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FAST INHOMOGENEOUS PLANE WAVE ALGORITHM FOR ANALYSIS OF COMPOSITE BODIES OF REVOLUTION

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Abstract—A fast inhomogeneous plane wave algorithm is developed for the electromagnetic scattering problem from the composite bodies of revolution (BOR). Poggio-Miller-Chang-Harrington-Wu (PMCHW) approach is used for the homogeneous dielectric objects, while the electric field integral equation (EFIE) is used for the perfect electric conducting objects. The aggregation and disaggregation factors can be expressed analytically by using the Weyl identity. Compared with the traditional method of moments (MoM), both the memory requirement and CPU time, are reduced for large-scale composite BOR problems. Numerical results are given to demonstrate the validity and the efficiency of the proposed method.

1. INTRODUCTION

Electromagnetic radiation and scattering from bodies of revolution (BOR) that consist of perfect electrical conductor (PEC), homogeneous dielectric media, composite PEC and dielectric and layered dielectric media have been widely discussed during last several decades. Because of the symmetry of the geometry, only the generatrix that forms the

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surfaces of the PEC part and dielectric part are needed for solving the BOR problem in a surface integral equation formulation [1–6]. Similarly, only the meridian cross section is needed for solving the BOR problem with the finite element method (FEM) [7–9]. Both the memory requirement and CPU time in BOR solvers are reduced compared with full three-dimensional methods. Other scattering problems for complex objects can be found in [10–12].

In a typical BOR solver in the cylindrical coordinate system, the electric and magnetic fields can be expanded as the summation of different Fourier series modes in ϕ (azimuth angle). Because of the orthogonality, each mode can be treated separately. In a surface integral equation formulation, the expansion functions chosen for the solution are harmonic in ϕ and sub-sectional in t (contour length variable). When the traditional method of moments (MoM) is used to solve a BOR integral equation, the memory requirement for BOR-MoM is $O(N^2)$, where N is the number of unknowns for each azimuthal mode. Thus, it is time consuming for large-scale composite BOR problems if the traditional MoM is used. The time for solving the integral equation of the BOR problem mainly depends on the evaluation of modal Green's function (MGF). Several works such as the fast Fourier transform (FFT), Bartkys transformation and spherical Bessel function expansion have been proposed to accelerate the evaluation of MGF [13–16]. More recently, the fast inhomogeneous plane wave algorithm is proposed to solve PEC and homogeneous dielectric BOR problems [17, 18]. Both the memory requirement and CPU time are reduced from the MoM.

In this work, we extend the fast inhomogeneous plane wave algorithm (FIPWA) to accelerate the computation of the MoM for composite bodies of revolution. PMCHW (Poggio, Miller, Chang, Harrington, Wu) integral equation [19–21] is used for solving the problem of a dielectric scatterer and the EFIE is used for solving the PEC part. The aggregation and disaggregation factors can be computed analytically. Both the memory requirement and the CPU time are saved for large-scale composite BOR problems. Numerical results are given to demonstrate the validity and the efficiency of the FIPWA method.

2. INTEGRAL EQUATION FOR BODY OF REVOLUTION

2.1. Integral Equations for the Composite Problem

The scattering problem of electromagnetic waves from a composite object is widely discussed recently [22–26]. The homogeneous dielectric

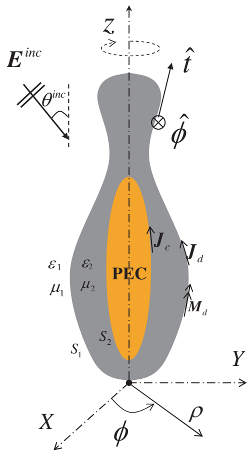

Figure 1. A composite body of revolution and the coordinate system, where θ_{inc} is the angle of incident wave, and $\hat{t}$ and $\hat{\phi}$ are the unit vectors.

