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ONE-DIMENSIONAL PHOTONIC HETEROSTRUCTURE WITH BROADBAND OMNIDIRECTIONAL REFLECTION

J. Manzanares-Martinez, R. Archuleta-Garcia P. Castro-Garay, D. Moctezuma-Enriquez and E. Urrutia-Banuelos

Departamento de Investigacion en Fisica
Universidad de Sonora
Apartado Postal 5-088, Hermosillo, Sonora, Mexico

Abstract—In this work we report the modeling of an one-dimensional photonic heterostructure which presents a giant omnidirectional photonic band gap. This omnidirectional reflector is made by the union of lattices with the same filling fraction and index contrast, but with different lattice periods. Using the scalability of the electromagnetic wave equation we present a simple manner to enlarge -as large as desired- the omnidirectional mirror. We apply our method to design an omnidirectional reflector for all the visible range.

1. INTRODUCTION

Photonic Crystals (PCs) are periodic dielectric structures in one, two or three dimensions which exhibit Photonic Bands Gaps (PBGs) [1]. These structures have been investigated intensively because of their ability to control the flux of light in a new manner opening the possibility to design optical and microwave devices [2–11]. The PBGs are the result of destructive interference of electromagnetic waves in PCs structures.

If a properly designed PC reflects electromagnetic waves impinging at any angle with any polarization, then an Omnidirectional Mirror (OM) can be achieved. The PC-based OMs have negligible loss compared with old metallic mirrors, specially at infrared, optical or higher frequencies [12, 13]. OMs have exciting applications, as for

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Corresponding author: Jesus Manzanares-Martinez (jmanza@cifus.uson.mx).

example perfect substrates for photonic antennas [14–19], filters in solars cells [20, 21] or enhancing the radiation emitted in planar cavities for the design of new PC-based low threshold lasers [22, 23].

It has been demonstrated that one-dimensional PC (1D-PC) structures (such as multilayer films) can exhibit an OM [12, 24–26]. It is also known that the range of the OM is determined by the contrast of dielectric functions between the PC composites; the larger the contrast, the wider the range. However, there are practical limitations for obtaining multilayers with a large dielectric contrast because the experimental techniques are nowadays limited to the fabrication of low dielectric contrast structures [27]. As a consequence of the narrow OM that can be achieved in 1D-PCs, only a limited applicability of these structures can be atttained in practical situations. Nevertheless, several authors have theorethically demonstrated that the frequency range of the OM can be enlarged by using a Photonic Heterostructure (PH), made by the combination of two or more 1D-PCs [28–36].

Some of the present authors have reported that it is possible to enlarge the total reflection range by using a PH in the case of normal incidence [37–39]. In this paper, based in a systematic investigation of the 1D-PCs properties, we present a method to enlarge -as much as desired- the frequency range of OMs. The central idea is that the OMs overlap each other. We design a PH with a giant OM composed by the union of several PCs lattices or submirrors of the same materials and filling fractions, but of different lattice periods. We present a simple rule to determine the lattice periods using the scalability of the eigenvalues obtained from the calculation of the electromagnetic wave equation, which to our knowledge seems not to have been considered in previous investigations. Our study has been carried out in two steps. First, in Section 2 we consider an OM and we present the mathematical formulation to determine the enlargement of the PBG. Second, we demonstrate the method for the design of an OM for the entire visible range. The results obtained are summarized in Section 4.

2. THEORY

We start our analysis by considering an unit cell composed of two materials of high (ε_H) and low (ε_H) dielectric function as illustrated in Fig. 1(a). The width of each layer is d_H and d_L, respectively. The repetition of the unit cell in the x direction gives rise to an infinite one-dimensional photonic crystal, as illustrated in Fig. 1(b). We consider two independent vibration modes for the propagation in the x-y plane. One is the Transversal Electric (TE) mode where the electric field is parallel to the z plane. The other is the Transversal Magnetic

