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APPLICATION OF THE GENERALIZED METHOD OF INDUCED EMF FOR INVESTIGATION OF CHARACTERISTICS OF THIN IMPEDANCE VIBRATORS

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Abstract—The authors suggest the generalized method of induced electromotive forces (EMF) for the investigation of the characteristics of single and systems of thin impedance vibrators at their arbitrary excitation and distribution of the surface impedance on the ground of the made analysis in the proposed paper. The distinctive peculiarity of this method is the use of the functional distributions, obtained as a result of the analytical solution of the integral equation for the current by the asymptotic averaging method before, as the basic approximations for the current along the impedance vibrator.

1. INTRODUCTION

Many publications are devoted to the investigation of the electrodynamic characteristics of material bodies of different configurations, on the surface of which impedance boundary conditions are set. The investigations of impedance thin vibrators and the systems from them, which had and have wide application in antenna-waveguide engineering (see, for example, [1–18]) take a special place among these publications. We should like to note, that new practical applications of vibrator structures are often based on their location in complex electrodynamic environment, which requires taking into account inhomogeneity of medium, boundaries of electrodynamic volumes, presence of vibrators with changing values of surface impedance and so on.

Direct numerical methods in all their variety are usually applied for mathematical analysis of functional characteristics of the devices, the combined components of which are thin impedance vibrators.

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We should note without the analysis of concrete realizations of these methods here, that they suppose minimization of residual of the function, dependent of the difference between precise and approximate solutions, in order that these two solutions differ from each other as less as possible, due to their essence. Really, identical transformation of the residual into zero at the problem numerical solution is not possible principally. So, independent of the choice of the method and the kind of basic functions, preciseness of the calculations in absolute majority of cases here is defined with the help of the analysis of inner convergence of the algorithms and (or) on the basis of comparison of the numerical results, obtained by different methods (including the experimental ones). At this, achievement of the required preciseness of the solution stipulates the necessity of the use of a rather large number of basic functions and thus of large volume of computer resources. From the other hand, the approximate analytical methods of the solution of the corresponding boundary problems of electrodynamics provide the set preciseness of the solution at minimal volume of the used computer resources. However, as it was shown in [18], it is necessary to obtain higher approximations (along the problem small parameter), except the first one, in order to increase preciseness of the calculated results in some case, for example, at calculation of the input characteristics of the radiating vibrator, at the solution of integral equations relative to the current in impedance vibrators by approximate analytical methods because of imperfection of the model of excitation. It leads to rather bulk formulas of small application in practical use. The finite expressions for the current are complicated sufficiently already in the first approximation at the analysis of the system of some impedance vibrators too. That is, the possibilities of analytical approaches are only for the cases of some model problems of the theory of impedance vibrators, and they turn out to be limited. Hence, an actual problem, arising before the investigators, is the development of numerical-analytical methods, which can combine in themselves the advantages of both analytical and numerical solutions.

The generalized method of induced EMF is proposed for the calculation of the electrodynamic characteristics of single vibrators and systems of vibrators at their arbitrary excitation and distribution of surface impedance in this paper. Efficiency of such an approach is described in the monograph of the authors [19] at the analysis of slot holes of coupling of electrodynamic volumes with the help of the method of induced magnetomotive forces. The essence of generalization of a well-known method of induced EMF here is the use of a new kind of basic functional distributions of a complete region, obtained as a result of preliminary analytical solution of key problems for the current in impedance vibrators by the asymptotic averaging method. Preciseness of the proposed method can be estimated with the help of the results of general investigations of the variational stability of the vibrator antenna calculated characteristics by the method of induced EMF, represented in [20]. For example, one can state on the basis of the results from [20], that deviation of the set current function from the real value on the magnitude of the first order of smallness leads to the magnitude of the second order of smallness in the definition of the vibrator input resistance. The latter is explained by that, that the vibrator input resistance for the method of induced EMF is a stationary parameter, because the first variation of the input resistance becomes zero at the precise set of the current distribution function. The aim of this paper is the description of the methodological bases of application of the generalized method of induced EMF for the investigation of the electrodynamic characteristics of thin impedance vibrators and also the ground of reliability and correctness of such an approach.

2. PROBLEM FORMULATION AND SOLUTION IN A GENERAL FORM

We shall use the results of the paper [18] in the very part, where the integral equation for the current has been solved by the asymptotic averaging method, as the problem key solution of the current definition in the impedance vibrator antenna. We should like to remind, that this analytical solution has been made in the frames of classical "thin-wire approximation", that is, in consideration of taking into account only the current longitudinal component along the vibrator axis and in neglect of its transverse component, including the neighborhood of the ends of its arms and on the ends of real radiating constructions.

Both the exciting field of impressed sources and the internal impedance per unit length z_i of the vibrator, being a rectilinear circular cylinder of the radius r and of the length $2L$ (Figure 1), can have two components — symmetrical (the index "s") and antisymmetrical (the index "a") relative to its geometrical centre: $E_{0s}(s) = E^s_{0s}(s) + E^a_{0s}(s)$, $z_i(s) = z^s_i(s) + z^a_i(s)$ in a general case. At this, naturally, the current in the vibrator will also consist of two components: $J(s) = J^s(s) + J^a(s)$ (at boundary conditions $J(\pm L) = 0$ [1,2]), and the initial equation concerning the current will have the following form [18]:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2} + k_1^2\right) \int\limits_{-L}^{L} \left[J^s\left(s'\right) + J^a\left(s'\right)\right] G_s\left(s, s'\right) \mathrm{d}s'$$
$$= -i\omega\varepsilon_1 \left\{\left[E^s_{0s}(s) + E^a_{0s}(s)\right] - \left[z^s_i(s) + z^a_i(s)\right]\left[J^s(s) + J^a(s)\right]\right\}. \quad (1)$$

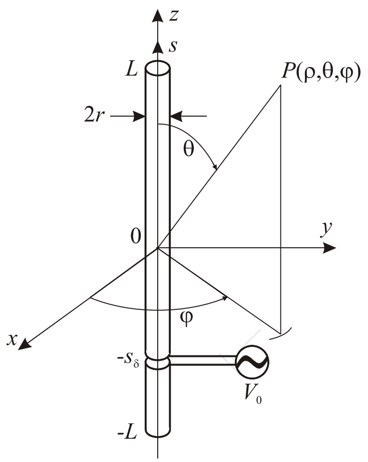

Figure 1. The problem geometry and the symbols used.

Here $E_{0s}(s)$ is the projection of the field of impressed sources on the vibrator axis, $G_s(s,s') = \frac{e^{-ik_1\sqrt{(s-s')^2+r^2}}}{\sqrt{(s-s')^2+r^2}}$, s and s' are the local coordinates, coupled with the axis and the surface of the vibrator, correspondingly, $k_1 = k_1' - ik_1'' = k\sqrt{\varepsilon_1\mu_1}$, $k = 2\pi/\lambda$, λ is the wavelength in free space, ε_1 and μ_1 are the permittivity and the permeability of the environment, $\omega = 2\pi f$ is the circular frequency, f is the frequency, measured in Hertz.

To solve the Equation (1), it is expedient to represent it in the form of the system of two coupled integral equations, concerning to the unknown currents $J^s(s)$ and $J^a(s)$, the first one of which is symmetrical, and the other — antisymmetrical relatively to the s variable:

$$\begin{cases} \left(\frac{d^2}{ds^2}+k_1^2\right)\int\limits_{-L}^{L} J^s(s')G_s(s,s')ds' \\ = -i\omega\varepsilon_1\left\{E_{0s}^s(s)-\left[z_i^s(s)J^s(s)+z_i^a(s)J^a(s)\right]\right\}, \\ \left(\frac{d^2}{ds^2}+k_1^2\right)\int\limits_{-L}^{L} J^a(s')G_s(s,s')ds' \\ = -i\omega\varepsilon_1\left\{E_{0s}^a(s)-\left[z_i^s(s)J^a(s)+z_i^a(s)J^s(s)\right]\right\}. \end{cases} \tag{2}$$

We represent the currents in the vibrator in the form of the product of the unknown complex amplitudes $J_n^{s,a}$ on the set functional distributions $f_n^{s,a}(s')$ $(n = 0, 1)$ further:

$$J^{s,a}(s') = J_0^{s,a}f_0^{s,a}(s') + J_1^{s,a}f_1^{s,a}(s'), \quad f_n^{s,a}(\pm L) = 0. \tag{3}$$

Let us multiply the left and right parts of the first equation in (2) on the weight function $f_n^s(s)$ and the second one — on $f_n^a(s)$ successively due to the general scheme of the solution of the equations system (2) by the method of induced EMF [20]. We obtain the algebraic equations system of the fourth order after integration of the obtained expressions along the vibrator length:

$$\begin{cases} J_0^s Z_{00}^{s\Sigma} + J_1^s Z_{01}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{00}^{sa} + J_1^a \tilde{\tilde{Z}}_{01}^{sa} = -(i\omega/2k) E_0^s, \\ J_0^s Z_{10}^{s\Sigma} + J_1^s Z_{11}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{10}^{sa} + J_1^a \tilde{\tilde{Z}}_{11}^{sa} = -(i\omega/2k) E_1^s, \\ J_0^a Z_{00}^{a\Sigma} + J_1^a Z_{01}^{a\Sigma} + J_0^s \tilde{\tilde{Z}}_{00}^{as} + J_1^s \tilde{\tilde{Z}}_{01}^{as} = -(i\omega/2k) E_0^a, \\ J_0^a Z_{10}^{a\Sigma} + J_1^a Z_{11}^{a\Sigma} + J_0^s \tilde{\tilde{Z}}_{10}^{as} + J_1^s \tilde{\tilde{Z}}_{11}^{as} = -(i\omega/2k) E_1^a. \end{cases} \tag{4}$$

The following symbols ($m = 0, 1;\ n = 0, 1$) are accepted in (4):

$$Z_{mn}^{s,a} = \frac{1}{2k}\left\{ -\frac{\mathrm{d}f_m^{s,a}(s)}{\mathrm{d}s} A_n^{s,a}(s)\Big|_{-L}^{L} + \int_{-L}^{L}\left[\frac{\mathrm{d}^2 f_m^{s,a}(s)}{\mathrm{d}s^2} + k_1^2 f_m^{s,a}(s)\right] A_n^{s,a}(s)\mathrm{d}s \right\},$$

$$\begin{aligned} A_n^{s,a}(s) &= \int_{-L}^{L} f_n^{s,a}(s') G_s(s,s')\mathrm{d}s', \\ \tilde{Z}_{mn}^{s,a} &= -\frac{i\omega}{2k}\int_{-L}^{L} f_m^{s,a}(s) f_n^{s,a}(s) z_i^s(s)\mathrm{d}s, \\ Z_{mn}^{(s,a)\Sigma} &= Z_{mn}^{s,a} + \tilde{Z}_{mn}^{s,a}, \\ \tilde{\tilde{Z}}_{mn}^{\left\{ {sa \atop as} \right\}} &= -\frac{i\omega}{2k}\int_{-L}^{L} f_m^{\left\{ {s \atop a} \right\}}(s) f_n^{\left\{ {a \atop s} \right\}}(s) z_i^a(s)\mathrm{d}s, \\ E_m^{s,a} &= \int_{-L}^{L} f_m^{s,a}(s) E_{0s}^{s,a}(s)\mathrm{d}s. \end{aligned} \tag{5}$$

Let us consider some particular solutions of the equations system (4).