object having permittivity ϵ_2 and permeability μ_2 and a perfect electric conducting (PEC) object in a homogeneous background medium (ϵ_1, μ_1) is shown in Figure 1. This problem can be solved by PMCHW (Poggio, Miller, Chang, Harrington, Wu) integral equations hybridized with the electric field integral equation (EFIE) [5, 6] as follows:

$$\begin{aligned} \hat{n}_1 \times \mathbf{E}_i(r) &= \hat{n}_1 \times [L_1(\mathbf{J}_d) + L_2(\mathbf{J}_d + \mathbf{J}_c)] \\ &\quad -\hat{n}_1 \times [K_1(\mathbf{M}_d) + K_2(\mathbf{M}_d)], \quad r \in S_1 \end{aligned} \tag{1}$$

$$\begin{aligned} \hat{n}_1 \times \mathbf{H}_i(r) &= \hat{n}_1 \times [K_1(\mathbf{J}_d) + K_2(\mathbf{J}_d + \mathbf{J}_c)] \\ &\quad -\hat{n}_1 \times \left[\frac{1}{\eta_1^2} L_1(\mathbf{M}_d) + \frac{1}{\eta_2^2} L_2(\mathbf{M}_d)\right], \quad r \in S_1 \end{aligned} \tag{2}$$

$$0 = \hat{n}_2 \times [L_2(\mathbf{J}_d + \mathbf{J}_c) - K_2(\boldsymbol{M}_d)], \quad r \in S_2 \tag{3}$$

where $\mathbf{J}_c$ is the induced electric current density on the PEC surface S_2; $\mathbf{J}_d$ is the induced electric current density on the dielectric surface S_1, and $\mathbf{M}_d$ is the induced magnetic current density on the dielectric surface S_1; $\eta_i = \sqrt{\mu_i/\varepsilon_i}$ is the wave impedance for region i $(i = 1, 2)$. $\mathbf{E}_i$ is the incident electric field, $\mathbf{H}_i$ is the incident magnetic field, and L_i and K_i are operators defined as

$$L_i(\mathbf{x}) = j\omega\mu_i \int_S \left[\mathbf{x}(\mathbf{r}') + \frac{1}{\omega^2 \mu_i \varepsilon_i} \nabla\nabla \cdot \mathbf{x}(\mathbf{r}')\right] G_i ds' \tag{4}$$

$$K_i(\mathbf{x}) = \int_S \mathbf{x}(\mathbf{r}') \times \nabla G_i ds' \tag{5}$$

Here G_i is the scalar Green's function of the background medium $(i = 1)$ or dielectric region $(i = 2)$. Using the Galerkin's method, we can rewrite Equations (1)–(3) as

$$
\begin{aligned}
(P_1 + P_2)[\mathbf{J}_d] + P_2[\mathbf{J}_c] - (Q_1 + Q_2)[\mathbf{M}_d] &= b^{TE} \\
(Q_1 + Q_2)[\mathbf{J}_d] + Q_2[\mathbf{J}_c] - \left(\frac{P_1}{\eta_1^2} + \frac{P_2}{\eta_2^2}\right)[\mathbf{M}_d] &= b^{TH} \\
P_2[\mathbf{J}_d] + P_2[\mathbf{J}_c] - Q_2[\mathbf{M}_d] &= 0
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
(P_i)_{pq} &= <\mathbf{f}_p, L_i(\mathbf{f}_q)> \\
(Q_i)_{pq} &= <\mathbf{f}_p, K_i(\mathbf{f}_q)> \\
b_p^{TE} &= <\mathbf{f}_p, \mathbf{E}_{inc}> \\
b_p^{TH} &= <\mathbf{f}_p, \mathbf{H}_{inc}>
\end{aligned}
$$

with $\mathbf{f}_q$ being the basis function and $\mathbf{f}_p$ being the testing function.

2.2. Body of Revolution

In the cylindrical coordinate system (ρ, ϕ, z) as shown in Figure 1, the electric and magnetic current densities on the surface S_1 and S_2 (which are generated by two curves around the z-axis) can be expanded as:

$$\mathbf{J} = \sum_{m,i}(J^t_{mi}\mathbf{f}^t_{mi} + J^\phi_{mi}\mathbf{f}^\phi_{mi}) \tag{7}$$

$$\mathbf{M} = \sum_{m,i}(M^t_{mi}\mathbf{f}^t_{mi} + M^\phi_{mi}\mathbf{f}^\phi_{mi}) \tag{8}$$

where $\mathbf{f}^\alpha_{mi} = \hat{\alpha} f_i(t)e^{jm\phi}$ is the basis function, $\alpha = t$ or ϕ, $\hat{t} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ (t-directed unit vector), $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ (ϕ-directed unit vector). Note here the definition of ϕ is the same as the azimuthal angle in the spherical coordinate system; but θ is different from the elevation angle of the position vector in the spherical coordinate system, and is defined as the angle between the tangent direction of the surface and the z axis, i.e., $\theta = \cos^{-1}(\hat{z}\cdot\hat{t})$. Function $f_i(t)$ is related to the triangular function T_i as $f_i(t) = \frac{1}{\rho}T_i(t)$, where

$$
T_i(t) = \begin{cases} \frac{t - t_{i-1}}{t_i - t_{i-1}} & \text{if } t \in [t_{i-1}, t_i] \\ \frac{t_{i+1} - t}{t_{i+1} - t_i} & \text{if } t \in [t_i, t_{i+1}] \\ 0 & \text{otherwise} \end{cases} \tag{9}
$$

