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MEASUREMENTS OF PLANAR MICROWAVE CIRCUITS USING AN IMPROVED TRL CALIBRATION METHOD

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Abstract—In this paper, an improved TRL (Thru-Reflect-Line) calibration method is presented. This method is based on ten-term error model of a two-port vector network analyzer(VNA) measurement system. Eight error terms induced by fixtures as well as two leakage errors are derived directly from the S parameters of the calibration standards measured from the coaxial reference plane without converting S parameters to T parameters. To validate our algorithm, a microstrip device with a via hole and a coplanar waveguide transmission line are fabricated and calibrated using the present TRL calibration method and Engen's algorithm, respectively. The magnitudes and phases of S_{11} and S_{21} of the devices are compared. The consistency of the de-embedded results with those calibrated by Engen's TRL algorithm illustrates the validity of the TRL algorithm in this paper.

1. INTRODUCTION

Planar microwave circuits such as microstrip lines, coplanar waveguides (CPW) are widely used in microwave and millimeter-wave circuits [1–5]. It is necessary to acquire the scattering parameters of the circuits to make sure that their performances satisfy the needs. Unlike coaxial devices, the planar circuits cannot be connected directly to the coaxial ports of the vector network analyzer (VNA). Therefore, two or more fixtures are needed to physically connect the non-coaxial devices under test (DUTs) to the coaxial test ports [2, 3]. When using vector network analyzer to obtain the scatter parameters of non-coaxial

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DUTs, however, the S parameters of those fixtures are consequently introduced into the measurement results as errors. A major problem encountered is the need to separate the errors of the fixtures [6] from the measured S parameters.

The TRL (Thru-Reflect-Line) calibration, a full two-port calibration method using three standards: THRU, REFLECT and LINE, is often employed to obtain the parameters of the fixtures which, in other words, are also called error terms [6–12]. The TRL calibration method was first presented by Engen and Hoer in 1979 [7]. In Engen's work, scattering parameters are converted to transfer scattering parameters (T parameter) in order to obtain the error terms of the fixtures. This method is mainly based on T parameters. Extended works in [8–10] are mainly focused on improving the accuracy of the Engen's TRL calibration using multiple LINE standards. This method is known as multiline calibration method. In [6], the fixtures as well as the DUT are viewed as two-port networks characterized by S parameters instead of T parameters. The measurement system is then denoted by the cascading of the three networks.

In order to connect to VNA, planar microwave circuits are usually designed as the devices mounted in test fixtures. Therefore, in-fixture calibration method [12] is often used. The scheme structures of microstrip circuit as well as calibration standards are shown in Fig. 1. Two drills at each side of the devices are needed to fix the launchers which can connect the microstrip devices to the coaxial ports of VNA. Therefore, the parts with drills as well as launchers are viewed as fixtures and need to be calibrated. Calibration standards are shown in Fig. 1(b). The calibration standards in Fig. 1(b) are designed to meet the needs for in-fixture calibration.

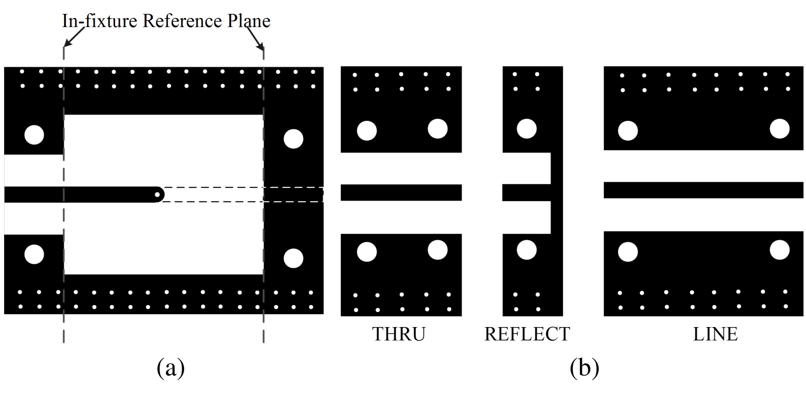


Figure 1. Scheme structures of microstrip device and calibration standards. (a) Microstrip device under test. (b) Calibration standards.

In this paper, eight error terms of fixtures as well as two leakage errors are considered when modeling the measurement system using signal flow networks. Then a TRL calibration procedure is performed. The ten error terms are solved directly from the raw S parameters of the calibration standards measured from coaxial reference plane. Unlike Engen's method, this technique is totally based on S parameters. In the procedure of error term computation, the problem of root selection [7, 13] is also encountered in our algorithm. A simple but useful strategy is proposed to select the proper roots. To validate our algorithm, a microstrip circuit and a coplanar waveguide circuit with calibration standards are designed and measured using PNA E8363B. The calibration standards are designed to meet the needs for in-fixture calibration. The de-embedded S parameters of the devices under test are compared with those calibrated by Engen's TRL method [7], and the agreements of the S parameters of DUTs show the good performance of the TRL technique in this paper.

