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ZERO REFLECTION FROM ANISOTROPIC METAMATERIAL STRATIFIED STRUCTURES

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Abstract—A method of solving the scattering problem for general multilayer anisotropic structures composed of conventional materials and metamaterial is presented. The analysis is based on calculation of the hybrid matrix of layers by means of a recursive algorithm. The method does not have the complexity and instability problems of other methods and is reliable in all cases. The zero reflection from stratified structures of conventional materials and metamaterials has then been introduced Various aspects of such a structure from the viewpoints of frequency and incident angle are presented and a rule for zero reflection from anisotropic medium is addressed. An interesting special case of total transparency is observed.

1. INTRODUCTION

The problem of interaction of electromagnetic waves with isotropic and anisotropic layers has long been a subject of interest due to its wide application in various areas such as geophysics, locating underground resources, microstrip radiators and absorbent coatings. The latter case is of remarkable significance, and has gained much more consideration from researchers. Although there have been important anisotropic materials like ferrites, recently, the problem is being reconsidered because of novel metamaterials and their applications in reduction of radar cross section (RCS).

Initial works on the subject were based on 4×4 characteristic matrix of a single anisotropic slab [1, 2]. Later efforts include

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generalization of the problem for stratified structures by different methods [3–5]. Morgan et al. paid attention to a numerical solution, and introduced an efficient and simple algorithm for this case [6]. Others proposed various techniques based on eigenvalue computation, Ricatti differential equation and transmission line method [7–11] which are more complex. The characteristic matrix algorithm [1, 2] had a serious drawback and showed instability for thick layers compared to wavelength. To avoid this instability which was due to the numerical finite difference algorithm, the use of hybrid matrix of the structure is suggested [12].

On the other hand, with realizing the negative permittivity and permeability, application of isotropic metamaterials was shown in reducing the electromagnetic scattering via coupling with conventional isotropic materials [13–16].

In the present work after explaining a modified numerical method, the interesting phenomena of zero reflection from anisotropic structures will be considered. This property can find many applications in realizing extremely wide-band non-reflecting invisible coatings, various angular filters, etc.

2. FORMULATION

The geometry of the problem to be discussed is shown in Figure 1. Regions (I) and (III) are free space and region (II) is composed of anisotropic layers. An incident plane wave traveling in air, encounters the boundary of a multilayered anisotropic planar structure at an angle θ_1. The wave interacts with the multilayered structure and after a series of reflections at discontinuities, part of its power is transmitted through the structure into air. The desired solution of the problem is the reflected wave from the boundary of regions (I) and (II), and transmitted wave into region (III).

Considering a single anisotropic layer and time dependence convention as $e^{j\omega t}$ one can write the electromagnetic propagating waves in the $(x\text{-}z)$ plane as follows:

$$\begin{cases} \bar{\mathcal{E}}(x,z) = \bar{E}(z)e^{-jk_x x} \\ \bar{\mathcal{H}}(x,z) = \bar{H}(z)e^{-jk_x x} \end{cases} \tag{1}$$

In a region with no sources, Maxwell's equations for electric and magnetic fields in general anisotropic environment can be written as

$$\begin{cases} \nabla \times \bar{E} = -j\omega\bar{\bar{\mu}} \cdot \bar{H} \\ \nabla \times \bar{H} = j\omega\bar{\bar{\varepsilon}} \cdot \bar{E} \end{cases} \tag{2}$$

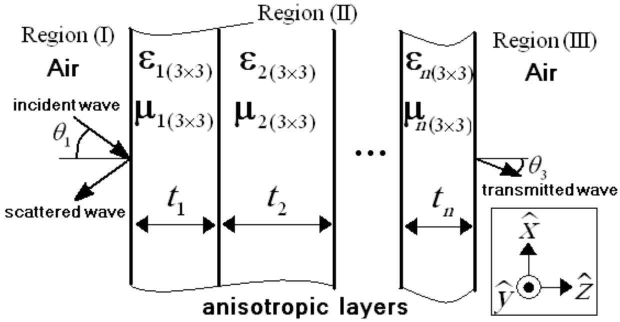


Figure 1. The geometry of a general multi layer structure.

with

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \quad \text{and} \quad \bar{\bar{\mu}} = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}. \tag{3}$$

In such an environment, the electric permittivity and magnetic permeability are tensors including complex elements. By expanding the curl equations, and considering y direction symmetry, and $\partial/\partial x \equiv -jk_x$, six scalar equations are concluded which include ten unknown parameters. k_x is the component of the wave vector along x direction ($k_x = k_1 \sin\theta_1$) and the unknowns are the fields components and the partial derivative of transverse components E_x, E_y, H_x and H_y with respect to z direction.

