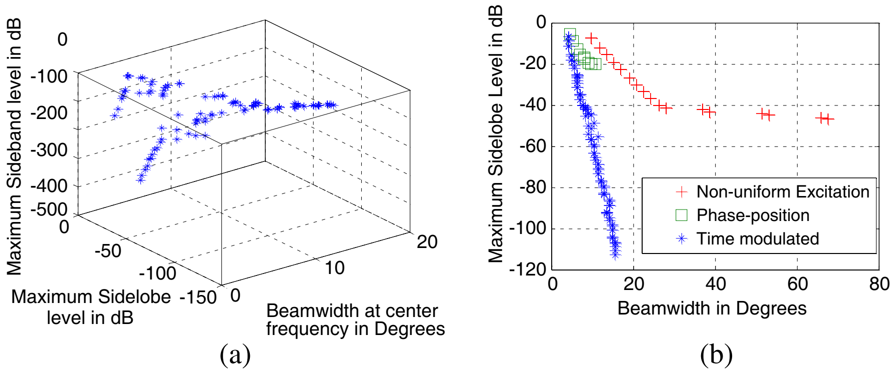


Figure 4. Best approximated PFs obtained with MOEA/D-DE over 32-element array design instance. (a) 3-dimensional PF for 32-element time-modulated array design. (b) 2-dimensional PF for three design methods over 32-element array.

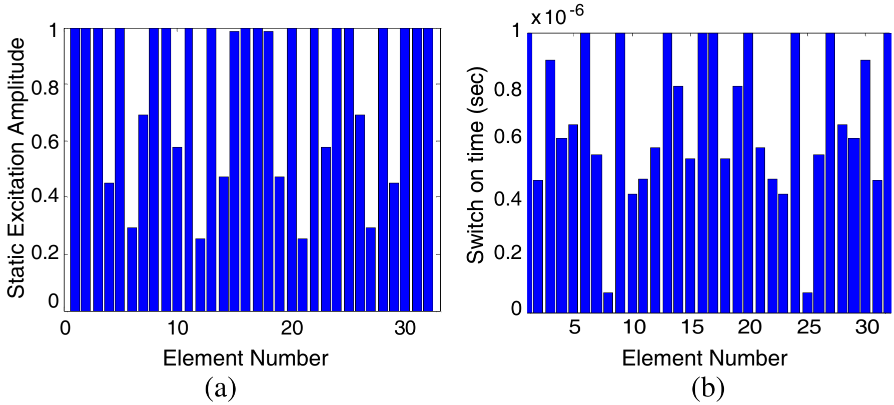
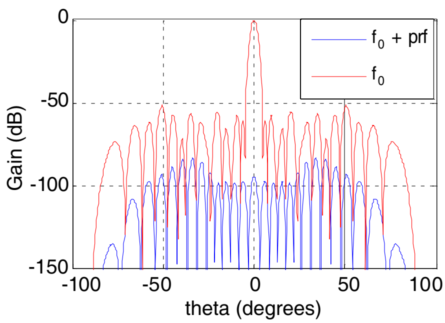


Figure 5. (a) Static excitations obtained by MOEA/D-DE. (b) Switch on time sequence obtained by MOEA/D-DE.

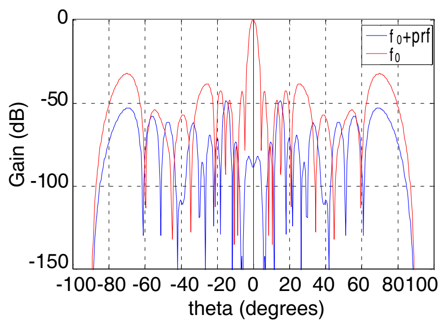
compromise solution found by MOEA/D-DE for the time-modulated array. BWFN and MSLL achieved by the best compromise solution for MOEA/D-DE corresponding to nonuniform excitation and phase-position based design methods have also been shown in the same table. Figure 4(a) presents the 3-dimensional approximated PF or trade-off curve obtained with MOEA/D-DE for the linear time-modulated array. In Figure 4(b) we show the 2-dimensional PF for all the three methods of linear array design. Table 8 presents values of the three design objectives finally achieved with MOEA/D-DE and two single-objective algorithms. The static excitation amplitudes and switch on time intervals are shown in Figure 5. Corresponding array patterns have been shown in Figure 6. Figure 4(b) reveals that the approximated PF obtained for time-modulated array contains far better solutions

Table 8. Best values of the three design-objectives achieved by best MO algorithm (MOEA/D-DE) and the two single-objective optimization algorithms over 32-element array design.