2. TRL CALIBRATION THEORY

2.1. Error Terms Computation

In a VNA measurement system, the fixtures and DUT are regarded as two-port networks. A signal flow diagram cascaded by the three networks is shown in Fig. 2. The S parameters subscripted by 'A' and 'B' denote the networks of the left and right fixtures respectively. And the S parameters subscripted by 'M' denote the parameters measured from the coaxial reference plane by vector network analyzer, and those with 'X' are the S parameters of DUT. C_F and C_R denote the forward and reverse leakage error, respectively.

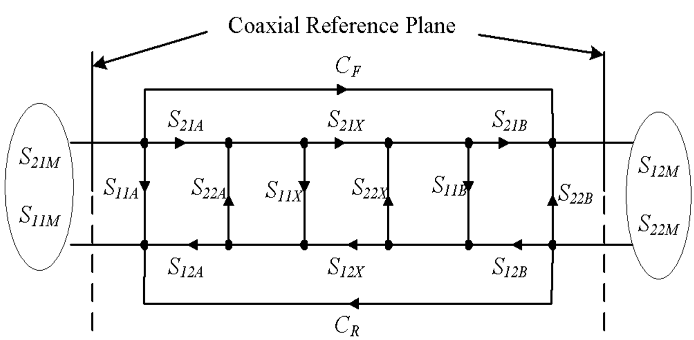


Figure 2. Two-port ten-term error model. S parameters are acquired by VNA from the coaxial reference plane.

Table 1. Scattering matrices of TRL calibration standards.

THRU	REFLECT	LINE
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix}$	$\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}$

The scattering matrices of the calibration standards are listed in Table 1, where Γ and X are the reflection coefficient of REFLECT and transmission coefficient of LINE respectively. Both Γ and X are unknown. Implementing TRL calibration procedure, three sets of S parameters are acquired from the coaxial reference plane. Let superscripts 'T', 'R' and 'L' denote the measured S parameters of THRU, REFLECT and LINE respectively. The mathematical expression of the error terms as well as Γ and X are solved and shown as follows.

$$C_F = S_{21M}^R, \qquad C_R = S_{12M}^R \tag{1}$$

$$S_{22A} = \frac{W}{\Gamma(1+W)}, \qquad S_{11B} = \frac{V}{\Gamma(1+V)} \tag{2}$$

$$S_{11A} = S_{11M}^T - \frac{\left(1 - AX^2\right)\left(S_{11M}^T - S_{11M}^L\right)}{1 - X^2} \tag{3}$$

$$S_{22B} = S_{22M}^T - \frac{\left(1 - AX^2\right)\left(S_{22M}^T - S_{22M}^L\right)}{1 - X^2} \tag{4}$$

$$T = S_{21A}S_{21B} = \left(S_{21M}^T - S_{21M}^R\right)(1 - A) \tag{5}$$

$$P = S_{12A}S_{12B} = \left(S_{12M}^T - S_{12M}^R\right)(1 - A) \tag{6}$$

$$Z = S_{21A}S_{12A} = \frac{S_{11M}^T - S_{11M}^L}{S_{11B}\left(\dfrac{1}{1-A} - \dfrac{X^2}{1-AX^2}\right)} \tag{7}$$

$$Y = S_{21B}S_{12B} = \frac{S_{22M}^T - S_{22M}^L}{S_{22A}\left(\dfrac{1}{1-A} - \dfrac{X^2}{1-AX^2}\right)} \tag{8}$$

where

$$X = -\frac{b}{2\left(S_{21M}^R - S_{21M}^T\right)\left(S_{12M}^L - S_{12M}^R\right)} \pm \frac{\sqrt{b^2 - 4\left(S_{21M}^R - S_{21M}^T\right)\left(S_{12M}^L - S_{12M}^R\right)\left(S_{21M}^L - S_{21M}^R\right)\left(S_{12M}^R - S_{12M}^T\right)}}{2\left(S_{21M}^R - S_{21M}^T\right)\left(S_{12M}^L - S_{12M}^R\right)} \tag{9}$$

$$b = \left(S_{21M}^R - S_{21M}^T\right)\left(S_{12M}^R - S_{12M}^T\right) + \left(S_{12M}^L - S_{12M}^R\right)\left(S_{21M}^L - S_{21M}^R\right) + \left(S_{22M}^L - S_{22M}^T\right)\left(S_{11M}^T - S_{11M}^L\right) \tag{10}$$

$$A = \frac{S_{22M}^T - S_{22M}^L}{\left(S_{12M}^T - S_{12M}^R\right) - \left(S_{12M}^L - S_{12M}^R\right)X} \cdot \frac{S_{11M}^T - S_{11M}^L}{\left(S_{21M}^T - S_{21M}^R\right) - \left(S_{21M}^L - S_{21M}^R\right)X} \tag{11}$$

