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ON THE VALIDITY OF BORN APPROXIMATION

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Abstract—Born approximation is widely used in (inverse) scattering problems to alleviate the computational difficulty, but its validity and applicability are not well defined. In this paper, a universal criterion to identify the validity of Born approximation is put forward based on applying the operator theory on the scattering integral equation. In comparison with the traditional criteria, the new one excels in its ability to give a wider and more rigorous valid frequency range, especially while non-uniform scatterers are under consideration. Numerical examples verify the validity and advantage of the new criterion.

1. INTRODUCTION

The IE (integral equation) method is widely used in computational electromagnetics to characterize the scattering behavior of a scatterer [12]. In the method, the integral equation to be solved is a 3D Fredholm integral equation of the 2nd kind. In most situations of practical interest, it is not possible to find a solution of this integral equation in a closed form. One must, therefore, employ a proper numerical scheme to solve it. Up to now, the most attractive numerical method for this kind of integral equation is the method of moment (MoM) [13, 14, 29, 35]. However, enormous computational cost is required by this method, especially for electrically large-scaled scatterers. In this sense, approximation techniques might be more advisable in some situations. Among the approximate methods, the Born approximation is the most commonly used one, which directly employs the incident field as the total field inside the scatterer [4]. This method is easy to use, and it is widely used in (inverse) scattering

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problems [8–10, 25, 26]. However, its applicability at high frequencies is believed to be unsatisfactory.

Some criteria have been developed in the past decades to identify the validity of this approximation, and they can generally be classified into the following three categories:

(i) The first one is like $\Delta\varepsilon_r kL \ll 1$, where L stands for the scale of the scatterer [6, 7, 16, 17]. This kind of criterion is generally concluded from some special numerical experiments where even scatterers are considered, so its rigour is limited.

(ii) The second one is the extension of the previous one. It follows $\Delta\varepsilon_r\{1, kL, (kL)^2\} \ll 1$ [28].

(iii) The third one is like $k^2\Delta\varepsilon_r V/(4\pi r) \ll 1$, where V is the volume of the scatterer and r is the detection distance [30–32]. This criterion is derived from the condition that the scattered field around the receiver is much smaller than the incident field. However, this comparison should actually be made in the scatterer [4], so the generality of this criterion is not satisfactory.

Conclusively, the validity of Born approximation has not been well specified. This paper intends to establish a new criterion to identify it.

2. BORN APPROXIMATION AND ITS VALIDITY

In light of the Maxwell equation, the scattering behavior of a scatterer can be characterized by an integral equation as follows [7]:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_{\mathrm{i}}(\mathbf{r}) + k^2 \iiint_{V'} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \Delta\varepsilon_r(\mathbf{r}')\mathbf{E}(\mathbf{r}')\mathrm{d}^3\mathbf{r}', \mathbf{r} \in V', \quad (1)$$

where k is the wave number of incident wave, $\mathbf{r}$ is the radius vector from the origin to the field point (receiver), $\mathbf{r}'$ is the radius vector from the origin to the source point, V' is the volume the scatterer occupies, $\Delta\varepsilon_r$ is the difference in relative dielectric constant between the scatterer and the ambient medium, $\mathbf{E}$ is the total electric field, $\mathbf{E}_{\mathrm{i}}$ is the incident electric field, $\overline{\mathbf{G}}$ is the dyadic Green function. The dyadic Green function is defined by

$$\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left(\bar{\mathbf{I}} + \frac{\nabla\nabla}{k^2}\right) G(\mathbf{r}, \mathbf{r}') \quad (2)$$

with $G(\mathbf{r}, \mathbf{r}') = \mathrm{e}^{\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}/(4\pi|\mathbf{r}-\mathbf{r}'|)$ being the free space Green function and $\bar{\mathbf{I}}$ being a unit dyadic.

As is well known, Equation (1) is very difficult to solve via classical algorithms (such as MoM), especially for electrically large-scaled scatterers. In order to alleviate the computational difficulty, Born proposed the Born approximation method [4]. The approximation is that if the contrast between the scatterer and the ambient medium is extremely small, the total field inside the scatterer can be well approximated by the incident field, and then the total field $\mathbf{E}$ at the receiver ($\mathbf{r}$ is located at the receiver) is approximately obtained as

$$\mathbf{E}(\mathbf{r}) \approx \mathbf{E}_{\mathrm{i}}(\mathbf{r}) + k^2 \iiint_{V'} \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \Delta\varepsilon_r(\mathbf{r}')\mathbf{E}_{\mathrm{i}}(\mathbf{r}')\mathrm{d}^3\mathbf{r}'. \tag{3}$$

This technique is easy to implement and it plays an important role in (inverse) scattering problems [8–10], but its validity needs more study. The following sections intends to put forward a new criterion to identify the validity of it.