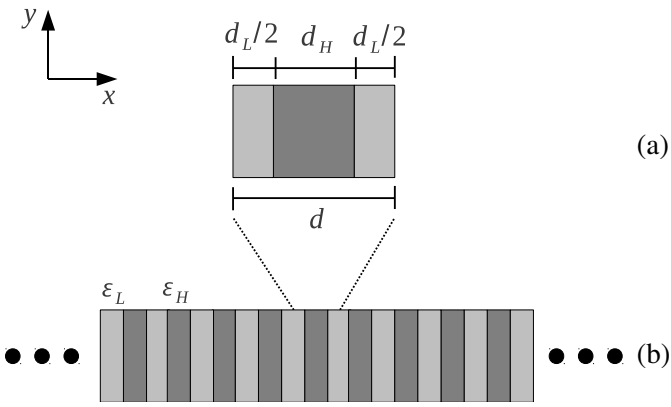

Figure 1. Geometry of the one dimensional photonic crystal. (a) Unit cell of width $d = d_H + d_L$. (b) Schematic of the infinite crystal.

(TM) mode with the electric field parallel to the x-y plane. The wave equations for the transverse fields $E_z(x, y)$ (TE) and $H_z(x, y)$ (TM) are [41]

$$\frac{1}{\varepsilon(x)}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)E_z(x,y) = -\frac{\omega^2}{c^2}E_z(x,y) \tag{1}$$

$$\left[\frac{\partial}{\partial x}\frac{1}{\varepsilon(x)}\frac{\partial}{\partial x}+\frac{1}{\varepsilon(x)}\frac{\partial^2}{\partial y^2}\right]H_z(x,y) = -\frac{\omega^2}{c^2}H_z(x,y) \tag{2}$$

To solve these equations we consider that the inverse of the dielectric function can be written in terms of a Fourier series in the form

$$\frac{1}{\varepsilon(x)} = \sum_n \alpha(n)e^{i2n\pi x/d} \tag{3}$$

where n is a integer number. The Fourier coefficients are [40]

$$\alpha(n) = \left\{\frac{1}{\varepsilon_L}+f\left(\frac{1}{\varepsilon_H}-\frac{1}{\varepsilon_L}\right)\right\}\delta_{n,0}+\left\{\left(\frac{1}{\varepsilon_H}-\frac{1}{\varepsilon_L}\right)\frac{\sin(f\pi n)}{n\pi}\right\}(1-\delta_{n,0}) \tag{4}$$

where the filling fraction is defined as space filled by the high dielectric function material in the unit cell, $f = d_H/d$. The periodic electric and magnetic fields are written as Fourier series in the form

$$E_z(x,y) = \sum_n E_z(n)e^{i(k_x+2n\pi/d)x+ik_y y} \tag{5}$$

$$H_z(x,y) = \sum_n H_z(n)e^{i(k_x+2n\pi/d)x+ik_y y} \tag{6}$$

The Equations (5) and (6) are substituted in Equations (1) and (2), and after some algebra one obtains the eigenvalue equations for the TE and TM polarizations [37, 40]

$$\sum_{n'} \alpha(n-n')[(k_x + 2n'\pi/d)^2 + k_y^2]E_z(n') = \frac{\omega^2}{c^2}E_z(n) \tag{7}$$

$$\sum_{n'} \alpha(n-n')[(k_x + 2n\pi/d)(k_x + 2n'\pi/d) + k_y^2]H_z(n') = \frac{\omega^2}{c^2}H_z(n) \tag{8}$$

If we multiply Equations (7) and (8) by the factor $[d/(2\pi)]^2$ we obtain

$$\sum_{n'} \alpha(n-n')[(\kappa_x + n')^2 + \kappa_y^2]E_z(n') = \Omega^2 E_z(n) \tag{9}$$

$$\sum_{n'} \alpha(n-n')[(\kappa_x + n)(\kappa_x + n') + \kappa_y^2]H_z(n') = \Omega^2 H_z(n) \tag{10}$$

where we have introduced the reduced wavevectors $\kappa_x = k_x d/2\pi$ and $\kappa_y = k_y d/2\pi$. The reduced frequency is

$$\Omega = \frac{\gamma d}{c}, \tag{11}$$

where $\gamma = \omega/2\pi$. The Equations (9) and (10) define two eigenvalue problems of the form $\mathbf{Ax} = \lambda\mathbf{x}$. The eigenvalues are obtained by standard numerical techniques. It is important to note that these eigenvalues equations are independent of the period d. The eigenvalues Ω are scalable and allow us to know the band structure for any 1D-PC of the same filling fraction and dielectric contrast.