$$W = \frac{S_{22M}^T - S_{22M}^L}{\left(S_{12M}^T - S_{12M}^R\right) - \left(S_{12M}^L - S_{12M}^R\right)X} \cdot \frac{S_{11M}^R - S_{11M}^T}{\left(S_{21M}^T - S_{21M}^R\right)(1-A)} + \frac{A}{1-A} \tag{12}$$

$$V = \frac{S_{11M}^T - S_{11M}^L}{\left(S_{21M}^T - S_{21M}^R\right) - \left(S_{21M}^L - S_{21M}^R\right)X} \cdot \frac{S_{22M}^R - S_{22M}^T}{\left(S_{12M}^T - S_{12M}^R\right)(1-A)} + \frac{A}{1-A} \tag{13}$$

$$\Gamma = \pm\sqrt{\frac{WV}{(1+W)(1+V)A}} \tag{14}$$

There are only three independent equations in (5)–(8). However, there is no need to solve the error terms S_{21A}, S_{12A}, S_{21B}, S_{12B}. The S parameters of DUT which are subscripted by 'X' can be calculated from the error terms, just as shown in (15)–(19).

$$S_{11X} = \frac{\frac{S_{11M}-S_{11A}}{Z} \cdot \left(1 + \frac{S_{22M}-S_{22B}}{Y} \cdot S_{11B}\right) - \frac{S_{21M}-C_F}{T} \cdot \frac{S_{12M}-C_R}{P} \cdot S_{11B}}{B} \tag{15}$$

$$S_{22X} = \frac{\frac{S_{22M}-S_{22B}}{Y} \cdot \left(1 + \frac{S_{11M}-S_{11B}}{Z} \cdot S_{22A}\right) - \frac{S_{21M}-C_F}{T} \cdot \frac{S_{12M}-C_R}{P} \cdot S_{22A}}{B} \tag{16}$$

$$S_{21X} = \frac{S_{21M} - C_F}{T \cdot B} \tag{17}$$

$$S_{12X} = \frac{S_{12M} - C_R}{P \cdot B} \tag{18}$$

where

$$B = \left(1 + \frac{S_{11M} - S_{11A}}{Z} \cdot S_{22A}\right) \cdot \left(1 + \frac{S_{22M} - S_{22B}}{Y} \cdot S_{11B}\right) - \frac{S_{21M} - C_F}{T} \cdot \frac{S_{12M} - C_R}{P} \cdot S_{11B} \cdot S_{22A} \tag{19}$$

2.2. Root Selection

In (9) and (14), both X and Γ contain two solutions. Since X is the transmission coefficient of LINE, the magnitude must satisfy the restriction as $|X| \leq 1$. And the phase should be continuous except at ± 180 degrees and keep decreasing from $+180$ degrees to -180 degrees and then jump back to $+180$ degrees again and repeat. Both the magnitude and phase change versus the frequency.

And for Γ, the magnitudes of the two roots at each frequency are equal to each other, and the phases have a difference of 180 degrees. Just as X, the angles of Γ must be continuous except ± 180 degrees. However, this is not enough for root selection of Γ. This restriction may induce two sets of roots, and only one should be chosen. The problem that we encountered now is which one to choose.

The calibration standard REFLECT can be viewed as a transmission line terminated with a resistance. The REFLECT can be described by a signal flow diagram shown in Fig. 3. The reflect coefficient Γ of REFLECT is given by

$$\Gamma = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \tag{20}$$

where Γ_L is reflection coefficient at the terminator of REFLECT and denoted as $|\Gamma_L|e^{j\theta}$. θ is the phase of Γ_L. Let the value of the resistance be R_L and the characteristic impedance of the transmission line be Z_0. Γ_L can be calculated by

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \tag{21}$$

We can choose the proper set of roots by estimating the phase of Γ at the start frequency according to (20) and (21). Suppose that the transmission line is ideal, that is $S_{11} = S_{22} = 0$ and $S_{21} = S_{12} = e^{j\beta l}$, where l and β are the length and phase constant of the transmission line component respectively. β can be calculated using EDA tools. Therefore,

$$\Gamma_L = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{21}S_{12}\Gamma_L = |\Gamma_L|\, e^{j(\theta + 2\beta l)} \tag{22}$$

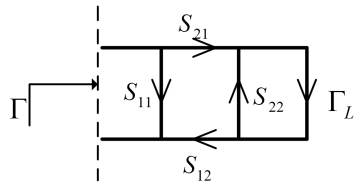


Figure 3. Signal flow diagram of REFLECT.

The proper set of roots is the one that the phase at the start frequency is closer to $(\theta + 2\beta l)$.