The current densities are divided into t and ϕ components in Equations (7) and (8); both parts are related to the azimuthal mode $e^{jm\phi}$ for the m-th mode. Note that different azimuthal modes are uncoupled to each other. Similarly, we choose the testing function as

$$\mathbf{W}^\alpha_{ni} = \hat{\alpha} f_i(t)e^{-jn\phi} \tag{10}$$

Here the testing function $\mathbf{W}_n$ is orthogonal to $\mathbf{f}_m$ and to the fields generated from $\mathbf{f}_m$, for example, $L(\mathbf{f}_m)$ and $K(\mathbf{f}_m)$. Then, each cylindrical harmonic can be treated separately by solving the matrix equation (6). The detail for solving the impedance matrix elements can be found in [3]. The key process is solving the modal Green's function (MGF) g_n^i, $(i = 1, 2)$ which can be expressed as

$$g_n^i = \int_0^\pi \frac{e^{-jk_i R_0}}{R_0} \cos n\phi d\phi \tag{11}$$

$$R_0 = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos\phi + (z - z')^2} \tag{12}$$

For the traditional MoM, the modal Green's function has to be evaluated by a numerical method, hence it is time consuming when the radius of the BOR is large. Some works based on the MoM have been done to improve the efficiency of the MGF in recent years. In the following part, the FIPWA method will be proposed to accelerate the MoM to solve the scattering problem of a composite BOR.

2.3. Fast Inhomogeneous Plane Wave Algorithm

As mentioned above, it is time consuming to solve the MGF, and the memory requirement of MoM is $O(N^2)$, where N is the number of the unknowns for one particular azimuthal mode m (note that different azimuthal modes are orthogonal to each other). The fast inhomogeneous plane wave algorithm (FIPWA) was first applied to accelerate the computation of the MoM for PEC bodies of revolution in [17] and homogeneous dielectric bodies of revolution in [18]. Here we extend this method to composite dielectric and PEC BOR.

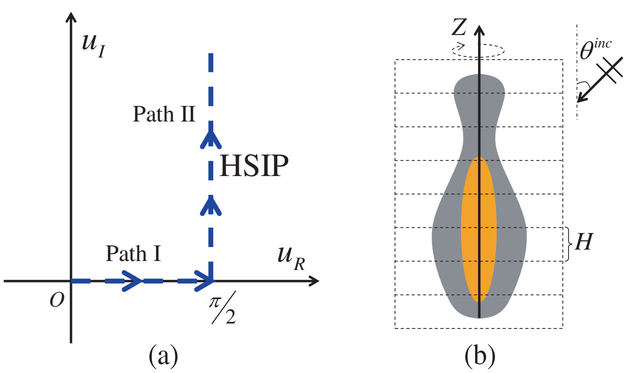

Figure 2. (a) The Sommerfeld integration path on the complex u plane. Path I is $(u_R = 0 \sim \frac{\pi}{2}, u_I = 0)$, the Path II is $(u_R = \frac{\pi}{2}, u_I = 0 \sim \infty)$. (b) The division of groups in the z direction, where H is the height of each group.

Based on Weyl identity [27–29], the Green's function can be rewritten as

$$\begin{aligned}
\frac{e^{-jkr_{pq}}}{r_{pq}} &= \frac{1}{j}\int_0^\infty dk_\rho \frac{k_\rho}{k_z} J_0(k_\rho \rho_{pq}) e^{-jk_z|z_{pq}|} \\
&= \frac{k}{j2\pi}\int_0^{2\pi} dv \int_{HSIP} du \sin u e^{-jk\hat{k}\cdot\mathbf{r}_{pq}} \\
&= \frac{k}{j2\pi}\int_0^{2\pi} dv \int_{HSIP} du \sin u e^{-jk\hat{k}\cdot(\mathbf{r}_{pm}+\mathbf{r}_{mm'}+\mathbf{r}_{m'q})} \\
&= \sum_{\Omega_s} B_{pm}(\Omega_s) B_{m'q}(\Omega_s) T_{mm'}(\Omega_s)
\end{aligned} \tag{13}$$

