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AMPLITUDE DISTRIBUTION SYNTHESIZE OF UNEQUALLY LINEAR AND PLANAR SPACED ARRAYS

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Abstract—An efficient hybrid method is presented to obtain the current distribution of both non-uniformly linear and planar arrays by sampling the array factor of a desired radiation pattern. The proposed method provides Fourier coefficients and uses the Least Mean Square method (LMS) to solve the system of equations in order to obtain current distribution in associate with the desired radiation pattern. The obtained level of first Peak Side Lobe Level (PSLL) is 3 dB lower than the level of first PSLL using conventional methods such as LMS method or Legendre function method.

1. NOMENCLATURE

LMS	Least Mean Square
SLL	Side Lobe Level
DFT	Discrete Fourier Transformation
PSO	Particle Swarm Optimization
GA	Genetic Algorithms
PGHA	Particle Swarm Optimization & Genetic Algorithms
FGA	Fuzzy Genetic Algorithms
PSLL	Peak Side Lobe Level

2. INTRODUCTION

Synthesis of unequally spaced array antenna by adjusting the amplitude or position of the elements has become an interesting category during recent years, because it reduces the size, weight and number of elements in array antennas [1–7].

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Unz obtained current distribution by finding the matrix relationship between arbitrary distributed elements of array and its far-zone pattern but not optimal SLL [8].

Harington, by adjusting the proper displacement of the array elements, tried to reduce SLL down to $2/N$ times of the field intensity of the main lobe, where N is the total number of elements [9]. Ishimaru used the Poisson sum expansion to design an unequally spaced uniform amplitude array with any desired SLL, resulting in grating lobe suppression [10]. Skolnik used dynamic programming approach to build arrays by evaluating optimal combinations of elements as the number of elements in the array was steadily increasing [11].

In addition to the mentioned methods, various numerical and analytical methods of pattern synthesis and optimizations of antenna arrays have been developed. Hejres presented a new and fast method of forming deep nulls in the antenna pattern in the directions of interferences by partially controlling the positions of a set of elements of a larger array [12]. Also a combined algorithm called PGHA which is a combination of Particle Swarm Optimization (PSO) and Genetic Algorithms (GA) is applied to design a linear array with ten elements and a circular array with thirty one elements to obtain desired beam forms [13].

Fuzzy Genetic Algorithms (FGAs) are algorithms that are used to synthesize planar antenna arrays by optimizing phase excitation coefficients in order to meet the desired radiation pattern perfectly [14]. On the other hand, using unequal spacing technique results in both reduced SLL and reduced number of elements in the linear array antennas [15]. This paper reassures that by using the unequal spacing technique, the linear array antenna would be realized to have less number of elements than equal spacing array with the same performance.

Kumar presented an analytical method based on Legendre transformation to obtain both the optimal current distribution with known radiation pattern and element spacing and the optimal spacing with known current distribution and desired pattern [16].

Recently, in a previous work of the authors, an LMS method has been presented to optimize SLL of the broadside and non-broadside pattern in which LMS was used to solve the system of M equations and N variables with $M > N$. It was shown that using LMS method can improve SLL 5 dB compared with using the Legendre method [17].

Extending our previous work, we present a new *hybrid* method based on both Discrete Fourier Transformation (DFT) and LMS methods and its ability to improve the first PSLL about 3 dB compared to the conventional methods of designing linear arrays, then the proposed method is applied to planar array, and PSLL of the proposed method is 3 dB lower than that of the Legendre method. In this method, after sampling the array factor at M points, DFT is used to find the Fourier coefficients. These coefficients are used to form a non-square matrix system, to be solved by LMS method at the end of Section 2. It is shown that this transformation improves the first PSLL about 3 dB compared to conventional methods [17–19]. The comparison among all radiation patterns is shown in Section 3.

The paper is organized as follows: Section 2 presents the problem formulation to describe the linear and planar array factor using proposed hybrid method. Section 3 presents the simulation results for the radiation pattern characteristic including SLL. Finally, Section 4 sums up the proposed work.