A noticeable point is the fact that only 2 of these unknowns E_z and H_z, are parallel to the stratification direction of the structure, z, and the other 8 are transverse components.

After some manipulations, a system of 4 equations is resulted which includes 8 unknowns i.e., E_x, E_y, H_x, H_y, $\partial E_x/\partial z$, $\partial E_y/\partial z$, $\partial H_x/\partial z$ and $\partial H_y/\partial z$. This system is shown as

$$\begin{bmatrix} dE_x/dz \\ dE_y/dz \\ dH_x/dz \\ dH_y/dz \end{bmatrix} = \bar{\bar{\Gamma}} \begin{bmatrix} E_x \\ E_y \\ H_x \\ H_y \end{bmatrix} \tag{4}$$

or equivalently,

$$\frac{d}{dz} \begin{bmatrix} \bar{E}_T \\ \bar{H}_T \end{bmatrix} = \bar{\bar{\Gamma}} \begin{bmatrix} \bar{E}_T \\ \bar{H}_T \end{bmatrix} \tag{5}$$

where,

$$\bar{E}_T = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \quad \text{and} \quad \bar{H}_T = \begin{bmatrix} H_x \\ H_y \end{bmatrix}. \tag{6}$$

The elements of the matrix Γ is given in Appendix A. Solving these linear first order differential equations, a matrix solution which relates the fields at two boundaries of the anisotropic layer is obtained; i.e.,

$$\begin{bmatrix} \bar{E}_{T(2\times1)}(t) \\ \bar{H}_{T(2\times1)}(t) \end{bmatrix} = \bar{\bar{\mathrm{A}}}_{(4\times4)} \begin{bmatrix} \bar{E}_{T(2\times1)}(0) \\ \bar{H}_{T(2\times1)}(0) \end{bmatrix}. \tag{7}$$

where $\{E_T(t), H_T(t)\}$ and $\{E_T(0), H_T(0)\}$ are the fields at $z = t$ and $z = 0$ respectively, and A is a (4×4) matrix as;

$$\bar{\bar{\mathrm{A}}} = \begin{bmatrix} \bar{\bar{A}}_{11(2\times2)} & \bar{\bar{A}}_{12(2\times2)} \\ \bar{\bar{A}}_{21(2\times2)} & \bar{\bar{A}}_{22(2\times2)} \end{bmatrix} = \exp(\bar{\bar{\Gamma}}\mathrm{t}). \tag{8}$$

To obtain the characteristic matrix one way is to find the eigenvalues of Γ matrix [11]. The method considered in this paper, is based on the finite difference technique and is more straightforward than the eigenvalue method.

Focusing on one of the anisotropic layers in region (II), the transition matrix of the layer must be calculated. Having the differential Equation (5) and denoting a thin portion of anisotropic layer with an incremental thickness, Δz, the fields at the two boundaries of the layer are related by

$$\begin{bmatrix} \bar{E}_T(z+\Delta z) \\ \bar{H}_T(z+\Delta z) \end{bmatrix} - \begin{bmatrix} \bar{E}_T(z) \\ \bar{H}_T(z) \end{bmatrix} = \frac{\Delta z}{2}\bar{\bar{\Gamma}} \left(\begin{bmatrix} \bar{E}_T(z+\Delta z) \\ \bar{H}_T(z+\Delta z) \end{bmatrix} + \begin{bmatrix} \bar{E}_T(z) \\ \bar{H}_T(z) \end{bmatrix} \right) \tag{9}$$