1. The vibrator with the impedance, constant along its length: $z_i^s(s) = const$, $z_i^a(s) = 0$. Then the equations system (4) is divided into two independent systems of equations, concerning to the unknown J_n^s and J_n^a, correspondingly:

$$\begin{cases} J_0^s Z_{00}^{s\Sigma} + J_1^s Z_{01}^{s\Sigma} = -(i\omega/2k) E_0^s, \\ J_0^s Z_{10}^{s\Sigma} + J_1^s Z_{11}^{s\Sigma} = -(i\omega/2k) E_1^s, \end{cases} \quad \begin{cases} J_0^a Z_{00}^{a\Sigma} + J_1^a Z_{01}^{a\Sigma} = -(i\omega/2k) E_0^a, \\ J_0^a Z_{10}^{a\Sigma} + J_1^a Z_{11}^{a\Sigma} = -(i\omega/2k) E_1^a, \end{cases} \tag{6}$$

and the expression for the current has the form $J(s) = J^s(s) + J^a(s)$,

$$J^{s,a}(s) = -\frac{i\omega\varepsilon_1}{2k}\left[\frac{E_0^{s,a}Z_{11}^{(s,a)\Sigma} - E_1^{s,a}Z_{01}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}}f_0^{s,a}(s) + \frac{E_1^{s,a}Z_{00}^{(s,a)\Sigma} - E_0^{s,a}Z_{10}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}}f_1^{s,a}(s)\right]. \tag{7}$$

2. The field of impressed sources has only the symmetrical component $E_{0s}^a(s) = 0$. It is natural to suppose in this case, that it is sufficient to use only the first addendum — $J^a(s) = J_0^a f_0^a(s)$ in the antisymmetrical component of the current in the vibrator (3). At this the equations system (4) transits into the coupled equations system of the third order:

$$\begin{cases} J_0^s Z_{00}^{s\Sigma} + J_1^s Z_{01}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{00}^{sa} = -(i\omega/2k)E_0^s, \\ J_0^s Z_{10}^{s\Sigma} + J_1^s Z_{11}^{s\Sigma} + J_0^a \tilde{\tilde{Z}}_{10}^{sa} = -(i\omega/2k)E_1^s, \\ J_0^a Z_{00}^{a\Sigma} + J_0^s \tilde{\tilde{Z}}_{00}^{as} + J_1^s \tilde{\tilde{Z}}_{01}^{as} = 0, \end{cases} \tag{8}$$

and the symmetrical and the antisymmetrical components of the current will be defined by the following expressions:

$$J^s(s) = -\frac{i\omega\varepsilon_1}{2k}\left[\frac{E_0^s Z_{11}^{sa\Sigma} - E_1^s Z_{01}^{sa\Sigma}}{Z_{00}^{sa\Sigma}Z_{11}^{sa\Sigma} - Z_{10}^{sa\Sigma}Z_{01}^{sa\Sigma}}f_0^s(s) + \frac{E_1^s Z_{00}^{sa\Sigma} - E_0^s Z_{10}^{sa\Sigma}}{Z_{00}^{sa\Sigma}Z_{11}^{sa\Sigma} - Z_{10}^{sa\Sigma}Z_{01}^{sa\Sigma}}f_1^s(s)\right], \tag{9a}$$

$$J^a(s) = -\frac{i\omega\varepsilon_1}{2k}\frac{E_0^s Z_{00}^{as\Sigma} + E_1^s Z_{01}^{as\Sigma}}{Z_{00}^{sa\Sigma}Z_{11}^{sa\Sigma} - Z_{10}^{sa\Sigma}Z_{01}^{sa\Sigma}}f_0^a(s). \tag{9b}$$

The symbols are accepted in the formulas (9):

$$\begin{aligned} Z_{00}^{sa\Sigma} &= Z_{00}^{s\Sigma} - \frac{\left(\tilde{\tilde{Z}}_{00}^{sa}\right)^2}{Z_{00}^{a\Sigma}}, & Z_{01}^{sa\Sigma} &= Z_{01}^{s\Sigma} - \frac{\tilde{\tilde{Z}}_{00}^{sa}\tilde{\tilde{Z}}_{01}^{as}}{Z_{00}^{a\Sigma}}, \\ Z_{10}^{sa\Sigma} &= Z_{10}^{s\Sigma} - \frac{\tilde{\tilde{Z}}_{00}^{as}\tilde{\tilde{Z}}_{10}^{sa}}{Z_{00}^{a\Sigma}}, & Z_{11}^{sa\Sigma} &= Z_{11}^{s\Sigma} - \frac{\tilde{\tilde{Z}}_{10}^{sa}\tilde{\tilde{Z}}_{01}^{as}}{Z_{00}^{a\Sigma}}, \\ Z_{00}^{as\Sigma} &= \frac{Z_{10}^{s\Sigma}\tilde{\tilde{Z}}_{01}^{as} - Z_{11}^{s\Sigma}\tilde{\tilde{Z}}_{00}^{as}}{Z_{00}^{a\Sigma}}, & Z_{01}^{as\Sigma} &= \frac{Z_{01}^{s\Sigma}\tilde{\tilde{Z}}_{00}^{as} - Z_{00}^{s\Sigma}\tilde{\tilde{Z}}_{01}^{as}}{Z_{00}^{a\Sigma}}. \end{aligned} \tag{10}$$

3. The vibrator excitation is symmetrical ($E_{0s}^a(s) = 0$), and its electrical length is close to half-wave ($0.4 \leq (2L/\lambda) \leq 0.6$):

$$J(s) = -\frac{i\omega\varepsilon_1}{2k} E_0^s \left[\frac{Z_{00}^{a\Sigma} f_0^s(s) - \tilde{\tilde{Z}}_{00}^{sa} f_0^a(s)}{Z_{00}^{s\Sigma} Z_{00}^{a\Sigma} - \left(\tilde{\tilde{Z}}_{00}^{sa}\right)^2} \right]. \tag{11}$$

Thus the obtained formulas allow to obtain the current in the single impedance vibrator under the condition of a correct set of the basic functional distributions $f_n^{s,a}(s')$ $(n = 0, 1)$. Certainly, the choice of the $f_n^{s,a}(s')$ functions, obtained in the analytical solutions of the same problem of current excitation in the vibrator with the already set degree of approximation is more natural. So, the functional dependence, obtained in the solution of the first approximation of the method of iterations along the small parameter [18], has been used as the basic function at the development of the classical method of induced EMF for perfectly conducting vibrators. We suggest to use the functional distributions, obtained in the solutions of the vibrator problem with the help of the asymptotic averaging method previously as $f_n^{s,a}(s')$ to analyze the impedance vibrators in our case. Let us consider some variants of the vibrator excitation and the distribution of impedance along its length in details further.

3. IMPEDANCE VIBRATOR WITH THE ARBITRARY EXCITATION POINT

Let the vibrator with the impedance, constant along its length ($z_i^s(s) = const$, $z_i^a(s) = 0$), be excited in the $s = -s_\delta$ point by the voltage generator V_0, as it is shown in Figure 1. Then

$$\begin{aligned} E_{0s}(s) &= V_0 \delta(s + s_\delta) = E_{0s}^s(s) + E_{0s}^a(s), \\ E_{0s}^s(s) &= \frac{V_0}{2} \left[\delta(s + s_\delta) + \delta(s - s_\delta) \right], \\ E_{0s}^a(s) &= \frac{V_0}{2} \left[\delta(s + s_\delta) - \delta(s - s_\delta) \right], \end{aligned} \tag{12}$$

The current in the vibrator will equal due to (7) in this case $J(s) = J^s(s) + J^a(s)$,

$$\begin{aligned} J^{s,a}(s) \;=\; -\frac{i\omega\varepsilon_1}{2k} V_0 \Bigg[& \frac{\tilde{E}_0^{s,a} Z_{11}^{(s,a)\Sigma} - \tilde{E}_1^{s,a} Z_{01}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma} Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma} Z_{01}^{(s,a)\Sigma}} f_0^{s,a}(s) \\ & + \frac{\tilde{E}_1^{s,a} Z_{00}^{(s,a)\Sigma} - \tilde{E}_0^{s,a} Z_{10}^{(s,a)\Sigma}}{Z_{00}^{(s,a)\Sigma} Z_{11}^{(s,a)\Sigma} - Z_{10}^{(s,a)\Sigma} Z_{01}^{(s,a)\Sigma}} f_1^{s,a}(s) \Bigg]. \end{aligned} \tag{13}$$

