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OPTIMAL SUB-ARRAYING OF COMPROMISE PLANAR ARRAYS THROUGH AN INNOVATIVE ACO-WEIGHTED PROCEDURE

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Abstract—In this paper, the synthesis of sub-arrayed monopulse planar arrays providing an optimal sum pattern and best compromise difference patterns is addressed by means of an innovative clustering approach based on the Ant Colony Optimizer. Exploiting the similarity properties of optimal and independent sum and difference excitation sets, the problem is reformulated into a combinatorial one where the definition of the sub-array configuration is obtained through the search of a path within a weighted graph. Such a weighting strategy allows one to effectively sample the solution space avoiding bias towards sub-optimal solutions. The sub-array weight coefficients are then determined in an optimal way by exploiting the convexity of the problem at hand by means of a convex programming procedure. Representative results are reported to assess the effectiveness of the weighted global optimization and its advantages over previous implementations.

1. INTRODUCTION

Monopulse radars present several advantages over other search-and-track systems [1] based on conical scan or lobe switching approaches [2]. Indeed, tracking the angular positions of high-speed targets is enabled just processing a single pulse echo (a *monopulse*). Moreover, range measurements are generally more reliable because of echo signals with higher signal-to-noise ratios are dealt with, the sum beam being always directed towards the target.

Monopulse radars require the generation of one sum pattern and a couple of spatially-orthogonal difference patterns to track

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targets both in azimuth and elevation [3]. Several implementations exploit reflectors or lens antennas [2], even if antenna arrays turn out to be more convenient for technological (e.g., the main beam can be electronically steered), implementative (e.g., heavy structures as reflectors are avoided), and applicative (e.g., arrays can be made conformal and installed on aircrafts) reasons. However, the complexity of the underlying beamforming network (BFN) must be properly taken into account since it unavoidably grows because of the need to generate more than one pattern and to use a large number of elements. To overcome these limitations, sub-arraying strategies (e.g., sub-array weighting [4] and overlapped sub-arrays [5]) as well as sharing common weights between the sum and difference channels [6, 7] have been proposed. The sub-array weighting technique has received the widest interest as confirmed by the large number of published research works [8–17]. Generally, the problem is formulated as the synthesis of an optimal sum beam and the "best" compromise difference patterns grouping the array elements into suitably weighted sub-arrays. Towards this purpose, several optimization strategies have been applied. More specifically, the Simulated Annealing (SA) has been used in [8] to compute the sub-array weights for *a-priori* fixed element groupings, while a Genetic Algorithm (GA) [9] and two different implementations of the Difference Evolution (DE) algorithm [10, 13] have been adopted to determine both weights and subarraying. Moreover, an effective hybrid method has been proposed in [11] to exploit the convexity of the problem with respect to the sub-array weights. Whether, on one hand, global optimization is mandatory to deal with the non-convex part of the problem, on the other, the "brute force" application of stochastic optimizers turns out being computationally cumbersome and inefficient because of the exponential growth versus the number of array elements of the admissible sub-array configurations. Such a bottleneck has been efficiently solved in [18] by means of an excitation matching strategy where the sub-arraying grouping is "guided" by the similarity properties between the excitations providing the sum pattern and a set of reference excitations generating an optimal (reference) difference pattern. The dimension of the solution space has been significantly reduced and the final partitioning has been obtained by choosing $Q-1$ cut points (Q being the number of sub-arrays) in a sorted list of real values each one related to an antenna element. In such a way, the admissible set of sub-array configurations grows polynomially versus the number of elements with a non-negligible reduction of the solution space if compared to standard approaches. Furthermore, the *essential* solution space has been represented by means of a non-complete binary tree [18] and,

successively, through a more compact and non-redundant direct acyclic graph (DAG) [19]. By virtue of its hill climbing behavior (mandatory for non-convex functionals), the Ant Colony Optimizer (ACO) [20] has been used to look for the optimal sub-array configuration both within the solution tree [21] as well as in the DAG [22]. Although the ACO has shown to outperform the ad-hoc deterministic method called Border Element Method (BEM) in both linear [18] and planar [19] problems, it still presents some inefficiencies when large-dimension problems as for planar architectures. It is worth pointing out that these drawbacks do not depend on the representation of the solution space or its dimension, but mainly on the control of the evolution process. Indeed, if all edges of the DAG have the same probability of being chosen at the initialization, some paths (i.e., sub-arraying solutions) turn out having less probability of being explored, while other paths are privileged. Such a bias is undesired and unavoidably limits the potentialities of the approach. On the other hand, although the non-complete binary tree [21] is not affected by such a drawback, it is not suitable for synthesizing large arrays because of high computational costs and memory storage requirements. In this work, a new synthesis approach based on an edge-weighting scheme is proposed to guarantee each path of the DAG be explored with an equal probability.

The rest of the paper is organized as follows. The synthesis problem is mathematically formulated in Section 2 where the edge-weighting scheme for graph searching is presented, as well. Section 3 is devoted to the numerical analysis aimed at describing the behavior of the proposed approach and assessing its advantages and enhanced potentialities over previous implementations. Eventually, conclusions are drawn (Section 4).

2. MATHEMATICAL FORMULATION

Let us consider a monopulse planar array of $2M \times 2N$ elements displaced on a regular lattice with inter-element spacing d_x and d_y along the x and y axes, respectively. The antenna aperture is subdivided into four symmetrical quadrants whose outputs are combined to generate the sum and difference mode signals (Fig. 1) for the estimation of the off-boresight angle (OBA), namely the direction of the target with respect to the electrical axis (i.e., the boresight direction) of the antenna [2, 3].

The sum mode, used both in transmission (i.e., for the generation of the radar pulses aimed at sensing the surrounding environment) and in reception (i.e., for detecting the presence and range of a target through a monopulse comparator), is obtained by summing the signal

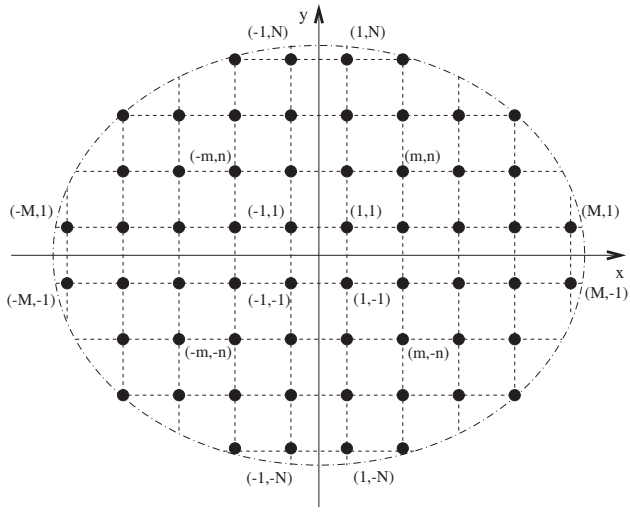

Figure 1. Sketch of a sub-arrayed monopulse array antenna.