By rearranging the above equation for the incremental layer, (9) can be rewritten in the discretized form, and thus the (4×4) transition matrix for thickness Δz is achieved as

$$\begin{bmatrix} \bar{E}_T(z+\Delta z) \\ \bar{H}_T(z+\Delta z) \end{bmatrix} = \bar{\bar{A}}_{(\Delta z)} \begin{bmatrix} \bar{E}_T(z) \\ \bar{H}_T(z) \end{bmatrix} \tag{10}$$

where,

$$\bar{\bar{A}}_{(\Delta z)} = \left(I_{(4\times4)} - \frac{\Delta z}{2}\bar{\bar{\Gamma}} \right)^{-1} \left(I_{(4\times4)} + \frac{\Delta z}{2}\bar{\bar{\Gamma}} \right). \tag{11}$$

To calculate the overall matrix of the whole layer, $i(= t/\Delta z)$ sub-layers can be simply cascaded, where 't' is the thickness of the whole layer. In case of transition matrix, this process is done by multiplying the matrix by itself, 'i' times. But a problem arises when the thickness of the layer is large in comparison to the wavelength, which is due to simultaneous existence of very large and very small exponential terms in the transition matrix that leads to an uncontrolled increase in error [11]. Alternative methods have been recommended to overcome this problem [11, 12]. The recursive hybrid matrix method is used here.

$$\begin{bmatrix} \bar{E}_T(0) \\ \bar{H}_T(t) \end{bmatrix} = \begin{bmatrix} H_{11(2\times2)} & H_{12(2\times2)} \\ H_{21(2\times2)} & H_{22(2\times2)} \end{bmatrix} \begin{bmatrix} \bar{H}_T(0) \\ \bar{E}_T(t) \end{bmatrix} \tag{12}$$

Equation (12) is the definition of the (4×4) hybrid matrix for fields perpendicular to z axis. Consequently, the matrix elements are (2×2) sub-matrices.

In view of Equation (10) and the calculation of hybrid matrix from transition matrix, the hybrid matrix of the thin sub-layer can be derived as

$$\begin{aligned} \bar{\bar{H}}_{(\Delta z)} = & \begin{bmatrix} I_{(2\times2)} + \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{11} & \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{12} \\ \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{21} & -I_{(2\times2)} + \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{22} \end{bmatrix}^{-1} \\ & \cdot \begin{bmatrix} -\frac{\Delta z}{2}\bar{\bar{\Gamma}}_{12} & I_{(2\times2)} - \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{11} \\ -I_{(2\times2)} - \frac{\Delta z}{2}\bar{\bar{\Gamma}}_{22} & -\frac{\Delta z}{2}\bar{\bar{\Gamma}}_{21} \end{bmatrix} \end{aligned} \tag{13}$$

Δz must be chosen such that by cascading the thin sub-layer with itself and repeating the same process recursively n times, the original layer with thickness 't' is made. Therefore, in this recursive method; $t = 2^n \Delta z$. For example if $n = 3$, the hybrid matrix of the whole layer is calculated through doing the recursive process 3 times (Figure 2).

Having the hybrid matrix of a sub-layer with thickness Δz as

$$\bar{\bar{H}}_{(\Delta z)} = \begin{bmatrix} \bar{\bar{H}}_{11(2\times2)} & \bar{\bar{H}}_{12(2\times2)} \\ \bar{\bar{H}}_{21(2\times2)} & \bar{\bar{H}}_{22(2\times2)} \end{bmatrix}, \tag{14}$$

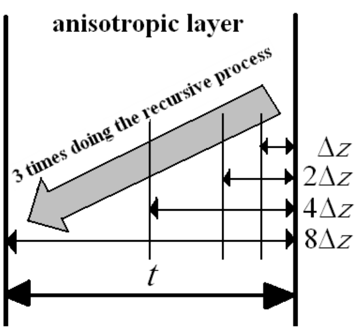


Figure 2. The recursive process for computing a layer's matrix.

