

A NOVEL INTEGRATED MATHEMATICAL APPROACH OF RAY-TRACING AND GENETIC ALGORITHM FOR OPTIMIZING INDOOR WIRELESS COVERAGE

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Abstract—While designing wireless networks, it is crucial to obtain the maximum coverage by using the minimum number of transmitting antennas. This paper proposes a new algorithm for determining the minimum number of transmitting antennas as well as their appropriate locations to provide the optimized wireless coverage in the indoor environment. The proposed algorithm uses a ray-tracing method to predict the signal distribution among the sampling points in the indoor area due to one or more transmitters and the genetic algorithm (GA) incorporated with the Breath First Search (BFS) terminology to determine the minimum number of transmitters and their corresponding locations to achieve the optimum wireless coverage. The proposed method outperforms the existing method in terms of both space and time complexities. The results obtained from this study also show that the computation time using the proposed algorithm is much less than that of the existing algorithm.

1. INTRODUCTION

The optimum coverage problem is a classical issue in computational complexity theory. Because of ever increasing demand of this field, the well-known brute force algorithms are systematically going to be replaced by the state-of-the-art optimization techniques. Here, several sets are given as input and a number k . The sets may contain some elements in common. The target is to select at most k of these sets so that the number of covered elements is the maximum. In general, the area coverage can be approximated as point coverage. That is,

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if all the sampling points are covered by the minimum number of transmitters, the entire area will be covered optimally. The problem of selecting the minimum number of transmitters, however, is NP-hard [1]. There are existing optimization methods to determine the locations of transmitting antennas [2,3] and ray-tracing [4–16] methods to design and optimize the base stations; however, none of them provides an efficient integrated approach that can lead to the achievement of an optimum wireless coverage. To achieve this goal, it is imperative to incorporate the ray-tracing technique with the proper optimization algorithm. In this regards, genetic algorithm (GA) is being successfully used in optimizing many antenna design and electromagnetic problems [17–25] over the years. In this study, the problem of selecting the minimum number of transmitters to cover the whole area has been addressed.

In [5,13,26], the multipath propagation and the resulting multipath fading in a sample indoor environment is taken into account. However, in this study, the proposed algorithm uses both ray-tracing as reported in [25] and GA incorporated with the Breath First Search (BFS) [27,28] to propose an efficient coverage model for optimizing indoor wireless coverage. Here, two types of parameters are considered: large scale path loss and small scale fading statistics due to time-varying channel. The path loss information is essential while optimizing the positions of the transmitters. Small scale fading is vital to get the information about local field variations that helps in site planning to improve the coverage and the interference, and to address the multipath fading issues for indoor wireless channel [24]. Each chromosome is represented by a coverage pattern that keeps the coverage information of the corresponding transmitter. In BFS, the live node refers to that node, which has been generated and all of whose children have not yet been generated. The live node, whose children are currently being generated, is called E-node. This study applies branch-and-bound terminology while generating search tree using BFS. Basically, branch-and-bound is a backtracking process, where bounding functions are used to help avoid the generation of sub-trees that do not contain an answer node. The proposed algorithm generates less number of nodes in the search tree by applying bounding functions that decreases both time and space complexities, which lead to less number of iterations as well as less computation time. The proposed algorithm also shows that, instead of rerunning the ray-tracing method for the same transmitter's position, an individual coverage pattern can be generated for each transmitter that will be reused for further iterations. This concept reduces the computation time as well. This study has been proved efficient in the subsequent sections by comparing with the

existing algorithm [25]. Finally, a conclusion has been presented at the end of the paper.

2. PROPOSED COVERAGE ALGORITHM

The basic idea of the proposed algorithm is to select k number of transmitters for k positions from n number of available positions, where each transmitter must have different coverage pattern and the resultant pattern formed by recombination, will cover all the sampling points. To achieve this goal, this study integrates ray-tracing with GA incorporated with the BFS to optimize the indoor wireless coverage. For simplicity of discussion, the following notations will be used in the subsequent discussion.