Let us choose the functions, which are obtained at the substitution of the expressions for the field of impressed sources (12) into the general solution of the equation for the current by the averaging method as $f_0^{s,a}(s)$ [18]:

$$
\begin{aligned}
f_0^s(s) =& \cos\tilde{k}_1 s_\delta \sin\tilde{k}_1 L \cos\tilde{k}_1 s \\
&- (1/2)\cos\tilde{k}_1 L \left(\sin\tilde{k}_1|s-s_\delta| + \sin\tilde{k}_1|s+s_\delta|\right), \\
f_0^a(s) =& \sin\tilde{k}_1 s_\delta \cos\tilde{k}_1 L \sin\tilde{k}_1 s \\
&+ (1/2)\sin\tilde{k}_1 L \left(\sin\tilde{k}_1|s-s_\delta| - \sin\tilde{k}_1|s+s_\delta|\right),
\end{aligned}
\tag{14}
$$

where $\tilde{k}_1 = k_1 - \frac{i\bar{Z}_S\sqrt{\varepsilon_1/\mu_1}}{2r\ln(2L/r)}$, $\bar{Z}_S = \bar{R}_S + i\bar{X}_S = 2\pi r z_i/Z_0$ is the distributed surface impedance, normalized on the wave impedance of free space $Z_0 = 120\pi$ Ohm. The expression for the function of the current distribution in the scattering impedance vibrator is used as the $f_1^s(s)$ function [18], and the formula, obtained in [21] at the investigation of the properties of the integral Equation (1) in the case, when $z_i = 0$, is used for $f_1^a(s)$:

$$f_1^s(s) = \cos\tilde{k}_1 s - \cos\tilde{k}_1 L, \tag{15a}$$

$$f_1^a(s) = \sin k_1 s - (s/L)\sin k_1 L. \tag{15b}$$

Substituting the expressions (14) and (15) into the ratios (5) now, we obtain all coefficients in the formula for the current (13) ($m = 0, 1$; $n = 0, 1$):

$$Z_{0n}^s = \frac{\tilde{k}_1}{k}\left[\cos\tilde{k}_1 s_\delta\, A_n^s(L) - \cos\tilde{k}_1 L\, A_n^s(s_\delta)\right] + \frac{\left(k_1^2 - \tilde{k}_1^2\right)}{2k}\int_{-L}^{L} f_0^s(s) A_n^s(s)\mathrm{d}s,$$

$$
\begin{aligned}
Z_{1n}^s =& \frac{\tilde{k}_1}{k}\sin\tilde{k}_1 L\, A_n^s(L) \\
&- \frac{1}{2k}\left[k_1^2\cos\tilde{k}_1 L \int_{-L}^{L} A_n^s(s)\mathrm{d}s - \left(k_1^2 - \tilde{k}_1^2\right)\int_{-L}^{L}\cos\tilde{k}_1 s A_n^s(s)\mathrm{d}s\right],
\end{aligned}
$$

$$
\begin{aligned}
Z_{0n}^a =& -\frac{\tilde{k}_1}{k}\left[\sin\tilde{k}_1 s_\delta\, A_n^a(L) - \sin\tilde{k}_1 L\, A_n^a(s_\delta)\right] \\
&+ \frac{\left(k_1^2 - \tilde{k}_1^2\right)}{2k}\int_{-L}^{L} f_0^a(s) A_n^a(s)\mathrm{d}s,
\end{aligned}
$$

$$Z_{1n}^a = \frac{k_1}{k}\left(\frac{\sin k_1 L}{k_1 L} - \cos k_1 L\right) A_n^a(L) - \frac{k_1^2}{2kL}\sin k_1 L \int_{-L}^{L} A_n^a(s) s ds,$$

$$\tilde{Z}_{mn}^{s,a} = \frac{\bar{Z}_S}{ir}\int_{-L}^{L} f_m^{s,a}(s) f_n^{s,a}(s) \mathrm{d}s,$$

$$\tilde{E}_0^s = \cos \tilde{k}_1 s_\delta \sin \tilde{k}_1 \left(L - |s_\delta|\right), \qquad \tilde{E}_1^s = \cos \tilde{k}_1 s_\delta - \cos \tilde{k}_1 L,$$

$$\tilde{E}_0^a = -\sin \tilde{k}_1 |s_\delta| \sin \tilde{k}_1 \left(L - |s_\delta|\right), \quad \tilde{E}_1^a = \sin k_1 s_\delta - (s_\delta/L)\sin k_1 L.$$

The expressions for the $Z_{in} = R_{in} + iX_{in}$ input impedance and the $Y_{in} = G_{in} + iB_{in}$ input admittance have the form in this case:

$$Z_{in}[\mathrm{Ohm}] = \frac{60i/\varepsilon_1}{J_0^s f_0^s(s_\delta) + J_1^s f_1^s(s_\delta) + J_0^a f_0^a(s_\delta) + J_1^a f_1^a(s_\delta)}, \quad Y_{in}[\mathrm{millimhos}] = \frac{10^3}{Z_{in}}. \tag{16}$$

The dependences of the input admittance real and imaginary parts (at $\varepsilon_1 = \mu_1 = 1$) of the perfectly conducting vibrator, excited in the centre, from its electrical length (Figure 2) and also for two cases of the surface impedance realization (Figures 3, 4, the experimental data from [4] and [9], correspondingly) have been calculated in order to check rightness of the obtained approximate expression for the current (13). The experimental data from [22] (the circles, $f = 663$ MHz) and the calculated values, obtained by the method of moments at the

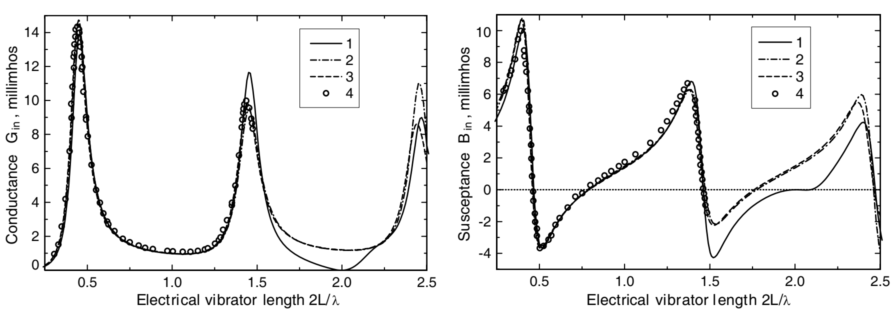

Figure 2. The input admittance of the perfectly conducting vibrator in dependence from its electrical length at $f = 663$ MHz, $r/\lambda = 0.007022$, $s_\delta = 0$: 1 — calculation (the functions (14) and (15)), 2 — calculation (the functions (14), (15) and (18)), 3 — calculation (the functions (17), $N = 24$), 4 — the experimental data [22].

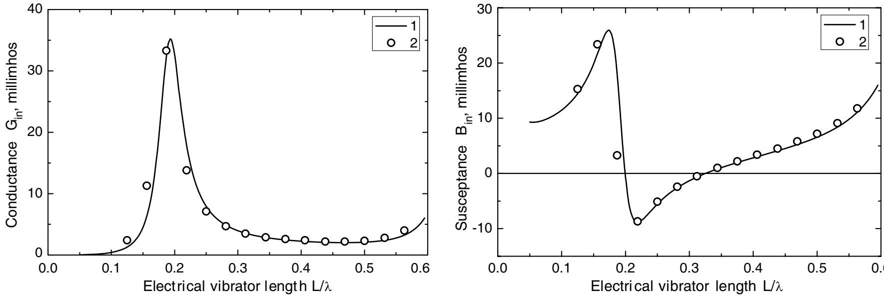

Figure 3. The input admittance of the metallic conductor of the radius $r_i = 0.3175\,\text{cm}$, covered by the dielectrical ($\varepsilon = 9.0$) shell of the radius $r = 0.635\,\text{cm}$ in dependence from its electrical length at $f = 600\,\text{MHz}$: 1 — calculation (the functions (14) and (15)), 2 — the experimental data [4].

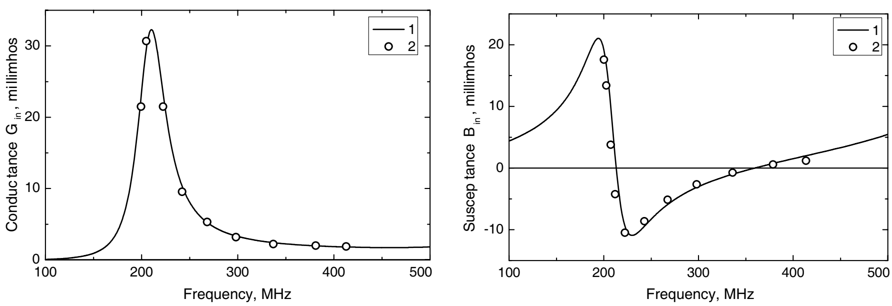

Figure 4. The input admittance of the metallic conductor of the radius $r_i = 0.5175\,\text{cm}$, covered by the ferrite ($\mu = 4.7$) shell of the radius $r = 0.6\,\text{cm}$ in dependence from the frequency at $2L = 30.0\,\text{cm}$: 1 — calculation (the functions (14) and (15)), 2 — the experimental data [9].

current approximation by the full region trigonometrical functions are dotted in Figure 2

$$J(s) = \sum_{n=1}^{N} J_n \sin \frac{n\pi(L+s)}{2L}, \tag{17}$$

what is more, it is necessary to increase a number of functions in the formula (17) at the increase of the vibrator electrical length to achieve suitable accuracy. The comparison of the calculated and the experimental curves in Figures 2–4 between each other allows to make the conclusion about adequacy of the chosen approximating functions for the current (14) and (15) to real physical process up to the

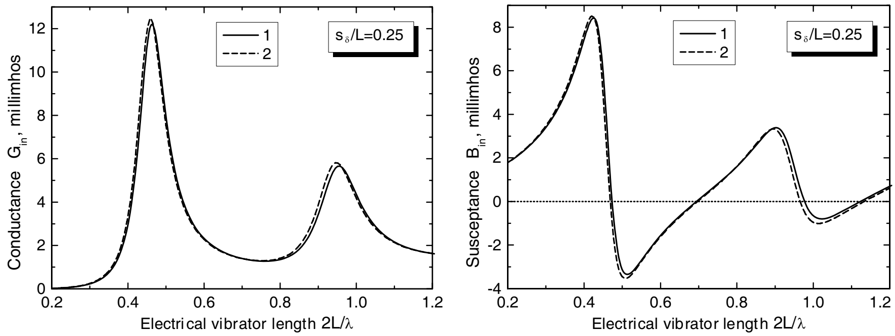

Figure 5. The input admittance of the perfectly conducting vibrator in dependence from its electrical length at $L/r = 75$, $s_\delta/L = 0.25$: 1— calculation (the functions (14) and (15)), 2 — calculation (the functions (17), $N = 24$).