from the four quadrants in phase. Under the assumption of quadrantal symmetry for the excitations [24], the sum pattern can be expressed as follows

$$\mathcal{S}(\theta,\phi) = 4\sum_{m=1}^{M}\sum_{n=1}^{N}\alpha_{mn}\cos\left(\frac{2m-1}{2}\psi_x\right)\cos\left(\frac{2n-1}{2}\psi_y\right) \quad (1)$$

where α_{mn}, $m = 1, \ldots, M$, $n = 1, \ldots, N$, are real excitation weights. Moreover, $\psi_x = kd_x\sin\theta\cos\phi$, $\psi_y = kd_y\sin\theta\sin\phi$, $k = \frac{2\pi}{\lambda}$ is the free-space wavenumber, λ being the wavelength.

The couple of difference mode signals used to determine the azimuthal and elevation OBA are generated summing in phase reversal pairs of quadrants of the optimal excitations β_{mn} that afford a desired difference pattern $\mathcal{D}(\theta,\phi)$. More specifically, the following difference pattern is synthesized

$$\mathcal{D}^{az}(\theta,\phi) = 4j\sum_{m=1}^{M}\sum_{n=1}^{N}\beta_{mn}\sin\left(\frac{2m-1}{2}\psi_x\right)\cos\left(\frac{2n-1}{2}\psi_y\right) \quad (2)$$

to track the target along the azimuthal plane [$\mathcal{D}^{az}(\theta,\phi) = \mathcal{D}(\theta,\phi)$], while the difference pattern for the elevation mode [$\mathcal{D}^{el}(\theta,\phi) = \mathcal{D}\left(\theta,\phi+\frac{\pi}{2}\right)$] is given by

$$\mathcal{D}^{el}(\theta,\phi) = 4j\sum_{m=1}^{M}\sum_{n=1}^{N}\beta_{mn}\cos\left(\frac{2m-1}{2}\psi_x\right)\sin\left(\frac{2n-1}{2}\psi_y\right). \quad (3)$$

According to the sub-arraying strategy [4], the excitations of the compromise difference patterns turn out to be

$$b_{m,n}=\alpha_{mn}\sum_{q=1}^{Q}\delta_{c_{mn}q}w_q;\ m=1,\ldots,M;\ n=1,\ldots,N;\ q=1,\ldots,Q \quad (4)$$

where $\mathbf{C}=\{c_{mn};\ m=1,\ldots,M;\ n=1,\ldots,N\}$ with $c_{mn}\in[0,Q]$ and $\mathbf{W}=\{w_q;\ q=1,\ldots,Q\}$ are the degrees of freedom of the problem at hand. They are two sets of integer values that code the element grouping and the weights of the corresponding clusters, respectively. In (4), $\delta_{c_{mn}q}$ is the Kronecker delta function defined as: $\delta_{c_{mn}q}=1$ if the element belongs to the q-th sub-array (i.e., $c_{mn}=q$) and $\delta_{c_{m,n}q}=0$, otherwise.

Following the guidelines described in [18], given a set of independent excitations $\mathbf{A}=\{\alpha_{mn};\ m=1,\ldots,M;\ n=1,\ldots,N\}$ affording an optimal sum pattern, the solution of the compromise between sum and difference patterns is obtained by minimizing the following cost function

$$\Psi(\mathbf{C})=\frac{1}{\Gamma}\sum_{m=1}^{M}\sum_{n=1}^{N}\alpha_{mn}^2\left\{g_{mn}-\sum_{q=1}^{Q}\delta_{c_{mn}q}w_q(\mathbf{C})\right\}^2 \quad (5)$$

where $g_{mn}\triangleq\frac{\beta_{mn}}{\alpha_{mn}}$, $m=1,\ldots,M$, $n=1,\ldots,N$ is the set of *optimal gains* and $\Gamma\leq M\times N$ is the number of radiating/active elements in each quadrant. Equation (5) defines a '*least square*' problem and its solution (i.e., the partition that minimizes the cost function) is a contiguous partition whose weighting vector $\mathbf{W}$ can be analytically computed as follows

$$w_q(\mathbf{C})=\frac{\sum_{m=1}^{M}\delta_{c_{mn}q}\alpha_{mn}\beta_{mn}}{\sum_{m=1}^{M}\delta_{c_{mn}q}\alpha_{mn}^2}. \quad (6)$$

since the value minimizing the sum of the square distances in a contiguous subset is the weighted arithmetic mean of the corresponding g_{mn} values. In order to determine the "optimal" sub-array configurations $\mathbf{C}^{opt}$, Eq. (5) is optimized according to the following procedure:

- *Step 1* — **Contiguous Partition Method (CPM)**. Exploiting the theory in [23] for the definition of contiguous partitions, $\mathbf{C}^{opt}$ is obtained by choosing Q subsets of the optimal gains g_{mn} sorted on a line [19]. Towards this end, a list $\mathbf{L}$ of Γ reference parameters is generated setting $l_1=\min_{m,n}\{g_{mn}\}$ and $l_\Gamma=\max_{m,n}\{g_{mn}\}$. In such a way, the number of admissible sub-array configurations (or

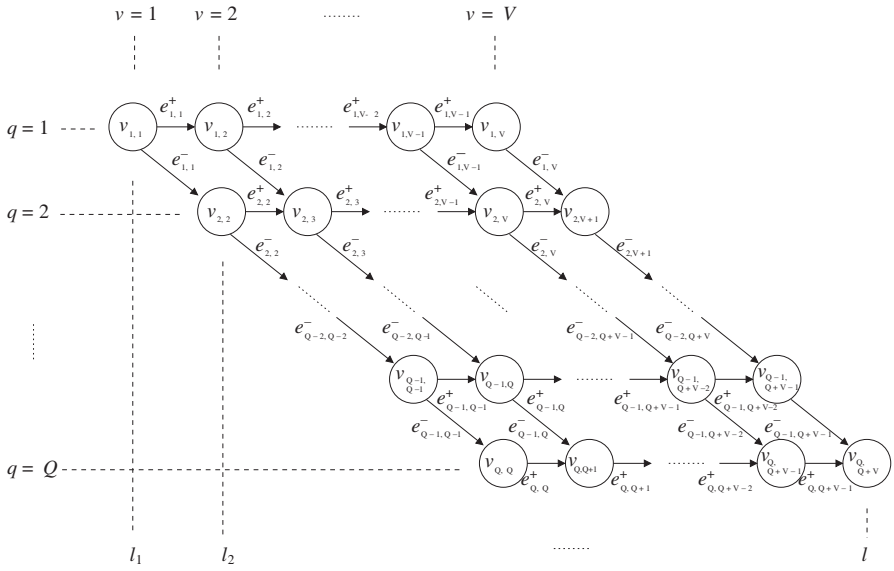

Figure 2. *DAG* representation of the solution space.

contiguous partitions) belonging to the so-called essential solution space $\Re^{(ess)\dagger}$ amounts to $U^{(ess)} = \begin{pmatrix} \Gamma - 1 \\ Q - 1 \end{pmatrix}$.