- (i) t_i is the i th transmitter where $1 \leq i \leq n$, if the number of sampling points in the indoor area is n .
- (ii) p_i is the coverage pattern of t_i .
- (iii) $G(i)$ is the number of good sampling point in p_i .
- (iv) $B(i)$ is the number of bad sampling point in p_i .
- (v) $N(g)$ is the number of good sampling point in a coverage pattern.
- (vi) $N(b)$ is the number of bad sampling point in a coverage pattern.

As for genetic calculation, each chromosome is represented by a coverage pattern to describe the coverage information of each transmitter. Before describing the proposed algorithm, the concept of the coverage pattern is needed to be explained. Suppose, a set of n sampling points $S = \{s_1, s_2, \dots, s_n\}$ has been deployed in the region to be covered by a set of k transmitters $T = \{t_1, t_2, \dots, t_k\}$, where $k \leq n$ and each transmitter has a coverage pattern $P_i = \{e_1, e_2, \dots, e_n\}$, where $1 \leq i \leq k$. Here, the value of the element e_j is either "0" or "1" and $1 \leq j \leq n$. The value $e_j = 1$ refers to the j th sampling point as good sampling point, where the j th sampling point is covered by the i th transmitter. Thus, the existence of the relationship between the transmitter t_i and the sampling point s_j can be expressed as follows:

$$e_{i,j} = \begin{cases} 1 & \text{Sampling point } s_j \text{ is covered by the transmitter } t_i \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

The union of any two coverage patterns P' and P'' generates the resultant pattern P^* . That is,

$$P^* = \left\{ P' \cup P'' \mid \sum_{i=1}^n e_i \geq 0 \right\} \quad (2)$$

Table 1. Truth table for bitwise OR operation.

1st value	2nd value	Result
0	0	0
0	1	1
1	0	1
1	1	1

In the optimal condition, the number of transmitter is minimized and the summation of the values of the elements of the resultant coverage pattern is n . That is,

$$P^* = \left\{ x \mid \sum_{i=1}^n e_i = n \right\} \quad (3)$$

For example, if there are 10 sampling points numbered from 1 to 10 and one transmitter covers the sampling points 1, 3, 4, 8, respectively, then the coverage pattern for that transmitter will be as follows:

$e1$	$e2$	$e3$	$e4$	$e5$	$e6$	$e7$	$e8$	$e9$	$e10$
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
1	0	1	1	0	0	0	1	0	0

And, for another transmitter covering the sampling points 2, 3, 7, 9, 10, the coverage pattern will be:

0	1	1	0	0	0	1	0	1	1
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Therefore, $P' \cup P'' = P^* =$

1	1	1	1	0	0	1	1	1	1
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Here, the resultant pattern P^* is created by merging both P' and P'' based on the concept of logical inclusive “OR” operation as shown in Table 1, where the result is “1”, if the first bit is “1” OR the second bit is “1” OR both bits are “1”. Otherwise, the result is “0”.

Thus, if the set of unique coverage patterns is $P = \{p_1, p_2, \dots, p_n\}$, the objective is to find a subset $P' \subseteq P$, where the number of covered

sampling points is $\left| \bigcup_{P_i \in P'} P_i \right| = n$.

In this study, the BFS uses branch-and-bound terminology. That is, bounding functions as well as termination criteria are applied while expanding the search space. For n number of sampling points in the indoor environment, the proposed bounding functions are as follows:

- (i) As a bounding function, the obvious criteria will be used such that if $T' = \{t_1, t_2, \dots, t_i\}$ is the set of transmitters that represents the path to the current E-node, then all children nodes with parent-child labeling t_{i+1} are $T'' = \{t_1, t_2, \dots, t_i, t_{i+1}\}$, where the coverage pattern p_{i+1} of t_{i+1} will not be covered by the resultant pattern $\bigcup_{1 \leq j \leq i} P_j$ of the set $T' = \{t_1, t_2, \dots, t_i\}$.
- (ii) The second bounding function for the proposed algorithm is $(N(b) \text{ in } \bigcup_{1 \leq j \leq i} P_j) \leq \sum_{i+1 \leq j \leq n} g(j)$, where n is the maximum number of transmitters that can be considered while exploring the solution space and i refers to the i th transmitter that corresponds to the current E-node. Hence, $(i+1)$ refers to the child node of the E-node. That is, the number of bad sampling points in the resultant coverage pattern generated from the set of transmitters that represents the path to the current E-node, should be less than or equal to the summation of the good sampling points of the subsequent coverage patterns that correspond to the sub-tree, where t_{i+1} forms the root node.

