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PARAMETER EXTRACTION FOR MICROWAVE COUPLED RESONATOR FILTERS USING RATIONAL MODEL AND OPTIMIZATION

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Abstract—A method is presented for the parameter extraction of microwave coupled resonator filters. The method is based on the estimation of a rational model of the filters. From these rational functions, a circuit network having the previously know topology is optimized. Two simple and efficient error functions are used to reduce the computational effort of the optimization while improving the speed and robustness of diagnosis process for lossless and lossy filters, respectively. Two numerical examples are presented to demonstrate the efficiency of the proposed technique. One deals with numerical simulation data from a full-wave electromagnetic simulation and the other one uses the measured data.

1. INTRODUCTION

The Tuning of filters is the last and important step in filter design procedures and dominates the performances of filters. Since the traditional tuning process is nontrivial, time-consuming, and very expensive, computer-aided tuning (CAT) of a microwave coupled resonator filter has drawn a great deal of attention in recent years [1–5].

Filter tuning may be carried out in the frequency and/or time domain. A time-domain filter tuning method is proposed in [1], which can indicate the inaccurate resonant frequencies of the resonators and coupling between them. In the frequency domain, a method for the determination of the individual resonant frequencies and coupling coefficients of a system consisting of cascaded coupled resonator was described by Atia and Yao [2]. The technique of computer diagnosis and tuning of microwave filters using model-based parameter

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estimation and optimization is described by Kahrizi [3]. Recently, an analytical approach to diagnosis and tuning of lossy microwave coupled resonator filters is proposed in [5]. The major difficulties that come with the CAT of microwave filters are 1) it is difficult to deal with cross coupling; 2) it is difficult to deal with the lossy systems.

In this paper, we present an efficient, robust and systematic algorithm to determine coupling coefficients, including adjacent and cross coupling, and individual resonator frequencies for each resonator. This algorithm consists of three steps. The first step is to convert the frequency sampled S-parameters, obtained from the simulation or measurement, into the rational functions given as a ratio of polynomial. In the second step, we use the rational functions to remove the loading effect and generate a rational model. In the third step, an efficient optimization technique is used to extract the parameters of the filter. For the method in step 1) had been described in detail in [6], we focused on the discussion of steps 2) and step 3) in this paper.

2. GENERATION OF RATIONAL MODEL

2.1. Filter Modeling and Synthesis

Figure 1 show the equivalent circuit of a multiple-coupled resonator filter [8]. The circuit consists of N asynchronously tuned resonators. All the resonators are represented by a LC circuit loop with their losses modeled by the resistance r_i. The coupling between the resonators i and j are modeled by M_{ij}. Under the assumption that all the coupling coefficients are frequency-invariant, the loop equations in matrix form can be written as

$$\left[\omega' U - jR - jR' + \tilde{M}\right] \cdot [I] = [Z] \cdot [I] = -j[E] \tag{1}$$

where U is the identity matrix, R is a $N \times N$ matrix with all entries zero, except $R_{11} = R_1$, $R_{NN} = R_N$, R_1 and R_N are the normalized input/output resistances of the filter. The diagonal matrix R' include the resistors r_i. M is the $N \times N$ coupling matrix where $\tilde{M}_{ij} = M_{ij}/FBW$, and the vectors $\{I\}$ and $\{E\}$ represent the loop current and voltage, respectively. It should be noted here that ω' in (1) represents the frequency variable in equivalent low-pass functions.

The scattering parameters for the input and output of the network are given by

$$S_{21} = 2\sqrt{R_1 R_2} i_N = -2j\sqrt{R_1 R_2}\left[Z^{-1}\right]_{N1} \tag{2}$$

$$S_{11} = 1 - 2R_1 i_1 = 1 + 2jR_1\left[Z^{-1}\right]_{11}. \tag{3}$$

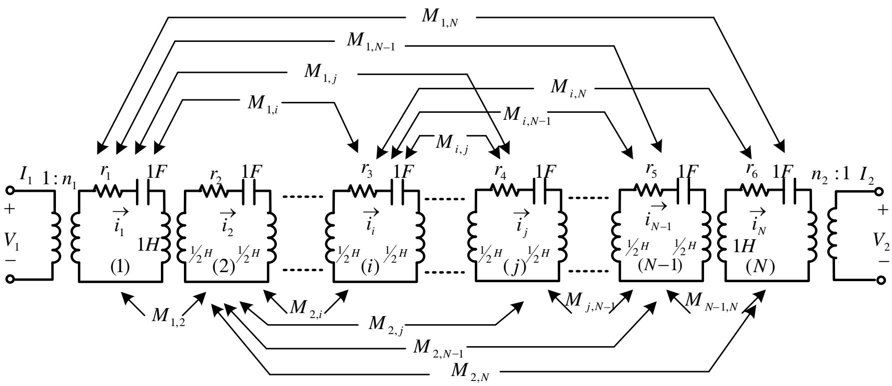

Figure 1. General equivalent circuit of coupled resonator filters.