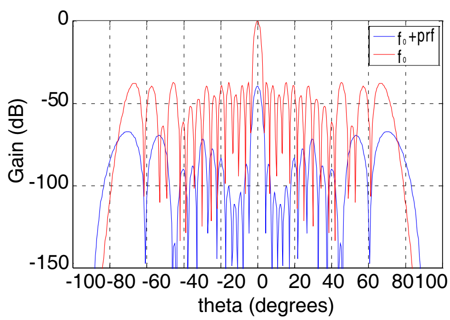
Algorithm	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)	Dynamic Range
MOEA/D	**9.684**	**−51.757**	**−83.64**	**3.96**
DEGL	9.231	−32.408	−48.85	4.00
CLPSO	7.561	−37.523	−39.56	4.00



(a) MOEA/D-DE



(b) CLPSO

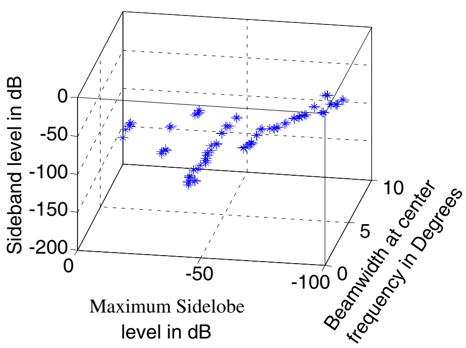


(c) DEGL

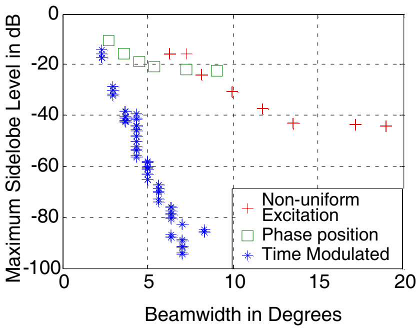
Figure 6. Normalized power patterns of the time-modulated linear array with optimized static excitations and switch-on time intervals: f_0 and $f_0 + prf$ for 32 element array.

Table 9. Best, worst, mean, and standard deviations of the performance metrics for comparing the MO algorithms (64 element array).

Performance Metric	Value type	MOEA/D-DE	NSGA-2	MODE	PEAS	SPEA2	NBI
R-indicator	Best	**6.12e-08**	1.19e-05	4.83e-05	5.12e-03	3.26e-04	9.92e-03
	Worst	**5.32e-05**	2.13e-03	7.03e-03	8.09e-02	5.47e-03	0.2192
	Mean	**8.93e-06**	2.67e-04	5.81e-04	1.42e-02	1.04e-03	8.71e-02
	Std. Dev.	**1.42e-06**	8.15e-05	8.97e-05	3.45e-03	7.09e-04	2.45e-02
Hypervolume-indicator	Best	**2.06e-05**	2.76e-04	7.04e-04	9.32e-03	1.05e-03	4.12e-02
	Worst	**6.73e-04**	4.78e-03	9.93e-03	4.01e-02	3.59e-02	0.2311
	Mean	**7.81e-05**	9.13e-04	2.04e-03	2.52e-02	8.81e-03	7.10e-02
	Std. Dev.	**1.98e-05**	3.98e-05	6.20e-05	6.12e-03	4.94e-03	1.21e-02



(a)



(b)

Figure 7. Best approximated PFs obtained with MOEA/D-DE over 64-element array design instance. (a) 3-dimensional PF for 64-element time-modulated array design. (b) 2-dimensional PF for three design methods over 64-element array.

(the knee-region being much closer to the utopia point) than the other methods.