To make sure the completeness of the proposed algorithm, the following termination criteria has been proposed. The algorithm will be terminated if any of the following conditions becomes true.

- (i) The set of transmitters $T' = \{t_1, t_2, \dots, t_i\}$ on the path to the current E-node have no bad sampling point in their resultant coverage pattern. That is, $P^* = (\bigcup_{1 \leq j \leq i} P_j)$ where $|p^*| = n$.
- (ii) There is no live node exists in the solution space to be explored.

To illustrate the proposed method, suppose, there are 6 sampling points labeled from 1 to 6 (i.e., $n = 6$) in the indoor propagation area as shown in Figure 1. The received power at each sampling point is calculated by using the ray tracer. The transmitters are supposed to be positioned to the left of the sampling points by an arbitrary small distance of 36 cm to avoid possible overlapping between the transmitters and the receiving points (sampling points) at which the powers are being calculated. That is, if the position of the transmitter is same as the sampling point, both positions of transmitter and sampling point will be overlapped and the proposed algorithm will skip that corresponding sampling point while calculating the received power for the sampling point.

Now, run the ray tracer individually for each of the 6 positions to calculate the received power at each sampling point and generate the coverage patterns for the corresponding transmitters. Suppose, the

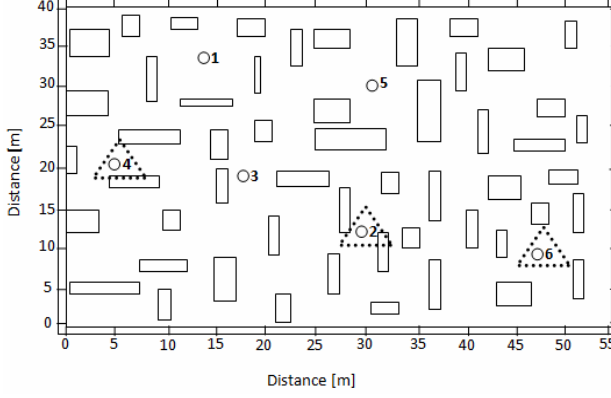


Figure 1. The floor plan of an office building including the locations of the sampling points (the optimized transmitter positions (Δ) when three transmitters are used).

coverage patterns of the transmitters t_1, t_2, \dots, t_6 are p_1, p_2, \dots, p_6 as follows:

$$\begin{aligned}
 p_1 &= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \\
 p_2 &= \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\
 p_3 &= \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \\
 p_4 &= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \\
 p_5 &= \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\
 p_6 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

Here, the cost function to measure the effectiveness of the coverage pattern is chosen as the number of bad sampling points. The lower the number of bad sampling points, the higher the effectiveness. From the above patterns, it is seen that the values of the cost functions are 3, 3, 3, 4, 3, and 5, respectively. Here, it is seen that both p_2 and p_5 are identical, that means, the sampling points covered by t_5 has already been covered by t_2 . Hence, the proposed algorithm skips the pattern p_5 of t_5 by marking it as a duplicate pattern. Thus, the proposed algorithm considers only 5 coverage patterns p_1, p_2, \dots, p_6 (except p_5) of their corresponding transmitters t_1, t_2, \dots, t_6 (except t_5) and selects the minimum number of patterns from them, whose resultant pattern covers the whole indoor propagation area. Figure 2 generates the state space search tree using different number of combinations of 5 transmitters to achieve the first optimal solution.