3. EXPERIMENTS AND RESULTS

3.1. Fixtures, Calibration Standards and Devices under Test

To validate the TRL calibration method proposed in this paper, two sets of planar microwave circuits are fabricated. In Fig. 4, the microstrip lines are built on NH9320 substrate of which the thickness is 0.381 mm and the relative dielectric constant is 3.2. The widths of the microstrip lines are 0.92 mm. The diameter of the via is 0.7 mm. The lengths of THRU, REFLECT and LINE are 10.4 mm, 7.0 mm and 21.4 mm, respectively. Two launchers are fixed on the test board through drills so that the test board can be connected to VNA. In Fig. 5, four coplanar waveguide transmission lines are fabricated. The lengths of the DUT, THRU, REFLECT and LINE are 40 mm, 5 mm, 3 mm and 10 mm, respectively. The substrate is alumina, of which the relative dielectric constant is 9.9 and the height is 0.127 mm. The widths of the center strip, the slot and the finite ground of the coplanar waveguide transmission lines are 0.35 mm, 0.1 mm and 20 mm, respectively. Since launchers cannot be fixed to the coplanar waveguide circuits through drills just as microstrip circuits, 3680 V, a universal test fixture of Anritsu [14], is used to complete the measurements.

3.2. Experiment Results

The microstrip DUT and calibration standards are measured from 10 MHz to 20 GHz, and the coplanar waveguide devices are measured from 10 MHz to 36 GHz using PNA E8363B. The REFLECT in Fig. 4

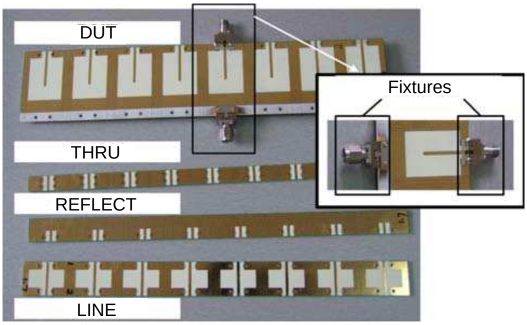
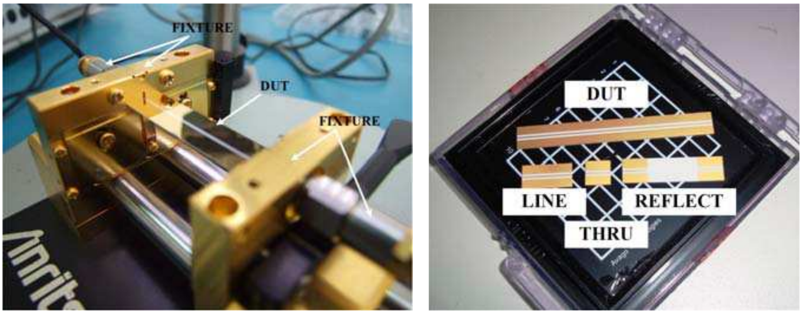
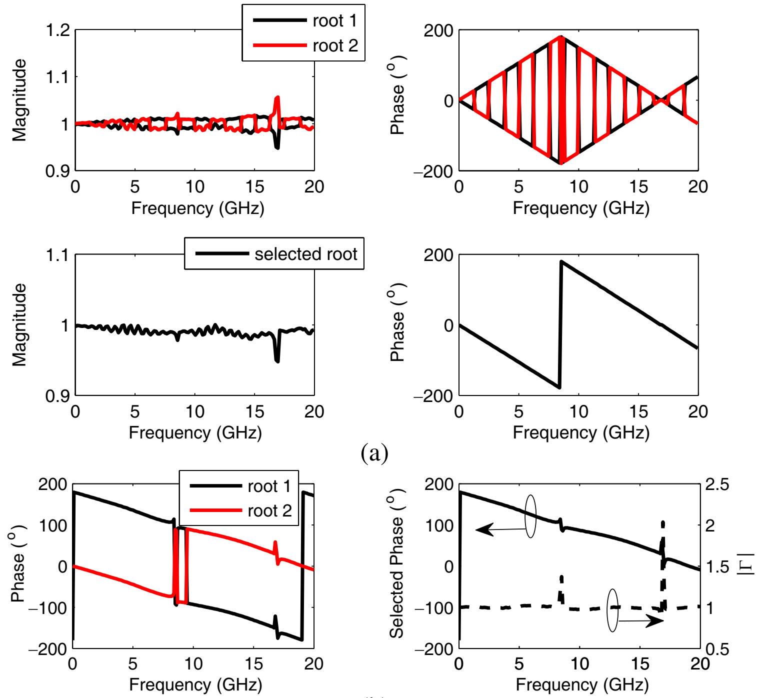


Figure 4. Fixtures, DUTs and calibration standards of microstrip devices.