Let us consider the projected band structure of a PC whith an OM. We consider a multilayer composed of two materials with dielectric functions of $\varepsilon_H = 6.76$ and $\varepsilon_L = 2.1$, which are typical values of ZnS/Si [24] or porous silicon [42] multilayers. We consider a filling fraction according to the quarter-wave condition [43] of $f = 0.36$. Fig. 2 shows the projected band structure, where the light gray zones indicate the allowed modes and the black region represents the OM. The upper and lower limits for the OM are $\Omega^+ = 0.32$ and $\Omega^- = 0.29$, respectively.

In Fig. 3, we present the variation of the OM limits as a function of d

$$\gamma^+(d) = \frac{c\Omega^+}{d} \tag{12}$$

$$\gamma^-(d) = \frac{c\Omega^-}{d} \tag{13}$$

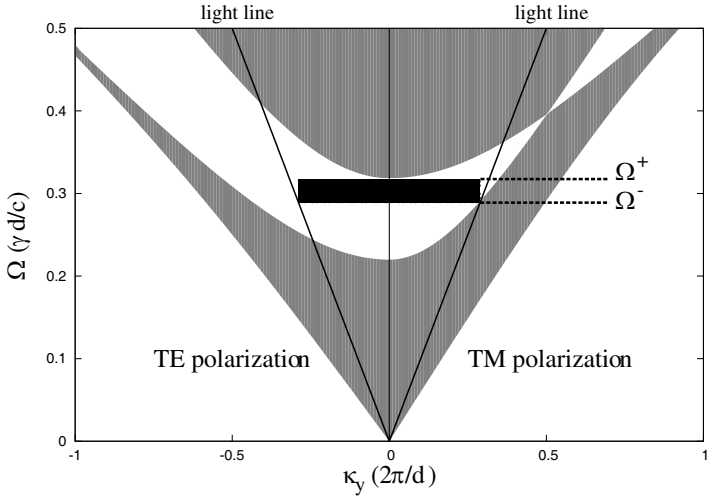

Figure 2. Photonic Band structure of an infinite multilayer. The light gray zones are the allowed frequencies and the black zone is the omnidirectional band gap.

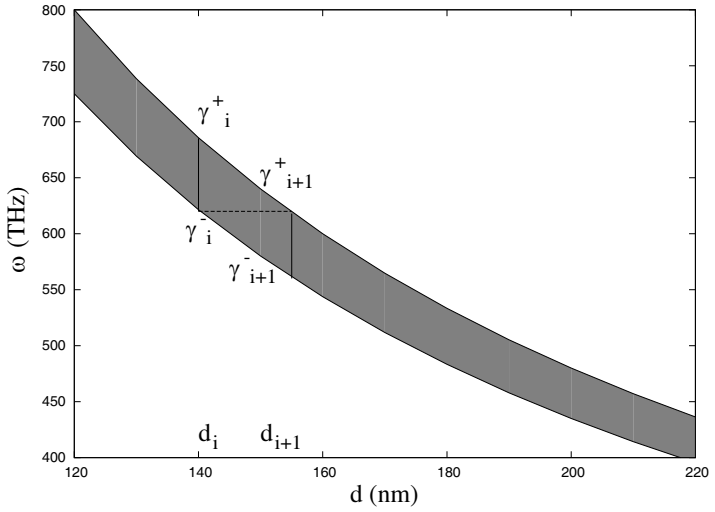

Figure 3. Variation of the omnidirectional mirror as a function of the thickness period d. In the graph we consider the OM limits for two submirrors of period d_i and d_{i+1}, respectively.