- *Step 2* — **Solution Space Representation**. Thanks to the sorted list defined at *Step 1*, the solutions in $\Re^{(ess)}$ are coded into a *Direct Acyclic Graph* (*DAG*) [28]. The graph $\mathcal{G}(\Gamma, Q, \Psi)$ represented in Fig. 2 is characterized by:
  - Q rows each one containing $V = (\Gamma - Q + 1)$ vertexes, V being the maximum number of elements that can be grouped in a sub-array;
  - a maximum depth Γ equal to the number of levels of the *DAG* and to the dimension of the list $\mathbf{L}$ as well as the number of vertexes along each r-th path $\mathbf{P}_r$, $r = 1, \ldots, U^{(ess)}$ in $\mathcal{G}$;
  - a suitability function Ψ (5) aimed at evaluating the goodness of each path $\mathbf{P}_r$, $r = 1, \ldots, U^{(ess)}$.

  The levels of the *DAG* map one-to-one the elements in $\mathbf{L}$. A vertex v_{q,l_q}, $q = 1, \ldots, Q$, $l_q = q, \ldots, q + V - 1$ is identified by its row index, q, and the depth index, l_q. Moreover, its argument,

† Essential with respect to the solution space which can be sampled using standard global optimizers whose dimension is $U = Q^{\Gamma}$.

$\arg\left(v_{q,l_q}\right) = q$, indicates the sub-array membership of each array element of the list $\mathbf{L}$. A path $\mathbf{P}$ of Γ vertexes and $\Gamma - 1$ edges codes a trial solution $\mathbf{C}$. As shown in Fig. 2, e^{+}_{q,l_q} is the edge (if present) connecting the vertexes v_{q,l_q} and v_{q,l_q+1} on the same row of the DAG, while e^{-}_{q,l_q} is the edge (if admissible) between the vertexes v_{q,l_q} and v_{q+1,l_q} on two different rows of the DAG;

- *Step 3* — **Edge Weighting**. In [22], the ACO was used to explore the DAG for identifying the best sub-array configuration $\mathbf{C}^{opt}$. Since the quantity of pheromone $\tau^{\pm}_{q,l_q}(0)$, $q = 1, \ldots, Q$, $l_q = q, \ldots, q + V - 1$ was uniformly set, the edges $e^{\pm}_{q,l_q}(0)$, $q = 1, \ldots, Q$, $l_q = q, \ldots, q + V - 1$ have at the initialization the same probability of being explored. Because of the DAG structure and the value of the ratio $\frac{V}{Q}$, such a choice affects in a non-negligible way the ACO-based sampling of the DAG. Indeed, some edges paths have a higher probability of being sampled since the vertexes could belong to a different number of paths. As representative examples, the DAGs of the cases ($\Gamma = 8$, $Q = 3$) and ($\Gamma = 8$, $Q = 6$), both having $U^{(ess)} = 21$, are reported in Fig. 3(a) and Fig. 3(b), respectively. By sake of clarity, the number of solutions to which edge belongs to is indicated.
  In order to assure a uniform probability of sampling to each solution/path and to allow an unbiased search a proper edge weighting scheme is necessary. Towards this end, the level of pheromone on each edge is increased/reduced proportionally to the number of different contiguous partition defined through that edge. Let us observe that the number of paths leaving the root vertex $v_{1,1}$ corresponds to the dimension of the whole solution space $\Omega_{1,1} = U^{(ess)} = \begin{pmatrix} \Gamma - 1 \\ Q - 1 \end{pmatrix}$, while those departing from the vertex $v_{1,2}$ [Fig. 4(a)] and $v_{2,2}$ [Fig. 4(b)] are $\Omega_{1,2} = \begin{pmatrix} \Gamma - 2 \\ Q - 1 \end{pmatrix}$ and $\Omega_{2,2} = \begin{pmatrix} \Gamma - 2 \\ Q - 2 \end{pmatrix}$, namely the number of path through $\mathcal{G}(\Gamma - 1, Q, \Psi)$ and $\mathcal{G}(\Gamma - 1, Q - 1, \Psi)$, respectively. Generalizing, the number Ω of paths/solutions available from the generic vertex v_{q,l_q} is equal to

$$\Omega_{q,l_q} = \begin{pmatrix} \Gamma - l_q \\ Q - q \end{pmatrix}. \tag{7}$$

  Therefore, the edge-weighting scheme is applied at the initialization ($j = 0$) as follows: the level of pheromone on edge e^{+}_{q,l_q} is set

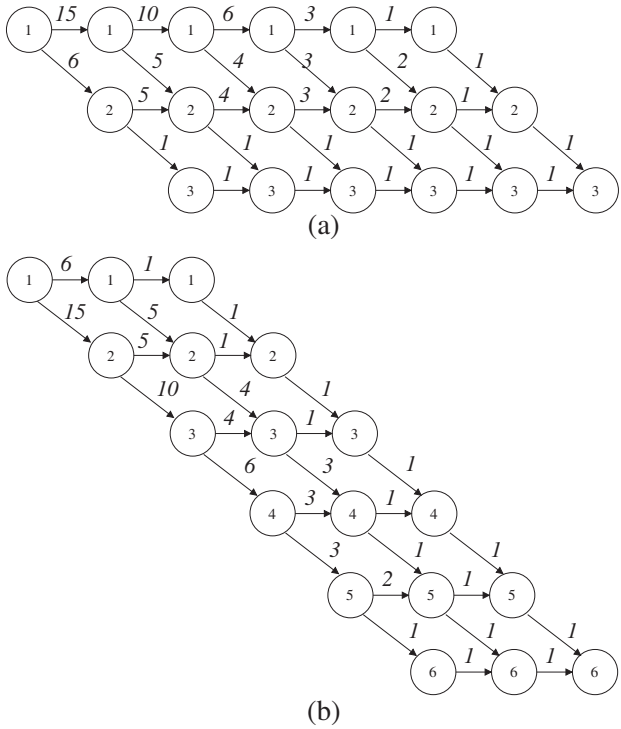

Figure 3. *DAG Analysis* ($\Gamma = 8$, $Q = \{3, 6\}$) — Number of trial solutions to which the *DAG* edges belong to when (a) $\Gamma = 8$, $Q = 3$ and (b) $\Gamma = 8$, $Q = 6$.

to

$$\tau^{+}_{q,l_q}(j) = \frac{\Omega_{q,l_q+1}}{\Omega_{q,l_q}} \tag{8}$$

while on the edge e^{-}_{q,l_q}

$$\tau^{-}_{q,l_q}(j) = \frac{\Omega_{q+1,l_q}}{\Omega_{q,l_q}}. \tag{9}$$

It is worth noting that $\Omega_{q,l_q} = \Omega_{q,l_q+1} + \Omega_{q+1,l_q}$.