$(2L/\lambda) \cong 1.4$ vibrator electrical lengths. It is also proved by the plots in Figure 5, where the input admittance dependences of the perfectly conducting vibrator from $2L/\lambda$ are represented at the shift of the point of excitation from the vibrator centre. Moreover, it is quite sufficient to add two functions, obtained at the investigation of the Equation (1) kernel by a natural way [22], in the expressions (3) for the electrically long $(1.4 < (2L/\lambda) \cong 2.5)$ vibrators (the corresponding curves are also represented in Figure 2):

$$f_2^s(s) = \cos\frac{k_1 s}{2} - \cos\frac{k_1 L}{2}, \quad f_2^a(s) = \sin\frac{k_1 s}{2} - \sin\frac{k_1 L}{2}. \tag{18}$$

Thus the generalized method of the induced EMF with the use of a minimal number of adequate approximating functions of the current distributions permits to calculate input characteristics of the vibrators with the electrical length to $(2L/\lambda) \leq 2.5$ (this ratio is also performed at $s_\delta \neq 0$) even at a non-perfect model of the vibrator excitation in the form of the point source.

It is known [22], that the model of description of the excitation source, which, as a rule, does not allow to take into account available constructions of real devices of excitation is "the narrowest place" in the vibrator problems. However, as it is seen in the case in question, the generalized method of induced EMF with the use of minimal quantity of adequate approximating functions of the current distribution permits to calculate the input characteristics of the vibrators with the electrical length $(2L/\lambda) \leq 2.5$ (this ratio is also performed at $s_\delta \neq 0$) rather precisely even at a non-perfect model of excitation of the vibrator as the point source.

If a vibrator is located in rather dense material medium, then large magnitudes of values of the surface impedance are required for sufficient change of its electrodynamic characteristics. So, it is necessary to cover the perfectly conducting vibrator of the radius r_i by the magnetodielectric shell of the radius r with some ε and large values of the permeability μ to obtain, for example, such values of the surface impedance of an inductive kind due to the formula [23]: $\bar{Z}_S = i\sqrt{\mu_1/\varepsilon_1}kr\mu\ln(r/r_i)$. Let us note, that the represented formula for the surface impedance is just at the fulfillment of the condition [3] $|(k\sqrt{\varepsilon\mu}r)^2\ln(k\sqrt{\varepsilon\mu}r_i)| \ll 1$, which is always performed in the cases in question.

Figure 6 gives the calculated dependences of the input impedance of the symmetrical ($s_\delta = 0$) vibrator, located in free space, at $r/r_i = 3$ and $\mu = 100$ in comparison with the experimental data (the circles) from [2] in a wide band of frequencies. These plots prove compatibility of the set problem solution, obtained by the generalized method of induced EMF, with real physical process for impedance vibrators, realized at the large values of μ practically. The dependences of the input characteristics from the electrical length of the perfectly conducting (the circles are the experimental data from [22]) and impedance asymmetrical vibrators (monopoles), located in the material mediums with different degrees of absorption: $k_1''/k_1' = 0.07$ and $k_1''/k_1' = 0.592$ ($\Delta = \lambda/\lambda_1$, λ_1 is the wavelength in medium, $f = 28.0$ MHz), are represented in Figures 7, 8. The parameters of the mediums are given in Table 1. As it is seen from the plots, the increase of the magnitudes of the surface impedance value of an inductive

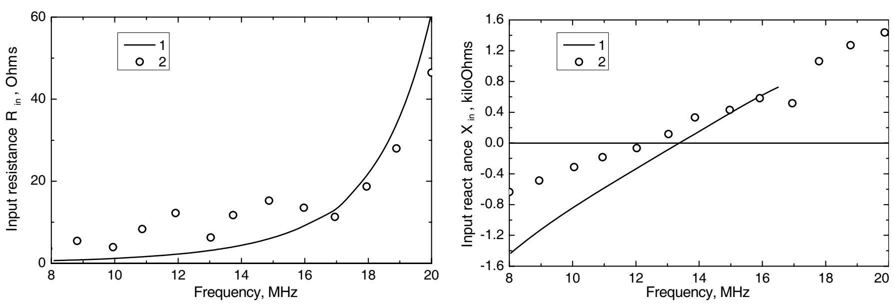

Figure 6. The input impedance of the metallic conductor of the radius $r_i = 0.007$ m, covered by the magnetodielectrical ($\mu = 100$, $\varepsilon = 10$) shell of the radius $r = 0.021$ m in dependence from the frequency at $2L = 2.0$ m: 1 — the calculation (the functions (14) and (15)), 2 — the experimental data [2].

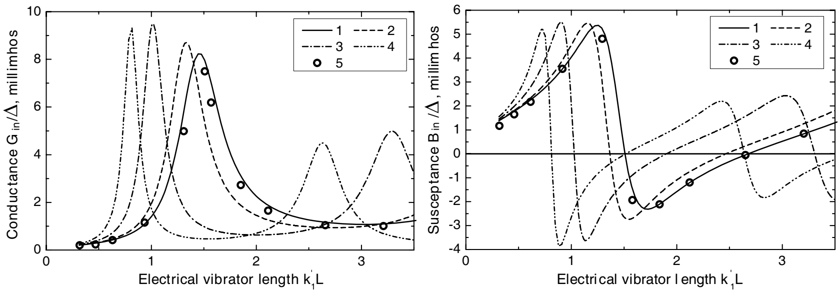

Figure 7. The input admittance of the perfectly conducting and impedance (the metallic cylinder with a ferrite shell) vibrators at $k_1''/k_1' = 0.07$, $r/\lambda_1 = 0.00265$, $r_i = r/2$, $\Delta = 8.96$: 1 — $\bar{Z}_S = 0$, 2 — $\mu = 10$, 3 — $\mu = 50$, 4 — $\mu = 100$, 5 — the experimental data [22] for $\bar{Z}_S = 0$.

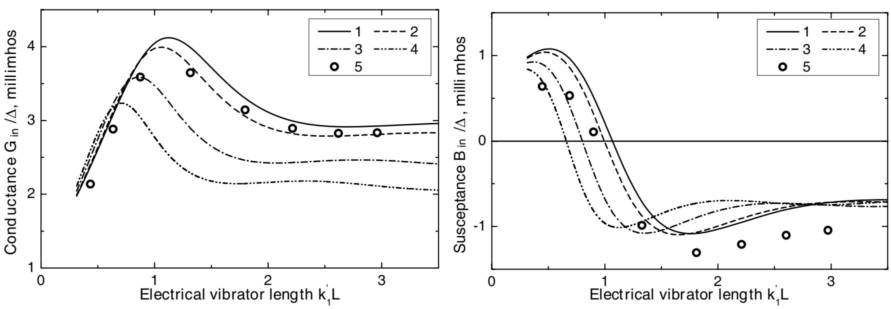

Figure 8. The input admittance of the perfectly conducting and impedance (the metallic cylinder with a ferrite shell) vibrators at $k_1''/k_1' = 0.592$, $r/\lambda_1 = 0.0037$, $r_i = r/2$, $\Delta = 12.54$: 1 — $\bar{Z}_S = 0$, 2 — $\mu = 10$, 3 — $\mu = 50$, 4 — $\mu = 100$, 5 — the experimental data [22] for $\bar{Z}_S = 0$.

kind allows to apply the impedance vibrators of a considerably less geometrical length in comparison with perfectly conducting vibrators in practice. This is a rather considerable factor, when making the process of non-destructive probing of different material mediums in order to define their electrophysical parameters [22]. We also note, that the resonant length of the vibrators (defined by the ratio $B_{in} = 0$) becomes rather small for making true measurements at the frequency increase, the probing is made on. It is expedient to use the vibrators with the distributed impedance of a capacitive kind, for example, metallic-dielectrical vibrators [23] in this case (see Figures 9, 10). Obviously, everything, written above, is just for the

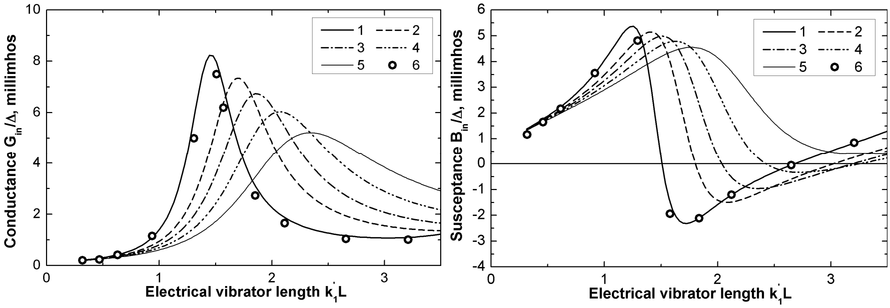

Figure 9. The input admittance of the perfectly conducting and impedance vibrators at $k_1''/k_1' = 0.07$, $r/\lambda_1 = 0.00265$, $\Delta = 8.96$: 1 — $\bar{Z}_S = 0$, 2 — $\bar{Z}_S = -i0.002$, 3 — $\bar{Z}_S = -i0.003$, 4 — $\bar{Z}_S = -i0.004$, 5 — $\bar{Z}_S = -i0.005$, 6 — the experimental data [22] for $\bar{Z}_S = 0$.

