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ELECTRIC CHARGES THAT BEHAVE AS MAGNETIC MONOPOLES

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Abstract—The memristor theory of Chua [1] provides a connection with the charge and magnetic flux in an electric circuit. We define a similar relation for the electric and magnetic flux densities in electromagnetism. Such an attempt puts forward interesting results. For example, the magnetic charges do not exist in nature however the electric charges behave as the magnetic monopoles in special media. We support our theory with the results of the recent experiments on materials named as spin ice.

1. INTRODUCTION

In 1971, Leon Chua proposed a fourth circuit element that connects the magnetic flux and electric charges in an electric circuit [1]. The importance of this attempt is the proposal of a relation between flux and charge. If we apply a voltage on a condenser, the electric charges will be collected. In a same manner, the flow of the electric current on an inductance causes the accumulation of the magnetic flux on this element. If the current flows on a resistance, a voltage will come into existence between the tips of the element. The inductance and capacitance deposit energy where the resistance changes the electrical energy into heat. These inferences of the circuit theory are in harmony with the electromagnetic theory of Maxwell [2]. The constitutive relations that combine the electric current, electric flux and magnetic flux densities with the electric and magnetic field intensities have a one to one correspondence with the equations of the circuit elements.

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Leon and recently Kavehei et al. [3] analyzed the memristor in terms of electromagnetism and they proposed a closed form relation between the magnetic and electric flux densities, but they did not mathematically formulize the proposal. The memristor could not have been realized till 2008. A group of researchers from HP obtained a nano-scale device that shows the hysteretic current-voltage graphic of a memristor [4, 5]. The studies on the memristor are generally focused on the current-voltage behavior of the element, but the relation between the flux and charges has not been fully taken into account [6–9]. As mentioned above, the electromagnetic side of memristor and its effects on the electromagnetic theory has not been sufficiently investigated yet.

The aim of this paper is to put forward an electromagnetic correspondent of the concept of memristor. The aspect of the relation between the charge and flux is the key point of our analysis. First of all, we will outline the correspondence between the circuit theory and electromagnetism. Based on this foundation, we will propose a constitutive relation between the magnetic and electric flux densities. Thus we will formulize the electromagnetic correspondent of memristor as a medium. Then we will derive the Maxwell's equations for the electromagnetic fields in the proposed medium. We will also support our theory by calling the reader's attention to the recent developments on the magnetic monopoles, observed in media, named as spin ice [10]. The magnetic (or Dirac) monopoles represent the magnetic charges that are only south or north poles. This concept was first suggested by Dirac in 1931 [11]. However, the magnetic charges could have not been observed in nature till 2008 [12–14]. Our theory will also suggest a solution to the problem of magnetic monopoles. An extended version of electromagnetism has also important effects on optics, because the optics is mainly based on the electromagnetic theory [15].

2. THE RELATION BETWEEN THE CIRCUIT THEORY AND CONSTITUTIVE EQUATIONS

Besides memristor, there are three circuit elements, namely resistance (R), inductance (L) and capacitance (C). The circuit equations of these elements can be given by the relations of $v = iR$, $q = Cv$ and $\phi = Li$, which connects the electrical and magnetic quantities to each other. v, i, q and ϕ represent the voltage, current, electric charge and magnetic flux, respectively. The constitutive equations of electromagnetism can be written as $\vec{J}_e = \sigma\vec{E}$, $\vec{D} = \varepsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ that construct relation between the conduction current ($\vec{J}_e$), electric flux ($\vec{D}$), magnetic flux ($\vec{B}$) densities and electric ($\vec{E}$) and magnetic

($\vec{H}$) field intensities. The conductivity (σ), permittivity (ε) and permeability (μ) of electromagnetism are the equivalents of resistance, conductance and inductance of the circuit theory, respectively. The units of σ, ε and μ are also in harmony with the circuit elements. For example, the permittivity is the correspondent of capacitance and its unit is Farad/meters. It is important to note that the quantities that correspond to the circuit elements of R, L and C represent the electromagnetic properties of the media. The fourth circuit element links the magnetic flux with the electric charges as

$$\phi = Mq \tag{1}$$

for $M$is the memristance.