- *Step 4* — *DAG ACO*-**Sampling**. Iteratively, the *ACO* [20, 25] explores the *DAG* to find $\mathbf{C}^{opt}$. Each ant of the colony $\mathbf{A}(j) = \{\mathrm{a}_t(j)\,;\, t = 1, \ldots, T\}$, T being the colony dimension, samples the graph starting from the root $v_{1,1}$ and choosing the next edge with

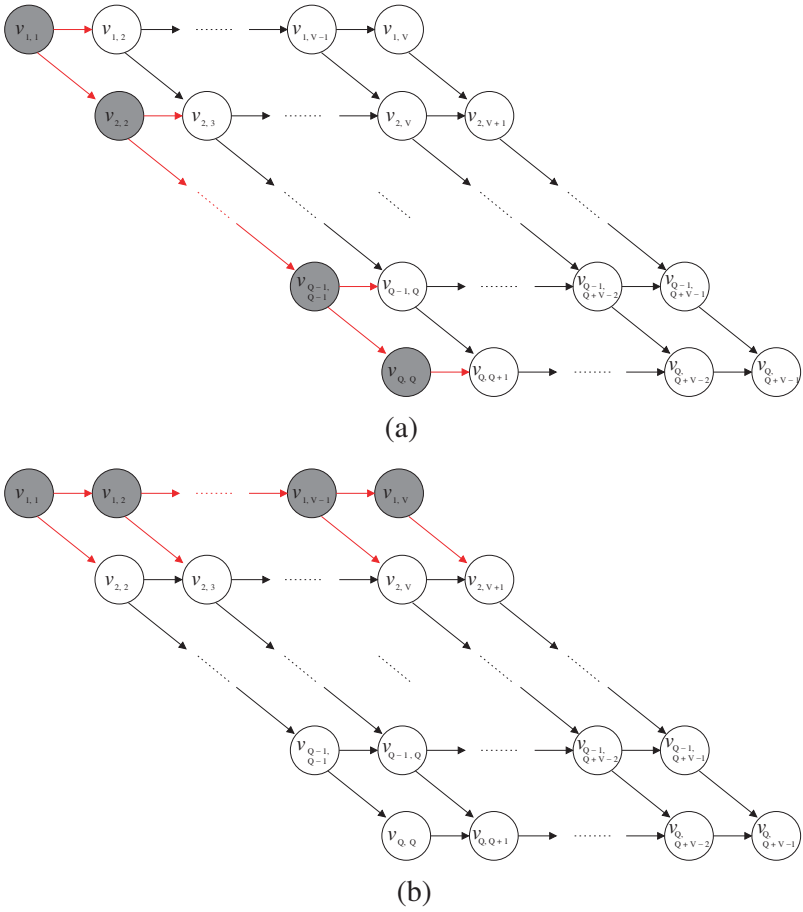

Figure 4. *Edge Weighting Approach— DAG* regions admissible from (a) the vertex $v_{1,1}$ and (b) the vertex $v_{2,2}$.

probability

$$\eta^{\pm}_{q,l_q}(j) = \frac{\tau^{\pm}_{q,l_q}(j)}{\tau^{+}_{q,l_q}(j) + \tau^{-}_{q,l_q}(j)},\ q = 1,\ldots,Q;\ l_q = q,\ldots,q+V-1. \quad (10)$$

The set of vertexes visited by an ant, $\mathrm{a}_t(j)$, from the root to the end of the graph codes a path $\mathbf{P}_t(j) = \left\{v_{q,l_q};\ q = 1,\ldots,Q;\ l_q = 1,\ldots,\Gamma\right\}$ of Γ vertexes composed by $\Gamma-1$ edges that identifies a trial sub-array configuration $\mathbf{C}_t(j) = \arg\{\mathbf{P}_t(j)\}$. The optimality of each trial solution is quantified by the value of the cost function in correspondence with

the corresponding subarray configuration, $\Psi(\mathbf{C}_t(j))$. Such an information is exploited to update the pheromone level on the edges of the DAG as

$$\tau^{\pm}_{q,l_q}(j+1) \leftarrow (1-\rho)\left[\tau^{\pm}_{q,l_q}(j) + \sum_{t=1}^{T} \frac{H \times \Psi_j^{\min}}{\Psi(\mathbf{C}_t(j))}\right] \tag{11}$$

where either e^{+}_{q,l_q} or $e^{-}_{q,l_q} \in \mathbf{P}_t(j)$ and $\Psi_j^{\min} = \min_{t=1,\ldots,T}\{\Psi(\mathbf{C}_t(j))\}$. Moreover, $\rho \in (0,1]$ and H are positive indexes that control the pheromone evaporation and deposition on the edges of the DAG. The algorithm stops when a maximum number of iterations $J_{\max}$ is reached or the minimization of the cost function reaches a stationary point ($j = J_{stat}$), then $\mathbf{C}^{opt}$ chosen as

$$\mathbf{C}^{opt} = \arg\left[\min_j \min_t \left\{\Psi(\mathbf{C}_t(j))\right\}\right]. \tag{12}$$

3. NUMERICAL RESULTS

A set of numerical experiments has been carried out to point out the potentialities of the proposed approach as well as its improvements over previous implementations.

The first example deals with the synthesis of a small array in order to detail in a comparative fashion the behavior of the edge-weighted approach versus the uniform technique [22]. The array elements are located on a regular lattice with $M = N = 3$ ($d_x = d_y = \frac{\lambda}{2}$) and belong to a circular support of radius $R = 1.5\lambda$ such that the resulting arrangement is composed by $\Gamma = 32$ radiators (8 for each quadrant). The excitations of the sum mode (Fig. 5) have been chosen to afford a Taylor pattern with $SLL = -35\,\mathrm{dB}$ and $\bar{n} = 6$ [24]. As far as the reference difference beam $\mathcal{D}(\theta,\phi)$ is concerned, a Bayliss pattern with $SLL = -30\,\mathrm{dB}$ and $\bar{n} = 7$ [24] has been used by setting the excitation distribution as in Fig. 6.