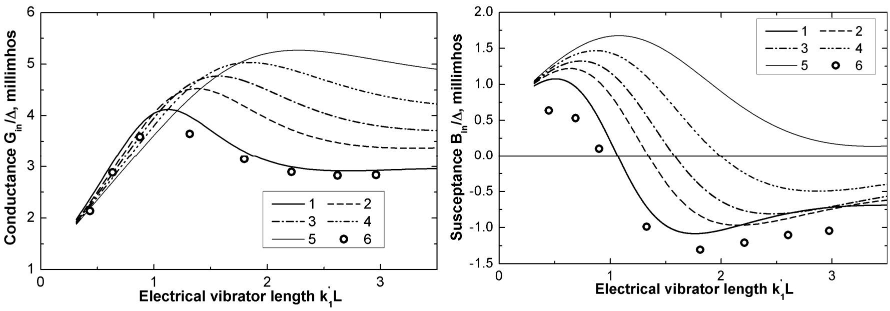

Figure 10. The input admittance of the perfectly conducting and impedance vibrators at $k_1''/k_1' = 0.592$, $r/\lambda_1 = 0.0037$, $\Delta = 12.54$: 1 — $\bar{Z}_S = 0$, 2 — $\bar{Z}_S = -i0.002$, 3 — $\bar{Z}_S = -i0.003$, 4 — $\bar{Z}_S = -i0.004$, 5 — $\bar{Z}_S = -i0.005$, 6 — the experimental data [22] for $\bar{Z}_S = 0$.

impedance vibrators with the arbitrary point of excitation, located in material mediums, too.

4. VIBRATOR WITH ASYMMETRICAL SURFACE IMPEDANCE ALONG ITS LENGTH IN FREE SPACE

The vibrators with the surface impedance, variable along their length, allow to widen the range of the electrodynamic characteristics change of antennas of fixed geometrical sizes. The investigations, made in [5, 6, 24–27], are devoted to the vibrators with the variable impedance, distributed symmetrically relatively to the vibrator centre. To our minds, the vibrators with the asymmetrically distributed impedance along their length represent indisputable interest from a practical point of view. The problem about the electromagnetic waves radiation by the thin vibrator with symmetrical and antisymmetrical components of the surface impedance relatively to its centre, where the point source of excitation is located, has been solved by the generalized method of the induced EMF in this section.

Let the vibrator, the length of which is close to the half-wave one $(0.4 \leq (2L/\lambda) \leq 0.6)$, be excited in the centre ($s_\delta = 0$ in Figure 1) by the hypothetical generator of voltage V_0: $E^s_{0s}(s) = V_0\delta(s)$, $E^a_{0s}(s) = 0$. Then the current symmetrical component can be approximated by the function $f^s(s) = \sin\tilde{k}(L - |s|)$, where $\tilde{k} = k - \frac{i\pi z_i^{av}}{Z_0 \ln(2L/r)}$, $z_i^{av} = \frac{1}{2L}\int\limits_{-L}^{L} z_i(s)\mathrm{d}s$ is the mean value of the internal impedance per unit length of the vibrator, rather precisely. We use the following expression for the current antisymmetrical component [27]: $f^a(s) = \sin 2ks - 2\sin ks \cos kL$, and we represent the $z_i^{s,a}(s)$ functions in the form of $z_i^{s,a}(s) = z_i^{s,a}\phi^{s,a}(s)$.

Let us consider the following simple functions of the impedance distribution (which are realized rather easily in practice) as an example: $\phi^s(s) = 1$ — the distribution, constant along the vibrator, $\phi^a(s) = \text{sign}s = (|s|/s)$ — the step-function alternating distribution. Substituting $f^{s,a}(s)$ and $\phi^{s,a}(s)$ into the expressions (5) and (11), we obtain the formula for the current in the vibrator with these laws of impedance distribution (the indices "0" and "00" in (11) are omitted and we put in $\varepsilon_1 = \mu_1 = 1$):

$$J(s) = -\frac{i\omega}{2k}V_0 \sin\tilde{k}L \left[\frac{(Z^a + \tilde{Z}^a)\sin\tilde{k}(L - |s|) - \tilde{\tilde{Z}}^{sa}(\sin 2ks - 2\sin ks\cos kL)}{(Z^s + \tilde{Z}^s)(Z^a + \tilde{Z}^a) - (\tilde{\tilde{Z}}^{sa})^2}\right], \quad (19)$$

where

$$Z^s = \frac{\tilde{k}}{k}[A^s(L) - \cos\tilde{k}LA^s(0)] + \frac{k^2 - \tilde{k}^2}{2k}\int\limits_{-L}^{L} f^s(s)A^s(s)\mathrm{d}s,$$

$$Z^a = 2\sin^2 kLA^a(L) - \frac{3}{2}k\int\limits_{-L}^{L} \sin 2ks\, A^a(s)\mathrm{d}s,$$

$$\tilde{Z}^s = \frac{\bar{Z}^s_S}{i2\tilde{k}r}(2\tilde{k}L - \sin 2\tilde{k}L),$$

$$\tilde{Z}^a = \frac{\bar{Z}_S^s}{ikr}\left(3kL - \frac{\sin 4kL}{12} - \frac{7}{3}\sin 2kL + 2kL\cos 2kL\right),$$
$$\tilde{\tilde{Z}}^{sa} = \frac{2\overline{Z}_s^a}{i\tilde{k}r}\left(\frac{2k^2\sin\tilde{k}L - k\tilde{k}\sin 2kL}{4k^2 - \tilde{k}^2} - 2\cos kL\frac{k^2\sin\tilde{k}L - k\tilde{k}\sin kL}{k^2 - \tilde{k}^2}\right).$$

As it is known, one of the main factors, defining the range of the symmetrical vibrators use in antenna practice, is the possibility of the agreement of its input resistance with the wave resistance of the feeding feeder line. The required agreement can be made for any ratio $2L/\lambda$ at the use of additional elements of tuning at the operation on the fixed wavelength. The surface impedance can be used successfully as such an effective "element of tuning", as it is shown in [16] for the case of the constant impedance, especially, when it is distributed along the vibrator length in a definite kind. We illustrate this possibility by the results of numerical calculations of the electrodynamic characteristics of the thin vibrators, having symmetrical excitation and asymmetrical distribution of the surface impedance. Figure 11 represents the dependences of the reflection coefficient module $|S_{11}| = \left|\frac{Z_{in}-W}{Z_{in}+W}\right|$ from the value of the imaginary part of the symmetrical component of the surface impedance $\bar{X}_S^s$ for different values of the wave resistance W ([Ohm], in abbreviated form Ω on graphs) of the feeding feeder and the vibrator electrical lengths $2L/\lambda$ at $\bar{Z}_S^a = 0$, here and further $\bar{R}_S^{s,a} = 0.0$, λ=10.0 cm, r/λ=0.0033. As it is seen, the $\bar{X}_S^s$ value at which the reflection coefficient is minimal, that is the vibrator is tuned into resonance, exists for each combination of the values $2L/\lambda$ and W.

The reflection coefficient (and the voltage standing wave ratio

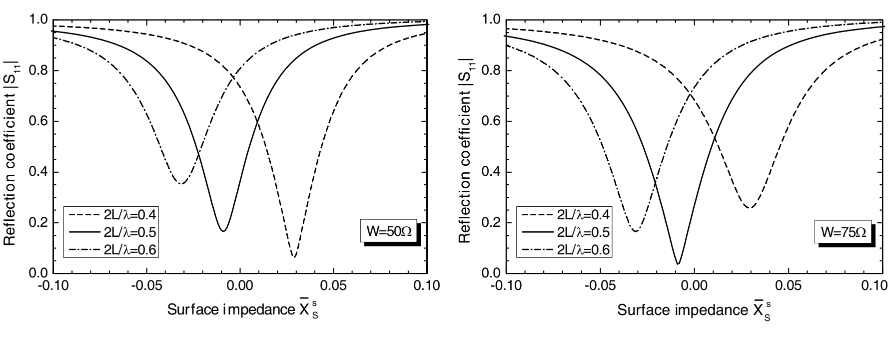

Figure 11. The $|S_{11}(\bar{X}_S^s)|$ dependences for the vibrators with the constant surface impedance.

VSWR $= \frac{1+|S_{11}|}{1-|S_{11}|}$, correspondingly) also attains the values, which are smaller than the perfectly conducting vibrator has, as it follows from the plots in Figures 12, 13, at availability of the antisymmetrical component of the surface impedance $\bar{X}_S^a$ in the vibrator for definite combinations of $\bar{X}_S^s$ and $\bar{X}_S^a$ (what is more, they are different for different $2L/\lambda$). Thus availability of different electrical length of the variable impedance of this or another kind ($\bar{X}_S < 0$ — the capacitive impedance, $\bar{X}_S > 0$ — the inductive impedance) in the vibrators allows

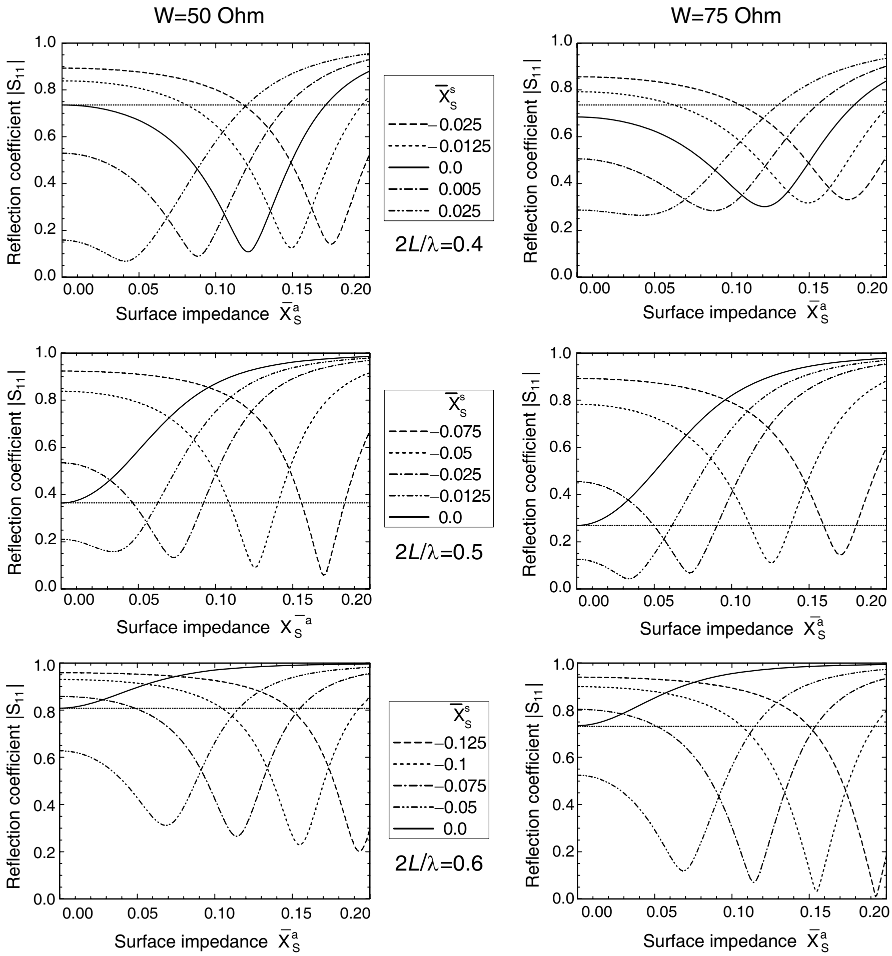

Figure 12. The $|S_{11}(\bar{X}_S^s, \bar{X}_S^a)|$ dependences for the vibrators with the variable surface impedance.

