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COMBINATION OF INVERSE FAST FOURIER TRANSFORM AND MODIFIED PARTICLE SWARM OPTIMIZATION FOR SYNTHESIS OF THINNED MUTUALLY COUPLED LINEAR ARRAY OF PARALLEL HALF-WAVE LENGTH DIPOLE ANTENNAS

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Abstract—In this paper, the authors propose a method based on the combination of inverse fast Fourier transform (IFFT) and modified particle swarm optimization for side lobe reduction of a thinned mutually coupled linear array of parallel half-wave length dipole antennas with specified maximum return loss. The generated pattern is broadside ($\phi = 90$ degree) in the horizontal plane. Mutual coupling between the half-wave length parallel dipole antennas has been taken care of by induced emf method considering the current distribution on each dipole to be sinusoidal. Directivity, first null beamwidth (FNBW), return loss of the thinned array is also calculated and compared with a fully populated array. Two cases have been considered, one with symmetric excitation voltage distribution and the other with asymmetric one. The method uses the property that for a linear array with uniform element spacing, an inverse Fourier transform relationship exists between the array factor and the element excitations. Inverse Fast Fourier Transform is used to calculate the array factor, which in turn reduces the computation time significantly. The element pattern of half-wave length dipole antenna has been

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assumed omnidirectional in the horizontal plane. Two examples are presented to show the flexibility and effectiveness of the proposed approach.

1. INTRODUCTION

Uniformly excited and equally spaced linear antenna arrays [1] have high directivity, but they usually suffer from high side lobe level. To reduce the side lobe level further, the array is thinned by switching off some of the antenna elements in a large array. The remaining elements are fed with uniform excitation voltage. It will reduce the complexity and fitness of feed networks by the use of parasitic arrays illuminated by active arrays as the switched off elements will act as parasitic. The switched off elements will have induced current because of mutual coupling.

Thinning a large array will reduce side lobe level further at a cost of reduced directivity. Due to the complexity in synthesis problem, analytical methods are not generally used in designing a thinned array. Therefore, evolutionary optimization tools such as Ant colony optimization [2], Genetic Algorithms [3, 4], Particle Swarm Optimization [5], Pattern search algorithm [6] etc.. are used to thin an array. Deterministic approach [7] has been applied for thinning a planar array.

Research article [8] deals with the synthesis of interleaved thinned array. Element behavior in a thinned array is described in [9].

PSO [10] is an evolutionary algorithm and has been successfully used in the design of antenna arrays [11–13].

Wang et al. [14] designed arrays by the combination of Genetic Algorithm (GA) and Fast Fourier Transform (FFT).

However, most of the authors used isotropic antennas for thinning a large array and neglected mutual coupling between the elements.

In this paper, we have proposed a method based on the combination of fast Fourier transform and modified particle swarm optimization for thinning a large linear array of mutually coupled parallel center-fed half-wave length dipole antennas that will reduce the side lobe level of the generated pattern in the horizontal plane. We also assumed that element pattern in the horizontal plane is omni-directional and is not affected by mutual coupling. The element pattern in the horizontal plane is very similar to an isotropic source. Computation time may be significantly reduced by using fast Fourier transform [14].

2. METHODOLOGY

Thinning an array means turning off some elements in a uniformly spaced or periodic array to generate a pattern with low side lobe level. In our proposed method, the positions of the elements are fixed and all the elements have two states either "on" or "off", depending on whether the element is connected to the feed network or not. In the "off" state, the element is passively terminated to a matched load.