2.1. Operator Expression of a Sufficient Condition for the Validity of Born Approximation

In (1), the presence of operator $\nabla\nabla$ results in a three-order singularity $1/|\mathbf{r}-\mathbf{r}'|^3$ when the field point coincides with the source point ($\mathbf{r}' = \mathbf{r}$). This makes the integral not integrable. In the present paper, we start our investigation with another expression of the original integral Equation (1), which is free of three-order singularity [7]:

$$\begin{aligned}\mathbf{E}(\mathbf{r}) \;=\; & \mathbf{E}_{\mathrm{i}}(\mathbf{r}) + k^2 \left\{ \iiint_{V'} G(\mathbf{r}, \mathbf{r}')\Delta\varepsilon_r(\mathbf{r}')\mathbf{E}(\mathbf{r}')\mathrm{d}^3\mathbf{r}' \right. \\ & \left. + \frac{1}{k^2} \iiint_{V'} \nabla' \cdot \left[\Delta\varepsilon_r(\mathbf{r}')\mathbf{E}(\mathbf{r}')\right] \nabla G(\mathbf{r}, \mathbf{r}')\mathrm{d}^3\mathbf{r}' \right\}. \end{aligned} \tag{4}$$

Define a vector $\mathbf{P}(\mathbf{r})$ to describe the total field such that

$$\mathbf{E}(\mathbf{r}) = \mathbf{P}(\mathbf{r})\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}, \tag{5}$$

then the vector integral Equation (4) can be transformed as

$$\begin{aligned}\mathbf{P}(\mathbf{r}) = & \mathbf{P}_{\mathrm{i}}(\mathbf{r}) + k^2 \left\{ \iiint_{V'} G(\mathbf{r}, \mathbf{r}')\Delta\varepsilon_r(\mathbf{r}')\mathbf{P}(\mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}' \right. \\ & \left. + \frac{1}{k^2} \iiint_{V'} \nabla' \cdot \left[\Delta\varepsilon_r(\mathbf{r}')\mathbf{P}(\mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}'}\right] \nabla G(\mathbf{r}, \mathbf{r}')\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{d}^3\mathbf{r}' \right\}, \end{aligned} \tag{6}$$

where $\mathbf{P}_{\mathrm{i}}(\mathbf{r}) \triangleq \left[P_{\mathrm{i}}^x(\mathbf{r}) \quad P_{\mathrm{i}}^y(\mathbf{r}) \quad P_{\mathrm{i}}^z(\mathbf{r})\right]^{\mathrm{T}}$ stands for the incident wave (the factor $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$ is excluded). Moreover, defined an operator $\mathcal{H}$ by

$$\begin{aligned}\mathcal{H}\mathbf{P} = & \iiint_{V'} k^2\Delta\varepsilon_r\mathbf{P}G(\mathbf{r}, \mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}' \\ & + \iiint_{V'} \left[\nabla'(\Delta\varepsilon_r)\cdot\mathbf{P} + \Delta\varepsilon_r\nabla'\cdot\mathbf{P} + \Delta\varepsilon_r\mathbf{P}\cdot(\mathrm{i}\mathbf{k})\right]\nabla G(\mathbf{r}, \mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}'. \end{aligned} \tag{7}$$

then the integral equation becomes

$$\mathbf{P} = \mathbf{P}_\mathrm{i} + \mathcal{H}\mathbf{P}. \tag{8}$$

In light of the operator theory [19], the solution of (8) can be expressed as

$$\mathbf{P} = (\mathcal{I} - \mathcal{H})^{-1}\,\mathbf{P}_\mathrm{i}, \tag{9}$$

where $\mathcal{I}$ is the identity operator. If $\|\mathcal{H}\| < 1$, the inverse operator can be expanded in a Neumann series [2, 5, 15], i.e.,

$$\mathbf{P} = \mathbf{P}_\mathrm{i} + \mathcal{H}\mathbf{P}_\mathrm{i} + \mathcal{H}^2\mathbf{P}_\mathrm{i} + \cdots = \mathbf{P}_\mathrm{i} + \sum_{n=1}^{\infty}\mathcal{H}^n\mathbf{P}_\mathrm{i}, \tag{10}$$

where $\mathcal{H}^0\mathbf{P}_\mathrm{i}, \mathcal{H}\mathbf{P}_\mathrm{i}, \mathcal{H}^2\mathbf{P}_\mathrm{i}, \cdots, \mathcal{H}^n\mathbf{P}_\mathrm{i}$ are the zeroth-order scattered field ($\mathbf{P}^0$), the first-order scattered field ($\mathbf{P}^1$), the second-order scattered field ($\mathbf{P}^2$), $\cdots$, and the nth-order scattered field ($\mathbf{P}^n$), respectively. Of course, if the high-order operators $\mathcal{H}^n\mathbf{P}_\mathrm{i}$ are available, the total field can be directly obtained by summing them. However, the calculation of high-order terms are generally very difficult because of the presence of highly oscillatory factors $G(\mathbf{r}, \mathbf{r}')$ [11]. This makes the applicability of series of (10) to be prohibitive.

In practice, if the contrast between the scatterer and the ambient medium is extremely small, the Born approximation can be used. This technique approximates the total field inside the scatterer with the incident field [4], i.e.,

$$\mathbf{P}(\mathbf{r}) \approx \mathbf{P}_\mathrm{i}(\mathbf{r}), \quad \mathbf{r} \in V'. \tag{11}$$

Obviously, this approximation takes only the first term on the right hand side of (10), and the left terms are truncated. In this manner, if the truncated terms are much smaller than the kept term, i.e.,

$$\left|\sum_{n=1}^{\infty}\mathcal{H}^n\mathbf{P}_\mathrm{i}\right| \ll |\mathbf{P}_\mathrm{i}|, \tag{12}$$

then the Born approximation is believed to be valid. This paper intends to study the validity of Born approximation based on this condition.