the resulting matrix from cascading the layer with itself would be

$$\bar{\bar{H}}_{(2\Delta z)}=\begin{bmatrix}\bar{\bar{H}}_{11}+\bar{\bar{H}}_{12}(I-\bar{\bar{H}}_{11}\bar{\bar{H}}_{22})^{-1}\bar{\bar{H}}_{11}\bar{\bar{H}}_{21} & \bar{\bar{H}}_{12}(I-\bar{\bar{H}}_{11}\bar{\bar{H}}_{22})^{-1}\bar{\bar{H}}_{12}\\ \bar{\bar{H}}_{21}(I-\bar{\bar{H}}_{22}\bar{\bar{H}}_{11})^{-1}\bar{\bar{H}}_{21} & \bar{\bar{H}}_{22}+\bar{\bar{H}}_{21}(I-\bar{\bar{H}}_{22}\bar{\bar{H}}_{11})^{-1}\bar{\bar{H}}_{22}\bar{\bar{H}}_{12}\end{bmatrix}. \tag{15}$$

Repeating the same process recursively, i.e., cascading the result of each stage with itself n time, results in the hybrid matrix of the whole layer. The advantage of recursive method is that by increasing n, the thickness of the first sub-layer decreases exponentially, leading to a significant improvement in accuracy.

After calculating characteristic matrices of all intermediate layers by the above method, one can compute the total characteristic matrix by cascading the layers' hybrid matrices.

Naming $\{E^i, H^i\}$, $\{E^r, H^r\}$ and $\{E^t, H^t\}$ as phasors of incident fields, scattered fields in region (I) and transmitted fields in region (III) respectively, the reflection and transmission coefficient matrices can be defined as

$$E_T^r=\begin{bmatrix}E_x^r\\ E_y^r\end{bmatrix}=\begin{bmatrix}r_{11} & r_{12}\\ r_{21} & r_{22}\end{bmatrix}\begin{bmatrix}E_x^i\\ E_y^i\end{bmatrix}=R\cdot E_T^i \tag{16}$$

and

$$E_T^t=\begin{bmatrix}E_x^t\\ E_y^t\end{bmatrix}=\begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix}\begin{bmatrix}E_x^i\\ E_y^i\end{bmatrix}=T\cdot E_T^i. \tag{17}$$

According to the definition, Equation (12) can be written as

$$\begin{bmatrix}E_T^i+E_T^r\\ H_T^t\end{bmatrix}=\begin{bmatrix}\bar{\bar{H}}_{11} & \bar{\bar{H}}_{12}\\ \bar{\bar{H}}_{21} & \bar{\bar{H}}_{22}\end{bmatrix}\begin{bmatrix}H_T^i+H_T^r\\ E_T^t\end{bmatrix}. \tag{18}$$

To compute the reflection matrix, all the unknowns in (18) must be eliminated except E_T^i and E_T^r. The same procedure can be done for E_T^i and E_T^t and for transmission matrix. To do this process, impedance relations between electric and magnetic fields in regions (I) and (III) are needed:

$$E_T^i=Z_1\cdot H_T^i;\quad E_T^r=-Z_1\cdot H_T^r;\quad E_T^t=Z_3\cdot H_T^t \tag{19}$$

with

$$Z_i=\begin{bmatrix}0 & \eta_i\cos\theta_i\\ -\eta_i\sec\theta_i & 0\end{bmatrix}. \tag{20}$$

θ_1 and θ_3 are the angles of the wave vector with normal to boundaries of the regions. These two angles satisfy Snell's law, $\sin\theta_3 \div \sin\theta_1 = \sqrt{\mu_1\varepsilon_1/\mu_3\varepsilon_3}$, which is still valid in anisotropic environment. 'η' is the wave impedance in each region. By doing some matrix calculations the reflection and transmission matrices can be computed using the hybrid matrix sub-matrices. i.e.,

$$\begin{aligned} R &= [-Z_1 + H_{11} + H_{12}Z_3(I - H_{22}Z_3)^{-1}H_{21}] \\ &\quad \cdot [Z_1 + H_{11} + H_{12}Z_3(I - H_{22}Z_3)^{-1}H_{21}]^{-1} \end{aligned} \tag{21}$$

and

$$\begin{aligned} T &= [Z_3^{-1} - H_{22} + H_{21}(Z_1 + H_{11})^{-1}H_{12}]^{-1} \\ &\quad \cdot [H_{21}Z_1^{-1}(I - (H_{11} - Z_1)(H_{11} + Z_1)^{-1})] \end{aligned} \tag{22}$$