(a) (b)

Figure 5. Fixture, DUT and calibration standards of coplanar waveguide circuits. (a) Universal test fixture of Anritsu, 3680 V. (b) DUT and calibration standards.



(b)

Figure 6. Root selection for parameters of microstrip calibration standards. (a) Root selection for parameter X of calibration standard LINE. (b) Root selection for parameter Γ of calibration standard REFLECT.

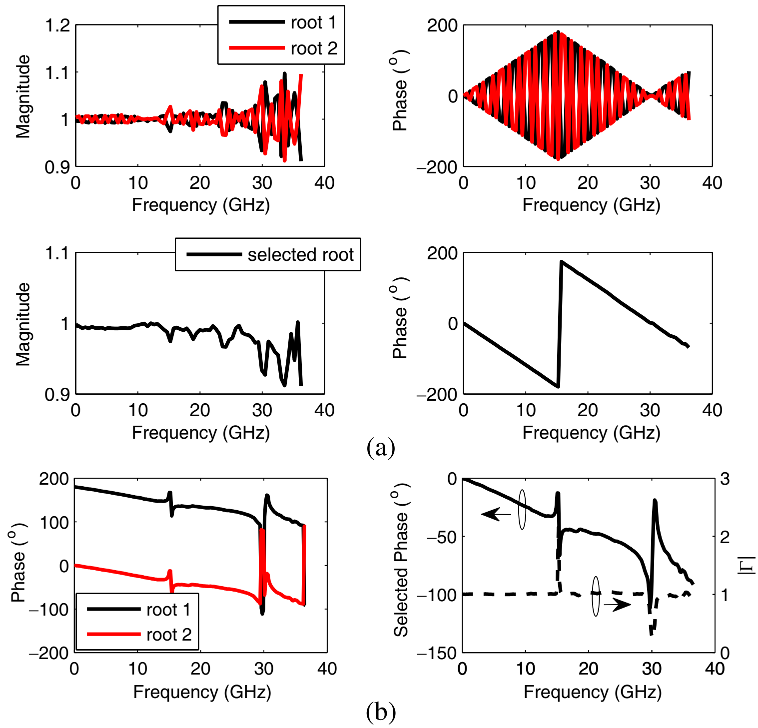


Figure 7. Root selection for parameters of coplanar waveguide calibration standards. (a) Root selection for parameter X of calibration standard LINE. (b) Root selection for parameter Γ of calibration standard REFLECT.

is 1.8 mm long with respect to half of the THRU. The terminated resistance is 0 (short), and the value of θ is -180 degrees. The phase βl at 10 MHz calculated by LineCalc utility of ADS is 0.035 degrees. Therefore, the phase of REFLECT at 10 MHz is approximately -179.93 degrees. The roots of X and those of Γ are shown in Fig. 6. For the coplanar devices, the proper roots of calibration standards are shown in Fig. 7.

The de-embedded results are shown in Fig. 8 and Fig. 9. Fig. 8 shows the S parameters of microstrip device under test compared with those calibrated using Engen's algorithm. Fig. 9 shows the S parameters of coplanar waveguide device under test compared with those calibrated using Engen's algorithm.

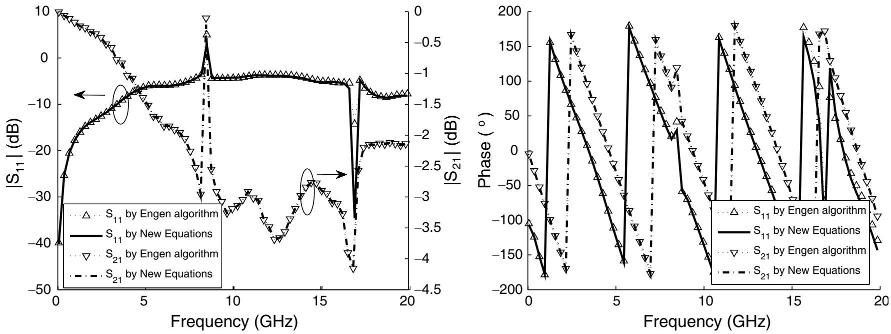


Figure 8. The S parameters of the microstrip device under test.