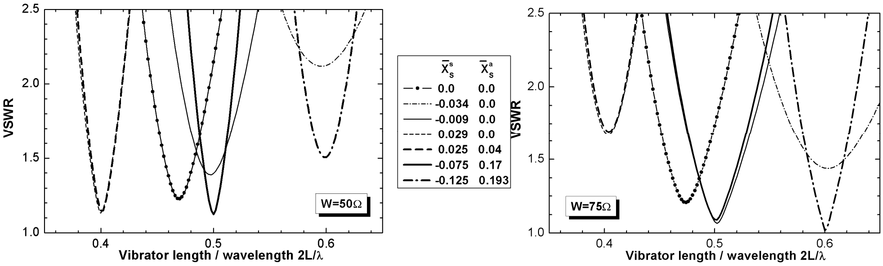

Figure 13. The dependences VSWR from the vibrator electrical length with the constant and variable surface impedance.

to tune them into the resonance with rather small values of the VSWR in the feeding feeders with the set wave resistance.

We note in the conclusion that the numerical results for simple laws of the change of the symmetrical and antisymmetrical components of the surface impedance along the vibrator length are given in this section. The problem obtained solution is suitable for any functional dependences of the impedance change, what creates wide possibilities for realization of the required characteristics of vibrator antennas in real problems, which are arisen for investigators by practice. At this, as it follows from Section 3, it is necessary to add the functions (15) and (18) into the current distribution for the increase of accuracy of the calculations and also in the case of electrically long vibrators.

5. SYSTEM OF IMPEDANCE VIBRATORS IN FREE SPACE

Let us consider the system, consisting of N of the parallel impedance vibrators, located in free space. We introduce numeration of the vibrators and designate the length and the radius of the vibrator with the number n via $2L_n$ and r_n, correspondingly, and the corresponding coordinates of the vibrator centre in the Decart's coordinate system — via z_n, x_n, y_n.

The projection of the $E_{0s_n}(s_n)$ impressed sources electrical field on the vibrator axis with the number n ($n = 1, 2 \dots N$) can be represented in the form of the sum of two components: symmetrical $E^s_{0s_n}(s_n)$ and antisymmetrical $E^a_{0s_n}(s_n)$ relatively to the vibrator geometrical centre (s_n is the local coordinate along the vibrator axis with the number n): $E_{0s_n}(s_n) = E^s_{0s_n}(s_n) + E^a_{0s_n}(s_n)$ as in the case of the single vibrator (see section 2). Satisfying the requirements of the impedance boundary condition for the electrical field on the surfaces of each of the vibrators, we obtain the integral-differential equations system relatively to the currents of the vibrators $J_n(s_n)$ in the form of $(m = 1, 2 \ldots N)$:

$$\sum_{n=1}^{N} \left(\frac{\mathrm{d}^2}{\mathrm{d}s_m^2} + k^2 \right) \int_{-L_n}^{L_n} J_n(s'_n) G_{s_m}(s_m, s'_n) \, \mathrm{d}s'_n$$
$$= -i\omega [E_{0s_m}^s(s_m) + E_{0s_m}^a(s_m)] + i\omega z_{i_m}(s_m) J_m(s_m), \tag{20}$$

where $z_{i_m}(s_m)$ is the internal impedance per unit length of the vibrator with the number m, changing along its length in a general case.

Because the impressed sources field is represented in the form of two components, the currents of each vibrator will also consist of two functions — symmetrical and antisymmetrical relatively to the centres of the vibrators — $J_n(s_n) = J_n^s(s_n) + J_n^a(s_n)$. Let us introduce the currents components in the vibrators in the form of the product of the $J_{nq}^{s,a}$ unknown complex amplitudes and the set scalar functions of distribution $f_{nq}^{s,a}(s_n)$ $(q = 0, 1 \ldots Q)$ farther:

$$J_n^{s,a}(s) = \sum_{q=0}^{Q} J_{nq}^{s,a} f_{nq}^{s,a}(s_n), \quad f_{nq}^{s,a}(\pm L_n) = 0. \tag{21}$$

Then the equations system (20) can be written in the form:

$$\sum_{n=1}^{N} \sum_{q=0}^{Q} \left(\frac{\mathrm{d}^2}{\mathrm{d}s_m^2} + k^2 \right) \int_{-L_n}^{L_n} \left[J_{nq}^s f_{nq}^s(s'_n) + J_{nq}^a f_{nq}^a(s'_n) \right] G_{s_m}(s_m, s'_n) \, \mathrm{d}s'_n$$
$$-i\omega z_{i_m}(s_m) \sum_{p=0}^{Q} \left[J_{mq}^s f_{mq}^s(s_m) + J_{mq}^a f_{mq}^a(s_m) \right]$$
$$= -i\omega [E_{0s_m}^s(s_m) + E_{0s_m}^a(s_m)]. \tag{22}$$

We multiply the left and the right parts of the Equations (22) on $f_{mp}^s(s_m)$ and $f_{mp}^a(s_m)$ $(p = 0, 1 \ldots Q)$ alternately and integrate them along the vibrators lengths due to the generalized method of the induced EMF. As a result, we obtain the system of the algebraic equations relatively to the J_{nq}^s and J_{nq}^a currents unknown amplitudes:

$$\begin{cases} \sum_{n=1}^{N} \sum_{q=0}^{Q} \left[J_{nq}^s \left(Z_{mn,pq}^{ss} + \delta_{mn} \tilde{Z}_{m,pq}^{ss} \right) + J_{nq}^a \left(Z_{mn,pq}^{sa} + \delta_{mn} \tilde{Z}_{m,pq}^{sa} \right) \right] \\ = -\frac{i\omega}{2k} E_{0\,mp}^s, \\ \sum_{n=1}^{N} \sum_{q=0}^{Q} \left[J_{nq}^s \left(Z_{mn,pq}^{as} + \delta_{mn} \tilde{Z}_{m,pq}^{as} \right) + J_{nq}^a \left(Z_{mn,pq}^{aa} + \delta_{mn} \tilde{Z}_{m,pq}^{aa} \right) \right] \\ = -\frac{i\omega}{2k} E_{0\,mp}^a. \end{cases} \tag{23}$$

The following symbols are accepted in (23):

$$Z_{mn,pq}^{\begin{Bmatrix} ss \\ aa \\ sa \\ as \end{Bmatrix}} = \frac{1}{2k} \int\limits_{-L_m}^{L_m} f_{mp}^{\begin{Bmatrix} s \\ a \\ s \\ a \end{Bmatrix}}(s_m) \left[\left(\frac{\mathrm{d}^2}{\mathrm{d}s_m^2} + k^2 \right) \int\limits_{-L_n}^{L_n} f_{nq}^{\begin{Bmatrix} s \\ a \\ a \\ s \end{Bmatrix}}(s'_n) G_{s_m}(s_m, s'_n) \mathrm{d}s'_n \right] \mathrm{d}s_m,$$

$$\tilde{Z}_{m,pq}^{\begin{Bmatrix} ss \\ aa \\ sa \\ as \end{Bmatrix}} = -\frac{i\omega}{2k} \int\limits_{-L_m}^{L_m} f_{mp}^{\begin{Bmatrix} s \\ a \\ s \\ a \end{Bmatrix}}(s_m) f_{mq}^{\begin{Bmatrix} s \\ a \\ s \\ a \end{Bmatrix}}(s_m) z_{i\,m}(s_m) \mathrm{d}s_m,$$

$$E_{0\,mp}^{s,a} = \int\limits_{-L_m}^{L_m} f_{mp}^{s,a} E_{0s_m}^{s,a}(s_m) \mathrm{d}s_m, \quad \delta_{mn} = \begin{cases} 1 & \text{at } m = n, \\ 0 & \text{at } m \neq n\,. \end{cases}$$

Let us consider the Yagi-Uda array, named after its inventors [28] and representing itself a linear system of the similar oriented vibrators, the axes of which are perpendicular to the line of their location, applied in practice widely [29, 30], in details further. Let us locate the vibrators in space so, that their central points will be on the $\{0z\}$ axis of the Decart's coordinate system, and the longitudinal axes of the vibrators will be oriented parallel to the $\{0x\}$ axis (Figure 14). The vibrator with the $n = 1$ number is active, with the $n = 2$ number is a reflector, and the rest vibrators ($n = 3, 4 \ldots N$) are directors. We consider, that the surface impedance of each of the vibrators is constant along its length, that is, $z_{i_n}(s_n) = z_{i_n} = const$. The active vibrator ($n = 1$) is excited in the central point ($s_1 = 0$) from the δ-generator of harmonic oscillations with the amplitude of voltage V_0. Thus the projection of the impressed sources electrical field on the longitudinal axis of the first vibrator has only a symmetrical component relatively to its center: $E_{0s_1}(s_1) = E_{0s_1}^s(s_1) = V_0\delta(s_1)$ and the field $E_{0s_n}(s_n) = 0$ at $n = 2, 3 \ldots N$.

