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TOTAL DIFFERENCE BASED PARTIAL SPARSE LCMV BEAMFORMER

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Abstract—Recent research demonstrates that sparse beam pattern constraint can suppress the sidelobe level of the linear constraint minimum variance beamformer. Here we improve the standard beam pattern by replacing it with a combination of a total difference minimization constraint on the mainlobe and a standard $\mathscr{C}_1$ norm minimization constraint on the sidelobe. As the new constraint matches the practical beam pattern better, the sidelobe level is further suppressed, while the robustness against the mismatch between the steering angle and the direction of arrival (DOA) of the desired signal, is maintained.

1. INTRODUCTION

Beamforming is used for enhancing a desired signal while suppressing noise and interference at the output of an array of sensors [1]. The linear constraint minimum variance (LCMV) beamformer is one of the most popular. It selects the weight vector to minimize the array output power subject to the linear constraint that the signal of interest (SOI) does not suffer from any distortion. It enjoys high resolution and interference rejection capability. To enhance the robustness in the presence of array steering vector errors, doubly constrained robust capon beamformer was proposed to use a norm constraint on the weight vector to improve the robustness [2]. To achieve a faster convergence speed and a higher steady state signal to interference plus noise ratio (SINR) [3] constrains its weight vector to a specific conjugate symmetric form. In [4], fully complex-valued radial basis function (RBF) network with the fully complex-valued activation function is used in LCMV beamformer to short the convergence period.

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However, high sidelobe is another drawback, which would result in deep degradations in the case of unexpected interferences or an increase in noise power [5]. In order to provide sidelobe suppression for an LCMV beamformer, a sparse constraint on the beam pattern was recently proposed in [6, 7]. The sparse beam pattern is added on all the array gains in the beam pattern. However, the expected beam pattern would enjoy most of the high array gains in the mainlobe. That is the array gains in the mainlobe is not sparse but dense. To match the practical beam pattern better, this letter proposed a new constraint on the beam pattern to encourage dense distribution in the mainlobe and sparse distribution in the sidelobe. The high sidelobe level problem can be alleviated.

2. MEASUREMENT MODEL

Assuming that the signal sources are narrowband, the signal received by a uniform linear array (ULA) with M sensors can be represented by an M-by-1 vector [1, 8]:

$$\mathbf{x}(k) = s(k)\mathbf{a}(\theta_0) + \sum_{j=1}^{J} \beta_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k) \tag{1}$$

where k is the index of time, J is the number of interference sources, s(k) and $\beta_i(k)$ (for $j = 1, \ldots, J$) are the amplitudes of the SOI and interfering signals at time instant k, respectively, θ_l (for $l = 0, 1, \ldots, J$) are the DOAs of the SOI and interfering signals, with d being the distance between two adjacent sensors and λ being the wavelength of the SOI, and $\mathbf{n}(k)$ is the additive white Gaussian noise (AWGN) vector at time instant k.

The output of a beamformer for the time instant k is then given by

$$y(k) = \mathbf{w}^H\mathbf{x}(k) = s(k)\mathbf{w}^H\mathbf{a}(\theta_0) + \sum_{j=1}^{J} \beta_j(k)\mathbf{w}^H\mathbf{a}(\theta_j) + \mathbf{w}^H\mathbf{n}(k) \tag{2}$$

where $\mathbf{w}$ is the M-by-1 complex-valued weighting vector of the beamformer.

3. THE PROPOSED BEAMFORMER

The sparse LCMV beamformer is designed to minimize the total array output energy, subject to a linear distortionless constraint on the SOI, and encourage spare distabution of the array gains in the beam patern, and the corresponding weighting vector of the is given by [6]

$$\mathbf{w}_S = \arg\min_{\mathbf{w}} \left(\mathbf{w}^H \mathbf{R}_x \mathbf{w} + \gamma \left\|\mathbf{w}^H \mathbf{A}\right\|_1\right), \text{s.t.} \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \quad (3)$$

where $\mathbf{R}_x$ is the M-by-M covariance matrix of the received signal vector $\mathbf{x}(k)$, and $\mathbf{w}^H \mathbf{a}(\theta_0) = 1$ is the distortionless constraint applied on the SOI. γ is the factor that controls the tradeoff between the minimum variance constraint on the total array output energy and the sparse constraint on the beam pattern, the M-by-N matrix $\mathbf{A}$ consists of N steering vectors for all possible interference with DOA in the range of $[-90°, \theta_0) \cup (\theta_0, 90°]$, with θ_0 being the DOA of the SOI as defined in (1), i.e.,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\varphi_1} & e^{j\varphi_2} & \cdots & e^{j\varphi_N} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M-1)\varphi_1} & e^{j(M-1)\varphi_2} & \cdots & e^{j(M-1)\varphi_N} \end{bmatrix} \quad (4)$$

$$\varphi_l = \frac{2\pi d}{\lambda} \sin\theta_l, \quad \text{for } l = 1, \ldots, N \quad (5)$$

and $\|\mathbf{x}\|_1 = (\sum_i |x_i|)$ is the $\mathscr{C}_1$ norm of a vector $\mathbf{x}$. It provides a measurement of sparsity for $\mathbf{x}$. In general, the $\mathscr{C}_1$ norm is approximate to the $\mathscr{C}_0$ norm which is the standard sparse constraint. For most of time, the smaller the value of the $\mathscr{C}_1$ norm is, the sparser the vector $\mathbf{x}$ is. It means that the number of trivial entries in $\mathbf{x}$ is larger [9]. The sparse LCMV beamformer (3) is a second order cone programming (SOCP), and can be solved efficiently.