3. THE MEMRISTIVE MEDIUM

We propose the constitutive relation of

$$\vec{B} = \gamma\vec{D} \tag{2}$$

as an electromagnetic equivalent of Equation (1). γ represents the memristivity of a medium. Equation (2) states that the electric flux density causes the creation of magnetic flux density in a memristive medium. If the divergence of the two sides of Equation (2) is taken, the expression that represents a new form of the Maxwell-Gauss equation will be obtained as

$$\nabla \cdot \vec{B} = \nabla \cdot \left(\gamma\vec{D}\right), \tag{3}$$

which yields

$$\nabla \cdot \vec{B} = \gamma\nabla \cdot \vec{D}, \tag{4}$$

for a homogeneous and isotropic memristive medium. Equation (4) becomes

$$\nabla \cdot \vec{B} = \gamma\rho_v, \tag{5}$$

because the divergence of the electric flux density is equal to the electric charge density. Equation (5) represents the effect of the memristivity of a medium to the Maxwell equations. In the actual form, the divergence of the magnetic flux density is equal to zero since there are not any magnetic charge density in nature. However, in the antenna theory, the right hand-side of the Maxwell-Gauss equation of the magnetic field is equated to an equivalent or fictitious magnetic charge density in order to model some types of antennas [16]. For example, the loop antennas can be represented by an equivalent dipole antenna, on which a magnetic current flows. The magnetic charge densities also oscillate on the tips of such an equivalent antenna. Such a configuration creates the same radiated electromagnetic field with an electrical loop antenna at the observation point. Another example is the usage of the magnetic current densities in the equivalent source theorem, which is used for the investigation of the aperture antennas [17–20]. But this mathematical model does not give any physical insight about the existence or nature of the magnetic monopoles. Equation (11) can be arranged as

$$\oiint_S \vec{B} \cdot \vec{n} dS = \iiint_V \gamma \rho_v dV \tag{6}$$

with the aid of the divergence theorem. V is a volume that surrounds the charges and S is the closed surface of the volume. $\vec{n}$ is the unit normal vector of the surface. The electromagnetic property of the inside of the volume is modeled by γ. Equation (6) states that the electric charges, collected in a memristive medium, creates a magnetic flux density that flows through the closed surface, which surround the volume. In fact Equation (5), is also valid for the static case when the charges and the field do not change with time.

As a second attempt, we will deal with the case when there is a current density in the memristive medium. First of all we introduce the continuity relations of

$$\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0 \tag{7}$$

and

$$\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0 \tag{8}$$

for $\sigma = 0$ and $\sigma \neq 0$, respectively. $\vec{J}$ is the electric current density. Equations (7) and (6) can be derived by taking the divergence of the Maxwell-Ampere equation of

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \sigma \vec{E} + \vec{J}. \tag{9}$$

We take into account the case of $\sigma = 0$. The Maxwell-Faraday equation can be written as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{M} \tag{10}$$

for $\vec{M}$ is an unknown vector. The expression of

$$-\gamma \frac{\partial \rho_v}{\partial t} + \nabla \cdot \vec{M} = 0 \tag{11}$$

can be obtained when the divergence of the two sides of Equation (10) is taken. Thus $\vec{M}$ is found to be

$$\vec{M} = -\gamma \vec{J} \tag{12}$$

when Equation (11) is compared with Equation (7). As a second step, we rewrite the Maxwell-Faraday equation as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \gamma \vec{J} - \eta \vec{H} \tag{13}$$

where η is the magnetic conductance. Equation (13) reads

$$-\gamma \frac{\partial \rho_v}{\partial t} - \gamma \nabla \cdot \vec{J} - \eta \nabla \cdot \vec{H} = 0 \tag{14}$$

when the divergence of the equation is taken. Equation (20) can be arranged as

$$\gamma \left(\frac{\partial \rho_v}{\partial t} + \nabla \cdot \vec{J} + \frac{\eta}{\mu} \rho_v \right) = 0 \tag{15}$$

by taking into account Equation (5). Thus the magnetic conductivity can be defined as

$$\eta = \frac{\sigma \mu}{\varepsilon} \tag{16}$$

if Equation (15) is compared with Equation (8). Equation (13) represents an extended version of the Maxwell-Faraday equation in a memristive medium. The integral form of this equation reads

$$\oint_C \vec{E} \cdot d\vec{l} = \iint_S \left(\frac{\partial \vec{B}}{\partial t} + \gamma \vec{J} + \eta \vec{H} \right) \cdot \vec{n} dS \tag{17}$$

with the aid of the Stokes theorem. Equation (17) states that the time harmonic variation of the magnetic flux density, the conductance and induction currents in a surface of the memristive medium creates an electric field along a contour C, which surrounds the surface.