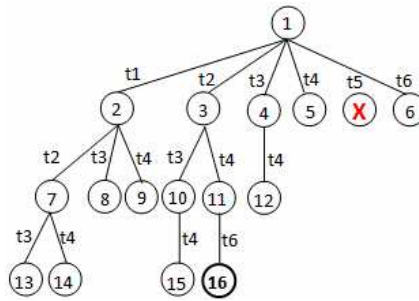


Figure 2. State space search tree generated by the proposed algorithm.

Initially, there is only one live node, node 1. This represents the case in which no transmitter has been placed in the propagation area. This node becomes the E-node. It is expanded and its children nodes 2, 3, 4, 5, and 6 are generated except the cross (X) marked node labeled by t_5 as its coverage pattern is already marked as duplicate. These nodes represent the solution space, where only one transmitter is considered at a time. The optimum solution is found in this state, which means, only one transmitter is sufficient for the optimum coverage of the corresponding area. Now, the only live nodes are 2, 3, 4, 5, and 6, respectively. If the nodes are generated in this order, then the next E-node is node 2 and the algorithm switches to the next level, where the optimization algorithm is based on the two transmitters. It is expanded and t_2 , t_3 , t_4 , and t_6 refer to the possible child nodes. The node for t_6 is ignored using the second bounding function. The set of transmitters that represents the path to the current E-node 2 is $\{t_1\}$. The set has only one transmitter t_1 , whose coverage pattern p_1 has 3 bad sampling points. The set of transmitters that form the sub-tree of root t_6 is $\{t_6\}$. This set contains only one transmitter t_6 , whose coverage pattern has 1 good sampling point. Therefore, according to the second bounding function:

$$N(b) \text{ in } \{t_1\} \leq N(g) \text{ in } \{t_6\} \quad (4)$$

$$\Rightarrow 3 \leq 1 \quad (5)$$

The condition of the above Equation (5) is incorrect. As a result, the proposed algorithm will not generate the child node for t_6 from node 2. Only nodes 7, 8, and 9 will be generated that correspond to the transmitters t_2 , t_3 , and t_4 . The node 3 of t_2 becomes the next E-node, which generates nodes 10 and 11 for t_3 and t_4 , respectively and skips node for t_6 because of the second bounding function. The next E-node 4 of t_3 generates only one node 12 for t_4 and skips node

for t_6 because of the second bounding function. The last node 6 on the same level cannot generate any node because of the second bounding condition as well. Now, the algorithm proceeds to the next level 2, where the E-node 7 of t_2 generates child nodes 13 and 14, respectively. The possible child nodes for the next E-node 8 of t_3 should be of the transmitters t_4 and t_6 . However, they cannot be generated because of the bounding functions. The set of transmitters that represents the path to the current E-node 8 is $\{t_1, t_3\}$, the resultant coverage pattern of which is as follows:

$$P^* = P_1 \cup P_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

The above pattern also covers p_4 that violates the first bounding function. On other hand, p_6 of t_6 violates the second bounding function. If the nodes are generated in this way, the tree in Figure 2 will be generated based on the proposed algorithm and the optimum solution will be formed by the transmitters t_2, t_4, t_6 , and the solution path will be formed by the nodes 1, 3, 11, and 16, respectively. The following recursive algorithm brings out the whole radio optimization technique for a typical indoor environment. The optimization algorithm is included as follows:

- (i) Suppose, there are n sampling points in the indoor environment. Therefore, in the worst case, maximum n transmitters are required for the optimum wireless coverage as sampling points are being used as the transmitter positions.
- (ii) Select i th sampling point as the i th transmitter position and run the ray tracer to calculate the received power at each sampling point and generate the coverage pattern for the i th transmitter. Here, the range of i is $1 \leq i \leq n$.
- (iii) If there is any duplicate coverage pattern, keep only one of them and skip others to handle redundancy issue. Hence, if k numbers of coverage patterns are skipped because of duplication, the number of accepted coverage pattern of the corresponding transmitters (for the generation of the solution space) will be $m = n - k$. Here, each or different combination of the m coverage patterns are possible solution candidates for the current optimization problem.
- (iv) Generate the search space tree based on the BFS. While expanding any live node of the search space, apply the proposed bounding functions as mentioned before to avoid generation of unnecessary sub-tree. Also, check the termination criterions as stated before to make sure proper termination as well as obtain the optimum solution.