As the system is symmetrical relatively to the $\{y0z\}$ plane, then the currents of each vibrator will have only symmetrical components: $J_n(s_n) = J_n^s(s_n) = \sum\limits_{q=0}^{Q} J_{nq}^s f_{nq}^s(s_n)$ and the equations system (23) is

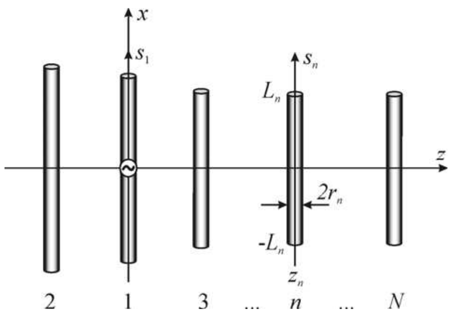

Figure 14. The configuration of the Yagi-Uda antenna.

simplified sufficiently:

$$\sum_{n=1}^{N}\sum_{q=0}^{Q} J_{nq}^{s}\left(Z_{mn,pq}^{s}+\delta_{mn}\tilde{Z}_{m,pq}^{s}\right)=-\frac{i}{60}E_{01p}^{s}\delta_{m1},\ m=1,2\ldots N;\quad p=0,1\ldots Q. \tag{24}$$

Here

$$Z_{mn,pq}^{s} = \frac{1}{2k}\int_{-L_m}^{L_m}\int_{-L_n}^{L_n} f_{mp}^{s}(s_m)f_{nq}^{s}(s_n')\tilde{G}_{s_m}(s_m,s_n')\mathrm{d}s_n'\mathrm{d}s_m,$$

$$\tilde{Z}_{m,pq}^{s} = \frac{\bar{Z}_{S_m}}{ir_m}\int_{-L_m}^{L_m} f_{mp}^{s}(s_m)f_{mq}^{s}(s_m)\mathrm{d}s_m,\quad \bar{Z}_{S_m} = \frac{r_m z_{i_m}}{60},$$

$$E_{0\,1p}^{s} = V_0 f_{1p}^{s}(0),\quad \tilde{G}_{s_m}(s_m,s_n') = \left(\frac{\mathrm{d}^2}{\mathrm{d}s_m^2}+k^2\right)G(s_m,s_n'),$$

$$G(s_m,s_n') = \frac{e^{-ikR(s_m,s_n')}}{R(s_m,s_n')},\quad R(s_m,s_n') = \sqrt{(s_m-s_n')^2+z_{mn}^2},$$

$$z_{mn} = r_m \text{ at } m=n \text{ and } z_{mn} = (z_m - z_n) \text{ at } m \neq n.$$

Let us make the following identical transformations for the convenience of calculations:

$$\tilde{G}(s_m,s_n') = \frac{G(s_m,s_n')}{R^4}\left\{(1+ikR)\left(2R^2-3z_{mn}^2\right)+k^2z_{mn}^2R^2\right\}.$$

The distributed surface impedance is a complex value $\bar{Z}_{S_n} = \bar{R}_{S_n}+i\bar{X}_{S_n}$ in a general case. We obtain the impedance of an inductive kind, which, as it is shown in [23], can be represented in the form of

$\bar{X}_{S_n} = kr_n C_{L_n}$ at $\bar{X}_{S_n} > 0$ and the impedance of a capacitive kind with its possible representation $\bar{X}_{S_n} = -C_{C_n}/(kr_n)$ — at $\bar{X}_{S_n} < 0$, where the C_{L_n} and C_{C_n} constants are defined by the vibrator geometrical sizes and electrophysical parameters of the material, it is made of.

We shall approximate the currents of the active vibrator (with the $n = 1$ number) by two functions due to the results of investigation of the characteristics of the single impedance vibrators, given in Sections 3, 4

$$f^s_{n0}(s_n) = \sin\tilde{k}(L_n - |s_n|), \quad f^s_{n1}(s_n) = \cos\tilde{k}s_n - \cos\tilde{k}L_n, \qquad (25)$$

and the currents of the passive vibrators ($n = 2, 3 \ldots N$) — by the functions:

$$f^s_{n1}(s_n) = \cos\tilde{k}s_n - \cos\tilde{k}L_n, \qquad (26a)$$

$$f^s_{n2}(s_n) = \cos\frac{ks_n}{2} - \cos\frac{kL_n}{2}. \qquad (26b)$$

If the length of a passive vibrator is close to $\lambda/2$, then it is possible to use only one function $f^s_{n1}(s_n)$ at approximation of its current.

The reflector of the antenna must be created so, that the current, occurring in it, will lead the current of the active vibrator along the phase, and the current of the nearest director ($n = 3$) will lag along the phase in respect to the current of the active vibrator in order to amplify energy flux in one direction and to attenuate it in an opposite one. It is also necessary, that the current of the director 4 will lag along the phase in respect to the current of the director 3, the current in the director 5 will lag along the phase in respect to the current of the director 4 and so on.

The analysis shows, that it is necessary, that the complete impedance of the reflector, concerning the current antinode, has a positive (inductive) reactive component, and the complete impedance of the directors has a negative (capacitive) reactive component to obtain the phases indicated ratio. One can obtain a suitable reactive component of the vibrator impedance by the corresponding fitting of the vibrator length and (or) its surface impedance value (Figure 15).

The calculated distributions of the currents of the vibrators in comparison with the experimental data [30] for the array, consisting of two elements — the active and passive vibrators ($2L_1 = 2L_2 = \lambda$, $r_1 = r_2 = 0.007022\lambda$), located at the distance $\Delta z = 0.25\lambda$ from each other, are given in Figure 16 in order to prove adequacy of the proposed mathematical model to real physical process. It is necessary to approximate the currents for such a length of the passive vibrators by two functions (26), at this one observes a rather satisfactory agreement of the calculated and experimental data.

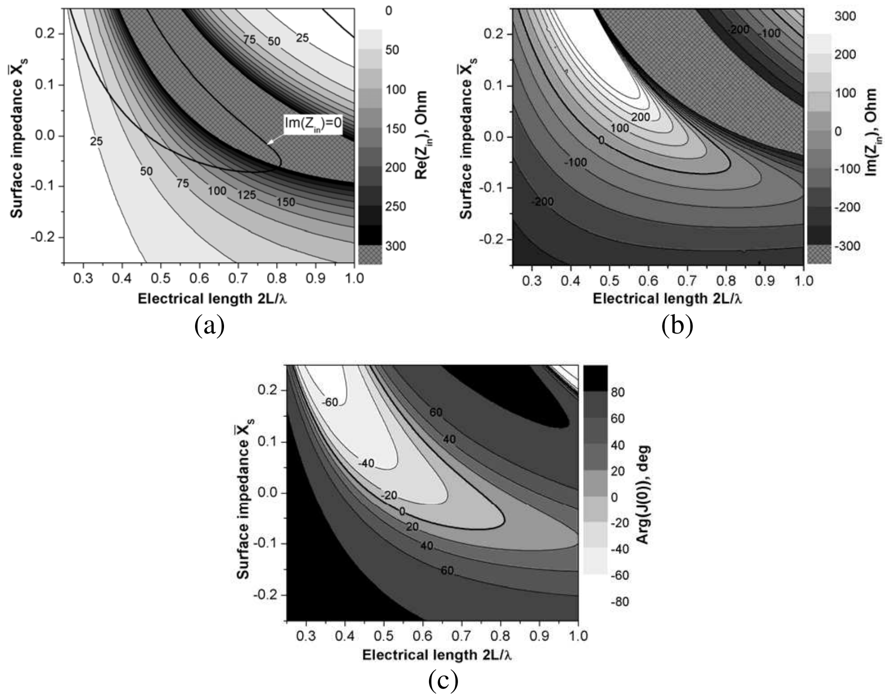

Figure 15. The active (a) and reactive (b) parts of the vibrator input impedance, and also the vibrator current phase in the feeding point (c) in dependence from the vibrator electrical length and the value of the surface impedance reactive part at $r = 0.01\lambda$, $\lambda = 100.0\,\mathrm{mm}$.

The use of the impedance vibrators in the Yagi-Uda antenna allows to increase the matching of the antenna with the feeding feeder. The change of the electrical length of the active vibrator with the corresponding change of $\bar{X}_S$ simultaneously gives the opportunity to vary the value of the active part of the input impedance R_{in} in wide ranges so, that at this the reactive part of the input resistance stays equal to null. At this the directed characteristics of the antenna radiation (directive gain D, the level of the side lobes) do not decrease. Figure 17 represents the dependance of the coefficients of reflection in the feeding feeder with different wave resistances from the wavelength for the array, consisting of 3 vibrators — an active vibrator, a reflector and a director. At this $2L_2 = 50.0\,\mathrm{mm}$, $2L_3 = 38.0\,\mathrm{mm}$, $r_{1,2,3} = 1.0\,\mathrm{mm}$, the distance between the vibrators $\Delta z = 25.0\,\mathrm{mm}$ ($z_3 = -z_2 = 25.0\,\mathrm{mm}$). The surface impedances of the reflector and the director equal to null, and the value of the surface impedance of

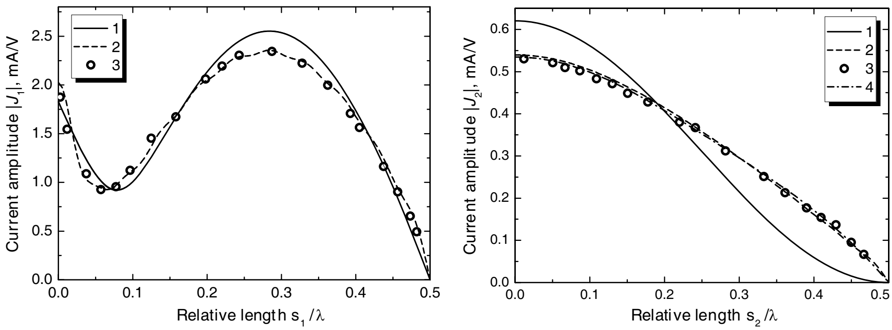

Figure 16. The currents distribution in the array, consisting of two vibrators, at $\Delta z = 0.25\lambda$: 1 — the calculation (the functions (25) and (26a)), 2 — the calculation (the functions (17), $N = 20$), 3 — the experimental data [30], 4 — the calculation (the functions (26a) and (26b)).