where $\hat{k} = \hat{x}\sin u \cos v + \hat{y}\sin u \sin v + \hat{z}\cos u$, $k_\rho = k\sin u$, $k_z = k\cos u$, and $\mathbf{r}_{pq} = \mathbf{r}_p - \mathbf{r}_q$. Here $\mathbf{r}_q$ is the source point and $\mathbf{r}_p$ is the field point; the integration of the variable u in Equation (13) is computed along the half Sommerfeld integration path (HSIP) in Figure 2. It is important to note that the variable $u = u_R + ju_I$ is complex here. The term $e^{-jk\hat{k}\cdot\mathbf{r}_{pq}}$ is called the inhomogeneous plane wave by Jackson [30]. The basis functions are divided into M groups along the z direction as shown in Figure 4, where $\mathbf{r}_m$ and $\mathbf{r}'_m$ are the centers of the groups which contain the source point $\mathbf{r}_q$ and field point $\mathbf{r}_p$ respectively, and $\mathbf{r}_{pq} = \mathbf{r}_{pm} + \mathbf{r}_{mm'} + \mathbf{r}_{m'q}$. Furthermore, $\mathbf{r}_{mm'}$ has $\hat{z}$ component only, with $\mathbf{r}_{mm'} = \hat{z}|z_m - z_{m'}|$. This property will make the integrand decay exponentially away from the real axis in the u plane. Note that the expansion in Equation (13) works only when $z_{mm'}$ is positive. If it is negative, we can easily rotate $z_{mm'}$ to make it positive. The detail can be found in [28, 29]. In Equation (13),

$$\begin{aligned}
\Omega_s &= (u_{s_1}, v_{s_2}) \\
B_{pm}(\Omega_s) &= e^{-jk\hat{k}\cdot\mathbf{r}_{pm}} \\
B_{m'q}(\Omega_s) &= e^{-jk\hat{k}\cdot\mathbf{r}_{m'q}} \\
T_{mm'}(\Omega_s) &= w_1 w_2 \frac{-jk}{2\pi} \sin u_{s_1} e^{-jkz_{mm'}\cos u_{s_1}}
\end{aligned}$$

u_{s_1} and v_{s_2} are the integration points for u and v, w_1 and w_2 are the weights for u and v, respectively. Equation (13) can be interpreted as the summation of inhomogeneous plane waves translated from the source group to the field group. Substituting Equation (13) into

Equation (6), we obtain

$$P^i_{pq} = \frac{k_i^2 \eta_i}{8\pi^2} \int_0^{2\pi} \int_{HSIP} \mathbf{V}^{iP}_{fmp} T^i_{mm'} \mathbf{V}^{iP}_{sm'q} du dv \tag{14}$$

$$Q^i_{pq} = -\frac{k_i^2}{8\pi^2} \int_0^{2\pi} \int_{HSIP} \mathbf{V}^{iQ}_{fmp} T^i_{mm'} \mathbf{V}^{iQ}_{sm'q} du dv \tag{15}$$

where the aggregation factors $\mathbf{V}^{iP}_{sm'q}$, $\mathbf{V}^{iQ}_{sm'q}$ and disaggregation factors $\mathbf{V}^{iP}_{fmp}$, $\mathbf{V}^{iQ}_{fmp}$ can be expressed as

$$\mathbf{V}^{iP}_{sm'q}(\Omega_s) = \int_S ds B^i_{m'q}(\Omega_s) \mathbf{f}_q(\mathbf{r}_{m'q}) \tag{16}$$

$$\mathbf{V}^{iP}_{fmp}(\Omega_s) = \int_S ds B_{pm}(\Omega_s) [\bar{\mathbf{I}} - \hat{k}\hat{k}] \cdot \mathbf{W}_p(\mathbf{r}_{pm}) \tag{17}$$

$\mathbf{V}^{iP}_{sm'q} = \mathbf{V}^{iQ}_{sm'q}$, $\mathbf{V}^{iQ}_{fmp} = \hat{k} \times \mathbf{V}^{iP}_{fmp}$. After substituting the basis and testing functions into Equation (16), the aggregation factors can be derived as

$$\mathbf{V}^{iP}_{sm'q}(\Omega_s) = \int dt \int_0^{2\pi} d\phi \rho_q(t) B_{m'q}(\Omega_s) \hat{a}_\alpha f_q(t) e^{jn\phi} \tag{18}$$