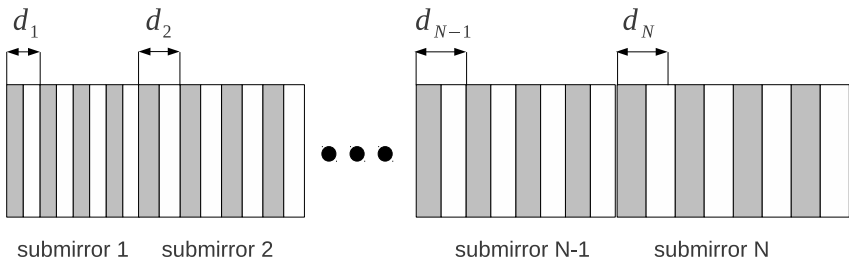

Figure 4. Schematic of a Photonic Heterostructure composed of N submirrors.

To determine a PH with enlarged OM, we consider two or more submirrors with thickness period d_i and d_{i+1}, such that their OM can be superposable to obtain an enlarged OM in the form

$$\gamma_i^- = \gamma_{i+1}^+ \tag{14}$$

The rule to determine the thickness for the periods is obtained from Equations (11) and (14)

$$d_{i+1} = \frac{\Omega^+}{\Omega^-} d_i \tag{15}$$

The PH is the result of stacking together several submirrors, as we illustrate in Fig. 4.

3. NUMERICAL RESULTS

The procedure to design a PH with an enlarged OM of arbitrary upper and lower limits γ^+ to γ^- can be summarized in four steps:

(i) First, we consider that the upper frequency limit of the first submirror is the upper frequency limit of the PH, $\gamma_1^+ = \gamma^+$. To obtain the first submirror period we use

$$d_1 = \frac{c\Omega^+}{\gamma_1^+} \tag{16}$$

(ii) Second, we determine the following submirrors periods using the recurrence relation of Equation (15)

$$d_{i+1} = \frac{\Omega^+}{\Omega^-} d_i \tag{17}$$

(iii) Third, we calculate the frequency limits of each submirror using Equations (12) and (13)

$$\gamma_i^+ = \frac{c\Omega^+}{d_i} \tag{18}$$

$$\gamma_i^- = \frac{c\Omega^-}{d_i} \tag{19}$$

(iv) Finally, we compare the lower limit of each submirror (γ_i^-) with the lower limit of the PH, (γ^-). We look for the condition where the lower limit of the submirror N (γ_N^-) be lower that the PH lower limit in the form

$$\gamma_N^- < \gamma^- \tag{20}$$

When we reach this condition, we take N submirrors for the PH.

Now, we apply our method to obtain a PH with an OM for entire the visible range which is defined from $\gamma^- = 400\,\text{THz}$ (THz=$10^{12}\,\text{s}^{-1}$) to $\gamma^+ = 790\,\text{THz}$. We consider the same 1D-PC than in the precedent section, where the dielectric contrast is $\varepsilon_H/\varepsilon_L = 6.76/2.1$ and the filling fraction is $f = 0.36$. The upper and lower reduced frequency limits are $\Omega^+ = 0.32$ and $\Omega^- = 0.29$, respectively.

In Table 1, we present the values of the periods and the frequency limits of the OM of each submirror. We observe that with only seven submirrors is possible to obtain a complete mirror in the visible.

The reflection of the PH is shown in Fig. 5. Each submirror is composed by 12 unit cells. In panels (a), (b) and (c) we present the reflectivity for an incident angle of 0°, 45° and 85°, respectively. We present with solid and dashed lines the results for TE and TM polarizations. The visible region is illustrated with a light gray zone.

Table 1. Table of the periods and OM limits for each submirror

i	d_i (nm)	γ_i^+ (THz)	γ_i^- (THz)
1	121.52	790.00	715.94
2	134.09	715.94	648.82
3	147.96	648.82	587.99
4	163.27	587.99	532.87
5	180.16	532.87	482.91
6	198.79	482.91	437.64
7	219.36	437.64	396.61

4. DISCUSSION

We have designed an all-dielectric reflector for TE and TM polarized light creating an omnidirectional reflection for any angle of incidence and any polarizations. The heterostructure reflector is composed of several submirrors in a manner that the heterostructure is configured such that

(i) a range of frequencies exists with a PBG for the normal incidence,
(ii) a range of frequencies with a PBG for electromagnetic energy incident along a direction approximately 90° from the normal direction.
(iii) the range of frequency of the PBG need to be common to both cases.