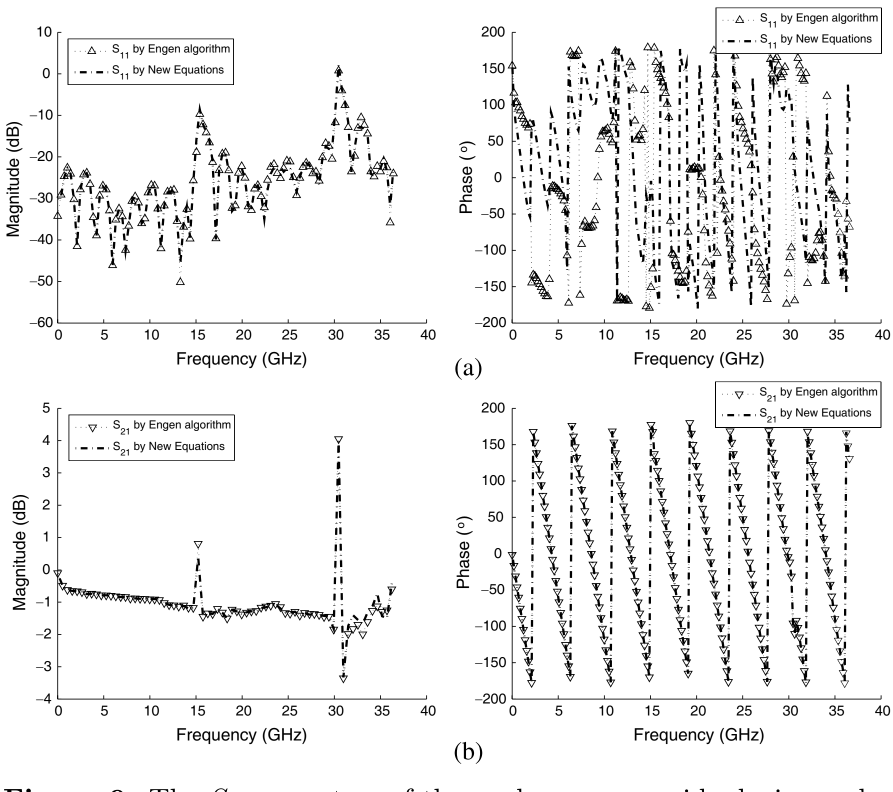


Figure 9. The S parameters of the coplanar waveguide device under test.

3.3. Discussion

In literatures [6] and [7], the insertion phase of LINE should be between 20 and 160 degrees with respect to the THRU, and the length should not be $\lambda/2$. That is the insertion phase of LINE which is approaching 0 degrees or ± 180 degrees with respect to the THRU can lead to bad calibration results. In Fig. 6, the insertion phase of LINE respected to THRU is -178 degrees at 8.4 GHz and 0.19 degrees at 16.95 GHz. Therefore, the calibration results in Fig. 8 at 8.4 GHz and 16.95 GHz are not reliable. This unfortunate situation also occurrs in Fig. 9 at 15.23 GHz and 30.28 GHz. In this case, multiple lines are needed [6] to solve the ill-conditioned problem.

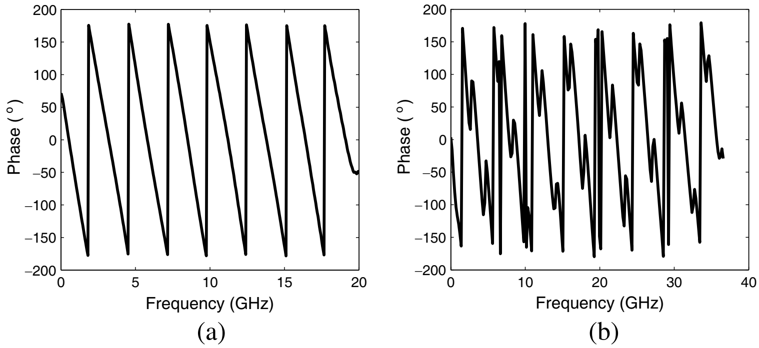


Figure 10. Raw phase of reflection coefficients of microstrip device and CPW device under test. (a) S_{11} of microstrip device. (b) S_{11} of CPW device.

The calibrated results agree well except the phase of S_{11} in Fig. 9(a). In the course of coplanar waveguide devices research, it is difficult for researchers to acquire accurate S_{11} [15–17]. As shown in [17], different calibration methods such as LRRM, LRM, SOLT and ML-TRL can induce large phase deviation for S_{11} of CPW transmission lines. The raw phases of S_{11} of the microstrip and the CPW device under test are shown in Fig. 10. For the CPW device with finite ground in this paper, the reason of the chaotic phase in Fig. 9(a) may be the bad performance of the coplanar waveguide device under test, which may be caused by the uncomplete ground connection between fixtures and CPW devices.

4. CONCLUSION

The good consistency of the de-embedded results with those calibrated by Engen's algorithm shows the validation of the TRL calibration method in this paper.

In Engen's algorithm, the raw S parameters of THRU, LINE and DUT are converted into T parameters firstly. Then the de-embedded T parameters of the DUT are derived using Engen's algorithm. An conversion algorithm is implemented again to transform the de-embedded T parameters of the DUT to the S parameters. Unlike Engen's method, the present method is directly based on S parameters. The ten error terms are directly solved from the raw S parameters of the calibration standards. And the real S parameters of the DUT are derived from the error terms and raw S parameters. Therefore, this calibration method is much easier for computation.