2.2. Estimation of the First-order Scattered Field

Without loss of generality, we assume a plane wave propagates to the scatterer in x direction and the polarization is in z direction, i.e.,

$$\mathbf{k} = [1, 0, 0]^\mathrm{T}, \quad \mathbf{P}^0 = \mathbf{P}_\mathrm{i} = [0, 0, 1]^\mathrm{T}.$$

In this situation, there should be $\nabla \cdot \mathbf{P}_{\mathrm{i}} = 0$, $\mathbf{P}_{\mathrm{i}} \cdot (\mathrm{i}\mathbf{k}) = 0$ and $\nabla(\Delta\varepsilon_r)\cdot\mathbf{P}_{\mathrm{i}} = \partial(\Delta\varepsilon_r)/\partial z$. Subsequently, substituting these expressions into (7) yields the first-order scattered field:

$$\begin{aligned}\mathbf{P}^1(\mathbf{r}) &= (\mathcal{H}\mathbf{P}_{\mathrm{i}})(\mathbf{r})\\ &= \begin{bmatrix}0\\0\\1\end{bmatrix} \iiint_{V'} k^2 \Delta\varepsilon_r(\mathbf{r}') G(\mathbf{r},\mathbf{r}') \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})} \mathrm{d}^3\mathbf{r}'\\ &\quad + \iiint_{V'} \frac{\partial(\Delta\varepsilon_r(\mathbf{r}'))}{\partial z'} \nabla G(\mathbf{r},\mathbf{r}') \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})} \mathrm{d}^3\mathbf{r}'. \end{aligned} \tag{13}$$

In (13), the presence of oscillatory factor $G(\mathbf{r},\mathbf{r}')\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$ and $\nabla G(\mathbf{r},\mathbf{r}')\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$ makes the computation of the two integrals very difficult; we shall make proper estimations on them.

Investigation of the Green function shows that if a plane incident wave propagates to the scatterer in x direction, there should be the following two approximations:

$$G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \approx \frac{\mathrm{i}}{2k} u(x-x')\delta(y-y')\delta(z-z'), \tag{14}$$

and

$$\begin{aligned}&\nabla G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\\ &\approx \frac{\mathrm{i}}{2k}\begin{bmatrix}\delta(x-x')\delta(y-y')\delta(z-z')\\ u(x-x')\delta'(y-y')\delta(z-z')\\ u(x-x')\delta(y-y')\delta'(z-z')\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\0\end{bmatrix} u(x-x')\delta(y-y')\delta(z-z'),\end{aligned} \tag{15}$$

where $\delta'(x-c) = \frac{\mathrm{d}}{\mathrm{d}x}\delta(x-c)$ is the unit impulse doublet function, $\delta(x-c)$ is the unit impulse function, and $u(x-c)$ is the unit step function [18]. For details of the derivation, please refer to Appendix A.

Substituting (14) and (15) into (13) yields an approximation of the first-order scattered field:

$$\begin{aligned}\mathbf{P}^1(\mathbf{r}) &\approx \underbrace{\frac{\mathrm{i}k}{2}\begin{bmatrix}0\\0\\1\end{bmatrix}\int_{x_{\min}}^{x} \Delta\varepsilon_r(x',y,z)\mathrm{d}x'}_{I_1} + \underbrace{\frac{\mathrm{i}}{2k}\begin{bmatrix}\frac{\partial\Delta\varepsilon_r(x,y,z)}{\partial z}\\ -\int_{x_{\min}}^{x}\frac{\partial^2\Delta\varepsilon_r(x',y,z)}{\partial y\partial z}\mathrm{d}x'\\ -\int_{x_{\min}}^{x}\frac{\partial^2\Delta\varepsilon_r(x',y,z)}{\partial z^2}\mathrm{d}x'\end{bmatrix}}_{I_2}\\ &\quad + \underbrace{\frac{1}{2}\begin{bmatrix}1\\0\\0\end{bmatrix}\int_{x_{\min}}^{x}\frac{\partial\Delta\varepsilon_r(x',y,z)}{\partial z}\mathrm{d}x'}_{I_3}.\end{aligned} \tag{16}$$

Here, we have considered that $\int_{-\infty}^{\infty} f(t)u(\tau-t)\mathrm{d}t = \int_{-\infty}^{\tau} f(t)\mathrm{d}t$, $\int_{-\infty}^{\infty} f(t)\delta(\tau-t)\mathrm{d}t = f(\tau)$ and $\int_{-\infty}^{\infty} f(t)\delta'(\tau-t)\mathrm{d}t = -f'(\tau)$.

In (16), the first-order scattered field is decomposed into three integrals $I_1,\ I_2,\ I_3$. Comparing the magnitudes of them shows that if the dielectric constant of the scatterer varies slowly, there should be

$$|I_1| \sim \mathcal{O}(kL\Delta\varepsilon_r), \quad |I_2| \sim \mathcal{O}\left(\frac{\Delta\varepsilon_r}{kL}\right), \quad |I_3| \sim \mathcal{O}(\Delta\varepsilon_r).$$

In (inverse) scattering study, the Born approximation for electrically large-scaled scatterers ($kL \gg 1$) attract much attention because the scattering integral equations for these scatterers are generally very difficult to solve. In this situation, the comparison of the above integrals gives

$$|I_2| \ll |I_3| \ll |I_1|.$$

So the first-order scattered field is mainly determined by I_1:

$$\mathbf{P}^1(\mathbf{r}) \approx I_1 = \frac{\mathrm{i}k}{2}\begin{bmatrix}0\\0\\1\end{bmatrix}\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x', \tag{17}$$

and its magnitude can be estimated as

$$\left|\mathbf{P}^1(\mathbf{r})\right| \approx \frac{k}{2}\left|\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x'\right| \leqslant \frac{k}{2}\left\|\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x'\right\| \triangleq \frac{k}{2}\|I\|, \tag{18}$$

where the norm is defined as $\|\cdot\| = \sup\limits_{\mathbf{r}\in V'} |\cdot|$. This norm is easy to obtain because only a one-dimensional non-oscillatory integral is involved. Specially, for a uniform scatterer ($\Delta\varepsilon_r(\mathbf{r})$ = constant), there should be $\|I\| = \Delta\varepsilon_r L$ with L being the scale of the scatterer in x direction. Obviously, the norm $\|I\|$ carries the units of meter, and the magnitude of $\mathbf{P}$ is unitless.