Using the integral representation of Bessel function, the above ϕ integration can be carried out analytically, then the aggregation factors of the t component can be simplified as

$$\begin{aligned} \mathbf{V}^{iPt}_{fmp} = & \int dt e^{-j\mathbf{k}_i \mathbf{r}_m} e^{-jk_i \cos u z_p} \\ & \cdot \left\{ \cos u \sin\theta \left[\frac{\pi}{j^{n+1}} J_{n+1}(\zeta_p) + \frac{\pi}{j^{n-1}} J_{n-1}(\zeta_p) \right] \hat{u} \right. \\ & - \sin u \cos\theta \frac{2\pi}{j^n} J_n(\zeta_p) \hat{u} \\ & \left. + \sin\theta \left[\frac{\pi}{j^n} J_{n-1}(\zeta_p) - \frac{\pi}{j^{n+2}} J_{n+1}(\zeta_p) \right] \hat{v} \right\} \end{aligned} \tag{19}$$

where $\zeta_i = k\rho_i \sin u$. This will greatly reduce the CPU time. From above it is clear that there is no variable v in the aggregation factor because of the use of azimuthal mode orthogonality. The other aggregation factors and disaggregation factors can also be derived analytically. The detail can be found in [18]. For numerical integration along the HSIP, Gauss-Legendre quadrature is used in Path I, while Gauss-Laguerre quadrature is used in Path II. The number of the sample points along Path I and Path II is decided by $M_u = kD + (kD)^{1/3}$. Numerical tests shows that $H = 0.3\lambda$ is a good dividing height for each group, see Figure 2.

3. NUMERICAL RESULTS

In this section, several numerical results are presented to show the validity of the proposed BOR-FIPWA. All problems are solved on the same computer (Intel Core2 Duo CPU P8400 @ 2.26 GHz with 1.92 GB RAM) in order to make a fair comparison, with only one core being used.

In order to test the accuracy of the proposed method, a coated sphere in free space is simulated by the BOR-FIPWA. The radius of the PEC sphere is 5 m, and the thickness of the homogeneous medium ($\epsilon_r = 4$, $\mu_r = 1$) is 3 m. The frequency of the incident plane wave is 150 MHz with horizontal polarization ($\theta_{inc} = 0°$, $\phi_{inc} = 0°$). In the discretization, there are 197 segments on the meridian line of the PEC surface and 315 segments on the meridian line of the dielectric surface. So there are 196 triangular functions for J_c^t, 196 triangular functions for J_c^ϕ, 314 triangular functions for J_d^t, 314 triangular functions for J_d^ϕ, 314 triangular functions for M_d^t, and 314 triangular functions for M_d^ϕ. So totally 1648 unknowns are involved for each azimuthal mode n. The number of basis functions per wavelength is about 12, which a reasonable sampling density. As shown in Figure 3, the bistatic RCS agrees well with Mie results [31]. The memory requirement is 20.7 MB, the CPU time for computing the impedance matrix is about 983 s for BOR-MoM [4, 6]. While for BOR-FIPWA, the memory requirement is 12.6 MB, and the CPU time is 139 s.

After the accuracy has been verified, four coated cylinders with the same radius ($R_{PEC} = 1$ m), same thickness ($t = 0.5$ m), same coated dielectric medium ($\epsilon_r = 4$, $\mu_r = 1$) but with different heights ($h_{PEC}^1 = 5$ m, $h_{PEC}^2 = 10$ m, $h_{PEC}^3 = 20$ m, $h_{PEC}^4 = 50$ m) are simulated by the BOR-FIPWA and BOR-MoM for testing the efficiency of the proposed

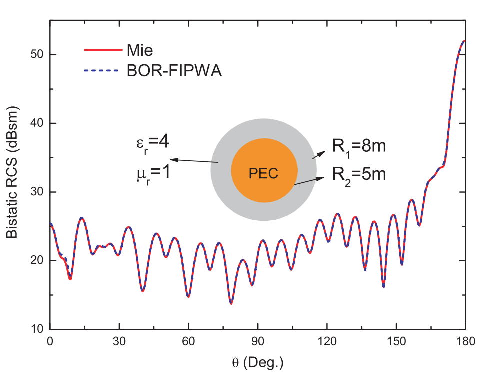

Figure 3. The bistatic RCS of the coated sphere with the inner radius $R_2 = 5$ m and outer radius $R_1 = 8$ m.