3. COMPLEXITY ANALYSIS AND COMPARISON

In this section, both space and time complexities of the proposed algorithm will be deduced and compared. The proposed algorithm will be compared with the existing algorithm as reported in [25]. Hence, before proceeding to the analysis of the proposed algorithm, a brief description is needed for the algorithm, proposed by Z. Yun et al. [25]. The method discussed in [25] is developed to optimize the wireless coverage in a typical indoor environment. Here, ray-tracing [24] algorithm is used to calculate the received power at each sampling point due to one to more transmitting antennas. Moreover, GA is used to determine the minimum number of transmitters as well as their corresponding locations. In each generation, for any generated chromosome, the algorithm reruns the ray tracer to calculate the field distribution to every sampling point from the location of the transmitter(s) as provided by the corresponding chromosome(s). At the beginning, the position of the transmitter is optimized using only one transmitter. If there is any bad sampling point for the obtained position of the transmitter, another GA optimization is applied with two transmitters to reduce the number of bad sampling points. The algorithm continues running GA optimization considering more transmitters until the optimum wireless coverage is achieved. According to this algorithm, a preorder (method to visit each node before its children) based search tree for a typical indoor environment having 6 sampling points can be generated as illustrated in Figure 3. Here, the labeling of the edge refers to the corresponding transmitter. For example, i refers to the i th transmitter.

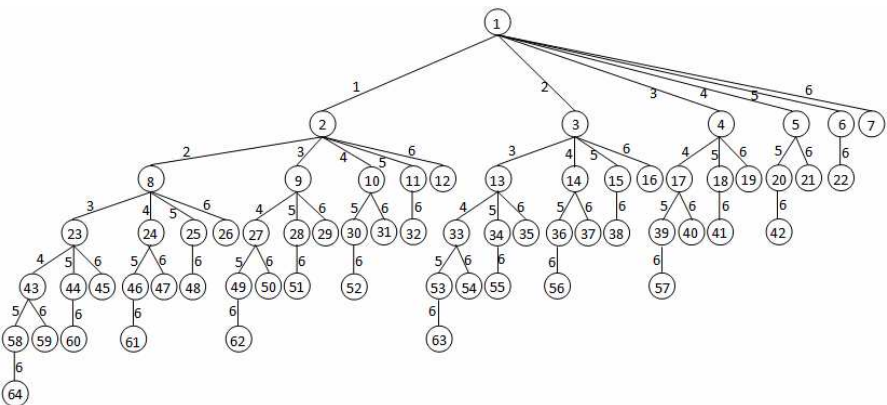


Figure 3. Tree organization for the existing coverage algorithm.

Table 2. Tabular format of a search tree.

$\begin{smallmatrix} l \\ Tx \end{smallmatrix}$	1	2	3	4	5	6
1	1					
2	1	1				
3	1	2	1			
4	1	3	3	1		
5	1	4	6	4	1	
6	1	5	10	10	5	1

The properties of the tree organization of Figure 3 can be described in Table 2, where l and Tx refer to the levels and transmitters, respectively. The node 1 at level 0 is not highlighting in Table 2 for the simplicity of analysis. The node 1 refers only the root node of the tree but it does not refer any transmitter. Therefore, node 1 can be referred as dummy node. Table 2 highlights of how many times a transmitter is being generated in each level.