The strategy for root selection is easy and valid. Using this strategy, the values of Γ and X can be selected properly. Because the difference of the two roots of Γ is 180 degrees, the approximation in (22) will not affect the results of root selection.

The fixtures in this TRL calibration theory are regarded as two arbitrary two-port nonreciprocal linear networks. Therefore, this calibration method is available for both coaxial and non-coaxial calibrations.

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APPENDIX A. DERIVATION OF ERROR TERMS

The signal flow diagram of a two-port VNA measurement system is shown in Fig. 2. According to the Mason's rule, the expression of the S parameters measured from the coaxial reference plane are shown in (A1)–(A3).

$$S_{11M} = \frac{S_{21A}S_{12A}S_{11X}\left(1-S_{11B}S_{22X}\right)+S_{21A}S_{12A}S_{21X}S_{12X}S_{11B}}{D} + S_{11A} \tag{A1}$$

$$S_{22M} = \frac{S_{21B}S_{12B}S_{22X}\left(1-S_{22A}S_{11X}\right)+S_{21B}S_{12B}S_{21X}S_{12X}S_{22A}}{D} + S_{22B} \tag{A2}$$

$$S_{21M} = C_F + \frac{S_{21B}S_{21A}S_{21X}}{D}, \quad S_{12M} = C_R + \frac{S_{12B}S_{12A}S_{12X}}{D} \tag{A3}$$

where

$$D = 1 - S_{22A}S_{11X} - S_{11B}S_{22X} - S_{22A}S_{11B}S_{12X}S_{21X} + S_{22A}S_{11B}S_{11X}S_{22X} \tag{A4}$$

From (A1)–(A4), the real S parameters of DUT which are subscripted by 'X' can be solved, just as shown in (A5)–(A8).

$$S_{11X} = \frac{1}{B} \cdot \left(\frac{S_{11M} - S_{11A}}{S_{21A}S_{12A}} \cdot \left(1 + \frac{S_{22M} - S_{22B}}{S_{21B}S_{12B}} \cdot S_{11B} \right) - \frac{S_{21M} - C_F}{S_{21A}S_{21B}} \cdot \frac{S_{12M} - C_R}{S_{12A}S_{12B}} \cdot S_{11B} \right) \tag{A5}$$

$$S_{22X} = \frac{1}{B} \cdot \left(\frac{S_{22M} - S_{22B}}{S_{21B}S_{12B}} \cdot \left(1 + \frac{S_{11M} - S_{11A}}{S_{21A}S_{12A}} \cdot S_{22A} \right) - \frac{S_{21M} - C_F}{S_{21A}S_{21B}} \cdot \frac{S_{12M} - C_R}{S_{12A}S_{12B}} \cdot S_{22A} \tag{A6}$$

$$S_{21X} = \frac{1}{B} \cdot \frac{S_{21M} - C_F}{S_{21A}S_{21B}} \qquad S_{12X} = \frac{1}{B} \cdot \frac{S_{12M} - C_R}{S_{12A}S_{12B}} \tag{A7}$$

where

$$B = \left(1 + \frac{S_{11M} - S_{11A}}{S_{21A}S_{12A}} \cdot S_{22A} \right) \cdot \left(1 + \frac{S_{22M} - S_{22B}}{S_{21B}S_{12B}} \cdot S_{11B} \right) - \frac{S_{21M} - C_F}{S_{21A}S_{21B}} \cdot \frac{S_{12M} - C_R}{S_{12A}S_{12B}} \cdot S_{11B} \cdot S_{22A} \tag{A8}$$

The scattering matrices of the calibration standards are listed in Table 1. Substituting the scattering matrix of THRU into (A5)–(A8), namely letting $S_{11X} = S_{22X} = 0$ and $S_{21X} = S_{12X} = 1$, four equations are established, just as shown in (A9)–(A12).

$$M_1 = S_{11M}^T = S_{11A} + \frac{S_{11B}S_{21A}S_{12A}}{1 - S_{22A}S_{11B}} \tag{A9}$$

$$M_2 = S_{22M}^T = S_{22B} + \frac{S_{21B}S_{12B}S_{22A}}{1 - S_{22A}S_{11B}} \tag{A10}$$

$$M_3 = S_{21M}^T = C_F + \frac{S_{21A}S_{21B}}{1 - S_{22A}S_{11B}} \tag{A11}$$

$$M_4 = S_{12M}^T = C_R + \frac{S_{12A}S_{12B}}{1 - S_{22A}S_{11B}} \tag{A12}$$

Then substituting the scattering matrix of REFLECT into (A5)–(A8), namely letting $S_{11X} = S_{22X} = \Gamma$ and $S_{21X} = S_{12X} = 0$, another four equations are established, just as shown in (A13)–(A16).