2.3. Estimation of Higher-order Scattered Fields

The second-order scattered field $\mathbf{P}^2(\mathbf{r})$ follows

$$\begin{aligned}\mathbf{P}^2(\mathbf{r}) \;=\; &\left(\mathcal{H}^2\mathbf{P}_\mathrm{i}\right) = \left(\mathcal{H}\mathbf{P}^1\right)(\mathbf{r}) = \iiint_{V'} k^2\Delta\varepsilon_r\mathbf{P}^1 G(\mathbf{r},\mathbf{r}')\mathrm{e}^{\mathbf{ik}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}'\\ &+\iiint_{V'}\left[\nabla'(\Delta\varepsilon_r)\cdot\mathbf{P}^1+\Delta\varepsilon_r\nabla'\cdot\mathbf{P}^1+\Delta\varepsilon_r\mathbf{P}^1\cdot(\mathrm{i}\mathbf{k})\right]\\ &\nabla G(\mathbf{r},\mathbf{r}')\mathrm{e}^{\mathbf{ik}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}'.\end{aligned} \tag{19}$$

According to (17), we have $\nabla' \cdot \mathbf{P}^1(\mathbf{r}') \approx 0$ and $\mathbf{P}^1(\mathbf{r}') \cdot \mathbf{k} \approx 0$. In this manner, (19) can be estimated as

$$\mathbf{P}^2(\mathbf{r})\approx\iiint_{V'} k^2\Delta\varepsilon_r \left\{\frac{\mathrm{i}k}{2}\begin{bmatrix}0\\0\\1\end{bmatrix}\int_{x'_{\min}}^{x'} \Delta\varepsilon_r(x'',y',z')\mathrm{d}x''\right\} G(\mathbf{r},\mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}'$$
$$+\iiint_{V'} \frac{\partial(\Delta\varepsilon_r)}{\partial z'}\left[\frac{\mathrm{i}k}{2}\int_{x'_{\min}}^{x'} \Delta\varepsilon_r(x'',y',z')\mathrm{d}x''\right]\nabla G(\mathbf{r},\mathbf{r}')\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\mathrm{d}^3\mathbf{r}'. \quad (20)$$

Substituting (14) and (15) into (20) and readopting the process of magnitude comparison in Section 2.2 yield the following approximation:

$$\mathbf{P}^2(\mathbf{r}) \approx \begin{bmatrix}0\\0\\1\end{bmatrix} \frac{\mathrm{i}k}{2}\int_{x_{\min}}^{x} \Delta\varepsilon_r(x',y,z)\left\{\frac{\mathrm{i}k}{2}\int_{x'_{\min}}^{x'} \Delta\varepsilon_r(x'',y',z')\mathrm{d}x''\right\}\mathrm{d}x'. \quad (21)$$

This approximation immediately leads to the following estimation of the second order scattered field:

$$\left|\mathbf{P}^2(\mathbf{r})\right| \lessapprox \left(\frac{k}{2}\left\|I\right\|\right)^2, \quad (22)$$

where $\lessapprox$ means "smaller than or approximate to". Moreover, the higher-order scattered field can be obtained in the same way:

$$\left|\mathbf{P}^n(\mathbf{r})\right| \lessapprox \left(\frac{k}{2}\left\|I\right\|\right)^n, \quad n = 3, 4, \ldots. \quad (23)$$

Therefore, the sum of the truncated terms shown in (12) can be estimated as

$$\left|\sum_{n=1}^{\infty}\mathcal{H}^n\mathbf{P}_{\mathrm{i}}\right| = \left|\sum_{n=1}^{\infty}\mathbf{P}^n\right| \lessapprox \sum_{n=1}^{\infty}\left(\frac{k}{2}\left\|I\right\|\right)^n. \quad (24)$$

If $\frac{k\|I\|}{2} < 1$ (this condition will be discussed later), (24) is simplified as

$$\left|\sum_{n=1}^{\infty}\mathcal{H}^n\mathbf{P}_{\mathrm{i}}\right| \lessapprox \frac{\frac{k}{2}\left\|I\right\|\left[1-\left(\frac{k}{2}\left\|I\right\|\right)^{\infty}\right]}{1-\frac{k}{2}\left\|I\right\|}. \quad (25)$$

Considering that the limit $\lim_{n\to\infty}\left(\frac{k}{2}\left\|I\right\|\right)^n = 0$ holds for $\frac{k\|I\|}{2} < 1$, estimation (25) can be simplified as

$$\left|\sum_{n=1}^{\infty}\mathcal{H}^n\mathbf{P}_{\mathrm{i}}\right| \lessapprox \frac{\frac{k}{2}\left\|I\right\|}{1-\frac{k}{2}\left\|I\right\|}. \quad (26)$$