The compromise difference beam has been synthesized varying the number of sub-arrays in the range $Q \in [2,6]$ to analyze the performance of the proposed method. First, the Γ optimal gains have been computed and the list $\mathbf{L}$ generated (Fig. 7) according to the CPM.

Figure 8 shows the values of the cost function for the best solutions found by the proposed weighted-graph ACO-based (WG-ACO) approach and the ACO version in [22] when running 10 different simulations for each value of Q. The ACO parameters have been set according to the outcomes from [22]: $T = 0.1 \times U^{(ess)}$ with a minimum value equal to $T_{\min} = 5$ to exploit the cooperative behavior of the

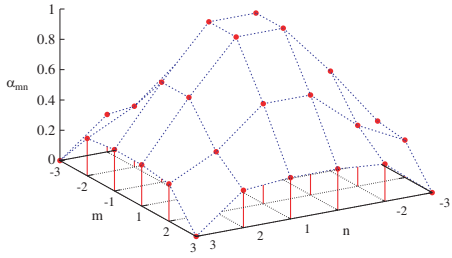

Figure 5. *WG-ACO Numerical Results* ($M = N = 3$, $\Gamma = 8$, $Q \in [2, 6]$) — Excitations of the optimal sum pattern (*Taylor*, $SLL = -35\,\mathrm{dB}$, $\bar{n} = 6$ [24]).

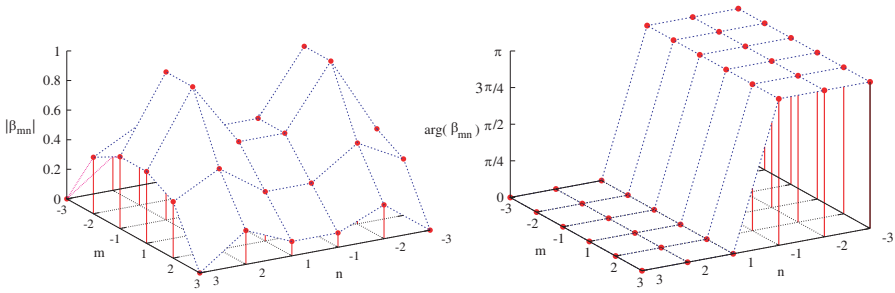

Figure 6. *WG-ACO Numerical Results* ($M = N = 3$, $\Gamma = 8$, $Q \in [2, 6]$) — Excitations of the reference difference pattern (*Bayliss*, $SLL^{ref} = -30\,\mathrm{dB}$, $\bar{n} = 7$ [24]): (a) amplitudes and (b) phase weights.

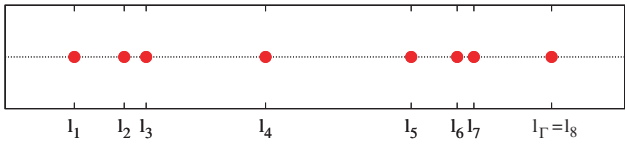

Figure 7. *WG-ACO Numerical Results* ($M = N = 3$, $\Gamma = 8$, *Taylor*, $SLL = -35\,\mathrm{dB}$, $\bar{n} = 6$, *Bayliss*, $SLL^{ref} = -30\,\mathrm{dB}$, $\bar{n} = 7$) — List $\mathbf{L}$ of the sorted optimal gains.

colony, $J_{\max} = 1000$, $H = 1$, and $\rho = 0.05$. It is worth noting that both methods find the global optimum when Q is smaller than Γ (e.g., $Q = \{2, 3, 4\}$) as confirmed by the plot in Fig. 9(a) that shows the cost function values for all the solutions belonging to $\Re^{(ess)}$ ($\Gamma = 8$, $Q = \{2, 3, 4\}$). Nevertheless, the bare *ACO* does not reach the global

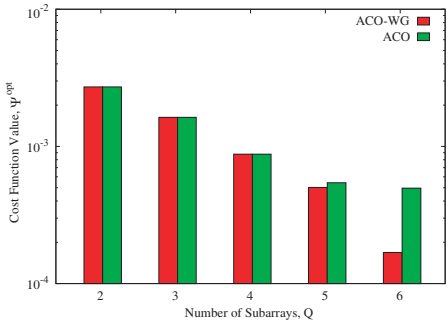

Figure 8. *Comparative Assessment* ($M = N = 3$, $\Gamma = 8$, $Q \in [2, 6]$, *Taylor*, $SLL = -35\,\text{dB}$, $\bar{n} = 6$, *Bayliss*, $SLL^{ref} = -30\,\text{dB}$, $\bar{n} = 7$) — Cost function values in correspondence with the optimal solutions found by the *ACO* and the *WG-ACO*.

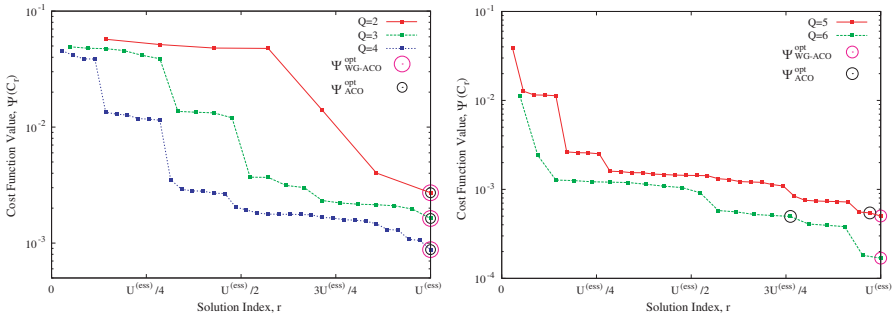

Figure 9. *Comparative Assessment* ($M = N = 3$, $\Gamma = 8$, $Q \in [2, 6]$, *Taylor*, $SLL = -35\,\text{dB}$, $\bar{n} = 6$, *Bayliss*, $SLL^{ref} = -30\,\text{dB}$, $\bar{n} = 7$) — Cost function values of the solutions coded within the *DAG* when (a) $Q \in [2, 5]$ and (b) $Q = \{5, 6\}$. The *ACO* and the *WG-ACO* solution are denoted by with a circle.

solution when $Q \simeq \Gamma$ ($\Gamma = 8$, $Q = \{5, 6\}$) since it gets stuck in a local minimum [Fig. 9(b)]. As a matter of fact, $.\Psi^{opt}_{WG-ACO}|_{Q=5} = 5.023 \times 10^{-4}$ vs. $.\Psi^{opt}_{ACO}|_{Q=5} = 5.438 \times 10^{-4}$ and $.\Psi^{opt}_{WG-ACO}|_{Q=6} = 1.685 \times 10^{-4}$ vs. $.\Psi^{opt}_{ACO}|_{Q=6} = 4.965 \times 10^{-4}$. The corresponding paths within the *DAG* are as follows: $.\underline{P}^{opt}_{WG-ACO}|_{Q=5} = \{11123445\}$ vs. $.\underline{P}^{opt}_{ACO}|_{Q=5} = \{12234445\}$ and $.\underline{P}^{opt}_{WG-ACO}|_{Q=6} = \{12234556\}$ vs. $.\underline{P}^{opt}_{ACO}|_{Q=6} = \{11123456\}$.