In case of a two-port network described by its scattering parameters S_{11} and S_{21}, three characteristic polynomials F, P and E completely define a rational model in the normalized low-pass domain s [7]

$$S_{11}(s) = \frac{F(s)}{E(s)} = \frac{\sum_{k=0}^{k=n} f_k \cdot s^k}{\sum_{k=0}^{k=n} e_k \cdot s^k}, \quad S_{21}(s) = \frac{P(s)}{E(s)} = \frac{\sum_{k=0}^{k=nz} p_k \cdot s^k}{\sum_{k=0}^{k=n} e_k \cdot s^k} \tag{4}$$

where n is the order of the filter, and nz is the number of transmission zeros.

From the data samples of the filter responses, the formulation of the Cauchy method described in [6] can be used to evaluate the coefficients of polynomials F, P and E.

2.2. Removal of Loading Effect

Basically, the method of model-based parameter estimation is based on the equivalent-circuit as shown in Fig. 1. However, in a physical filter model, there is always an unwanted length of transmission line at the physical reference plane and the port of the corresponding inverter in the circuit model [5], which results in a constant phase loading. The modified equivalent-circuit of the physical filter model is shown in Fig. 2.

Consider the ideal equivalent-circuit, the phase of S_{11} is near to zero when the points are far away from the center frequency. Therefore,

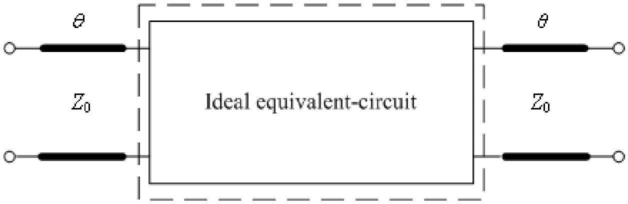

Figure 2. Modified equivalent-circuit with transmission lines.

as $\Omega \to \pm\infty$,

$$\phi(S_{11}) \approx \phi\left(\frac{f'_n}{e'_n}\right) \approx 0 \tag{5}$$

where f'_n and e'_n are the highest degree coefficients of ideal equivalent-circuit.

As to the physical filter model, it is easy to obtain an equation

$$\phi\left(\frac{f_n}{e_n}\right) = 2\theta + \phi\left(\frac{f'_n}{e'_n}\right) = 2\theta \tag{6}$$

where θ is the electrical length of transmission line.

Therefore, θ can be expressed as

$$\theta = \frac{1}{2} * \phi\left(\frac{f_n}{e_n}\right). \tag{7}$$

Consequently, it can be removal of the phase loading form $S_{11}(s)$ and $S_{21}(s)$ by simply multiplying $e^{-j2\theta}$.

3. ERROR FUNCTIONS FOR OPTIMIZATION

3.1. Lossless Filters

We first consider the lossless systems. In this case, all the resistances r_i in Fig. 1 are equal to zero. (1) can be rewritten as

$$\left[\omega' U - jR + \widetilde{M}\right] \cdot [I] = [Z] \cdot [I] = -j\,[E]. \tag{8}$$

Using (2), (3) and (4),

$$\frac{P(s)}{E(s)} = S_{21} = -2j\sqrt{R_1 R_2}\left[Z^{-1}\right]_{N1} = -2j\sqrt{R_1 R_2} \cdot \frac{\det(Z')}{\det(Z)} \tag{9}$$

$$\frac{F(s) - E(s)}{E(s)} = S_{11} - 1 = 2jR_1\left[Z^{-1}\right]_{11} = 2jR_1 \cdot \frac{\det(Z'')}{\det(Z)} \tag{10}$$

where Z' is the sub-matrix obtained by deleting the last row and first column of the matrix Z, Z'' is the sub-matrix obtained by deleting the first row and column of matrix Z. From Feldkeller's equation [7], it is easy to draw the conclusion that $f_n = e_n$. Therefore, the number of roots of $F(s) - E(s)$ is equal to the order of Z''.