Though the final approximated PF corresponding to phase-position synthesis has the least diversity the best compromise solution obtained could be better than non-uniform excitation method of design. Table 6 indicates that the best compromise solution of MOEA/D-DE is considerably superior in comparison to the best results obtained with DEGL and CLPSO.

4.3. Design Results for 64 Element Array

Next the algorithms have been applied to a 64 element time-modulated linear array with an equal spacing of $\lambda/2$. Table 9 shows that once again the best R-indicator and hypervolume indicator values are obtained for MOEA/DDE. Table 10 shows three design objectives of

Table 10. Optimal compromise table for 64-element antenna array design.

Method of Design	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)
Time-modulated	**5.677**	**−67.08**	**−91.12**
Non-uniform Excitation	13.57	−43.21	NA
Phase-Position	7.236	−21.95	NA

Table 11. Best values of the three design-objectives achieved by best MO algorithm (MOEA/D-DE) and the two single-objective optimization algorithms over 64-element array design.

Algorithm	BWFN (degrees)	MSLL (dB)	$SBL_{\max}$ (dB)	Dynamic Range
MOEA/D	**5.677**	**−67.08**	**−91.12**	**3.57**
DEGL	7.039	−35.73	−83.82	3.85
CLPSO	5.415	−56.22	−63.12	

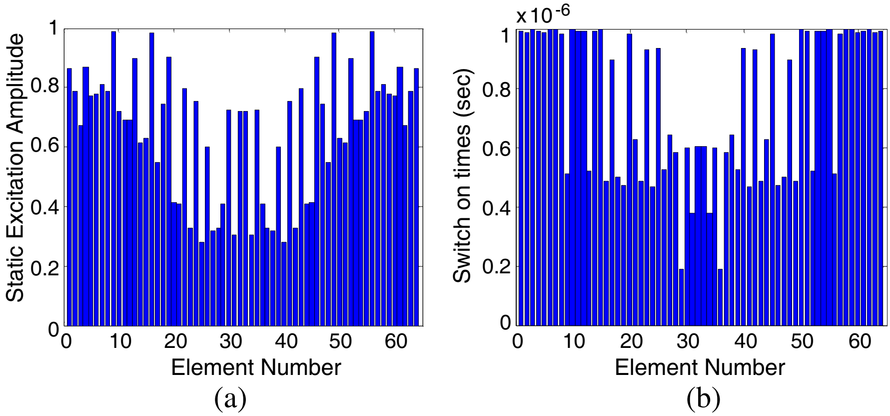
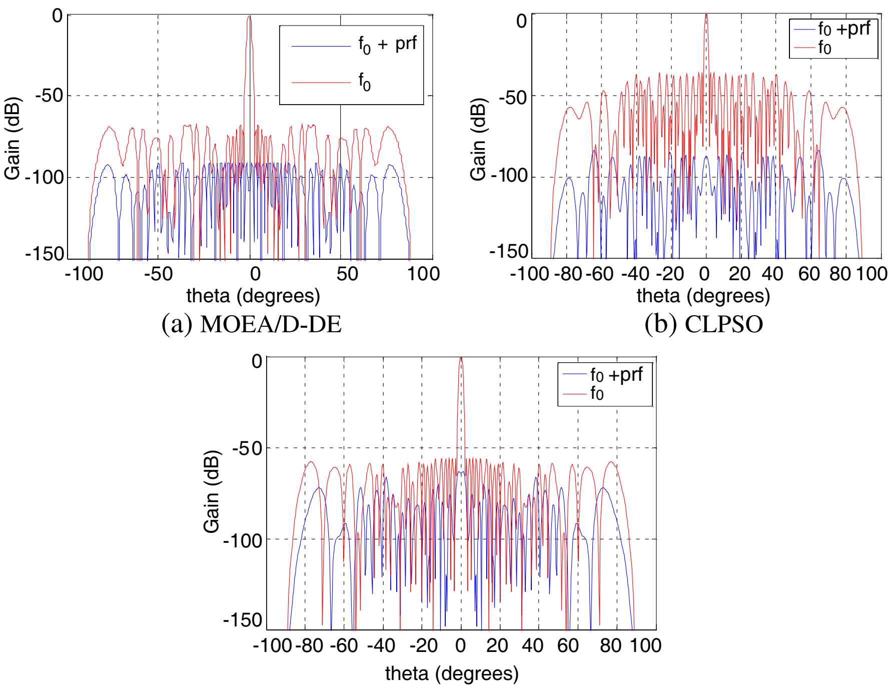