Table 1. *WG-ACO Numerical Results* ($M = N = 3$, $\Gamma = 8$, $Q \in [2, 6]$) — Dimension of the solution space $U^{(ess)}$.

Γ	8				
Q	2	3	4	5	6
U^{ess}	7	21	35	35	21

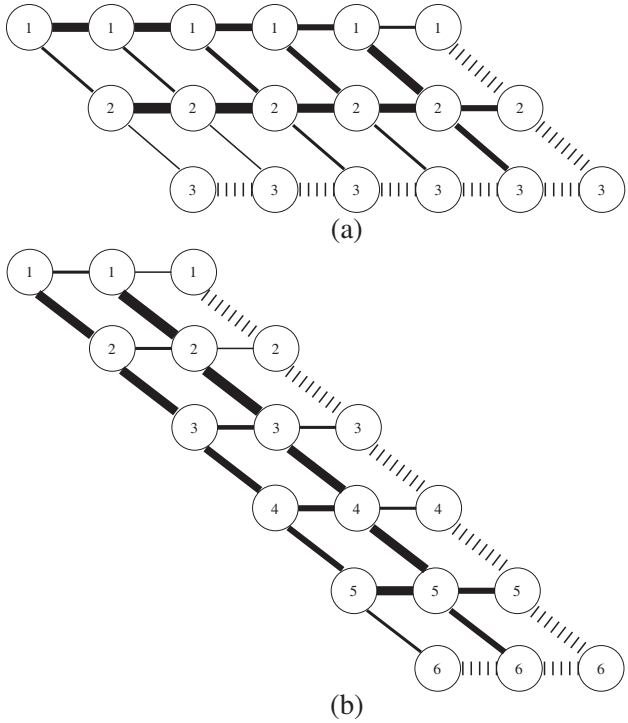

Figure 10. *WG-ACO Numerical Results* ($\Gamma = 8$, $Q = \{3, 6\}$) — Edge weighting approach as applied to the *DAG* sampling when (a) $\Gamma = 8$, $Q = 3$ and (b) $\Gamma = 8$, $Q = 6$. The dotted lines indicates mandatory choices.

Let us notice that, despite the dimension of the solution space does not vary from $Q = 3$ up to $Q = 6$ (see Table 1), the uniform *ACO* is able to get the "best" compromise solution only in the former case, while sub-optimal solutions are found otherwise. Such a result is not due to the *DAG* representation of the solution space, but on the "control level" of the *ACO* [25] (i.e., control parameters, initialization

criteria, constraints, and termination conditions) which exploits the *pheromone update mechanism* to sample the solution space looking for the global optimum. As a matter of fact, still keeping the same *ACO* structure presented in [22], but initializing the pheromone levels through the weighted approach, the reliability of the *DAG* sampling has been improved. As an illustrative example, Figure 10 gives a representation of the relative amount of pheromone on the edges of the *DAG* for the case ($\Gamma = 8$, $Q = 3$) [Fig. 10(a)] and the case ($\Gamma = 8$, $Q = 6$) [Fig. 10(b)]. More in detail, the thickness of the segments between two vertexes is proportional to the amount of pheromone on the corresponding edge. Moreover, the dotted lines indicate obliged choices when only the corresponding path is admissible.

The inefficiencies of the uniform-weight approach is more evident when $U^{(ess)}$ grows as pointed out by the plots of Ψ^{opt} in Fig. 11. The test case is here concerned with a lattice of dimension $2M \times 2N = 20 \times 20$, a circular boundary $R = 5.0\lambda$ in radius, and a number of active elements for each quadrant equal to $\Gamma = 75$. The number of sub-arrays has been varied between $Q = 2$ and $Q = 20$. As for the excitations, the sum excitations was chosen to afford a Taylor pattern with $SLL = -35\,\text{dB}$ and $\bar{n} = 6$ [24], while reference excitations was used to generate a Bayliss pattern with $SLL = -30\,\text{dB}$ and $\bar{n} = 7$ [24]. Concerning the parameters of the *ACO*, the same setting of the previous experiment has been used also introducing a maximum threshold $T_{\max} = 1000$ (when $T = 0.1 \times U^{(ess)} > T_{\max}$) on the number of ants for each iteration to limit the computational time. As expected

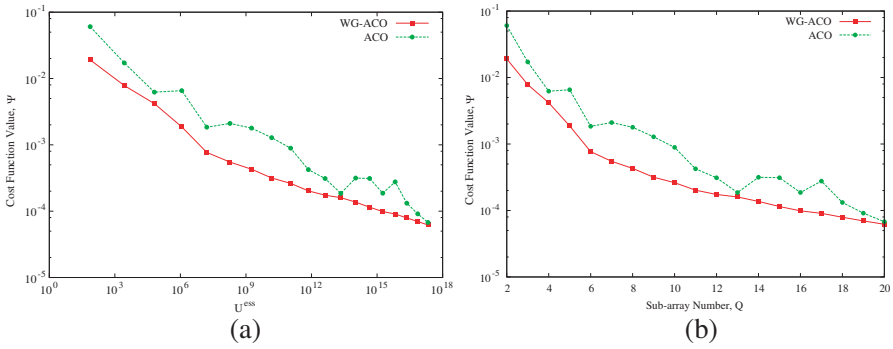

Figure 11. *Comparative Assessment* ($M = N = 20$, $\Gamma = 75$, $Q \in [2, 20]$, *Taylor*, $SLL = -35\,\text{dB}$, $\bar{n} = 6$, *Bayliss*, $SLL^{ref} = -30\,\text{dB}$, $\bar{n} = 7$) — Cost function values in correspondence with the optimal solutions found by the *ACO* and the *WG-ACO* versus (a) the dimension of the solution space, $U^{(ess)}$, and (b) the number of sub-arrays, Q.

(Fig. 11), the weighted approach always outperforms the previous implementation and, for each example (i.e., U^{ess} value or Q value), solutions with lower cost function values have been determined.