Using (8), (10) can be rewritten as

$$\frac{F(s) - E(s)}{E(s)} = -2jR_1 \cdot \frac{\mathrm{Det}\left[\left(jR'' - \tilde{M}''\right) - \omega' U''\right]}{\mathrm{Det}\left[\left(jR - \tilde{M}\right) - \omega' U\right]}. \tag{11}$$

Here we set $A = (jR - \tilde{M})$, $A'' = (jR'' - \tilde{M}'')$. From the above equation it is seen that roots of polynomial $E(s)$ are eigenvalues of matrix A, and the roots of polynomial $F(s) - E(s)$ are the eigenvalues of matrix A''.

Let us denote the roots of polynomials $E(s)$ and $F(s) - E(s)$ by λ_i^e and λ_i^t, respectively. For the rational functions are uniquely specified by the location of their poles and zeros, the parameters extraction problem can be seen as an optimization problem with the error function defined as

$$\varepsilon\left(\tilde{M}, R_1, R_N\right) = \sum_{k=1}^{n} \left|\lambda_k^m - \lambda_k^e\right|^2 + \sum_{k=1}^{n-1} \left|\lambda_k^{m''} - \lambda_k^t\right|^2 + \sum_{k=1}^{n} \left|S_{11}\left(\omega_k^z\right)\right|^2 + \sum_{k=1}^{nz} \left|S_{21}(\omega_k^p)\right|^2 \tag{12}$$

where λ_i^m are the eigenvalues of A, $\lambda_i^{m''}$ are the eigenvalues of A'', ω_k^z and ω_k^p are the roots of F and P in domain Ω, respectively. The functions S_{11} and S_{21} are evaluated from the current trial matrix M. The error functions in (12) and (13) ensure the responses of functions S_{11} and S_{21} a good agreement with the rational model.

3.2. Lossy Filters

In the case of lossy filters ($r_i \neq 0$), the highest degree coefficients of polynomials $E(s)$ and $F(s)$ are not equal. Therefore, the number of λ_i^t are not equal to the number of $\lambda_i^{m''}$. Take the resistance r_i into account, we modify the error function as follows

$$\varepsilon\left(\widetilde{M}, R_1, R_N, R'\right) = \sum_{k=1}^{n} \left|\lambda_k^m - \lambda_k^e\right|^2 + \sum_{k=1}^{n} \left|S_{11}\left(\omega_k^z\right)\right|^2 + \sum_{k=1}^{nz} \left|S_{21}\left(\omega_k^p\right)\right|^2 + \sum_{k=1}^{m} \left|S_{21}\left(\omega_k^q\right) - S'_{21}\left(\omega_k^q\right)\right| \tag{13}$$

where m is the number of frequency samples, $S'_{21}(\omega_k^q)$ represent the value of polynomial function S_{21} in low-pass frequency point ω_k^q. Here we choose $\omega^q = [-1, 0, 1]$. The first three terms in (13) specify the location of zeros and poles of the rational functions, and the losses of filter is specified by the last term in (13).

The Nelder-Mead optimization algorithm can be used to solve this problem. This algorithm is one of the best known algorithms for multidimensional unconstrained optimization without derivatives. It is widely used to solve parameter estimation and similar statistical problems. The design values for coupling matrix and input/output resistances are chosen as the initial points in optimization. The initial values for the resistance r_i in lossy case are zero. It should be noted here that the undesired cross coupling due to leakage between the resonators can be modeled through the additional nonzero elements of coupling matrix. By comparing the final results and the initial design, the amount of mistuning for all the coupling and the offset frequencies is obtain and the tuning procedure can be performed.

4. APPLICATION

4.1. EM Design of a Six-pole Cross Coupling Filter

This new formulation discussed above has been applied first to a set of simulation data samples of a cross-coupled filter response. The filter structure is shown in Fig. 3, which is designed to have Chebyshev response having two transmission zeros located at 1.795 GHz and 1.825 GHz with a return loss of −22 dB, and the central frequency at 1.81 GHz with a bandwidth of 18 MHz. This device can be seen as a synchronously tuned filter, and the position of transmission zeros in low-pass frequency are located at −1.67 and 1.67. The coupling matrix elements and the normalized input/output resistances of the filter as obtained from synthesis applying the theory of Cameron [9] are given in (14).

$$\widetilde{M}_{12} = \widetilde{M}_{56} = 0.8703, \quad \widetilde{M}_{23} = \widetilde{M}_{45} = 0.6057, \quad \widetilde{M}_{34} = -0.6756, \quad \widetilde{M}_{25} = 0.1062, \quad R_1 = R_N = 1.0775. \tag{14}$$

The dot line in Fig. 4 show the narrow-band system response obtain by a full-wave electromagnetic simulation (CST). However, the initial response is far from the ideal response.