The free space far-field pattern $F(\phi)$ in azimuth plane (x-y plane) [1] for a linear array of parallel half-wavelength dipole antennas equally spaced a distance d apart along the x-axis is given by Eq. (1). This is shown in Fig. 1 below.

$$F(\phi) = AF(\phi) \times EP(\phi) = \left[\sum_{n=1}^{N} I_n e^{j(n-1)kd\cos\phi} \right] \times EP(\phi) \quad (1)$$

where n = element number, j = imaginary quantity, d = inter-element spacing = 0.5λ, $k = 2\pi/\lambda$, being wave number, λ = wavelength, ϕ being azimuth angle of the far-field point measured from x-axis, I_n = excitation current amplitude of the n-th element ($I_n = I_{N-n+1}$, for symmetric excitation). In our case, V_n, excitation voltage, is 1 if the n-th element is turned "on" and 0 if it is "off." All the elements have same voltage excitation phase of zero degree. $AF(\phi)$ is the array factor. $EP(\phi)$ is element pattern of z-directed vertical half-wave length dipole

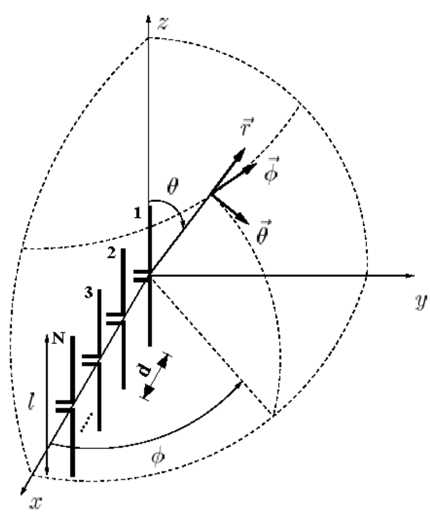

Figure 1. Geometry of linear array of parallel half-wave length dipole antennas.

antenna in x-y plane ($\theta = 90$ degree plane) and the pattern is assumed omnidirectional, i.e., $EP(\phi) = 1$.

The vector representing the current distribution on the antenna is given by

$$I = Z^{-1}V$$

where V is the vector of voltages applied to dipole antennas ($V_n = 1$ if element n is switched on, $V_n = 0$ if it is switched off.) and Z is the impedance matrix.

Self-impedances Z_{nn} and mutual impedances Z_{nm} of Z matrix are calculated by induced emf method [1], which assume the current distribution on the dipoles to be sinusoidal.

The voltage across the n-th dipole is given by:

$$V_n = I_n Z_{nn} + \sum_{m \neq n} I_m Z_{nm} \tag{2}$$

where Z_{nn} is the self-impedance of dipole n and Z_{nm} is the mutual impedance between dipoles n and m. The active impedance of dipole n, Z_n^A is given by:

$$Z_n^A = V_n / I_n = Z_{nn} + \sum_{m \neq n} \left(I_m / I_n\right) Z_{nm} \tag{3}$$

Considering that the characteristic impedance of the feeding network is 50 Ohm, the return loss, RL in dB at the input of each dipole antenna [1] is given by

$$RL = 20 \log_{10} \left[\frac{Z^A - Z_0}{Z^A + Z_0} \right] \tag{4}$$

Finally, the maximum return loss among all elements, $RL_{\max}$, is derived. A low value of $RL_{\max}$ ensures that the impedance matching condition is well satisfied for all the elements of the array.

The problem is now to find the set of on and off elements with the proposed modified PSO that will minimize the following fitness function

$$Fitness = \max SLL + C_r \tag{5}$$

where

$$C_r = \begin{cases} \left| RL_{\max}^o - RL_{\max}^d \right|, & \text{if } RL_{\max}^o > RL_{\max}^d \\ 0, & \text{if } RL_{\max}^o \leq RL_{\max}^d \end{cases} \tag{6}$$

where max SLL is the maximum value of side lobe level, $RL_{\max}^o$ and $RL_{\max}^d$ are obtained and desired value of maximum return loss respectively. Each component of fitness in Eq. (5) is equally weighted.

