ELECTROMAGNETIC FIELDS IN A CIRCULAR WAVEGUIDE CONTAINING CHIRAL NIHILITY METAMATERIAL

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Abstract—Propagation of electromagnetic fields and power in a circular waveguide containing chiral nihility metamaterial is studied. Space inside the waveguide is divided into two circular regions. One region contains chiral nihility metamaterial while other region is of free space. Two cases of the waveguide, in this regard, are considered for analysis. For the case of perfect electric conductor (PEC) waveguide, there is no net electric field and power propagation in chiral nihility region of the guide whereas both fields and power exist in non-nihility region (which is free space in our cases) of the guide. For perfect electromagnetic conductor (PEMC) waveguide, both electric and magnetic fields exist in the chiral nihility and non-nihility regions.

1. INTRODUCTION

Recent scientific investigation has revealed that, in chiral medium, the phenomena of negative refraction and reflection of electromagnetic/optical waves can be achieved by controlling the chirality parameter [1–7]. Study of reflection properties of electromagnetic/optical waves in isotropic chiral media, when the chiral parameter is strong enough, has shown that unusual negative reflection occurs at the interface between a chiral medium and a perfectly conducting plane. That is, the incident wave and one of the reflected eigenwaves lie in the same side of the interface normal [6, 7]. Phenomena of negative reflection and refraction is also observed by considering the nihility in chiral medium [8]. In electromagnetics, the concept of nihility medium was introduced in 2001 by Lakhtakia [9] as a dielectric medium in which both permittivity and permeability are zero simultaneously. It is shown

that no propagation is possible in nihility medium due to zero value of wavenumber of the corresponding medium. Later, in 2003, Tretyakov et al. [8] applied the nihility concept to isotropic chiral medium and resulting medium was termed as chiral nihility medium. They concluded that an unbounded chiral nihility medium can also support two circularly polarized modes; one forward and one backward mode.

Although chiral nihility medium has not been realized yet, the chiral nihility has attracted the attention of many researchers [10–14]. Chiral nihility is an emerging area, and chiral nihility still imposes great requirement on permittivity and permeability being very small. The nonreciprocal chiral nihility and gyrotropic chiral (G-chiral) material have also been addressed for optical applications in this regard [15, 16]. An interesting and valuable characteristic is the phenomena of negative reflection of uniform plane wave from a planner PEC interface placed in chiral nihility medium [6]. In this arrangement, electric field of incident plane wave and electric field of reflected plane wave cancel each other yielding net zero electric field. That is, there is no propagation of power in chiral nihility medium if PEC interface is affecting the propagation of fields in chiral nihility medium. Knowing this fact, waveguide with chiral nihility metamaterial was introduced for the purpose of confinement of power in non-nihility region of the guide. Various papers have been published on this topic and parallel and rectangular waveguides [17–20] have been studied up till now.

In present manuscript, propagation of fields and power in a circular waveguide containing chiral nihility metamaterial is studied. Two cases are considered one by one: first case deals with a PEC circular waveguide coated with chiral nihility metamaterial whereas other is PEMC circular waveguide coated with chiral nihility metamaterial. Behavior of fields and power in chiral nihility region and ordinary dielectric region of the guide is investigated.

2. FORMULATION

Consider a circular waveguide having infinite extent and is of radius b. The space inside the waveguide is divided into two circular regions, i.e., $r \leq a$ and $a < r < b$. The region $r \leq a$, inside the circular guide, is free space (μ_0, ϵ_0) while the region $a < r < b$ is of chiral nihility metamaterial. The chiral nihility medium is characterized by the constitutive parameters $(\epsilon = 0, \mu = 0, \kappa \neq 0)$. The z-axis of cylindrical coordinate system (r, ϕ, z) is assumed to coincide with the axis of the waveguide. The time dependency is of the form $\exp(j\omega t)$, which is suppressed throughout the paper.