It is necessary to consider that the range of the PBG for the normal incidence is wider than in the case of an incident angle of 90°, as it can be observed in Fig. 5.

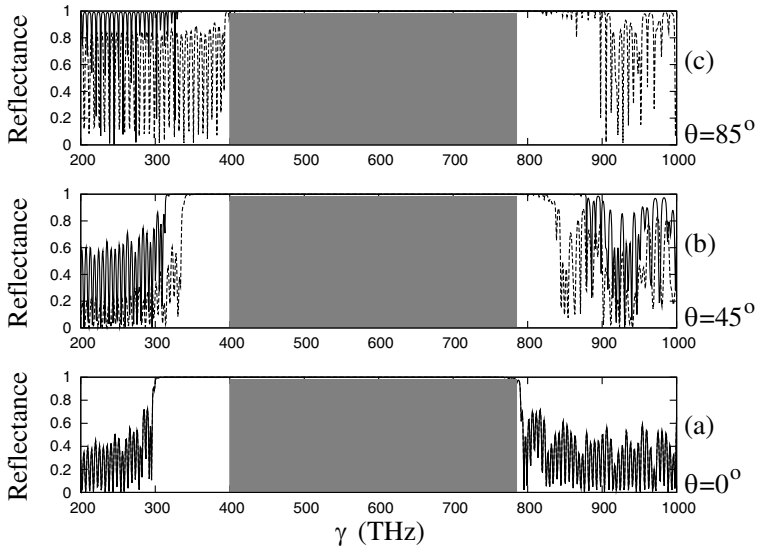

Figure 5. Reflectance of a Photonic heterostructure composed by seven submirrors. Each submirror have 12 unit cells. We present with solid and dashed lines the results for the TE and TM polarizations. In panels (a), (b) and (c) we have the reflectance for the incidence angle of $\theta = 0°$, $\theta = 45°$ and $\theta = 85°$, respectively.

5. CONCLUSIONS

We present a simple method for designing a photonic heterostructure with an omnidirectional mirror as large as desired. The photonic heterostructure is made by combining several submirrors, whose are determined by using an analytical expression. We apply our procedure to obtain a photonic heterostructure with an omnidirectional mirror for the whole visible range.

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REFERENCES

1. Yablonovitch, E., "Inhibited spontaneous emission in solid-state physics and electronics," *Phys. Rev. Lett.*, Vol. 58, 2059–2062, 1987.
2. Ozbay, E., B. Temelkuran, and M. Bayindir, "Microwave applications of photonic crystals," *Progress In Electromagnetics Research*, Vol. 41, 185–209, 2003.
3. Guida, G., A. de Lustrac, and A. Priou, "An introduction to photonic band gap (PBG) materials," *Progress In Electromagnetics Research*, Vol. 41, 1–20, 2003.
4. Banerjee, A., "Enhanced temperature sensing by using one-dimensional ternary photonic band gap structures," *Progress In Electromagnetics Research Letters*, Vol. 11, 129–137, 2009.
5. Awasthi, S. K., U. Malaviya, S. P. Ojha, N. K. Mishra, and B. Singh, "Design of a tunable polarizer using a one-dimensional nano sized photonic bandgap structure," *Progress In Electromagnetics Research B*, Vol. 5, 133–152, 2008.
6. Hsu, H.-T., T.-W. Chang, T.-J. Yang, B.-H. Chu, and C.-J. Wu, "Analysis of wave properties in photonic crystal narrowband filters with lefthanded defect," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 2285–2298, 2010.
7. Dai, G.-L. and M.-Y. Xia, "An investigation of quarter-wavelength square-spiral resonator and its applications to miniaturized bandpass filters," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 1303–1313, 2010.