3. ARRAY FACTOR AND INVERSE FAST FOURIER TRANSFORM

Array factor (AF) in X-Y plane is given by

$$\begin{aligned} &AF(\phi) = \sum_{n=1}^{N} I_n e^{j(n-1)kd\cos\phi} \\ &\text{Let } p = 1 + \frac{N}{2\pi} kd\cos\phi, \\ &\text{then } AF(p) = \sum_{n=1}^{N} I_n e^{j(2\pi/N)(n-1)(p-1)} \end{aligned} \tag{7}$$

Through this mapping procedure, the sampling in ϕ domain is transformed into p domain. Note that Eq. (7) has the same form with the standard definition of one dimensional inverse fast Fourier transform (IFFT), which indicates that the array pattern can be directly computed through an IFFT operation on the excitations I_n and the computational complexity can be reduced significantly. IFFT used in the program is 4096-point IFFT padded with zeros if excitation current has less than 4096 points.

As compared with the conventional element-by-element superposition method, an obvious advantage of this new approach is that the overall computational complexity is determined by the sampling density rather than the actual array size itself.

4. MODIFIED PARTICLE SWARM OPTIMIZATION

Particle swarm optimization [10–13] emulates the swarm behavior of insects, animals herding, birds flocking, and fish schooling where these swarms search for food in a collaborative manner. Each member in the swarm adapts its search patterns by learning from its own experience and other member's experiences. These phenomena are studied and mathematical models are constructed. In PSO, a member in the swarm, called a *particle*, represents a potential solution, which is a point in the search space. The global optimum is regarded as the location of food. Each particle has a fitness value and a velocity to adjust its flying direction according to the best experiences of the swarm to search for the global optimum in the D-dimensional solution space. The PSO algorithm is easy to implement and has been empirically shown to perform well on many optimization problems.

The PSO algorithm is an evolutionary algorithm capable of solving difficult multidimensional optimization problems in various fields.

Since its introduction in 1995 by Kennedy and Eberhart [10], the PSO has gained an increasing popularity as an efficient alternative to GA and SA in solving optimization design problems in antenna arrays. As an evolutionary algorithm, the PSO algorithm depends on the social interaction between independent agents, here called particles, during their search for the optimum solution using the concept of fitness.

PSO emulates the swarm behavior and the individuals represent points in the D-dimensional search space. A particle represents a potential solution. The particle swarm optimization used in this paper is a real-coded one.

The steps involved in modified PSO (MPSO) are given below:

***Step1*:** Initialize positions and associated velocity of all particles (potential solutions) in the population randomly in the D-dimension space.

***Step2*:** Evaluate the fitness value of all particles.

***Step3*:** Compare the personal best (*pbest*) of every particle with its current fitness value. If the current fitness value is better, then assign the current fitness value to *pbest* and assign the current coordinates to *pbest* coordinates.

Step 4: Determine the current best fitness value in the whole population and its coordinates. If the current best fitness value is better than global best (*gbest*), then assign the current best fitness value to *gbest* and assign the current coordinates to *gbest* coordinates.

***Step5*:** Update velocity (V_{id}) and position (X_{id}) of the d-th dimension of the i-th particle using the following equations:

$$V_{id}^{t} = sign(r1) * w(t) * V_{id}^{t-1} + c_1 * rand1_{id}^{t} * \left(pbest_{id} - X_{id}^{t-1}\right) + c_2 * (1 - rand1_{id}^{t}) * \left(gbest_d - X_{id}^{t-1}\right) \quad (8)$$

where $sign(r1) = \begin{cases} -1, & \text{if } r1 \leq 0.05 \\ +1, & \text{if } r1 > 0.05 \end{cases}$, $r1$ is a random number between 0 and 1.

$$V_{id}^{t} = \min\left(V_{\max}^{d}, \max\left(V_{\min}^{d}, V_{id}^{t}\right)\right) \quad (9)$$

$$X_{id}^{t} = X_{id}^{t-1} + V_{id}^{t} \quad (10)$$

$$\begin{aligned} &\text{If } X_{id}^{t} > X_{\max}^{d} \\ &\text{then } X_{id}^{t} = X_{\min}^{d} + rand2_{id}^{t} * \left(X_{\max}^{d} - X_{\min}^{d}\right) \\ &\text{If } X_{id}^{t} < X_{\min}^{d} \\ &\text{then } X_{id}^{t} = X_{\min}^{d} + rand3_{id}^{t} * \left(X_{\max}^{d} - X_{\min}^{d}\right) \end{aligned} \quad (11)$$

where c_1, c_2 = acceleration constants = 1.4945, $w(t)$ = inertia weight changed linearly from 0.9 at the start of generation to 0.4 at 0.8 times the maximum generation number and thereafter it is fixed to 0.4 for the rest of the generation, $rand1$, $rand2$ and $rand3$ are uniform random numbers between 0 and 1, different value in different dimension, t is the current generation number.