The constitutive relations of an isotropic, lossless and reciprocal

chiral medium are given below [21]

$$\mathbf{D} = \epsilon\mathbf{E} - j\kappa\sqrt{\epsilon_0\mu_0}\mathbf{H}$$
$$\mathbf{B} = \mu\mathbf{H} + j\kappa\sqrt{\epsilon_0\mu_0}\mathbf{E} \tag{1}$$

where ϵ, μ, and κ are permittivity, permeability, and chirality parameter of chiral material, respectively. Chiral nihility is considered as the special case of a chiral medium under the condition ($\epsilon \to 0$, $\mu \to 0$, and $\kappa \neq 0$). The constitutive relations for a chiral nihility metamaterial are

$$\mathbf{D} = -j\kappa\sqrt{\epsilon_0\mu_0}\mathbf{H}$$
$$\mathbf{B} = j\kappa\sqrt{\epsilon_0\mu_0}\mathbf{E} \tag{2}$$

In chiral nihility metamaterial, two modes propagate, one is right circularly polarized (RCP,+) and other is left circularly polarized (LCP,−) with wavenumbers $k_\pm = \pm\kappa k_0$, respectively. The field in chiral nihility metamaterial may be decomposed as [8, 21]

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$$
$$\mathbf{H} = \frac{j}{\eta}(\mathbf{E}_+ + \mathbf{E}_-) \tag{3}$$

where $\eta = \lim_{\epsilon\to 0,\mu\to 0}\sqrt{\frac{\mu}{\epsilon}}$ is the wave impedance of chiral nihility matamaterial. The wave is assumed to propagate in the positive z direction and field component varies as $\exp(-j\beta z)$.

Consider a TE^z mode is excited in region $r \leq a$, the solution to the wave equation in this region is written as

$$E_{z0} = 0$$
$$H_{z0} = B_{1m}J_m(k_{0r}r)\exp(jm\phi)$$
$$E_{\phi 0} = \frac{j\omega\epsilon_0}{k_{0r}}B_{1m}J_m'(k_{0r}r)\exp(jm\phi)$$
$$H_{\phi 0} = \frac{\beta m}{k_{0r}}B_{1m}J_m(k_{0r}r)\exp(jm\phi) \tag{4}$$
$$E_{r0} = \frac{\omega\epsilon_0}{k_{0r}}B_{1m}J_m(k_{0r}r)\exp(jm\phi)$$
$$H_{r0} = -\frac{j\omega}{k_{0r}}B_{1m}J_m'(k_{0r}r)\exp(jm\phi)$$

where $k_{0r} = \sqrt{k_0^2 - \beta^2}$ and $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. $J_m(.)$ is Bessel function of order m and prime denotes the derivative with respect to the argument.

In the chiral nihility layer, the longitudinal components of the electromagnetic fields $\mathbf{E}_\pm$ can be written as

$$E_{+z1} = C_{1m}J_m(k_{r+}r)\exp(jm\phi) + C_{2m}Y_m(k_{r+}r)\exp(jm\phi)$$
$$E_{-z1} = D_{1m}J_m(k_{r-}r)\exp(jm\phi) + D_{2m}Y_m(k_{r-}r)\exp(jm\phi) \tag{5}$$

where

$$k_{r\pm} = \sqrt{k_\pm^2 - \beta^2} = k_0\sqrt{\kappa^2 - (\beta/k_0)^2} = k_r$$

Transverse components are

$$E_{r1} = (C_{1m} + D_{1m})\left[\frac{jm\kappa k_0}{k_r^2 r}J_m(k_r r) - \frac{j\beta}{k_r}J'_m(k_r r)\right]\exp(jm\phi)$$

$$+ (C_{2m} + D_{2m})\left[\frac{jm\kappa k_0}{k_r^2 r}Y_m(k_r r) - \frac{j\beta}{k_r}Y'_m(k_r r)\right]\exp(jm\phi)$$

$$H_{r1} = \frac{j}{\eta}(C_{1m} - D_{1m})\left[\frac{jm\kappa k_0}{k_r^2 r}J_m(k_r r) - \frac{j\beta}{k_r}J'_m(k_r r)\right]\exp(jm\phi)$$

$$+ \frac{j}{\eta}(C_{2m} - D_{2m})\left[\frac{jm\kappa k_0}{k_r^2 r}Y_m(k_r r) - \frac{j\beta}{k_r}Y'_m(k_r r)\right]\exp(jm\phi)$$

$$E_{\phi 1} = (C_{1m} + D_{1m})\left[\frac{\beta m}{k_r^2 r}J_m(k_r r) - \frac{\kappa k_0}{k_r}J'_m(k_r r)\right]\exp(jm\phi) \tag{6}$$

$$+ (C_{2m} + D_{2m})\left[\frac{\beta m}{k_r^2 r}Y_m(k_r r) - \frac{\kappa k_0}{k_r}Y'_m(k_r r)\right]\exp(jm\phi)$$

$$H_{\phi 1} = \frac{j}{\eta}(C_{1m} - D_{1m})\left[\frac{\beta m}{k_r^2 r}J_m(k_r r) - \frac{\kappa k_0}{k_r}J'_m(k_r r)\right]\exp(jm\phi)$$

$$+ \frac{j}{\eta}(C_{2m} - D_{2m})\left[\frac{\beta m}{k_r^2 r}Y_m(k_r r) - \frac{\kappa k_0}{k_r}Y'_m(k_r r)\right]\exp(jm\phi)$$