A careful study of Table 2 reveals the criteria of Pascal's triangle, where any number is made up of the sum of the number above it and the one to the left. It can be expressed as the following symbol:

$$\binom{r}{k} = \binom{r-1}{k} + \binom{r-1}{k-1} \quad (6)$$

where r is the number of row and k is the number of column. The value of each cell can also be calculated using the combination formula $r^{-1}C_{k-1}$, where the values of r and k start from 1. Thus, in general, the total number of nodes generated by the tree can be calculated as the following level-wise formula:

$$1 + \sum_{1 \leq r \leq n} r^{-1}C_0 + \sum_{2 \leq r \leq n} r^{-1}C_1 + \dots + \sum_{n-1 \leq r \leq n} r^{-1}C_{n-2} + \sum_{r=n} r^{-1}C_{n-1} \quad (7)$$

$$= 1 + \left(\sum_{1 \leq l \leq d} \sum_{l \leq r \leq n} r^{-1}C_{l-1} \right) \quad (8)$$

$$= 2^n \quad (9)$$

where d is the maximum depth of the tree and n is the number of sampling points in the indoor environment. For algorithm by Yun et al. [25], the value of d equals to n in the worst case and the number of sampling point equals to the number of transmitters. Equation (9) represents the time complexity of the algorithm as in the worst case, all the nodes are generated until the required solution is found. As all the generated nodes stay in memory, therefore, the space

complexity is also same as the time complexity. It is computationally more expensive due to the exponential complexity that may lead to combinatorial explosion. According to this algorithm [25], each node in the solution space is represented by a chromosome and for each generated chromosome, the algorithm reruns the ray-tracing program to calculate the field distribution at every sampling point from the given position of the transmitter. Hence, the algorithm [25] runs the ray tracer maximum $2^n - 1$ times. As the node 1 at level 0 is a dummy node, thus, the ray tracer will not run for this case. If the indoor environment having 6 sampling points is covered optimally using 3 transmitters t_2 , t_4 , and t_6 , respectively, then the reduced tree view of the solution space of the existing algorithm [25] will be as in Figure 4, where the optimum solution path consists of the nodes 1, 3, 14, and 37, respectively.

As the space complexity refers to the number of nodes generated until the deepest level and the time complexity depends on the number of nodes generated or expanded until the required solution has been found; the proposed algorithm can be expressed as follows by modifying Equation (9):

$$2^{n-m} - k \tag{10}$$

where n is the number of sampling points in the selected indoor environment, m is the number of coverage patterns that has been rejected because of duplication, and k is the number of nodes that has been unexplored because of the proposed bounding functions as stated in Section 2. In the worst case, the value of both m and k will be 0. The space and time complexities of the existing algorithm (Yun et al. [25]) and the proposed algorithm derived from this study are shown in Table 3.

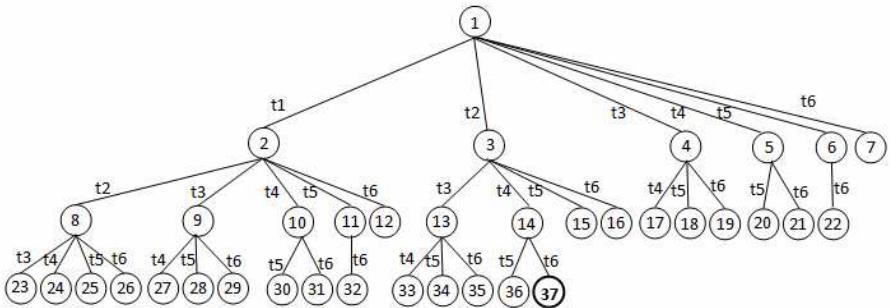


Figure 4. Tree organization of the solution space of the existing algorithm where the transmitters t_2 , t_4 , and t_6 provide the optimum coverage.

Table 3. Complexities of the existing and the proposed methods.

Complexity	Yun et al. [25]	Proposed
Space	2^n	$2^{n-m} - k$
Time	2^n	$2^{n-m} - k$

From Table 3, it is seen that both time and space complexities of the existing algorithm [25] are similar. In case of the proposed algorithm, both time and space complexities are also similar. Let, the time or space complexity of the existing algorithm [25] is $C_1 = 2^n$ and the proposed algorithm is $C_2 = 2^{n-m} - k$. Now, mathematically we prove as follows:

$$\begin{aligned}
& C_1 - C_2 \\
&= 2^n - 2^{n-m} + k \\
&= 2^m \cdot 2^{n-m} - 2^{n-m} + k \\
&= 2^{n-m} (2^m - 1) + k \\
&\geq 0
\end{aligned}$$

Thus, it can be written that $C_1 \geq C_2$. Therefore, both input and space complexities of the proposed algorithm are better than that of the existing algorithm as reported in [25], which is also highlighted in Figure 5, where the values of m and k are assigned to 1 and 2, respectively. That is, it has been considered that the number of rejected duplicate pattern is $m = 1$ and the number of unexplored node is $k = 2$. From Figure 5, it is seen that if the values of m and k are increased, the complexity difference will be much larger, which indicates better performance of the proposed algorithm in terms of both space and time complexities.