3. THEORETICAL BACKGROUND

The structure of the symmetric linear and planar antenna array is shown in Figs. 1(a) and 1(b). Linear array has odd elements $(2N+1)$ in which d_n and I_n represent element spacing and currents respectively. Also, planar array has $N \times N$ elements with $I_{n1,n2}$ current distribution and d_{n1} spacing in x-plane and d_{n2} spacing in y-plane.

In the next two sections, we intend to represent the detail of mathematical method for linear and planar array.

3.1. synthesis of Linear Unequally Array

The theory behind the proposed method is based on the previous work [17, 18], but using Fourier coefficients instead. First, the array factor of pattern is sampled at M points,

$$E\left(u_m\right) = \sum_{n=0}^{N} I_n \cos\left(k_0 d_n u_m\right) \tag{1}$$

where

$$u_m = \cos\left(\theta_m\right) = \frac{m}{M-1} \quad m = 0, 1, \ldots, M-1 \tag{2}$$

Since the pattern is symmetric, i.e., $E(u) = E(-u)$, the synthesis problem is addressed only over the interval of $0 < u < 1$. This pattern is equalized with desired pattern E_d at sampled points,

$$E\left(u_m\right) = E_d\left(u_m\right) \tag{3}$$

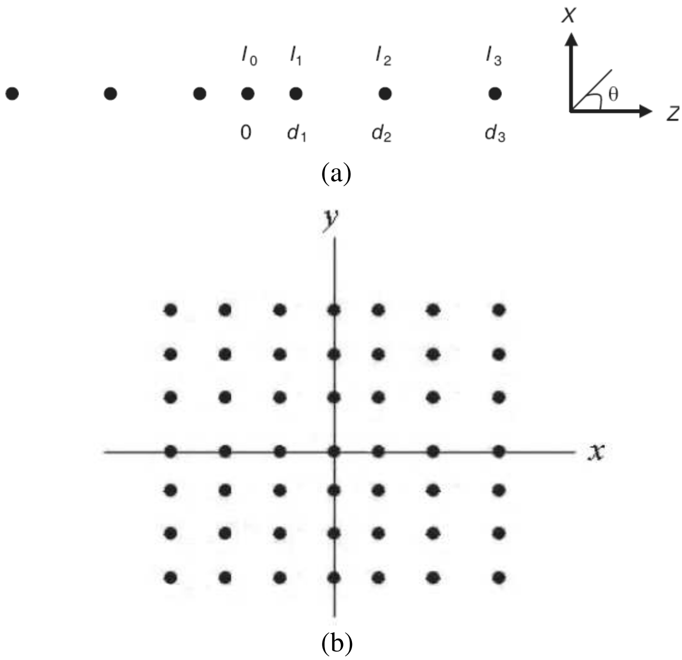


Figure 1. Geometry of non-uniformly spaced (a) Linear symmetric array with $2N+1$ elements and (b) Planar array with $N \times N$ elements.

At this stage, DFT from both sides of this equation is calculated,

$$a_k = \frac{1}{M}\sum_{m=0}^{M-1} E_d\left(u_m\right) e^{-jk\omega_0 m} = \frac{1}{M}\sum_{m=0}^{M-1}\sum_{n=0}^{N} I_n \cos\left(k_0 d_n u_m\right) e^{-jk\omega_0 m} \tag{4}$$

where $\omega_0 = \frac{2\pi}{M}$.

The second side can be written as

$$\begin{aligned} & \frac{1}{M}\sum_{m=0}^{M-1}\sum_{n=0}^{N} I_n \cos(k_0 d_n u_m) e^{-jk\omega_0 m} \\ = & \frac{1}{M}\sum_{n=0}^{N} I_n \sum_{m=0}^{M-1} \cos(k_0 d_n u_m) e^{-jk\omega_0 m} \end{aligned} \tag{5}$$