Total axial fields in the chiral nihility region are

$$E_{z1} = (C_{1m} + D_{1m})J_m(k_r r)\exp(jm\phi)$$
$$+ (C_{2m} + D_{2m})Y_m(k_r r)\exp(jm\phi)$$

$$H_{z1} = \frac{j}{\eta}(C_{1m} - D_{1m})J_m(k_r r)\exp(jm\phi) \tag{7}$$

$$+ \frac{j}{\eta}(C_{2m} - D_{2m})Y_m(k_r r)\exp(jm\phi)$$

Geometry of the guiding structure, except the nature of outer wall located at $r = b$, has been described in the beginning of this section. Two cases with different properties of the outer wall of the guide are being considered below.

2.1. PEC Circular Waveguide

First of all assume that the wall of the guide located at $r = b$ is perfect electric conductor (PEC). By using boundary conditions at interfaces

$r = a$ and $r = b$, unknown coefficients can be obtained. The boundary conditions

$$
\begin{aligned}
E_{z1} &= 0, & E_{\phi 1} &= 0, & r &= b \\
E_{z0} &= E_{z1}, & E_{\phi 0} &= E_{\phi 1}, & r &= a \\
H_{z0} &= H_{z1}, & H_{\phi 0} &= H_{\phi 1}, & r &= a
\end{aligned}
\tag{8}
$$

yield the following results for the coefficients in Equations (6) and (7)

$$
\begin{aligned}
C_{1m} &= \frac{\eta}{2jk_{0r}k_r\kappa k_0}\left[\frac{S_1 + S_2}{J'_m(k_r a)Y_m(k_r a) - Y'_m(k_r a)J_m(k_r a)}\right]B_{1m} \\
B_{1m} &= 2j\frac{J_m(k_r a)C_{1m} + Y_m(k_r a)C_{2m}}{\eta J_m(k_{0r}a)} \\
C_{1m} &= -D_{1m} \\
C_{2m} &= -D_{2m}
\end{aligned}
$$

where

$$
\begin{aligned}
S_1 &= \left(ak_r^2 - k_{0r}\right)\beta m Y_m(k_r a)J_m(k_{0r}a) \\
S_2 &= ak_{0r}\kappa k_0 Y'_m(k_r a)J_m(k_{0r}a)
\end{aligned}
$$

Relationship between unknown coefficients shows that total electric field inside the chiral nihility region is zero whereas total magnetic field inside the chiral nihility region is nonzero.

The energy flux along z-axis, in the waveguide, may be obtained using the following relation

$$
S_z = \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*) \cdot \hat{z} = \frac{1}{2}\text{Re}(E_{r1}H_{\phi 1}^* - E_{\phi 1}H_{r1}^*)
\tag{9}
$$

It is obvious that no power propagates in the chiral nihility region and power is confined only to the free space region of the guide.

2.2. PEMC Circular Waveguide

Now assume that the wall of the waveguide located at $r = b$ is perfect electromagnetic conductor (PEMC). The concept of PEMC metamaterial is introduced by Lindell and Sihvola [22] and perfect electric conductor (PEC) and perfect magnetic conductor (PMC) may be considered as special cases of PEMC. A PEMC interface is defined through admittance parameter M. For example; $M \to \infty$ corresponds to PEC case whereas $M \to 0$ corresponds to PEMC case. Electromagnetic fields satisfy the following boundary condition at the PEMC interface

$$
\hat{\mathbf{n}} \times (\mathbf{H} + M\mathbf{E}) = 0
\tag{10}
$$

Using the boundary conditions at interfaces $r = a$ and $r = b$, unknown coefficients can be obtained by solving the following set of equations

$$C_{1m} = -B_{1m}\left[\frac{T_1 - F_2 F_1}{2MP_0}\right]$$

$$B_{1m} = 2j\frac{J_m(k_r a)C_{1m} + Y_m(k_r a)C_{2m}}{\eta J_m(k_{0r})} \qquad (11)$$