The comparison based on the computation time between the existing [25] and the proposed algorithm has been shown in Table 4. Here, 10 different scenarios having different number of sampling points have been considered. The number of transmitters in Table 4 refers how many transmitters are needed to cover all the sampling points. From Table 4, it is seen that the computation time difference is becoming larger as the number of sampling points is increasing. Thus, from Table 4, it is seen that 99% reduction of the computation time is possible by the proposed algorithm.

Moreover, for each generated chromosome (node), the existing algorithm [25] reruns the ray-tracing method each time. On the other hand, the proposed algorithm runs the ray tracer only once for one transmitting position and generates a coverage pattern that is used for further iterations. Therefore, if there is n number of sampling points

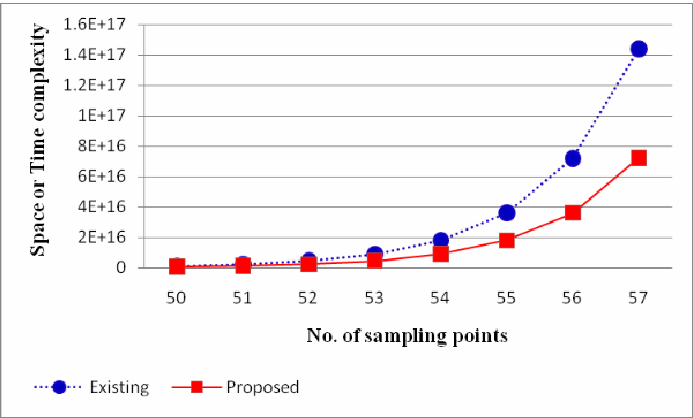


Figure 5. Comparison between the existing and the proposed method in terms of space or time complexity.

Table 4. Comparison between the existing and the proposed algorithm in terms of computation time.

No. of sampling points	No. of transmitters	Time for the existing algorithm [25] in seconds	Time for the proposed algorithm in seconds	Reduction (%)
5	2	2.63	1.87	28.9
7	3	14.33	2.71	81.09
10	3	25.01	3.64	85.45
12	4	118.79	4.5	96.21
15	3	89.24	5.36	93.99
18	4	386.56	6.26	98.38
20	4	584.2	6.84	98.83
22	4	719.86	7.56	98.95
24	4	928.14	8.4	99.09
25	4	1134.19	8.72	99.23

in the indoor environment, the proposed algorithm will run the ray tracer maximum of n times that refer to the remarkable reduction of the computation time as the ray tracer can be a costly service for a complex indoor environment. Moreover, the proposed algorithm is capable of ignoring the generation of unnecessary nodes, thus, it can find out the optimum solution with less number of iterations than that of the existing algorithm [25].

4. CONCLUSION

In this paper, a novel algorithm for wireless indoor coverage has been presented that has lower time and space complexities. The complexity difference between the existing and the proposed algorithm would be even larger, if the number of sampling points in the indoor environment increases. It is also shown that the proposed algorithm is capable of reducing the computation time as high as 99% because of strong bounding functions as well as the concept of coverage pattern. Therefore, it can be summarized that the proposed algorithm outperforms the existing algorithm in terms of space or time complexity and computation time. Although this paper describes the coverage optimization technique based on the indoor environment, it is actually a generalized algorithm that can be also applied for the outdoor wireless coverage in future. However, in that case, the prerequisite is to develop a suitable radio signal prediction algorithm. Moreover, the outcome of this study will be helpful while analyzing the radio networks and personal communication systems.

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