By calculating M discrete Fourier coefficients from Eq. (4), a matrix system of $M \times (N+1)$ equations can be formed as $A = BI$, in which

$$A = [a_1 \quad a_2 \quad \ldots \quad a_M]^T \tag{6}$$

$$B = [b_{ks}]_{M\times(N+1)}, \quad b_{ks} = \frac{1}{M}\sum_{m=0}^{M-1}\cos(k_0 d_{s-1} u_m) e^{-jk\omega_0 m} \tag{7}$$

$$I = [I_0 \quad I_1 \quad I_2 \quad \dots \quad I_N]^T \tag{8}$$

Non-square matrix equation ($A = BI$) has to be solved to obtain unknown elements current distribution. So, LMS method is applied to solve matrix system to minimize the inevitable error. In LMS method, both sides of the equations are multiplied by B^T, then the square $B^T B$ is inverted to obtain matrix I.

$$B^T A = B^T B I \tag{9}$$

$$I = \left(B^T B\right)^{-1} B^T A \tag{10}$$

3.2. Synthesis of Planar Unequally Array

The array factor of the planar array (Fig. 1(b)) is,

$$E(u,v) = \sum_{n_1=0}^{N}\sum_{n_2=0}^{N} I_{n_1,n_2}\cos(k_0 d_{n_1} u)\cos(k_0 d_{n_2} v) \tag{11}$$

In which,

$$u = \cos(\theta_1), \qquad v = \cos(\theta_2) \tag{12}$$

We sample the desired pattern at L by M points,

$$E(l,m) = \sum_{n_1=0}^{N}\sum_{n_2=0}^{N} I_{n_1,n_2}\cos(k_0 d_{n_1} u_l)\cos(k_0 d_{n_2} v_m) \tag{13}$$

$$u_l = \cos\left(\theta_{1(l)}\right) = \frac{l}{L-1}, \quad v_m = \cos\left(\theta_{2(m)}\right) = \frac{m}{M-1} \tag{14}$$

And then the array factor is equalized with the desired pattern at the sample points,

$$E(l,m) = E_d(l,m) \tag{15}$$

As a reminder, note that Fourier coefficients for a discrete function as $x(l,m)$ are calculated from Eq. (16),

$$a(k_1,k_2) = \frac{1}{L\times M}\sum_{l=0}^{L-1}\sum_{m=0}^{M-1} x(l,m) e^{-jk_1\omega_0 l} e^{-jk_2\omega_0 m} \tag{16}$$

If the number of sample points is set equal to T, then we have,

$$L = M = T,\ a(k_1, k_2) = \frac{1}{T^2} \sum_{l=0}^{T-1} \sum_{m=0}^{T-1} x(l, m) e^{-jk_1\omega_0 l} e^{-jk_2\omega_0 m} \tag{17}$$

Similar to linear array, the discrete Fourier transformation (16) is applied to both sides of Eq. (15) to calculate the discrete Fourier coefficients as below,

$$\begin{aligned} a(k_1, k_2) &= \frac{1}{T^2} \sum_{l=0}^{T-1} \sum_{m=0}^{T-1} E_d(l, m) e^{-jk_1\omega_0 l} e^{-jk_2\omega_0 m} \\ &= \frac{1}{T^2} \sum_{l=0}^{T-1} \sum_{m=0}^{T-1} \left[\sum_{n_1=0}^{N} \sum_{n_2=0}^{N} I_{n_1,n_2} \cos(k_0 d_{n_1} u_l) \cos(k_0 d_{n_2} v_m) \right] \\ &\quad e^{-jk_1\omega_0 l} e^{-jk_2\omega_0 m} \end{aligned} \tag{18}$$

The right hand side of (18) can be written as,

$$\begin{aligned} a(k_1, k_2) \ &= \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} I_{n_1,n_2} \left[\frac{1}{T} \sum_{l=0}^{T-1} \cos(k_0 d_{n_1} u_l)\, e^{-jk_1\omega_0 l} \right] \\ &\quad \left[\frac{1}{T} \sum_{m=0}^{T-1} \cos(k_0 d_{n_2} v_m) e^{-jk_2\omega_0 m} \right] \end{aligned} \tag{19}$$