2.4. A New Validity Criterion for Born Approximation

Substituting the estimation of the truncated terms (26) to condition (12) shows that the inequality

$$\frac{\frac{k}{2}\left\|I\right\|}{1-\frac{k}{2}\left\|I\right\|} \ll 1 \tag{27}$$

can serves as a sufficient condition to ensure the validity of Born approximation. Generally, a positive number a is believed to be much smaller than another positive number b if the ratio a/b is smaller than a small threshold q_{th} such as $q_{\mathrm{th}} = 0.1$. Therefore, introducing a threshold q_{th} to (27) gives the following sufficient condition for Born approximation:

$$\frac{k\left\|I\right\|}{2} \leqslant \frac{q_{\mathrm{th}}}{1+q_{\mathrm{th}}}. \tag{28}$$

From (28), the upper bound of valid frequency is yielded as

$$f \leqslant \frac{cq_{\mathrm{th}}}{\pi\left\|I\right\|(1+q_{\mathrm{th}})}, \tag{29}$$

where $c = 3\times10^8$ m/s is the velocity of light in free space. Considering that the threshold q_{th} is very small, it is also advisable to simplify the valid frequency as

$$f \leqslant \frac{cq_{\mathrm{th}}}{\pi\left\|I\right\|}. \tag{30}$$

It should be noted that during the derivation from (24) to (26), we have assumed $\frac{k\|I\|}{2} < 1$. Actually, this assumption is weaker than the sufficient condition (28) when a small threshold q_{th} is taken into account, so the assumption is acceptable and (29) can really serve as a sufficient condition to identify the validity of Born approximation.

Table 1. Parameters for the dielectric sphere.

Parameters	Value
Outmost radius of the sphere	1 m
Radial profile of $\Delta\varepsilon_r$	$10^{-2}/(1+30r^2)$
Number of layers	200, equal thickness
Detection distance R	500 m
Amplitude of incident wave $\lvert\mathbf{E}_{\mathrm{i}}\rvert$	1 V/m
Frequency f	10 MHz–10 GHz, 400 equi-spaced samples in the logarithmic scale

3. VERIFICATION OF THE CRITERION

In this section, we test the validity of the proposed criterion. The scattering characteristics of (homogenous or layered) dielectric spheres have been elaborately studied via the Mie theory [3, 33, 34]. In this work, we adopt a multi-layered dielectric sphere to verify the present criterion. The parameters for the dielectric sphere and the incident wave are listed in Table 1.

According to the parameters, the scattered electric field can be

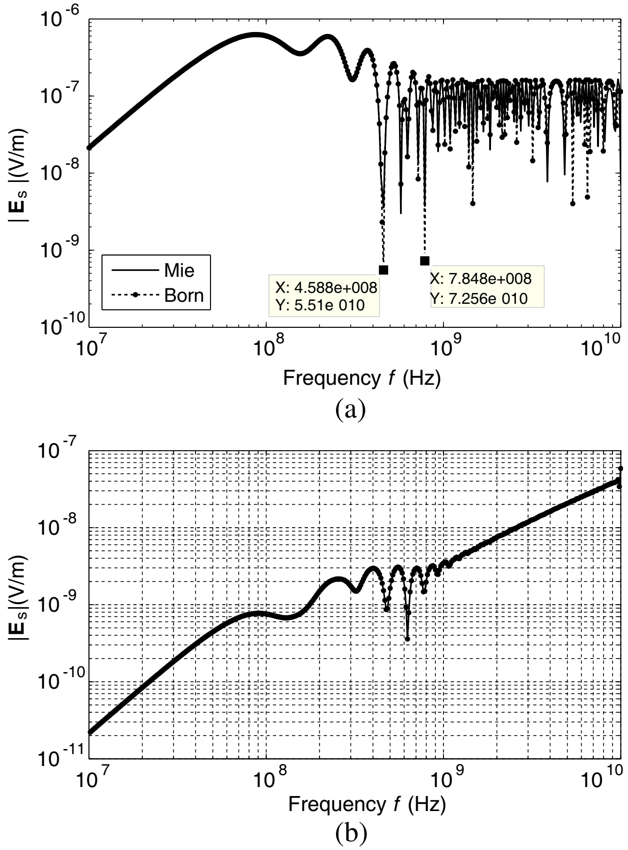

Figure 1. Relationship between the back-scattered field and the frequency while a multi-layered dielectric sphere is under consideration. (a) Scattered field obtained by the Born approximation and the Mie theory. (b) Difference in scattered electric field between the Born approximation and the Mie solution.

worked out via the Mie theory. For more details please refer to Appendix B. At the same time, the scattered field based on the Born approximation can be computed according to formula (3). During the processing, the oscillatory integral caused by the Green function $G(\mathbf{r}, \mathbf{r}')$ is computed with an accurate and efficient quadrature method developed by the present authors [20–22].

The relationship between the scattered field amplitude and the frequency are presented in Figure 1(a), and the difference between the two solutions (Mie theory and Born approximation) is presented in Figure 1(b). As shown in Figure 1(a), the two curves are almost coincident (the difference between them can only be observed at high frequencies); this indicates that the Born approximation can in fact obtain good results if weak scatterers are considered.

For better description of the solution error of Born approximation, the relative error of the scattered field at the receiver is considered, which follows

$$E_r = \left| \frac{E_{\text{s,Born}} - E_{\text{s,Mie}}}{E_{\text{s,Mie}}} \right|. \tag{31}$$

Figure 2 presents the relationship between the relative error and the frequency. It is observed that the relative error increases as a whole with the increase of frequency. In the low frequency band, E_r is very small, so the Born approximation is believed to be valid. As the frequency increases, E_r increases (although not smooth enough), and then the validity of Born approximation becomes worse and worse. At a very high frequency, E_r would be very large, and then the Born approximation becomes invalid.