In the perspective of the beam pattern, all the array gains $\mathbf{w}^H \mathbf{A}$ in all the possible values of DOA are encouraged to be sparse by $\mathscr{C}_1$ norm minimization. However, the array gains are not in common sparse distribution, but in dense distribution in the mainlobe and in sparse distribution in the sidelobe. To let the beam pattern more properly, the constraint is refined to encouraged dense cluster distribution in the mainlobe and sparse distribution in the sidelobe. As the total difference minimization can enforce a dense cluster structure, we incorporate it and improve the sparse constraint only on the sidelobe. Then we can obtain the total difference base linear constraint minimum variance (TD-LCMV) beamformer as:

$$\mathbf{w}_{TD} = \arg\min_{\mathbf{w}} \left[\mathbf{w}^H \mathbf{R}_x \mathbf{w} + \gamma_2 \sum_{i=1}^{I} \left\|\mathbf{D}_i \left(\mathbf{w}^H \mathbf{A}_M\right)^T\right\|_1 + \gamma_3 \left\|\mathbf{w}^H \mathbf{A}_S\right\|_1\right]$$
$$\text{s.t.} \mathbf{w}^H \mathbf{a}(\theta_0) = 1 \quad (6)$$

where

$$\mathbf{D}_i = \begin{bmatrix} \mathbf{D}_{i,F} \\ \mathbf{D}_{i,B} \end{bmatrix} \tag{7}$$

$$\mathbf{D}_{i,F} = \begin{bmatrix} -\mathbf{1} & \mathbf{1} & 0 & \dots & 0 & 0 & 0 \\ 0 & -\mathbf{1} & \mathbf{1} & \dots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & -\mathbf{1} & \mathbf{1} \\ 0 & 0 & 0 & \dots & 0 & 0 & -\mathbf{1} \end{bmatrix} \tag{8}$$

$$\mathbf{D}_{i,B} = \begin{bmatrix} \mathbf{1} & -\mathbf{1} & 0 & \dots & 0 & 0 & 0 \\ 0 & \mathbf{1} & -\mathbf{1} & \dots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \mathbf{1} & -\mathbf{1} \\ 0 & 0 & 0 & \dots & 0 & 0 & \mathbf{1} \end{bmatrix} \tag{9}$$

$$\mathbf{A}_M = [\ \mathbf{a}(\theta_{-b}) \ \ \dots \ \ \mathbf{a}(\theta_0) \ \ \dots \ \ \mathbf{a}(\theta_{+b})\ ] \tag{10}$$

$$\mathbf{A}_S = [\ \mathbf{a}(\theta_{-90}) \ \ \dots \ \ \mathbf{a}(\theta_{-b-1}) \ \ \mathbf{a}(\theta_{+b+1}) \ \ \dots \ \ \mathbf{a}(\theta_{+90})\ ] \tag{11}$$

$\mathbf{D}_{i,F}$ and $\mathbf{D}_{i,B}$ are the i-th forward and backward differential matrix; $\mathbf{1}$ is a 1-by-i row vector with all the elements being ones; and $-\mathbf{1}$ is a 1-by-i row vector with all the elements being -1. $\mathbf{A}_M$ and $\mathbf{A}_S$ are sub-matrices of the steering matrix $\mathbf{A}$. $\mathbf{A}_M$ is composed of $2b + 1$ steering vectors with the sampled angles in the mainlobe; while $\mathbf{A}_S$ is constituted with the rest of the steering vectors in $\mathbf{A}$. The product $\mathbf{w}^H\mathbf{A}_M$ indicates array gains of the mainlobe in the beam pattern, and $\mathbf{w}^H\mathbf{A}_S$ indicates array gains of the sidelobe. The minimization of the total difference $\sum_i \left\|\mathbf{D}_i\left(\mathbf{w}^H\mathbf{A}_M\right)^T\right\|_1$ enforces dense cluster distribution, while the minimization of $\left\|\mathbf{w}^H\mathbf{A}_S\right\|_1$ enforces sparse distribution. γ_2 and γ_3 are the weighting factors that controls the tradeoff among the minimum variance constraint on the total array output energy and the dense cluster mainlobe constraint and the sparse sidelobe constraint on the beam pattern; b is an integer representing the bounds of the mainlobe width. Since the objective function of (6) is convex too, the optimal $\mathbf{w}$ can be solved out by standard software packages [10, 11].