Considering the fact that the 2-D discrete Fourier coefficients are known, Eq. (19) can be formulated as a matrix system with unknown matrix I, as following

$$A = UIV \tag{20}$$

$$A = [a_{pq}]_{T\times T}, \quad a_{pq} = a(p, q) \tag{21}$$

$$U = [u_{pq}]_{T\times(N+1)}, \quad u_{pq} = \frac{1}{T} \sum_{l=0}^{T-1} \cos\left(k_0 d_{(q-1)_1} u_l\right) e^{-jp\omega_0 l} \tag{22}$$

$$I = [I_{pq}]_{(N+1)\times(N+1)}, \quad I_{pq} = I_{p-1,q-1} \tag{23}$$

$$V = [v_{pq}]_{(N+1)\times T}, \quad v_{pq} = \frac{1}{T} \sum_{m=0}^{T-1} \cos\left(k_0 d_{(p-1)_2} v_m\right) e^{-jq\omega_0 m} \tag{24}$$

Unknown matrix, I, can be calculated in terms of known A, U, and V matrixes using LMS method and the same process that was applied to linear arrays. Therefore, the calculated pattern is in good correspondence with the desired pattern. The procedure of calculating the matrix I is described in (25)–(27).

$$AV^T = UIVV^T \tag{25}$$

$$AV^T (VV^T)^{-1} = UI \tag{26}$$

$$U^T AV^T (VV^T)^{-1} = U^T UI \tag{27}$$

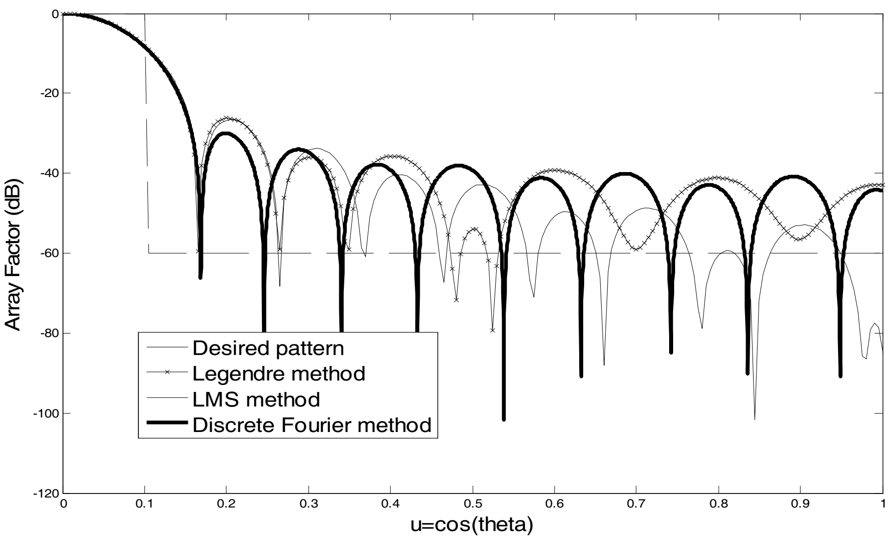
$$(U^T U)^{-1} U^T AV^T (VV^T)^{-1} = I \tag{28}$$

4. SIMULATION RESULTS

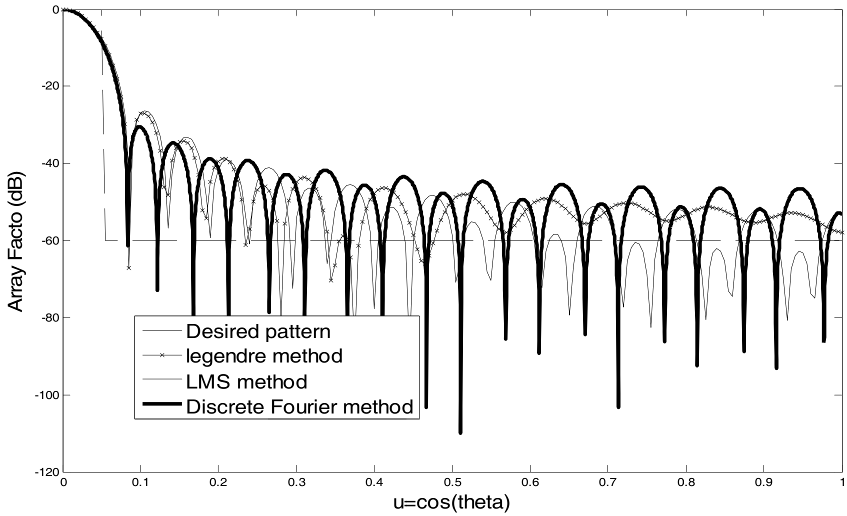
In this section, results of the methods are compared, e.g., Legendre, LMS, and Hybrid Fuorier Transformation, which has been applied to the previously-described linear and planar arrays.