In the proposed modified PSO, velocity-clipping technique is applied with time-varying maximum velocity, which decreases linearly from $V_{\max}^d$ to $0.1V_{\max}^d$ over the full range of search, because as the particles approach the optimal result it is preferred to have them move with lower velocities.

Equations (9) and (11) have been introduced to clamp the velocity and position along each dimension to $(V_{\max}^d, X_{id}^t)$ and $(V_{\min}^d,\ X_{id}^t)$ value if they try to cross the desired domain of interest. These clipping techniques are sometimes necessary to prevent particles from explosion. The maximum velocity is set to the upper limit of the dynamic range of the search $(V_{\max}^d = 0.5X_{\max}^d)$ and the minimum velocity is set to $-0.5X_{\max}^d$.

Step 6: Repeat Steps 2–5 until a stop criterion is satisfied, usually it is stopped when there is no further update of best fitness value for 150 or 250 generation.

5. RESULTS

We consider a linear array of 100 parallel half-wave length center-fed dipole antennas uniformly spaced 0.5λ apart along x-axis in order to generate a broadside pattern in azimuth plane (x-y plane) with minimum side lobe level and specified maximum return loss of -10 dB or less. Two cases have been studied and presented with results. In the first case, the excitation voltage amplitude distribution is symmetric with respect to the center of the array. In the second case, it is asymmetric. The diameter of each dipole is 0.01λ and the current distribution on each dipole is assumed to be sinusoidal.

IFFT used in the program is 4096-point IFFT padded with zeros if excitation current has less than 4096 points.

The computation time is measured with a PC with Intel core2 duo processor of clock frequency 1.83 GHz and 1 GB of RAM. Program is written in Matlab. Synthesis using 4096-point IFFT takes only 78.53 or 301.78 seconds where the same without IFFT takes 912.48 seconds.

For the first case (case1) of symmetric excitation $(V_n = V_{N-n+1})$, PSO is run for 700 generations with particle size of 50. The algorithm is stopped before as there is no further update of best fitness value for 150 generation. Here, the number of variables is 50.

For the second case (case2) of asymmetric excitation $(V_n \neq V_{N-n+1})$, PSO is run for 700 generations with particle size of 100.

Table 1. Switched off element numbers for both the cases.

Category	**Switched off element numbers**
Case 1 (First 50) Symmetric excitation	2, 3, 4, 6, 8, 10, 11, 12, 14, 16, 19, 21, 23, 25, 30
Case 2 Asymmetric excitation	1, 3, 5, 7, 8, 11, 13, 15, 18, 20, 26, 70, 75, 77, 79, 81, 84, 86, 87, 88, 89, 91, 92, 94, 96, 97, 98, 100

Table 2. Comparative results.

Design Parameters	**Proposed thinned array**		**Fully populated array**
	Case 1 (Symmetric excitation)	**Case 2 (Asymmetric excitation)**	
Percentage of thinning	30.00	28.00	0
Side lobe level (dB)	−23.15	−24.35	−13.26
Directivity (dB)	19.31	19.27	19.99
First null beamwidth (degree)	5.18	5.00	3.60
Maximum Return loss (dB)	−10.22	−10.42	−16.15
Minimum return loss (dB)	−36.12	−38.67	−40.50
Computation time (Seconds)	78.53	301.78	

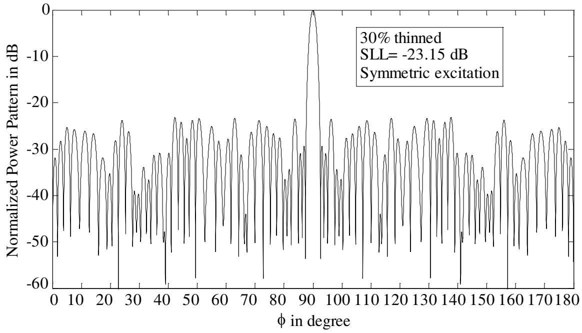

Figure 2. Normalized power pattern in dB for thinned array with symmetric excitation.