$$(W - X)C_{1m} = -(Y - Z)D_{1m}$$

$$\frac{J_m(k_r a)}{Y_m(k_r a)}C_{1m} + \frac{J_m(k_r a)}{Y_m(k_r a)}D_{1m} + C_{2m} + D_{2m} = 0$$

where

$$
\begin{aligned}
F_1 &= \eta M(J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)) \\
F_2 &= (ak_r^2 - k_{0r})\beta m Y_m(k_r a)J_m(k_{0r}a) + ak_{0r}\kappa k_0 Y_m'(k_r a)J_m(k_{0r}a) \\
W &= \frac{P_1 - P_2 + T_2}{\eta b k_r^2 Y_m(k_r a)} \\
X &= \frac{U_1 U_2}{2jb\eta k_r^2 Y_m(k_r a)Y_m(k_r b)} \\
Y &= \frac{P_3 - P_4 + T_3}{\eta b k_r^2 J_m(k_r a)} \\
Z &= \frac{V_1 V_2}{2jb\eta k_r^2 Y_m(k_r a)Y_m(k_r b)} \\
T_1 &= [jak_{0r}k_r\kappa k_0 J_m(k_{0r}a)Y_m(k_r b)] \\
&\qquad [J_m'(k_r a)Y_m(k_r a) - J_m(k_r a)Y_m'(k_r a)] \\
T_2 &= \eta M\beta m[J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)] \\
&\qquad + \eta Mbk_r[J_m(k_r a)Y_m'(k_r b) - J_m'(k_r b)Y_m(k_r a)] \\
U_1 &= j[J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)] \\
&\qquad - \eta M[J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)] \\
U_2 &= 2j\beta m Y_m(k_r b) - 2jk_r\kappa k_0 Y_m'(k_r b) + 2M\beta m Y_m(k_r b) \\
&\qquad - 2\eta bk_r\kappa k_0 Y_m'(k_r b) \\
T_3 &= \eta M\beta m(J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)) \\
&\qquad + \eta Mbk_r(J_m'(k_r b)Y_m(k_r a) - J_m(k_r b)Y_m'(k_r b)) \\
P_0 &= \left\{J_m'(k_r a)Y_m(k_r a) - J_m(k_r a)Y_m'(k_r a)\right\} \\
&\qquad \left\{J_m(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m(k_r b)\right\} \\
P_1 &= j\beta m[J_m(k_r b)Y_m(k_r a) + J_m(k_r a)Y_m(k_r b)] \\
P_2 &= jbY_m(k_r a)k_r\kappa k_0[(J_m'(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m'(k_r b))] \\
P_3 &= j\beta m(J_m(k_r a)Y_m(k_r b) - J_m(k_r b)Y_m(k_r a)) \\
P_4 &= jbk_r\kappa k_0(J_m'(k_r b)Y_m(k_r a) - J_m(k_r a)Y_m'(k_r b)) \\
V_1 &= [2j\beta m Y_m(k_r b) - 2jbk_r\kappa k_0 Y_m'(k_r b) \\
&\qquad + 2\eta M\beta m Y_m(k_r b) - 2\eta bk_r\kappa k_0 Y_m'(k_r b)]
\end{aligned}
$$

$$V_2 = j[J_m(k_ra)Y_m(k_rb) - J_m(k_rb)Y_m(k_ra)]$$
$$+ \eta M[J_m(k_rb)Y_m(k_ra) - J_m(k_ra)Y_m(k_rb)]$$

Both electric and magnetic fields exist in the chiral nihility and free space regions of the circular waveguide under consideration.

By substituting $M = \infty$ in above expressions, same results as derived in Section 2.1 for a PEC circular waveguide are obtained

$$C_{1m} = \frac{\eta}{2jk_{0r}k_r\kappa k_0} \left[\frac{S_1 + S_2}{J'_m(k_ra)Y_m(k_ra) - Y'_m(k_ra)J_m(k_ra)} \right] B_{1m}$$
$$B_{1m} = 2j \frac{J_m(k_ra)C_{1m} + Y_m(k_ra)C_{2m}}{\eta J_m(k_{0r}a)}$$
$$C_{1m} = -D_{1m}$$
$$C_{2m} = -D_{2m}$$

3. CONCLUSION

Expressions for fields inside a circular waveguide, containing chiral nihility metamaterial, are derived. It is concluded that for a PEC circular waveguide, net electric field is zero inside chiral nihility region whereas both electric and magnetic fields exist in achiral region of the guide. For a PEMC circular waveguide both electric and magnetic fields exist in chiral nihility and non-nihility regions of the guide.

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