Figure 8. (a) Static excitations obtained by MOEA/D-DE. (b) Switch on time sequence obtained by MOEA/D-DE.

the best compromise solution found by MOEA/D-DE for the time-modulated array. BWFN and MSLL achieved by the best compromise solution for MOEA/D-DE corresponding to non-uniform excitation and phase-position based design methods have also been shown in the same table. Figure 7(a) presents the 3-dimensional approximated PF or trade-off curve obtained with MOEA/D-DE for the linear time-modulated array. In Figure 7(b), we show the 2-dimensional PF for all the three methods of linear array design. Table 11 presents values of the three design objectives finally achieved with MOEA/D-DE and two single-objective algorithms. Figure 8 shows the static excitation amplitudes and switch on time intervals obtained by the MOEA/D-DE algorithm. Corresponding array patterns are presented in Figure 9.

Finally in Table 12, we show the mean CPU time taken by the eight algorithms compared over the three design instances. As it is evident from the table, among the MO algorithms, MOEA/D-DE is the fastest. However, the MO algorithms take marginally greater time



(a) MOEA/D-DE

(b) CLPSO

(c) DEGL

Figure 9. Normalized power patterns of the time-modulated linear array with optimized static excitations and switch-on time intervals: f_0 and $f_0 + prf$ for 64 element array.

Table 12. Mean CPU time taken (per run) by the compared algorithms over three instances of the design problem.

Problem	MOEA/D-DE	NSGA2	MODE	PEAS	SPEA2	NBI	DEGL	CLPSO
16-Element	156.54 sec	171.44 sec	163.80 sec	172.72 sec	178.20 sec	181.56 sec	129.16 sec	144.52 sec
32-Element	409.24 sec	457.12 sec	441.32 sec	462.60 sec	473.92 sec	480.42 sec	405.08 sec	421.48 sec
64-Element	832.04 sec	923.64 sec	916.16 sec	926.08 sec	934.24 sec	941.40 sec	862.76 sec	879.32 sec

as compared to the single-objective ones. This can be attributed to the complicated sorting and selection techniques employed by the MO algorithms. However, when accuracy is the major bottleneck, since the design process is off-line, we must choose an MO algorithm like MOEA/D-DE in order to achieve the best trade-off among all the objectives concerned.

5. CONCLUSION

In this article, we demonstrated a new approach to the design of time-modulated linear antenna arrays that provide an attractive means for synthesis of low/ultra-low sidelobes, in the framework of multi-objective optimization. One of the most recent and best-known MO algorithms, called MOEA/D-DE, has been applied over three different instances of the design problem, keeping minimum maximum sidelobe level, maximum sideband level and the beamwidth between the first nulls at the center frequency as three design-objectives to be achieved simultaneously. Through extensive simulation experiments, we illustrated that the MO design method is more suitable for time-modulated antenna arrays because as evident from the approximated PFs provided in Figures 1, 4, and 7, the PF for time-modulated arrays are more diverse producing a much better trade-off among the design-objectives considered here, in comparison to the non-uniform excitation and phase-position synthesis based methods for linear arrays. Unlike the single-objective approaches, the MO approach provides greater flexibility in the design by yielding a set of equivalent final solutions from which the user can choose one that attains a suitable trade-off margin as per requirements.We illustrated that the best compromise solution returned by MOEA/D-DE was able to comfortably outperform the best results obtained with two powerful single-objective optimization algorithms CLPSO and DEGL over three significant design instances.

Our research indicates that powerful multi-objective optimization algorithms can be applied to obtain better results over many problems of electromagnetics where there are two or more conflicting design objectives that are to be achieved simultaneously. A few examples of such problems are like Ultra wideband TEM horn antenna design, Wire antenna geometry design, difference pattern synthesis for monopulse antenna arrays, radio network optimization, etc.

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