8. He, Y., L. Li, C.-H. Liang, and Q. H. Liu, "EBG structures with fractal topologies for ultra-wideband ground bounce noise suppression," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 1365–1374, 2010.
9. Zhang, X. F., L. F. Shen, J.-J. Wu, and T.-J. Yang, "Backward guiding of terahertz radiation in periodic dielectric waveguides," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 557–564, 2010.
10. Wu, C.-J. and Y.-L. Chen, "Effective surface impedance for a superconductor-semiconductor superlattice at mid-infrared frequency," *Journal of Electromagnetic Waves and Applications*, Vol. 23, 1441–1453, 2009.
11. Wu, C.-J., C.-L. Liu, and W.-K. Kuo, "Analysis of thickness-dependent optical properties in a one-dimensional superconducting photonic crystal," *Journal of Electromagnetic Waves and Applications*, Vol. 23, 1113–1122, 2009.
12. Winn, J. N., Y. Fink, S. Fan, and J. D. Joannopoulous, "Omnidirectional reflection from a one-dimensional photonic crystal," *Opt. Lett.*, Vol. 23, 1573–1575, 1998.
13. Banerjee, A., "Enhanced incidence angle based spectrum tuning by using one-dimensional ternary photonic band gap structures," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 1023–1032, 2010.
14. Kuzma-Filipek, I. J., F. Duerinckx, E. van Kerschaver, K. van Nieuwenhuysen, G. Beaucarne, and J. Poortmans, "Chirped porous silicon reflectors for thin-film epitaxial silicon solar cells," *J. Appl. Phys.*, Vol. 104, 073529–073536, 2008.
15. Biancotto, C. and P. Record, "Dielectric EBG corner reflector antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 2107–2118, 2010.
16. Chau, Y.-F., H.-H. Yeh, and D. P. Tsai, "A new type of optical antenna: Plasmonics nanoshell bowtie antenna with dielectric hole," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 1621–1632, 2010.
17. Zhou, H., S. Qu, Z. Pei, Y. Yang, J. Zhang, J. Wang, H. Ma, C. Gu, and X. Wang, "A high-directive patch antenna based on all-dielectric near-zero-index metamaterial superstrates," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 1387–1396, 2010.
18. Curel, C. S., H. Cosar, and O. Akalm, "Accurate resonant frequency computation of multisegment rectangular dielectric resonator antennas," *Journal of Electromagnetic Waves and*

Applications, Vol. 24, 839–847, 2010.

19. Masri, T., M. K. A. Rahim, H. A. Majid, O. Ayop, F. Zubir, and M. N. A. Karim, "Electromagnetic band gap structure for planar ultra wide band antenna," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 229–239, 2010.
20. Dominici, L., L. Vesce, D. Colonna, F. Michelotti, T. F. Brown, A. Reale, and A. Di Carlo, "Angular and prism coupling refractive enhancement in dye solar cells," *Appl. Phys. Lett.*, Vol. 96, 103302–103304, 2010.
21. Liu, C.-C., Y.-H. Chang, and C.-J. Wu, "Refractometric optical sensing by using a multilayer reflection and transmission narrowband filter," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 293–305, 2010.
22. Manzanares-Martinez, J. and P. Castro-Garay, "Modeling the tuning of lasing in liquid crystal based one-dimensional photonic crystals using the Finite Difference Time-Domain method," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 14/15, 1867–1875, 2010.
23. Wu, C.-J., J.-J. Liao, and T.-W. Chang, "Tunable multilayer fabry-perot resonator using electro-optical defect layer," *Journal of Electromagnetic Waves and Applications*, Vol. 24, 531–542, 2010.
24. Deopura, M., C. K. Ullal, B. Temelkuran, and Y. Fink, "Dielectric omnidirectional visible reflector," *Optics Letters*, Vol. 26, 1197–1199, 2001.
25. Chigrin, D. N., A. D. Lavrinenko, D. A. Yarotsky, and S. V. Gaponenko, "All-dielectric one-dimensional periodic strcutures for total omnidirectional reflection and partial spontaneous emission control," *J. Lightwave Technol.*, Vol. 17, 2018–2024, 1999.
26. Fink, Y., J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos, and E. L. Thomas, "A dielectric omnidirectional reflector," *Science*, Vol. 282, 1679–1682, 1998.
27. Istrate, E. and E. H. Sargent, "Photonic crystal heterostructures and interfaces," *Rev. of Mod. Phys.*, Vol. 78, 455–481, 2006.
28. Banerjee, A., "Enhanced refractometric optical sensing by using one-dimensional ternary photonic crystals," *Progress In Electromagnetics Research*, Vol. 89, 11–22, 2009.
29. Awasthi, S. K. and S. P. Ojha, "Wide-angle broadband plate polarizer with 1d photonic crystal," *Progress In Electromagnetics Research*, Vol. 88, 321–335, 2008.