Extraction of the features of TE and TM polarizations needs more consideration. TE polarization consists of $\{E_y, H_x, H_z\}$ field components and TM polarization consists of $\{H_y, E_x, E_z\}$ components, therefore, Equation (16) transforms to

$$TE: \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{12} \\ r_{22} \end{bmatrix} E_y^i \quad \text{and} \quad TM: \begin{bmatrix} E_x^r \\ E_y^r \end{bmatrix} = \begin{bmatrix} r_{11} \\ r_{21} \end{bmatrix} E_x^i. \tag{23}$$

The returned power of the TM polarization is therefore proportional to $(r_{11}^2 + r_{21}^2)$ and in case of TE polarization it is proportional to $(r_{12}^2 + r_{22}^2)$. It is clear that if the electric and magnetic tensors be diagonal matrices, r_{21} and r_{12} are zero and the returned power will be proportional to r_{11}^2 and r_{22}^2 for TM and TE modes respectively.

To check the validity of the given method, a magneto-plasma layer coated on PEC ($Z_3 = 0$) is considered. Analytical solution for reflection from anisotropic magneto-plasma coated on PEC can be found in [11]. The structure's specifications and reflection are shown in Figure 3. It can be seen that for thick layers compared to

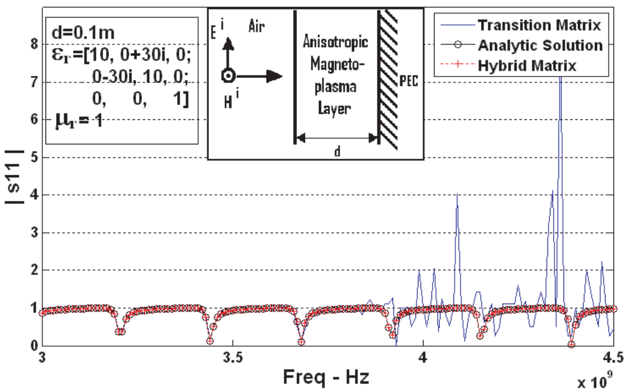


Figure 3. Checking the validity of transition and hybrid matrix methods.

wavelength, transition matrix solution loses its stability while hybrid matrix solution is still valid and carefully matches the analytical solution.

3. RESULTS

The geometry being considered here is a general two layer anisotropic structure perpendicular to z direction and illuminated by an electromagnetic plane wave with a specified frequency. The layers are assumed to be lossless in all examples and the outside region of layered anisotropic structure is free space.

By choosing the permittivity and permeability tensors as

$$\begin{cases} \varepsilon_{2(3\times3)} = \frac{-1}{m}\boldsymbol{\varepsilon}_{1(3\times3)} \\ \mu_{2(3\times3)} = \frac{-1}{m}\mu_{1(3\times3)} \end{cases} ; \qquad t_2 = m \times t_1 \tag{24}$$

The incident wave will not experience any reflections while traveling through the structure. In the following, we consider five special cases related to Equation (24).

Case I. Coupling of DPS-DNG materials with diagonal characteristic tensors: With the relative permittivity and permeability tensors given in Figure 4, $m = 3$ is chosen and the result is practically zero reflection from the structure in the frequency range of 1 to 100 GHz at normal incidence. Thicknesses of the layers are stated in Meters.

The negligible fluctuations are due to errors related to the finite difference calculations. It is evident that the accuracy of the algorithm can be conveniently set by the appropriate selection of the segmentation parameter n.

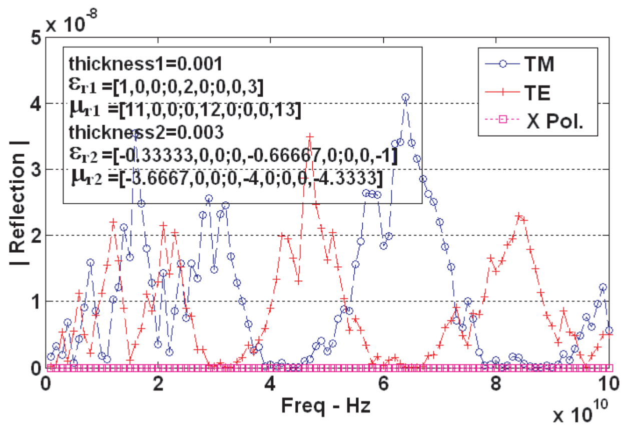


Figure 4. Coupling of DPS-DNG with $m = 3$.