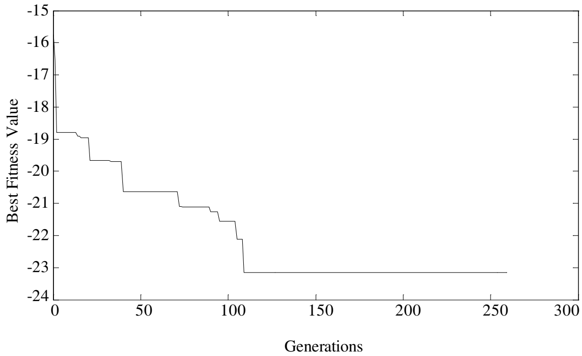

Figure 3. Best fitness value versus generation for thinned array with symmetric excitation.

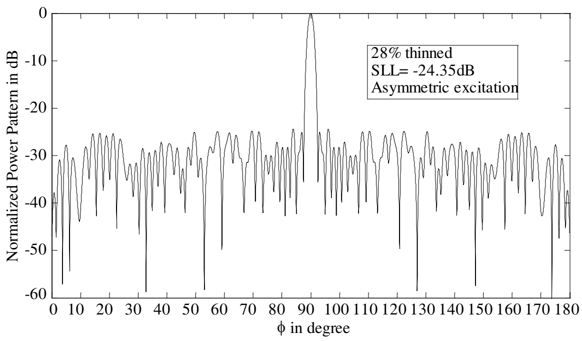

Figure 4. Normalized power pattern in dB for thinned array with asymmetric excitation.

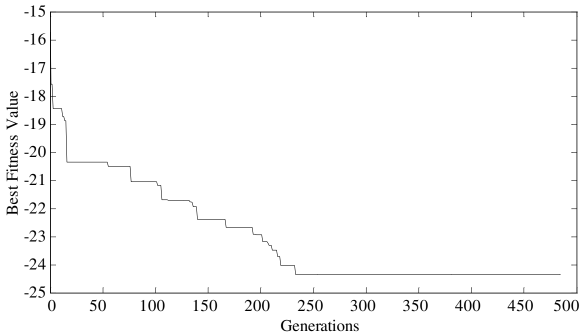

Figure 5. Best fitness value versus generation for thinned array with asymmetric excitation.

The algorithm is stopped before as there is no further update of best fitness value for 250 generation. Here the number of variables is 100.

The details of switched off elements for both the cases are shown in Table 1. Results of proposed thinned array design parameters and its comparison to a fully populated array of 100 parallel half-wave length dipole antennas are shown in Table 2.

Figures 2 and 4 show normalized power pattern in dB for thinned array with symmetric and asymmetric excitation respectively. Figs. 3 and 5 show best fitness value versus generation for thinned array with symmetric and asymmetric excitation respectively.

6. CONCLUSION

This paper presents a technique based on modified particle swarm optimization combined with inverse fast Fourier transform for thinning a large linear array of parallel half-wave length dipole antennas to generate a pencil beam in the horizontal plane with minimum side lobe level and fixed return loss.

The method uses IFFT to reduce computation time significantly. Realistic antenna with mutual coupling is used in the simulation in place of isotropic element considering the practical scenario.

Results clearly show a very good agreement between the desired and synthesized specifications.

The paper also shows that the array with asymmetric excitation gives better results than the array with symmetric excitation in terms of side lobe level. A significant improvement in SLL value is observed compared to that of a fully populated array.

Results for a thinned large array of parallel half-wave length dipole antennas have illustrated the performance of this proposed technique. This method is very simple and can be applied in practice to thin a planar array.

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