30. Wang, Z. Y., X. M. Cheng, X. Q. He, S. L. Fan, and W. Z. Yan, "Photonic crystal narrow filters with negative refractive index structural defects," *Progress In Electromagnetics Research*, Vol. 80, 421–430, 2008.
31. Kumar, N. and S. P. Ojha, "Photonic crystals as infrared broadband reflectors with different angles of incidence: A comparative study," *Progress In Electromagnetics Research*, Vol. 80, 431–445, 2008.
32. Srivastava, S. K. and S. P. Ojha, "Omnidirectional reflection bands in one-dimensional photonic crystal structure using fullerene films," *Progress In Electromagnetics Research*, Vol. 74, 181–194, 2007.
33. Srivastava, S. K. and S. P. Ojha, "Enhancement of omnidirectional reflection bands in one-dimensional photonic crystals with left-handed materials," *Progress In Electromagnetics Research*, Vol. 68, 91–111, 2007.
34. Srivastava, R., K. B. Thapa, S. Pati, and S. P. Ojha, "Design of photonic band gap filter," *Progress In Electromagnetics Research*, Vol. 81, 225–235, 2008.
35. Wu, C. J. and Z. H. Wang, "Properties of defect modes in one-dimensional photonic crystals," *Progress In Electromagnetics Research*, Vol. 103, 169–184, 2010.
36. Wu, C.-J., B.-H. Chu, and M.-T. Weng, "Analysis of optical reflection in a chirped distributed bragg reflector," *Journal of Electromagnetic Waves and Applications*, Vol. 23, 129–138, 2009.
37. Archuleta-Garcia, R., M. B. Manzanares-Martinez, and J. Manzanares-Martinez, "Giant photonic band gaps in photonic heterostructures," *Revista Mexicana de Fisica*, Vol. 53, 371–375, 2007.
38. Archuleta-Garcia, R. and J. Manzanares-Martinez, "Slow group velocity in photonic heterostructures," *Superficies y Vacio*, Vol. 21, 18–20, 2008.
39. Archuleta-Garcia, R., D. Moctezuma-Enriquez, and J. Manzanares-Martinez, "Enlargement of photonic band gap in porous silicon dielectric mirrors," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 2/3, 351–361, 2010.
40. Archuleta-Garcia, R., M. B. Manzanares-Martinez, and J. Manzanares-Martinez, "A description of the plane wave method for the calculation of photonic bands," *Revista Boliviana de Fisica*, Vol. 13, 79–85, 2007.
41. Ramos-Mendieta, F. and P. Halevi, "Electromagnetic surface

modes of a dielectric superlattice: The supercell method," *J. Opt. Soc. Am. B*, Vol. 14, 370–381, 1997.

42. Xifre-Perez, X., L. F. Marshal, J. Pallares, and J. Ferre-Borull, "Porous silicon mirrors with enlarged omnidirectional band gap," *J. Appl. Phys.*, Vol. 97, 064503–064508, 2005.
43. Sozuer, H. S. and S. Kevim, "Robustness of one-dimensional photonic band gaps under random variations of geometrical parameters," *Phys. Rev. B.*, Vol. 72, 195101–195110, 2005.