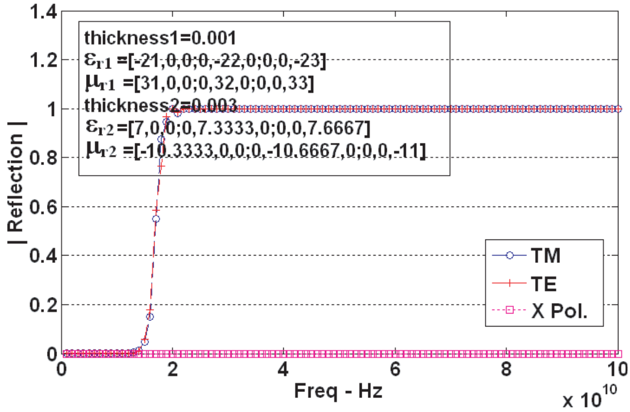


Figure 5. Coupling of ENG-MNG with $m = 3$.

Case II. Coupling of ENG-MNG metamaterials with diagonal characteristic tensors: It can be seen that this combination leads to a low-pass filter (Figure 5). With the parameter values shown on the figure and $m = 1$, the structure is transparent to the wave up to 13 GHz and totally blocks it from about 20 GHz. Like the previous example, cross polarization reflection coefficient is zero due to diagonally assumed electromagnetic tensors.

Case III. Coupling of layers with non-diagonal characteristic tensors: Figure 6 shows that even with non-diagonal entries of permittivity and permeability tensors, the zero reflection is still present and no power returns to region (I). In this example $m = 5$ and in contrast to prior examples, some fluctuations can be observed in cross polarization coefficient which is a direct consequence of adding the non-diagonal elements. One should note that a medium with a general non-diagonal tensor is not necessarily realizable.

Case IV. The effect of illumination angle: Up to now, normal incidence was considered in all examples. In fact zero reflection explained so far, can be seen solely in this case. The reflection coefficient vs. different incident angles is illustrated in Figure 7 at 20 GHz.

What if we want a zero reflection at other angles? Such a case may have applications in angular filters, polarization filters and treating RCS of moving objects. It is possible to shape the angular pattern of the reflection, or to place a zero at a specific angle.

For example with arbitrary thicknesses of the layers, ignoring the second condition of (24), the zero reflection angle can be changed. For example in Figure 8(a), $m = 5$, has led to zero reflection for TE polarization at about 40 degrees.

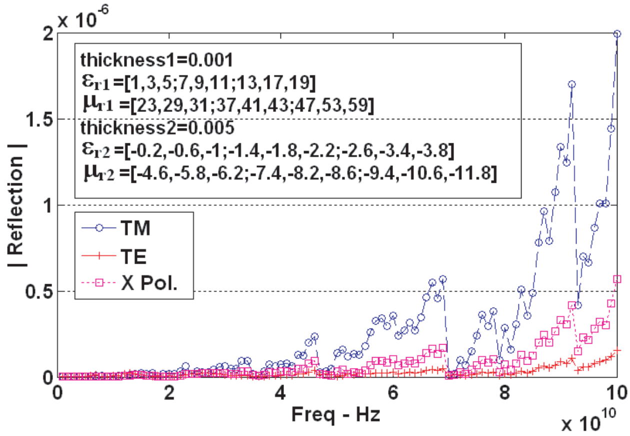


Figure 6. Zero reflection for non-diagonal matrices with $m = 5$.