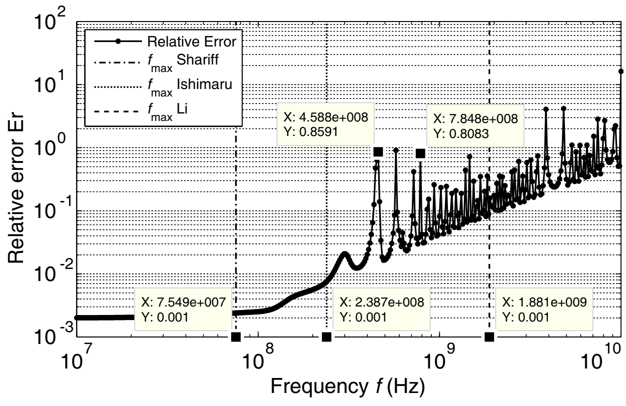

Figure 2. Relative error of Born approximation for a multi-layered dielectric sphere.

At the same time, although the error of scattered field is smooth (see Figure 1(b)), some special peak values are observed in the relative error curve (see Figure 2). The main reason for this phenomenon is as follows: the scattered field E_s serves as the denominator in the relative error (31), so the presence of valley values of scattered field may result in peak values on the relative error curve even if the error curve is smooth. For example, two valley values at $f = 4.588 \times 10^8$ Hz and $f = 7.848 \times 10^8$ Hz are indicated in Figure 1(a); they correspond to the two peak values as indicated in Figure 2. In this manner, employing the average values (not the peak values) to verify the present criterion would make more sense.

Based on Figure 2, we now intend to give a comparison of the existing criteria and the newly proposed one.

(i) For the criterion like $\Delta\varepsilon_r kL \ll 1$ [6, 16, 17], the valid frequency range follows

$$f \leqslant f_{\max}^{\text{Ishimaru}} = \frac{cq_{\text{th}}}{2\pi\|\Delta\varepsilon_r\|L}. \tag{32}$$

Here we have assumed that $a/b \leqslant q_{\text{th}}$ serves as a sufficient condition for $a \ll b$.

(ii) For the criterion like $\Delta\varepsilon_r\{1, kL, (kL)^2\} \ll 1$ [28], the dominant condition is $\Delta\varepsilon_r(kL)^2 \ll 1$ while electrically large-scaled scatterers are considered, so the valid frequency range follows

$$f \leqslant f_{\max}^{\text{Shariff}} = \frac{c\sqrt{\frac{q_{\text{th}}}{\|\Delta\varepsilon_r\|}}}{2\pi L}. \tag{33}$$

Obviously, this bound is smaller than (32) because $\Delta\varepsilon_r < q_{\text{th}}$ is involved in the criterion.

(iii) For the present criterion (see (30)), the valid frequency range follows

$$f \leqslant f_{\max}^{\text{Li}} = \frac{cq_{\text{th}}}{\pi\,\|I\|}, \tag{34}$$

where $\|I\| = \max_{\mathbf{r}\in V'}\left|\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x'\right|$ is an integral norm.

As is known, $\|I\| = \max_{\mathbf{r}\in V'}\left|\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x'\right| \leqslant \|\Delta\varepsilon_r\|L$, so we have

$$f_{\max}^{\text{Li}} > 2f_{\max}^{\text{Ishimaru}}.$$

Moreover, if $\Delta\varepsilon_r$ is with an evident peak distribution ($\Delta\varepsilon_r$ in a small region is much larger than that in other regions), then there should be $\|I\| = \max_{\mathbf{r}\in V'}\left|\int_{x_{\min}}^{x} \Delta\varepsilon_r(x', y, z)\mathrm{d}x'\right| \ll \|\Delta\varepsilon_r\|L$. This means that the following relationship will hold:

$$f_{\max}^{\text{Li}} \gg 2 f_{\max}^{\text{Ishimaru}}.$$

In this sense, our criterion can give much wider valid frequency in comparison with the traditional ones, especially when a non-uniform scatterer is under consideration.

In Figure 2, we have marked the valid frequency boundaries ($f_{\max}^{\text{Ishimaru}}, f_{\max}^{\text{Shariff}}, f_{\max}^{\text{Li}}$) with three vertical lines, where the threshold $q_{\text{th}} = 0.1$ is assumed. From this figure, we know:

(i) On the one hand, it is seen that the relative errors around the upper bound valid frequencies are all smaller than 0.1 (some peak values are larger than 0.1, but they are caused by dividing small scattered field E_s), so the Born approximation is believed to be valid there.

(ii) On the other hand, the valid frequency boundaries are $f_{\max}^{\text{Shariff}} = 75.5\,\text{MHz}$, $f_{\max}^{\text{Ishimaru}} = 238\,\text{MHz}$, and $f_{\max}^{\text{Li}} = 1.88\,\text{GHz}$, so we have $f_{\max}^{\text{Shariff}} < f_{\max}^{\text{Ishimaru}} < f_{\max}^{\text{Li}}$. In this sense, the present criterion can give a much wider valid frequency range, and it is more attractive.

4. CONCLUSIONS

A universal criterion to identify the validity of the Born approximation is proposed. Compared with the existing criteria, the new one can provide a much wider valid frequency range. This could provide help to the (inverse) scattering studies while the Born approximation is employed to alleviate the computational difficulty of solving the scattering integral equations.

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APPENDIX A. APPROXIMATION OF SPECTRAL GREEN FUNCTION

This appendix presents the derivation of approximation (14) and (15).