As far as the computational issues are concerned, let us consider that the CPU-time required to complete an ACO iteration is the same for both the weighted and uniform scheme. It is also worth noticing that the improvements from the weighted scheme are not concerned with the convergence speed, but rely in a more reliable search of the optimal solution. For completeness and as a representative example, the case $Q = 5$ needs $J_{stat} = 85$ iterations of the WG-ACO (i.e., a total CPU-time of 16.34 sec) to sample the solution space of dimension $U^{(ess)} = 1150626$, while the uniform approach with the same ACO parameter setting stops after $J_{stat} = 122$ iterations (i.e., a total CPU-time of 23.28 sec).

3.1. The Hybrid Extension

Although the WG-ACO proved its effectiveness, the computation of the sub-array weights through (6) does not guarantee the retrieval of the global optimum solution. Moreover, it does not allow to set constraints on the desired radiation pattern in a direct fashion [26, 27]. The hybrid method in [11, 29] overcomes such a limitation. Once $\mathbf{C}^{opt}$ was defined by means of the WG-ACO, the optimal weights $\mathbf{W}^{opt}$ can be computed by means of a convex programming (CP) strategy [30], aimed at minimizing

$$\Phi(\mathbf{W}) = -\mathrm{Im}\left\{\frac{\partial \mathcal{D}(\theta,\phi)}{\partial \gamma}\right\}_{\gamma=\{\theta,\phi\}}\Bigg|_{\substack{\theta=\theta_0\\ \phi=\phi_0}} \tag{13}$$

to the maximize the slope along the boresight direction (θ_0, ϕ_0) of the difference pattern $\mathcal{D}(\theta,\phi)$, subject to $\mathrm{Re}\left\{\frac{\partial \mathcal{D}(\theta,\phi)}{\partial \gamma}\right\}_{\gamma=\{\theta,\phi\}}\Big|_{\substack{\theta=\theta_0\\ \phi=\phi_0}} = 0$, $\mathcal{D}(\theta_0,\phi_0) = 0$, and $|\mathcal{D}(\theta,\phi)|^2 \leq \mathcal{M}(\theta,\phi)$, $\mathcal{M}(\theta,\phi)$ being a positive upper bound function on the power radiated in the sidelobe region. Moreover, $\mathrm{Re}\{\,\}$ and $\mathrm{Im}\{\,\}$ indicate the real and imaginary part, respectively. Furthermore, $\theta \in \left[0, \frac{\pi}{2}\right]$ and $\phi \in [0, 2\pi]$.

To show the behavior of the hybrid method (H-WG-ACO), an array with $M = N = 5$ elements located on a square grid with uniform spacing $d = \frac{\lambda}{2}$ is used as benchmark geometry. The aperture radius has been set to $R = 2.5\lambda$ such that $\Gamma = 19$. The same sum pattern of the previous examples has been kept, while the reference difference excitations β_{mn} [Figs. 12(a) and (b)] have been generated

by applying the procedure in [30] to synthesize the optimal difference pattern $D(\theta, \phi)$ with $SLL^{ref} = -25\,\mathrm{dB}$ shown in Fig. 12(c). In order to design the compromise difference pattern, $Q = 5$ sub-arrays have been used for each quadrant.

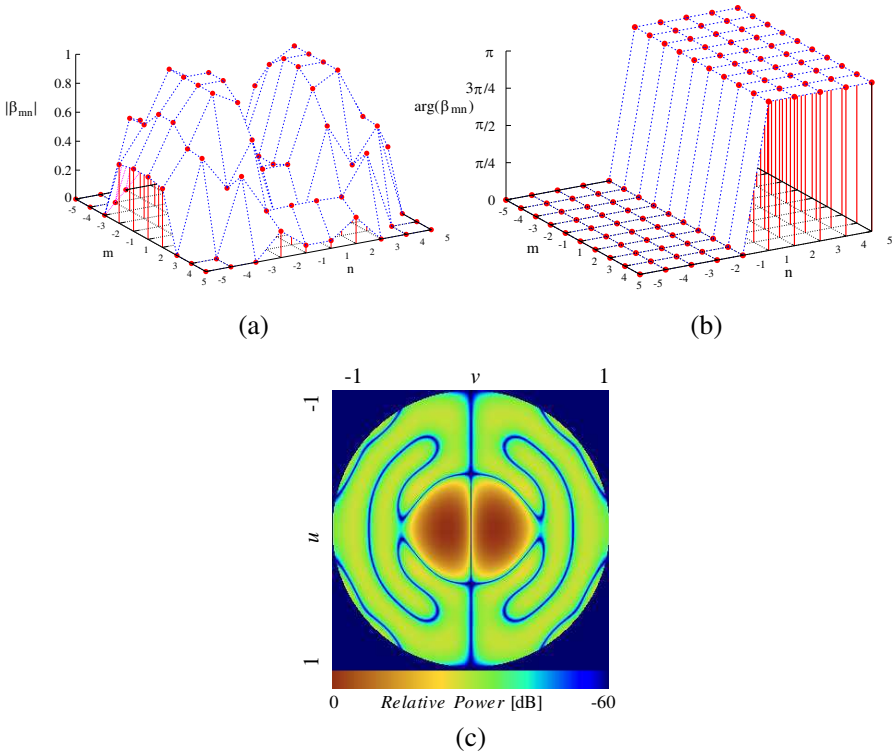

Figure 12. *Hybrid Extension* ($M = N = 5$, $\Gamma = 19$, *Reference difference* [30] — $SLL^{ref} = -25\,\mathrm{dB}$) — Reference difference excitations: (a) amplitudes and (b) phase weights. Power pattern of the reference mode (c).

Table 2. *Hybrid Approach* ($M = N = 5$, $\Gamma = 19 \times 4$, $Q = 5$) — Values of the sub-array weights.

	w_1	w_2	w_3	w_4	w_5
H-WG-ACO	1.0942	2.0305	2.9870	4.5573	5.6723
H-BEM	1.0488	2.7605	4.2845	4.8999	5.5077