Spin ice is a crystal structure of phyrochlore lattice, which is formed of corner sharing tetrahedra. There are two kinds of spin ice as $Dy_2Ti_2O_7$ and $Ho_2Ti_2O_7$ where Dy and Ho represent dysprosium and holmium respectively. These crystals have an analogous characteristic, which makes them candidates for the sources of magnetic dipoles, with the water ice. The elementary construction of the spin ice crystal obeys the ice rule according to which the four spins, at the corners, are directed along the center of the tetrahedron. The two of these turn towards the inward direction whereas the other two point through outwards. These spins can be separated under thermal energy and the single spins forms equivalent magnetic monopoles [10, 21]. One monopole has three spins and other has one spin. This behavior of spin ice is the result of the thermal defects. The interesting part of this phenomenon is the particle that causes the spin location in the tetrahedral. These are the magnetic moments of the positive ions of

Ho^{3+} and Dy^{3+} that behave as two states [22–24]. The existence of the Dirac monopoles in spin ice is detected by the experiments of neutron scattering [25, 26]. The spins that create the equivalent magnetic monopoles are the results of the positive ions.

It is interesting to note that the movement of the equivalent magnetic charges does not create an electric field. The construction of the magnetic currents in spin ice puts forward the reason of this statement. The Dy or Ho ions, located at the corners of the tetrahedron lattice, have a spin arrangement of two-inwards and two-outwards. Thus the net magnetic field is neutral in normal conditions. However, this arrangement can be distorted as one-inwards, three-outwards or three-inwards, one outwards because of structural defects. This construction creates a magnet, composed of two tetrahedron crystals. Each of these crystals represents a pole of the magnet. When a magnetic field is applied to such a structure, the poles will be separated and move in the spin ice thus creating a magnetic current [27, 28]. However the Dy or Ho ions do not move. Only their spin polarizations (directions) change from crystal to crystal. Hence the movement of the spin polarizations does not induce an electric current. This process is similar to the propagation of electron spins instead of electrons in spintronics [29–31]. The movement of the spins creates a magnetic current, not an electric current.

Another important point is the memory property of the memristive systems. A memristor satisfies the relation of

$$\int vdt = M \int idt \tag{18}$$

that can be arranged as

$$\int (v - Mi)\, dt = 0. \tag{19}$$

A suitable expression that satisfies Equation (19) can be written as

$$v = Mi + w \tag{20}$$

for w is the integration constant. Mathematically, the memristor remembers its previous state because of w [8]. Now, we take into account Equation (2) that defines the memristive medium. It can be rewritten as

$$\frac{\partial \vec{B}}{\partial t} = \gamma \frac{\partial \vec{D}}{\partial t} \tag{21}$$

by taking the time derivative of two sides. Equation (21) leads

$$\nabla \times \vec{E} = -\gamma \nabla \times \vec{H}, \tag{22}$$

which can be arranged as

$$\nabla \times \left(\vec{E} + \gamma \vec{H}\right) = 0. \tag{23}$$

The equation of

$$\vec{E} = -\gamma \vec{H} + \nabla \varphi. \tag{24}$$

satisfies Equation (23). φ is a scalar function. It is apparent that Equation (24) is analogous to Equation (20). The memristive medium remembers its latest state according to $\nabla \varphi$ as in a memristor [8].

4. CONCLUSION

In this paper, we extended the electromagnetic theory by the aid of the concept of memristor in the circuit theory. A memristor defines a relation between the magnetic flux and electric current. In a similar way, we proposed a relation between the magnetic and electric flux densities in the electromagnetic theory. This attempt led us to two equations of

$$\nabla \cdot \vec{B} = \gamma \rho_v \tag{25}$$

and

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \gamma \vec{J} - \eta \vec{H}, \tag{26}$$

which are the extended versions of the Maxwell-Gauss and Maxwell-Faraday equations in a memristive medium, respectively. It is also mentioned that the equivalent magnetic monopoles are the results of the spin arrangements of the positive ions in spin ice. This experimental invention offers a realistic basis for our theory. Equation (18) states that the electric charge density creates a magnetic flux density instead of an electric flux density in a memristive medium. Equation (19) has an interesting result that the electric current density will induce an electric field intensity that surrounds its path of flow in a memristive material. We also propose that the current-voltage graphics of a spin ice material can be studied for the realization of the memristor in the circuit theory.

As a result, our theory has four important results;

1) The magnetic charges and currents do not exist.
2) The electric charges and currents behave as magnetic monopoles and currents in a memristive medium.
3) The electric current density creates electric field intensity in a memristive material.

4) The magnetic field intensity induces an electric current density in a memristive medium.

The combined investigation of **equivalent magnetic monopoles** and memristor may provide interesting inventions on the electromagnetic and circuit theory. For example the relation between the magnetic flux and electric charges can be examined in the nanoscale systems of Strukov et al. [4, 5]. Also the voltage-current characteristics of samples, made of spin ice materials, can be studied experimentally.

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