A.1. Proof of Approximation (14)

It is known that the spectral-domain Green function follows [1, 27]

$$G\left(\mathbf{r},\mathbf{r}'\right)=\left(\frac{1}{2\pi}\right)^3\lim_{\varepsilon\to0^+}\iiint\frac{\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}}{q^2-k^2-\mathrm{i}\varepsilon}\mathrm{d}^3\mathbf{q}.$$

According to this expression we have

$$G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}=\left(\frac{1}{2\pi}\right)^3\lim_{\varepsilon\to0^+}\iiint\frac{\mathrm{e}^{\mathrm{i}(\mathbf{q}-\mathbf{k})\cdot(\mathbf{r}-\mathbf{r}')}}{q^2-k^2-\mathrm{i}\varepsilon}\mathrm{d}^3\mathbf{q}.$$

Let $\mathbf{s}=\mathbf{q}-\mathbf{k}$, then

$$G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}=\left(\frac{1}{2\pi}\right)^3\lim_{\varepsilon\to0^+}\iiint\frac{\mathrm{e}^{\mathrm{i}\mathbf{s}\cdot(\mathbf{r}-\mathbf{r}')}}{s^2+2\mathbf{s}\cdot\mathbf{k}-\mathrm{i}\varepsilon}\mathrm{d}^3\mathbf{s},$$

where $s=|\mathbf{s}|$. Because the factor $\mathrm{e}^{\mathrm{i}\mathbf{s}\cdot(\mathbf{r}-\mathbf{r}')}$ is a highly oscillatory function, the integral result is chiefly determined by the region of $\mathbf{s}$ around $\mathbf{s}=0$, then we have the following approximation:

$$G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}\approx\left(\frac{1}{2\pi}\right)^3\lim_{\varepsilon\to0^+}\iiint\frac{\mathrm{e}^{\mathrm{i}\mathbf{s}\cdot(\mathbf{r}-\mathbf{r}')}}{2\mathbf{s}\cdot\mathbf{k}-\mathrm{i}\varepsilon}\mathrm{d}^3\mathbf{s}.$$

Assume the incident wave propagates to the scatterer in a fixed direction (x direction for example), there will be $\mathbf{s}\cdot\mathbf{k}=s_xk$ and the above-mentioned expression is further transformed as

$$\begin{aligned}G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}&\approx\left(\frac{1}{2\pi}\right)^3\lim_{\varepsilon\to0^+}\iiint\frac{\mathrm{e}^{\mathrm{i}\mathbf{s}\cdot(\mathbf{r}-\mathbf{r}')}}{2s_xk-\mathrm{i}\varepsilon}\mathrm{d}^3\mathbf{s}\\&=\frac{1}{2\pi}\lim_{\varepsilon\to0^+}\int_{-\infty}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_xk-\mathrm{i}\varepsilon}\mathrm{d}s_x\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}s_y(y-y')}\mathrm{d}s_y\cdot\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}s_z(z-z')}\mathrm{d}s_z\\&\approx\delta(y-y')\delta(z-z')\cdot\frac{1}{2\pi}\lim_{\varepsilon\to0^+}\int_{-\infty}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_xk-\mathrm{i}\varepsilon}\mathrm{d}s_x,\end{aligned}\tag{A1}$$

where we have considered $\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}s_y(y-y')}\mathrm{d}s_y=\delta(y-y')$ and $\frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}s_z(z-z')}\mathrm{d}s_z=\delta(z-z')$.

Obviously, the third integral on the right-hand side of (A1) has a pole $s_x^{\mathrm{p}}=\mathrm{i}\varepsilon/2k$ on the upper half plane (see Figure A1).

In complex analysis theory, this kind of integral can be computed with the residue theorem by introducing a particular integral path [24]. Figure A1 provides two candidate paths (C and C') and we are trying to choose a proper one according to the characteristic of the integral. These two paths are both semicircles with infinite radius, i.e.,

$$C:s_x=\lim_{R_c\to\infty}\mathrm{e}^{\mathrm{i}\theta}=\lim_{R_c\to\infty}\left(\cos\theta+\mathrm{i}\sin\theta\right),\theta\in(\pi,2\pi),$$

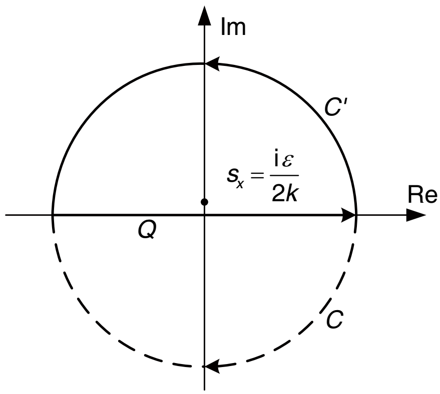

Figure A1. Cauchy integtral path for the integral with a pole.

and

$$C' : s_x = \lim_{R_c\to\infty} \mathrm{e}^{\mathrm{i}\theta} = \lim_{R_c\to\infty} (\cos\theta + \mathrm{i}\sin\theta), \theta \in (0, \pi).$$

Then we have

$$\mathrm{e}^{\mathrm{i}s_x(x-x')} = \lim_{R_c\to\infty} \mathrm{e}^{[-R_c(x-x')\sin\theta + \mathrm{i}R_c(x-x')\cos\theta]}.$$