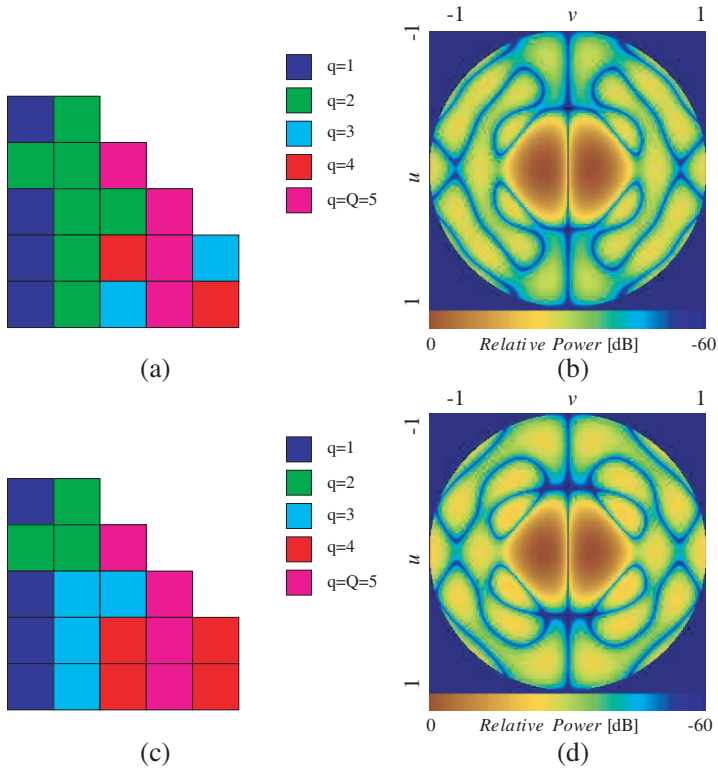

Figure 13. *Hybrid Extension* ($M = N = 5$, $\Gamma = 19$, *Reference difference* [30] — $SLL^{ref} = -25\,\mathrm{dB}$, $Q = 5$) — Plots of (a) (c) the sub-array configurations and of (b) (d) the relative power pattern determined with (a) (b) the *H-WG-ACO* and (c) (d) the *H-BEM*.

The array clustering found by the *WG-ACO* when exploring the solution graph with $T = 30$ ants is shown in Fig. 13(a). Successively, the convex programming procedure has been applied by constraining the pattern to the same mask used to determine the optimal difference pattern [Fig. 12(c)]. The values $\mathbf{W}^{opt}$ are then given in Table 2, while the corresponding pattern is shown in Fig. 13(b). For comparison, the same synthesis problem has been addressed with the hybrid-*BEM* (*H-BEM*) approach [19] and the results are reported in Fig. 13 and Table 2, as well. For completeness, Fig. 14 plots the level of the secondary lobe normalized to the maximum of the power pattern for each ϕ-cut, $\phi \in [0 : 80°]$ [Fig. 14(a)] and the sidelobe ratio defined as $SLR(\phi) = \frac{SLL(\phi)}{\max_{0 \le \theta < \frac{\pi}{2}} [\mathcal{D}(\theta,\phi)]}$, $\phi \in [0 : 80°]$ [Fig. 14(b)]. As it can be observed, the *H-WG-ACO* solution improves that obtained with

the *H-BEM* in terms of maximum *SLL* ($SLL_{H\text{-}BEM} = -21.3\,\mathrm{dB}$ vs. $SLL_{H\text{-}WG\text{-}ACO} = -25.4\,\mathrm{dB}$) and *SLR* value, which turns out to be smaller in a large part (i.e., almost 90%) of the angular range. The reliability of the new hybridization in better matching the reference pattern $\mathcal{D}(\theta,\phi)$ [Fig. 12(c)] is further pointed out in Fig. 15 where the mismatch index $\Xi(\theta,\phi) \triangleq \left|\mathcal{D}_{\mathrm{dB}}(\theta,\phi) - \mathcal{D}^{H}_{\mathrm{dB}}(\theta,\phi)\right|$ is shown for both hybrid methods.

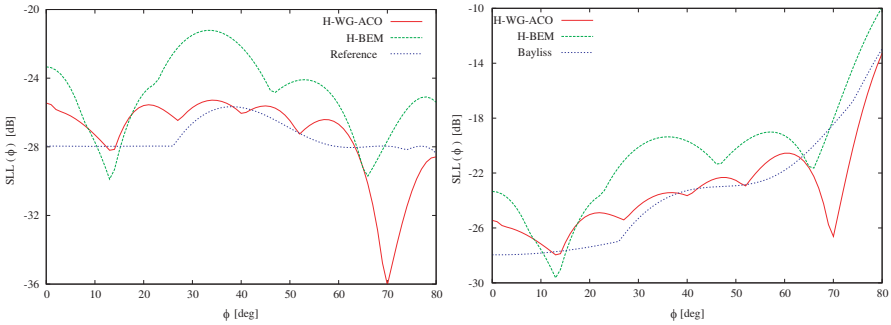

Figure 14. *Hybrid Extension* ($M = N = 5$, $\Gamma = 19$, *Reference difference* [30] — $SLL^{ref} = -25\,\mathrm{dB}$, $Q = 5$) — Plots of (a) the *SLL* and (b) the *SLR* of the solutions found by the *H-WG-ACO* and the *H-BEM*.

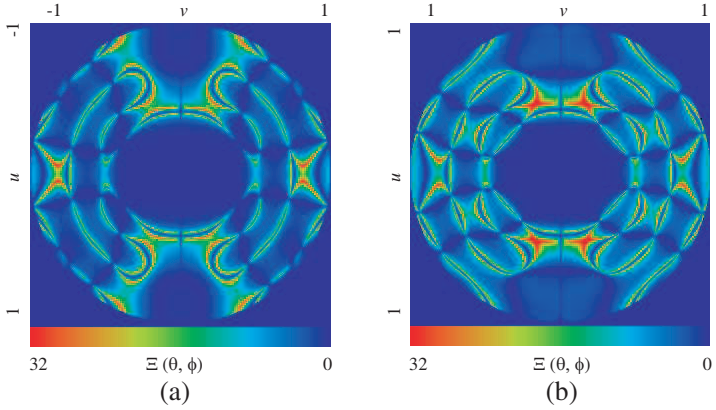

Figure 15. *Hybrid Extension* ($M = N = 5$, $\Gamma = 19$, *Reference difference* [30] — $SLL^{ref} = -25\,\mathrm{dB}$, $Q = 5$) — Plot of the mismatch function $\Xi(\theta,\phi)$ when applying the (a) *H-WG-ACO* and the (b) *H-BEM*.

Finally, in order to give some indications on the computational costs of the hybrid *ACO*-based approach, let us consider that sampling the solution space of dimension $U^{(ess)} = 3060$ requires 133 *ACO* iterations and $11CP$ iterations when using the *H-WG-ACO* [i.e., 5.8×10^{-2} sec (*WG-AGO*) and 850 sec (*CP*)], while the *H-BEM* performs 22 *BEM* iterations and 17 *CP* iterations [i.e., $< 10^{-6}$ sec (*BEM*) and 1370 sec (*CP*)].

4. CONCLUSIONS

In this work, an edge weighting technique has been proposed for the effective *ACO*-based sampling of the graph architecture coding the admissible clustering configurations of a sub-arrayed monopulse planar array. The advantages of the *ACO* in dealing with the non-convexity of the problem at hand and to explore graph representations of the solution space have been further and better exploited for enabling the synthesis of large-scale planar arrangements. Representative results have demonstrated the enhancement of the synthesis performance with respect to previous methods (e.g., *BEM*) and implementations (i.e., uniform *ACO*).

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