The formulation of Cauchy method described in [6] then be applied to a set of simulated data samples. It should be noted, because the accuracy of the model may be reduce by second order effects such as the frequency dependence of the coupling and high order poles, the frequency points should not too much distant from the passband. So the data set here employ refers to frequency interval 1.79 GHz–1.83 GHz. Table 1 contains the polynomial coefficients of the model which had been removal of the phase loading. The elements of coupling matrix M and the normalized input/output resistances are extracted by applying the optimization algorithm based on the error function in (12), and are given in (15). It should be noted here that the structure of the filter is symmetric, and we can reduce the optimized variables

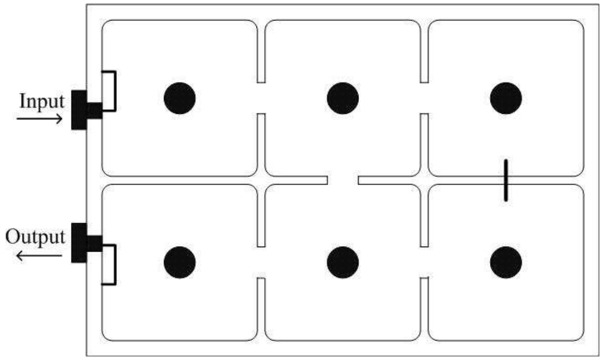

Figure 3. Filter of degree 6 with one cross coupling.

Table 1. Coefficients of polynomials for cross-coupling filter.

k	e_k	f_k	p_k
0	$1.3035 + 0.8203i$	$-0.0243 + 0.0000i$	$0.0000 - 1.5588i$
1	$3.8578 + 2.6415i$	$0.0000 + 1.1482i$	$0.1416 + 0.0000i$
2	$6.2157 + 4.0973i$	$0.4736 + 0.0001i$	$0.0000 - 0.3189i$
3	$6.4859 + 4.2197i$	$0.0000 + 2.3183i$	
4	$5.0743 + 2.4423i$	$1.7179 + 0.0001i$	
5	$2.5093 + 1.1781i$	$0.0000 + 1.3007i$	
6	1.0000	1.0000	

by setting $\widetilde{M}_{12} = \widetilde{M}_{56}$, $\widetilde{M}_{23} = \widetilde{M}_{45}$, $R_1 = R_N$. The S-parameters are computed from the extracted parameters and the result, in comparison with the simulated one, is shown in Fig. 4. It is obvious that both the amplitude and the phase are well matched. By complying the extracted results and the initial design, the difference between them can guide the tuning process. The filter can be fine tuned with explicit adjustments, and the final result is shown in Fig. 5. Apparently, the response of the

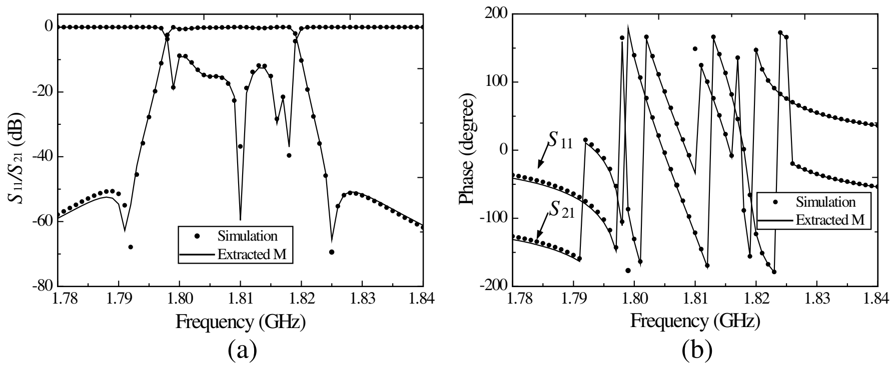

Figure 4. The simulated (dot line) S-parameter and the one calculated from extracted M (solid line) for the cross coupling filter. (a) Amplitude. (b) Phase.