$$M_5 = S_{11M}^R = S_{11A} + \frac{\Gamma S_{21A} S_{12A}}{1 - \Gamma S_{22A}} \tag{A13}$$

$$M_6 = S_{22M}^R = S_{22B} + \frac{\Gamma S_{21B} S_{12B}}{1 - \Gamma S_{11B}} \tag{A14}$$

$$M_7 = S_{21M}^R = C_F \tag{A15}$$

$$M_8 = S_{12M}^R = C_R \tag{A16}$$

Finally substituting the scattering matrix of LINE into (A5)–(A8), namely letting $S_{11X} = S_{22X} = 0$ and $S_{21X} = S_{12X} = X$, another four equations are established, just as shown in (A17)–(A20).

$$M_9 = {S^L}_{11M} = S_{11A} + \frac{S_{21A} S_{12A} S_{11B} X^2}{1 - S_{22A} S_{11B} X^2} \tag{A17}$$

$$M_{10} = {S^L}_{22M} = S_{22B} + \frac{S_{21B} S_{12B} S_{22A} X^2}{1 - S_{22A} S_{11B} X^2} \tag{A18}$$

$$M_{11} = {S^L}_{21M} = C_F + \frac{S_{21A} S_{21B} X}{1 - S_{22A} S_{11B} X^2} \tag{A19}$$

$$M_{12} = {S^L}_{12M} = C_R + \frac{S_{12A} S_{12B} X}{1 - S_{22A} S_{11B} X^2} \tag{A20}$$

To simplify the equations above, let

$$A = S_{22A} S_{11B} \tag{A21}$$

$$D = \frac{S_{22A}}{S_{21A} S_{12A}}, \qquad E = \frac{S_{11B}}{S_{21B} S_{12B}} \tag{A22}$$

$$W = \frac{\Gamma S_{22A}}{1 - \Gamma S_{22A}}, \qquad V = \frac{\Gamma S_{11B}}{1 - \Gamma S_{11B}} \tag{A23}$$

$$K = S_{21A} S_{12A} S_{11B}, \qquad H = S_{21B} S_{12B} S_{22A} \tag{A24}$$

$$T = S_{21A} S_{21B}, \qquad P = S_{12A} S_{12B} \tag{A25}$$

$$Z = S_{21A} S_{12A}, \qquad Y = S_{21B} S_{12B} \tag{A26}$$

The variables in (A21)–(A26) as well as Γ can be solved from (A9)–(A20), just as shown in (A27)–(A32).

$$A = \frac{M_2 - M_{10}}{(M_4 - M_8) - (M_{12} - M_8)\, X} \cdot \frac{M_1 - M_9}{(M_3 - M_7) - (M_{11} - M_7)\, X} \tag{A27}$$

$$D = \frac{M_2 - M_{10}}{(M_4 - M_8) - (M_{12} - M_8)\, X} \cdot \frac{1}{(M_3 - M_7)\,(1 - A)} \tag{A28}$$

$$K = \frac{M_1 - M_9}{\dfrac{1}{1 - A} - \dfrac{X^2}{1 - AX^2}}, \qquad H = \frac{M_2 - M_{10}}{\dfrac{1}{1 - A} - \dfrac{X^2}{1 - AX^2}} \tag{A29}$$

$$E = \frac{M_1 - M_9}{(M_3 - M_7) - (M_{11} - M_7)\, X} \cdot \frac{1}{(M_4 - M_8)\,(1 - A)} \tag{A30}$$

$$W = (M_5 - M_1)\, D + \frac{A}{1 - A}, \qquad V = (M_6 - M_2)\, E + \frac{A}{1 - A} \tag{A31}$$

$$\Gamma = \pm\sqrt{\frac{WV}{(1 + W)\,(1 + V)\, A}} \tag{A32}$$

According to (A11), (A12), (A15), (A16), (A19) and (A20), the relations in (A33) are established.

$$\frac{M_{11} - M_7}{M_3 - M_7} = \frac{M_{12} - M_8}{M_4 - M_8} = \frac{(1 - A)\, X}{1 - AX^2} \tag{A33}$$

The transmission coefficient X of LINE is solved according to (A27) and (A33), just as shown in (A34).

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{A34}$$

where

$$a = (M_7 - M_3)\,(M_{12} - M_8)\,, \quad c = (M_{11} - M_7)\,(M_8 - M_4) \tag{A35}$$

$$\begin{aligned} b = {} & (M_7 - M_3)\,(M_8 - M_4) + (M_{12} - M_8)\,(M_{11} - M_7) \\ & + (M_{10} - M_2)\,(M_1 - M_9) \end{aligned} \tag{A36}$$

Now ten error terms are solved from (A21)–(A26) according to (A27)–(A32) and shown in (1)–(8). The solutions of Γ and X are in (A32) and (A34). Substituting (1)–(8) into (A5)–(A7), the S parameters of DUT can be calculated, just as shown in (15)–(19).

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