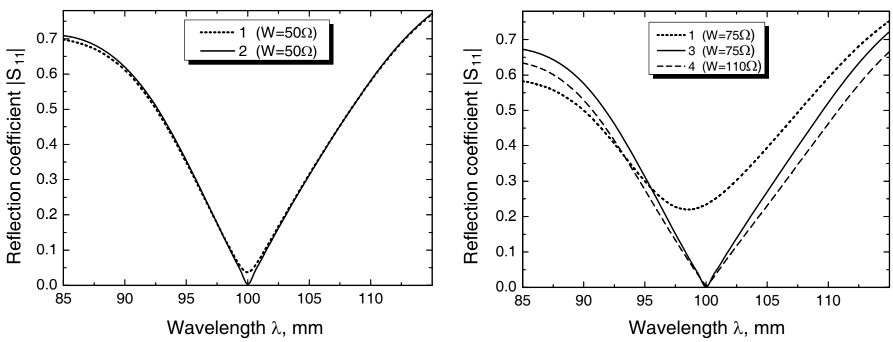

Figure 17. The reflection coefficients dependences in the feeding feeder of the Yagi-Uda arrays with the impedance active vibrator from the wavelength for the feeders with different wave resistance W.

the active vibrator and its length are fitted so, that the antenna input impedance is active and equals to the wave resistance W of the feeder (50, 75 or 110 Ohm) on the set wavelength ($\lambda = 100.0$ mm in this case). The parameters, corresponding to the system in question, are represented in Table 1.

We note that, if the decrease of the value of the active vibrator input impedance real part is observed at the tuning of the Yagi-Uda antenna, then application of the capacitive impedance (and at this longer vibrators) permits to increase the R_{in} value. The tuning of the antenna on the set wavelength or the retuning of the antenna on another wavelength is also possible by means of the surface impedance change.

Table 1. The parameters of the Yagi-Uda arrays with the impedance active vibrator.

No.	$2L_1$, mm	$\bar{X}_{S_1}$	$C_{C_1} \times 10^3$	R_{in}, Ohm	D
1.	43.8	0.0	0.0	46.6	6.4
2.	45.75	−0.0145	0.911	50.0	6.4
3.	56.9	−0.07075	4.445	75.0	6.5
4.	70.0	−0.106	6.66	110.0	6.6

Table 2. The parameters of the Yagi-Uda arrays from $N = 3$ elements.

No.	$2L_n$, mm	$\bar{X}_{S_n}$	$C_{C_n} \times 10^3$	C_{L_n}	R_{in}, Ohm	D
1.	44.85 50.0 39.0	0.0	- - -	- - -	43.37	6.36
2.	50.0	−0.0303 0.0 −0.09	1.9 0.0 5.655	- - -	52.62	6.5
3.	35.0	0.091 0.122 0.054	- - -	1.448 1.942 0.859	26.2	6.46
4.	65.0	−0.084 −0.053 −0.016	5.28 3.3 0.01	- - -	84.6	6.78
5.	44.7 35.0 35.0	0.0 0.122 0.054	- - -	0.0 1.942 0.859	42.03	6.44

The required currents phases of the vibrators in the array to provide with the axial radiation can be obtained by fitting the suitable value of the surface impedance of each of the vibrators of the array at the unchangeable length of the vibrators. Figure 18(a) gives radiation patterns ($|F_E|$) of the arrays, consisting of three vibrators ($z_2 = 0.25\lambda$, $z_3 = 0.2\lambda$, $r_{1,2,3} = 1.0$ mm, $\lambda = 100.0$ mm): 1 — the surface impedances of all vibrators equal to null; 2 — the lengths of all vibrators equal to $2L_n = 0.5\lambda$; 3 — the lengths of all vibrators equal to $2L_n = 0.35\lambda$; 4 — the lengths of all vibrators equal to $2L_n = 0.65\lambda$; 5 — the length of the reflector and the length of the director equal to $2L_{2,3} = 0.35\lambda$. The values of the vibrators surface impedances, providing a minimal level of the back lobe of $|F_E|$, the values of the input resistances and D of the arrays are represented in Table 2 for

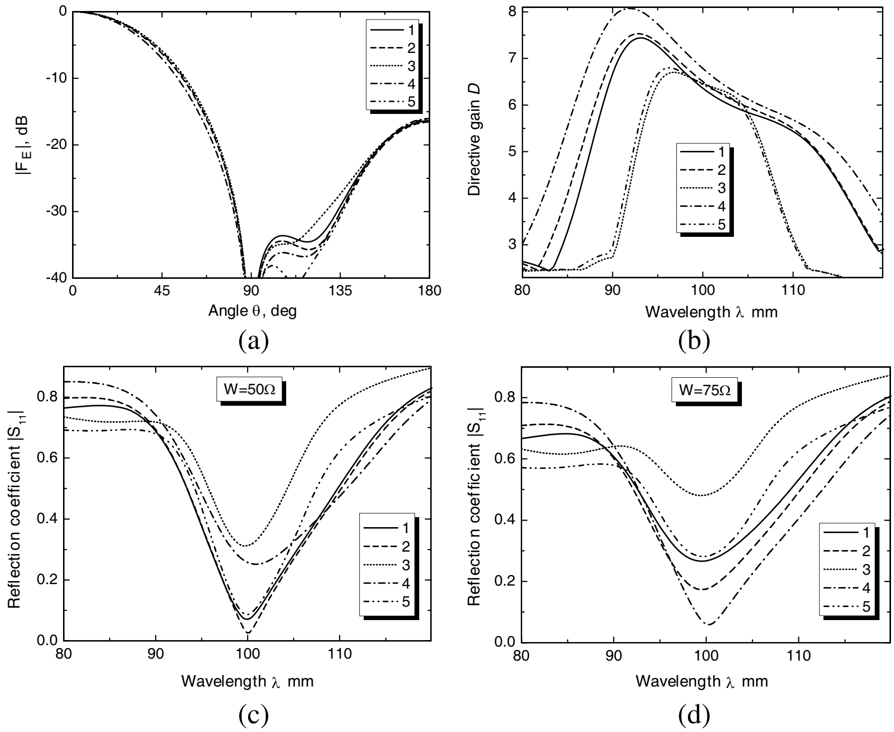

Figure 18. (a) $|F_E|$, (b) D and (c), (d) the coefficients of reflection in the feeding feeder of the Yagi-Uda arrays from $N = 3$ elements.

these cases. The energetic characteristics and the characteristics of directivity of these arrays in dependence from the wavelength are represented in Figures 18(b), (c), (d).

As it is seen from the plots, application of the vibrators with the inductive impedance (a variant 3) in the Yagi-Uda array allows to decrease the vibrators sizes, but at this the value of the input resistance R_{in} decreases, which can be increased by the length increase of either only the active vibrator (a variant 5) or at the length increase of all vibrators with simultaneous increase of the capacitive impedance values (the variants 2 and 4). The increase of the antenna band properties is observed at the use of longer vibrators with capacitive impedance. One would note, that it is possible to change the direction of the antenna radiation (a function of the reflector is performed by the director, and of the director — by the reflector), having changed the surface impedances values of passive radiators suitably.

Figure 19 give the band characteristics of the arrays, consisting of

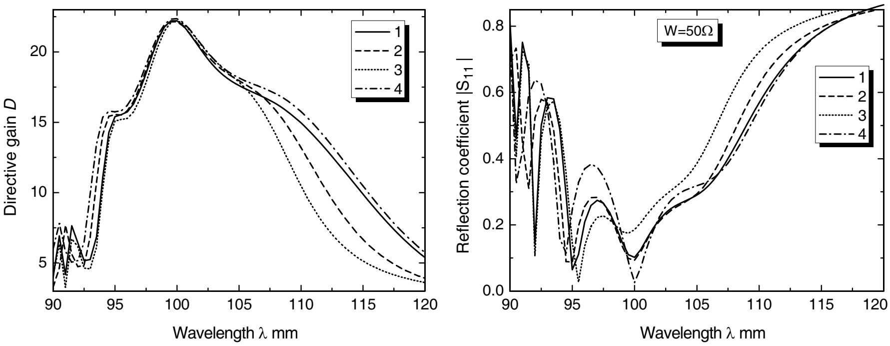

Figure 19. D and the $|S_{11}|$ coefficients of reflection in the feeding feeder of the Yagi-Uda arrays from $N = 10$ elements.

Table 3. The parameters of the Yaga-Uda arrays from $N = 10$ elements.

No.	$2L_1$, mm $2L_2$, mm $2L_{3-10}$, mm	$\bar{X}_{S_1}$ $\bar{X}_{S_2}$ $\bar{X}_{S_{3-10}}$	R_{in}, Ohm	D
1.	43.6 50.0 39.6	0.0	40.76	22.2
2.	43.6	0.0 0.038 −0.0394	51.5	22.1
3.	39.6	0.0335 0.07 0.0	34.4	22.2
4.	50.0	−0.0415 0.0 −0.086	52.6	22.4

$N = 10$ elements (the distance between the vibrators $\Delta z = 0.25\lambda$ and their radiuses $r_{1-10} = 0.01\lambda$) at $\lambda = 100.0$ mm as an example of the use of the distributed surface impedance for the arrays with a large quantity of vibrators. The lengths of the vibrators and the values of their surface impedances, chosen from the condition of attainment of a maximal D, are represented in Table 3.

6. CONCLUSION

The methodological grounds of application of the generalized method of induced EMF to investigate the electrodynamic characteristics of thin impedance vibrators are represented in the paper. The distinctive peculiarity of the method, proposed by the authors, is the use of the approximating functions (two and more), resulted from the integral equation solution for the current by the asymptotic averaging method, in the current distribution along the impedance vibrator. The ground of rightness and correctness of such an approach is represented in the format of comparative analysis with the known published calculated and experimental results. One would note, that the new conception of the generalized method of induced EMF, keeping all known advantages of numerical-analytical methods in comparison with direct numerical methods, extends to the cases of the vibrator with the impedance, variable along its length, and the impedance vibrators systems rather simply. Thus the proposed generalized method of induced EMF allows to widen the boundaries of numerical-analytical investigations of practically significant problems of the impedance vibrators application sufficiently, and it is a natural next step in the development of the general fundamental theory of thin vibrators.

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