Obviously, the proper integral path should be a path where the real part of the exponent is less than 0 (otherwise the integrand will becomes infinity), so the following condition has to be satisfied:

$$\left(x - x'\right)\sin\theta > 0.$$

If this condition holds, then there should be

$$\lim_{R_c\to\infty} \mathrm{e}^{[-R_c(x-x')\sin\theta + \mathrm{i}R_c(x-x')\cos\theta]} = 0.$$

In this manner, the integral result can be obtained as follows using the residual theorem:

(i) If $x - x' > 0$, we should choose C' as the proper integral path, and then the pole is located in the closed integral path $Q + C'$. Subsequently, according to the Cauchy's theorem and the residue theorem [24], we have

$$\frac{1}{2\pi} \lim_{\varepsilon\to 0^+} \int_{-\infty}^{+\infty} \frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_x k - \mathrm{i}\varepsilon} \mathrm{d}s_x$$
$$= \frac{1}{2\pi} \lim_{\varepsilon\to 0^+} \left\{ 2\pi\mathrm{i} \lim_{s_x\to\frac{\mathrm{i}\varepsilon}{2k}} \left[\frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_x k - \mathrm{i}\varepsilon} \left(s_x - \frac{\mathrm{i}\varepsilon}{2k} \right) \right] \right\} = \frac{\mathrm{i}}{2k},$$

(ii) otherwise if $x - x' < 0$, the proper integral path should be C, then the pole is excluded from the closed integral path $Q + C$. Subsequently, we have

$$\frac{1}{2\pi}\lim_{\varepsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_xk-\mathrm{i}\varepsilon}\mathrm{d}s_x=0.$$

In light of these two situations, the integral is finally obtained as

$$\frac{1}{2\pi}\lim_{\varepsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{\mathrm{e}^{\mathrm{i}s_x(x-x')}}{2s_xk-\mathrm{i}\varepsilon}\mathrm{d}s_x=\frac{\mathrm{i}}{2k}u(x-x'). \tag{A2}$$

Substituting (A2) into (A1) yields the following approximation:

$$G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\approx\frac{\mathrm{i}}{2k}u(x-x')\delta(y-y')\delta(z-z').$$

A.2. Proof of Approximation (15)

According to (14), the gradient of $G(\mathbf{r},\mathbf{r}')\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$ follows

$$\nabla\left[G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\right]\approx\frac{\mathrm{i}}{2k}\begin{bmatrix}\delta(x-x')\delta(y-y')\delta(z-z')\\u(x-x')\delta'(y-y')\delta(z-z')\\u(x-x')\delta(y-y')\delta'(z-z')\end{bmatrix},$$

where we have considered the relationship $u'(x-c)=\delta(x-c)$. Moreover, the expansion of this gradient follows

$$\nabla\left[G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\right]=\nabla G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}+\mathrm{i}\mathbf{k}G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}, \tag{A3}$$

so, there should be

$$\begin{aligned}\nabla G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\ \approx\ &\frac{\mathrm{i}}{2k}\begin{bmatrix}\delta(x-x')\delta(y-y')\delta(z-z')\\u(x-x')\delta'(y-y')\delta(z-z')\\u(x-x')\delta(y-y')\delta'(z-z')\end{bmatrix}\\&-\mathrm{i}\mathbf{k}G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}.\end{aligned} \tag{A4}$$

Considering that $\mathbf{k}=[1,0,0]^{\mathrm{T}}$, we then have the following approximation after substituting approximation (14) into (A4):

$$\begin{aligned}\nabla G\left(\mathbf{r},\mathbf{r}'\right)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}\ \approx\ &\frac{\mathrm{i}}{2k}\begin{bmatrix}\delta(x-x')\delta(y-y')\delta(z-z')\\u(x-x')\delta'(y-y')\delta(z-z')\\u(x-x')\delta(y-y')\delta'(z-z')\end{bmatrix}\\&+\frac{1}{2}\begin{bmatrix}1\\0\\0\end{bmatrix}u(x-x')\delta(y-y')\delta(z-z').\end{aligned}$$

This finishes the proof of (15).

APPENDIX B. COMPUTATION OF THE SCATTERED FIELD BY MIE THEORY

For a plane wave propagating to a dielectric sphere, the scattered field expressed in the spherical coordinate system reads [3, 23]

$$\begin{aligned} E_{s\theta} &= \frac{e^{ikR}}{ikR}\cos\phi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}(a_n\tau_n+b_n\pi_n), \\ E_{s\phi} &= \frac{e^{ikR}}{-ikR}\sin\phi\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}(a_n\pi_n+b_n\tau_n) \end{aligned} \tag{B1}$$

where $E_{s\theta}$ is the scattered far-field component in the scattering plane (defined by the incident and scattered direction), $E_{s\phi}$ is the orthogonal component, ϕ is the angle between the incident electric field and the scattering plane. For back-scattering problems, there should be

$$\pi_n = -(-1)^n\frac{n(n+1)}{2}, \quad \tau_n = (-1)^n\frac{n(n+1)}{2}, \quad \phi = \pi/2. \tag{B2}$$

At the same time, it is generally accurate enough to obtain the scattered field by summing only the first $N_c = x_L + 4x_L^{1/3} + 2$ terms in (B1). Therefore, the scattered field components can be simplified as

$$E_{s\theta} = 0, \quad E_{s\phi} = \frac{e^{ikR}}{ikR}\sum_{n=1}^{N_c}\frac{2n+1}{2}(-1)^n(a_n-b_n) \tag{B3}$$

In (B3), only the coefficients a_n and b_n are undetermined. They can be yielded with a very stable recurrence algorithm developed in [33], then the scattered field can be easily worked out .

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