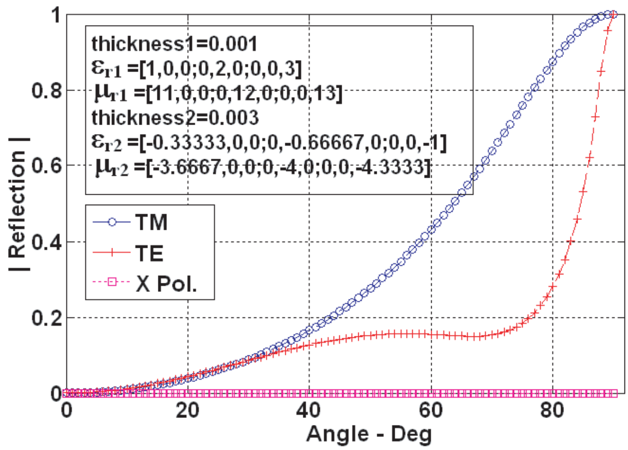
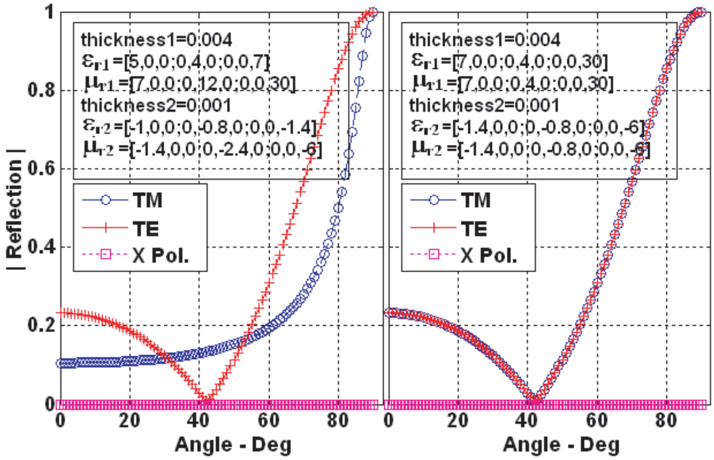


Figure 7. Reflectance vs. different incident angles.

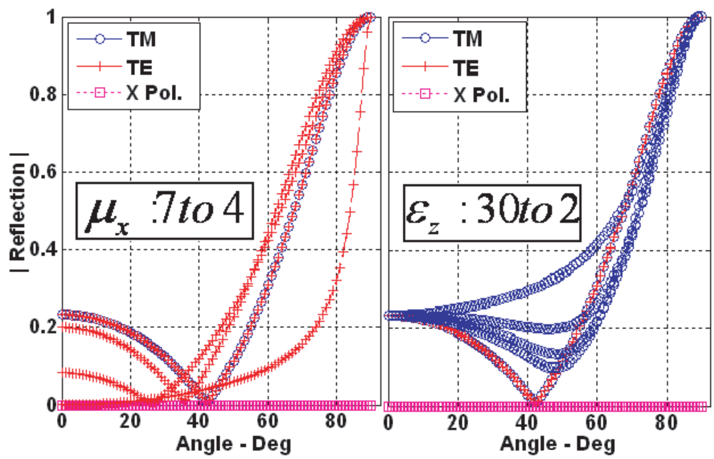
It is notable that in all cases of diagonal tensors as well as special non-diagonal tensors, TE and TM reflection curves can be controlled separately and shaped independently. For diagonal tensors, TE polarization is related only to ε_y, μ_x and μ_z and TM polarizations is related only to μ_y, ε_x and ε_z. Therefore, if $\varepsilon_y = \mu_y$, $\mu_x = \varepsilon_x$ and $\mu_z = \varepsilon_z$, the structure affects TE and TM polarizations in a similar manner.

By changing the elements of the tensors stated in Figure 8(b), one can set the location of zero reflection and change the behavior of the reflection curve (Figures 9(a) and (b)).



(a) (b)

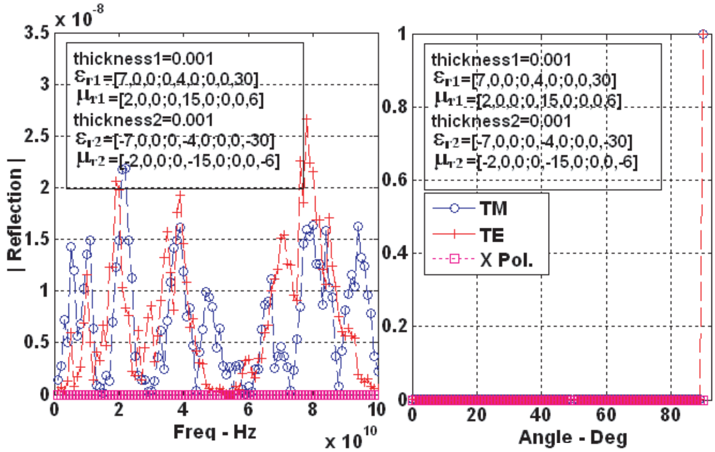
Figure 8. The same curve for TE and TM polarizations.