4.1. Linear Array

In this section, the Fourier method is applied to synthesize the linear array. The simulation results of Fig. 2 are obtained from three methods



(a)



(b)

Figure 2. Comparison of array factors calculated with three different methods for linear array, (a) $N = 9$, (b) $N = 19$.

Table 1. The synthesized array current distributions (a) $N = 9$ and (b) $N = 19$.

(a)

	Discrete Fourier Method	Legendre Method	LMS method
n	I_n	I_n	I_n
0	1	1	1
1	0.9501	0.9854	0.9548
2	0.8805	0.9155	0.8873
3	0.7952	0.8272	0.8059
4	0.6891	0.7214	0.7011
5	0.5696	0.6000	0.5810
6	0.4455	0.4704	0.4502
7	0.3248	0.3374	0.3166
8	0.2158	0.2087	0.1872
9	0.1253	0.0908	0.0689

(b)

	Discrete Fourier Method	Legendre Method	LMS method
n	I_n	I_n	I_n
0	1	1	1
1	0.9548	0.9953	0.9593
2	0.9246	0.9632	0.9286
3	0.9002	0.9311	0.9051
4	0.8689	0.8972	0.8725
5	0.8329	0.8575	0.8354
6	0.7900	0.8132	0.7911
7	0.7425	0.7632	0.7414
8	0.6891	0.7085	0.6854
9	0.6323	0.6481	0.6266
10	0.5731	0.5858	0.5624
11	0.5128	0.5198	0.4973
12	0.4524	0.4538	0.4286
13	0.3910	0.3858	0.3608
14	0.3329	0.3189	0.2929
15	0.2773	0.2528	0.2269
16	0.2251	0.1887	0.1627
17	0.1763	0.1283	0.1022
18	0.1334	0.0708	0.0443
19	0.0963	0.0179	0.0081

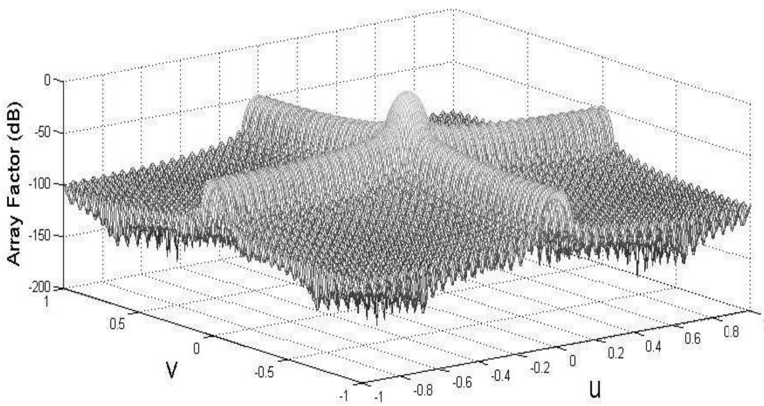
for two arrays with $N = 9$ and $N = 19$, and the element spacing d is determined as [18],