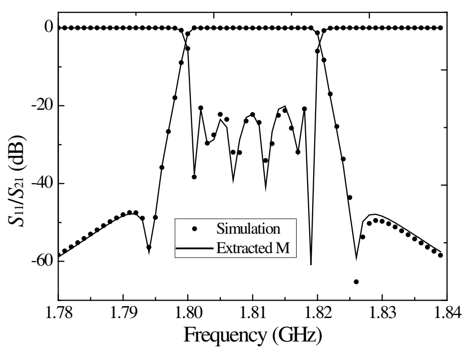

Figure 5. The simulated and extracted passband responses of the cross coupling filter after tuning.

tuned filter is well with the specification.

$$\widetilde{M} = \begin{bmatrix} 0.2503 & 0.8722 & 0 & 0 & 0 & 0 \\ 0.8722 & 0.0841 & 0.6237 & 0 & 0.0948 & 0 \\ 0 & 0.6237 & 0.2011 & -0.7227 & 0 & 0 \\ 0 & 0 & -0.7227 & 0.2011 & 0.6237 & 0 \\ 0 & 0.0948 & 0 & 0.6237 & 0.0841 & 0.8722 \\ 0 & 0 & 0 & 0 & 0.8722 & 0.2503 \end{bmatrix}$$

$$R_1 = R_N = 1.0778 \tag{15}$$

4.2. Cascaded Coupled Resonators Filter

As a second example on the application of this new technique, a set of measured responses of cascaded coupled resonators filter are tested to validate the proposed approach. This device is a sixth-order Chebyshev filter with no finite transmission zeros, tuned in the band 157 MHz–163 MHz, with a return loss of −25 dB; the passband losses are −1 dB. The dot line in Fig. 6 shows the narrow-band system response of the measured data. The elements of coupling matrix M and the normalized input/output resistances are extracted by applying the optimization algorithm based on the error function in (13), and are given in (16). It should be noted that here we assume $r_1 = r_6$ and $r_2 = r_3 = r_4 = r_5$. The response of the extracted result is also reported in Fig. 6 with solid line. It can be observed the very good agreement with the measurements. The detailed comparison of the insertion losses in the pass band is zoomed in, demonstrating the accuracy of the extracted r_i

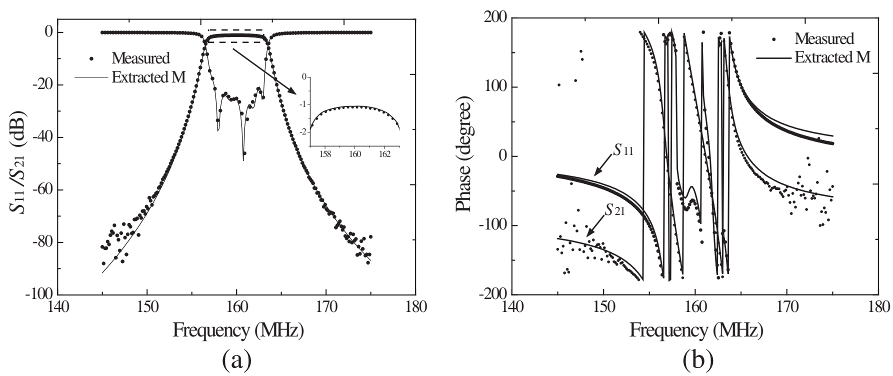

Figure 6. The measured (dot line) S-parameter and the one calculated from extracted M (solid line) for the cascaded coupled resonators filter. (a) Amplitude. (b) Phase.

values. It should be noted here that, due to the effect of measurement noise, there is a slight disagreement on the out-band.

$$M = \begin{bmatrix} -0.0632 & 0.9179 & 0 & 0 & 0 & 0 \\ 0.9179 & -0.0099 & 0.6303 & 00 & 0 & \\ 0 & 0.6303 & -0.0078 & 0.6097 & 0 & 0 \\ 0 & 0 & 0.6097 & 0.0303 & 0.6586 & 0 \\ 0 & 0 & 0 & 0.6586 & 0.0525 & 0.9291 \\ 0 & 0 & 0 & 0 & 0.9291 & 0.0893 \end{bmatrix}$$

$$R_1 = 1.1972 \quad R_N = 1.2245 \quad r_1 = r_6 = 0.0254$$
$$r_2 = r_3 = r_4 = r_5 = 0.0321 \tag{16}$$

5. CONCLUSION

A method of model-based parameter estimation for microwave filters is proposed in this paper. This technique can be used to estimate the coupling coefficients, resonant frequencies of the filters. The relationship between the polynomials coefficients and phase loading has been analyzed for the first time. This loading effect can be effectively removed by the proposed method. Optimization based on two novel error functions adds robustness to the approach for the lossless and lossy filters. The eigenvalues of M is introduced to the error functions. As a result, it reduces the steps of iterations and save the computing time. Finally, two numerical examples, including an electromagnetic design cross-coupled filter and a measured lossy filter, are provided to show the validity of this new technology. The extracted result show good agreement simultaneously with respect to both the amplitude and the phase.

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