(a) (b)

Figure 9. Changing the angle of zero and shaping the curve of reflectance.

Case V. The special case of $m = 1$: If the parameter m is set equal to 1 in any of prior examples, and the two thicknesses are equal, we encounter an interesting phenomena. Reflection would be zero for all angles of incidence in all frequencies and the structure would be totally transparent, even though we have an anisotropic medium (Figure 10).



(a) (b)

Figure 10. The special case of $m = 1$ for frequency and angle variations.

4. DISCUSSION

Materials used in these cases show the same electromagnetic properties in all frequencies and therefore are not dispersive. In reality, we cannot find such materials and their presence violates the thermodynamic principles governing the universe [17]. Also the materials were assumed lossless in a wide range in frequency domain which is in contrast to the dependency of real and imaginary parts of ε and μ via Kramers-Kronig relations.

Since, the interaction of a medium and a monochromatic EM wave depends on ε and μ tensor values at the corresponding frequency, zero reflection is conceivable for dispersive materials only when the tensors satisfy (24) in every frequency for a specified m.

5. CONCLUSION

With the help of recursive algorithm and hybrid matrix computation for anisotropic slabs with free space outside the slabs, a powerful, fast and yet reliable procedure for solving the scattering problem was presented The method is also convenient for an optimization procedure. The generality of the method makes it suitable for all kinds of anisotropic materials containing even negative tensor elements i.e., metamaterials. Using this method the possibility of zero reflection from stratified anisotropic medium was demonstrated which includes a special case of transparency, in which the total power is transmitted for all incident angles and all frequencies. This occurs when the electromagnetic parameters are related by (24) with $m = 1$.

APPENDIX A.

The matrix Γ defined in (4) is derived from Maxwell's equations:

$$\Gamma = \begin{bmatrix} jk_x\frac{\varepsilon_{31}}{\varepsilon_{33}} & jk_x\left(\frac{\varepsilon_{32}}{\varepsilon_{33}}-\frac{\mu_{23}}{\mu_{33}}\right) & j\omega\left(\frac{\mu_{23}\mu_{31}}{\mu_{33}}-\mu_{21}\right) & j\left\{\frac{k_x^2}{\omega\varepsilon_{33}}+\omega\left(\frac{\mu_{23}\mu_{32}}{\mu_{33}}-\mu_{22}\right)\right\} \\ 0 & jk_x\frac{\mu_{13}}{\mu_{33}} & j\omega\left(\mu_{11}-\frac{\mu_{13}\mu_{31}}{\mu_{33}}\right) & j\omega\left(\mu_{12}-\frac{\mu_{13}\mu_{32}}{\mu_{33}}\right) \\ j\omega\left(\varepsilon_{21}-\frac{\varepsilon_{23}\varepsilon_{31}}{\varepsilon_{33}}\right) & -j\left\{\frac{k_x^2}{\omega\mu_{33}}+\omega\left(\frac{\varepsilon_{23}\varepsilon_{32}}{\varepsilon_{33}}-\varepsilon_{22}\right)\right\} & jk_x\frac{\mu_{31}}{\mu_{33}} & jk_x\left(\frac{\mu_{32}}{\mu_{33}}-\frac{\varepsilon_{23}}{\varepsilon_{33}}\right) \\ j\omega\left(\frac{\varepsilon_{13}\varepsilon_{31}}{\varepsilon_{33}}-\varepsilon_{11}\right) & j\omega\left(\frac{\varepsilon_{13}\varepsilon_{32}}{\varepsilon_{33}}-\varepsilon_{12}\right) & 0 & jk_x\frac{\varepsilon_{13}}{\varepsilon_{33}} \end{bmatrix}$$

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