$$d(k) = d(k-1) + \ 0.5 + 0.05e^{(1-k)} \tag{29}$$

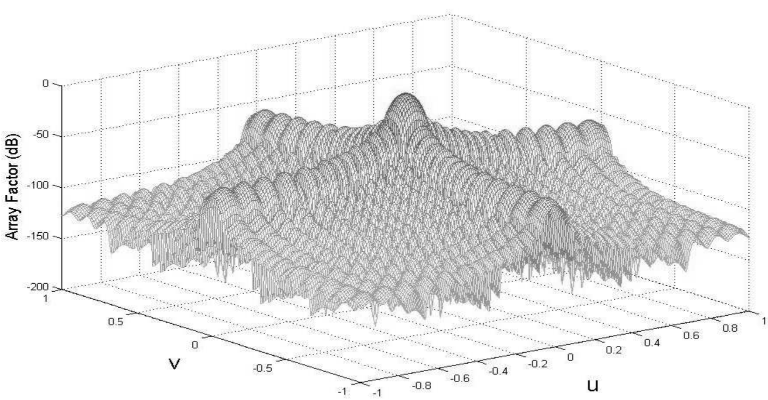
As can be seen in this figure, the first PSLL of the proposed hybrid method is almost 3 dB lower than that of Legendre and LMS methods. The synthesized array current distributions with three methods are presented in Tables 1(a) and (b) for cases (a) $N = 9$ and (b) $N = 19$, respectively.

4.2. Planar Array

In the second case, two planar arrays with 39 by 39 and 19 by 19 elements are investigated with element spacings using Eq. (29) in two directions, X and Y. As shown in Fig. 3 and Fig. 4, the Fourier Transformation method presents $\sim 3\,\mathrm{dB}$ lower in the first PSLL than Legendre method [19] because this method is based on the lowest error in the process of solving the equations systems.

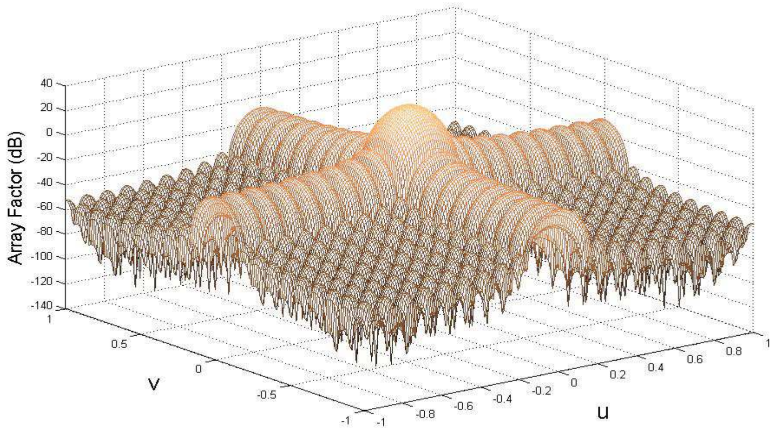


(a)

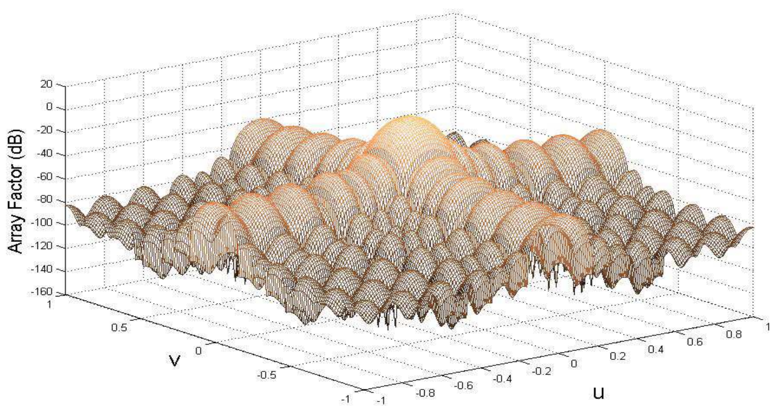


(b)

Figure 3. Comparision of array factors, calculated with (a) Fourier and (b) Legendre methods for planar array with $39 * 39$ elements.



(a)



(b)

Figure 4. Comparision of array factors, calculated with (a) Fourier and (b) Legendre methods for planar array with $19 * 19$ elements.

5. CONCLUSION

In this paper, a novel efficient hybrid mathematical method is presented based on the Fourier transformation and LMS method to synthesize current distribution of both linear and planar arrays. Legendre, LMS, and Fourier transformation methods have been used to design the array antenna, and it is shown that 3 dB improvement in the first PSLL can be achieved by using the hybrid method in comparison with other two methods.

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