4. SIMULATION RESULTS

In the simulations, a ULA with 8 half-wavelength spaced antennas is considered. The AWGN at each sensor is assumed spatially uncorrelated. The DOA of the SOI is set to be 0°, and the DOAs of three interfering signals are set to be −30°, 30°, and 70°, respectively.

The signal to noise ratio (SNR) is set to be 10 dB, and the interference to noise ratios (INRs) are assumed to be 20 dB, 20 dB, and 40 dB in $-30°$, $30°$, and $70°$, respectively. 100 snapshots are used for each simulation. Without loss of generality, here we only use the first order differential matrix, i.e., $I = 1$; b is set to be 15; and γ_1, γ_2, γ_3 are all set to be 10. The matrix $\mathbf{A}$ consists of all steering vectors in the DOA range of $[-90°, 90°]$ with the sampling interval of $1°$.

To evaluate the performance in detail. The signal to interference and noise ratio (SINR) is calculated via the following formula:

$$SINR = \frac{\sigma_s^2 \mathbf{w}^H \mathbf{a}(\theta_0)\mathbf{a}^H(\theta_0)\mathbf{w}}{\mathbf{w}^H \left(\sum_{j=1}^{J} \sigma_j^2 \mathbf{a}(\theta_j)\mathbf{a}^H(\theta_j) + \mathbf{Q} \right) \mathbf{w}} \quad (12)$$

where σ_s^2 and σ_j^2 are the variances of the SOI and j-th interference, $\mathbf{Q}$ is a diagonal matrix with the diagonal elements being the noise's variances.

Figure 1 shows beam patterns of the standard LCMV beamformer, the sparse LCMV beamformer (3), and the TD-LCMV beamformer (6) of 1000 Monte Carlo simulations. It is obvious that the best sidelobe suppression performance is achieved by the TD-LCMV beamformer (6). the TD-LCMV beamformer has the lowest array gain level in sidelobe area, and provides the deepest nulls in the directions of interference, i.e., $-30°$, $30°$ and $70°$. The average received SINR by the standard LCMV beamformer, the sparse LCMV beamformer (3) and the TD-LCMV beamformer (6) are 1.8461 dB, 4.2603 dB and 5.3487 dB.

Figure 2 shows beam patterns of the mentioned beamformers, with each beamformer having a $4°$ mismatch between the steering angle and the DOA of the SOI [12]. we can see that the TD-LCMV beamformer (6) further suppresses sidelobe levels and deepens the nulls for interference avoidance, and has almost the same robustness against mismatch as the sparse LCMV beamformer. The average received SINR by the sparse LCMV beamformer (3) and the TD-LCMV beamformer (6) are 0.0011 dB, 2.1277 dB and 3.77181 dB respectively.

Thus, our proposed beamformer provides improvements in terms of sidelobe suppression, nulling for interference avoidance, and the robustness against the DOA estimation errors, with respect to existing beamformers.

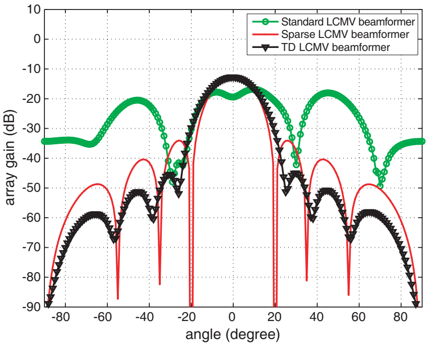

Figure 1. Normalized beam patterns of the standard LCMV beamformer, the sparse LCMV beamformer and the TD-LCMV beamformer, without mismatch between the steering angle and the DOA of the SOI.

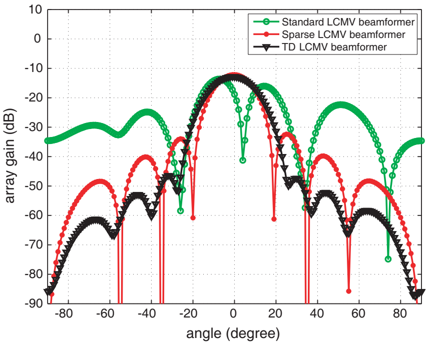

Figure 2. Normalized beam patterns of the standard LCMV beamformer, the sparse LCMV beamformer and the TD-LCMV beamformer, each having a 4° mismatch between the steering angle and the DOA of the SOI.

5. CONCLUSION

The proposed robust block sparse beamformer shows improvement to the sparse LCMV beamformer. It outperforms the standard LCMV beamformer and the sparse LCMV beamformer in terms of sidelobe suppression, nulling for interference avoidance.

In the future, the efficient way to solve the proposed TD-LCMV can be investigated. Furthermore, This work may be extended into a more practical OFDMA scenario with multi-clusters. Then, the objective is modified to select the best subcarrier. OFDM resource allocation and/or scheduling algorithms may also come into play.

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