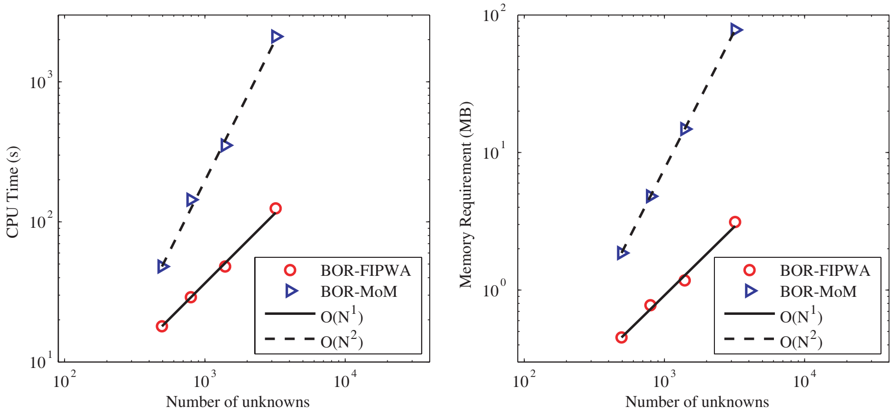

Figure 4. The CPU time and memory requirement for system creation of the BOR-FIPWA and BOR-MoM.

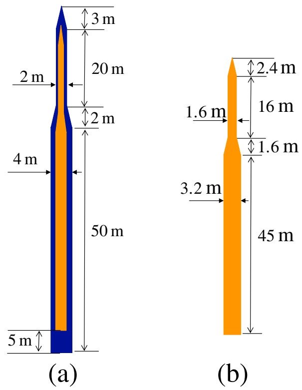

Figure 5. The geometry of the coated rocket. (a) The geometry of dielectric part. (b) The geometry of PEC part.

method. The frequency of the incident plane wave is 150 MHz with horizontal polarization. The CPU time and memory requirement for system creation of the BOR-FIPWA and BOR-MoM are shown in Figure 4. It is clear that the complexity of memory requirement for BOR-MoM and BOR-FIPWA are $O(N^2)$ and $O(N)$. Note that in this case D (diameter) does not change, so the number of sample points $M_u = kD + (kD)^{1/3}$ does not change in BOR-FIPWA; thus, the memory in BOR-FIPWA is only $O(N)$. The CPU time complexity for BOR-MoM and BOR-FIPWA are $O(N^2)$ and $O(N)$, respectively.

Finally, a rocket coated by a dielectric ($\epsilon_r = 4$, $\mu_r = 1$) is simulated by the BOR-FIPWA. The geometry is shown in Figure 5. The frequency of the incident plane wave is 150 MHz with horizontal

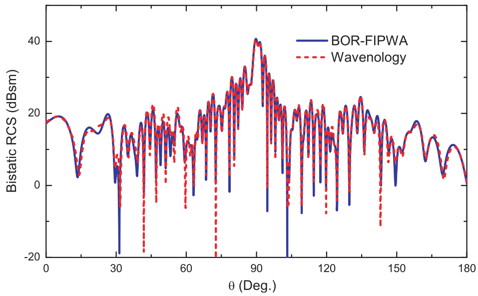

Figure 6. The bistatic RCS of the coated rocket in Fig. 5.

polarization ($\theta_{inc} = 90°$, $\phi_{inc} = 0°$). The total number of unknowns is 5802. As shown in Figure 6, the bistatic RCS agrees well with that from Wavenology EM [32], a software tool based on the enlarged cell technique [33, 34] in a conformal finite-difference time-domain method (rather than MoM). The memory requirement is 256.8 MB, and the CPU time for computing the impedance matrix is about 7956 s for each mode by BOR-MoM. In contrast, for BOR-FIPWA, the memory requirement is 36.5 MB, and the CPU time is 591 s.

4. CONCLUSION

In this paper, a fast inhomogeneous plane wave algorithm is applied to solve the composite BOR scattering problem. Analytical expressions for the aggregation and disaggregation factors are derived to save a lot of CPU time over the BOR-MoM. Both CPU time and memory requirement are saved by using BOR-FIPWA. For a coated rocket, the improvement factors for memory and CPU time over the MoM are 7.03 and 13.5, respectively.

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