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THE MICROSTRUCTURE DESIGN OPTIMIZATION OF NEGATIVE INDEX METAMATERIALS USING GENETIC ALGORITHM

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Abstract—In recent years, metamaterials have been the subject of research interest for many investigators worldwide. However, most of reported metamaterial microstructures are obtained based on human intuition, experience or large numbers of simulation experiments which were time-consuming, ineffective or expensive. In this paper, we propose a novel negative index metamaterial microstructure design methodology that uses a FDTD solver optimized by genetic algorithm (GA) technique in order to achieve a simultaneously negative permeability and permittivity. Firstly, an novel genetic algorithm optimization model for wide frequency band of negative refraction was proposed. Then the effectiveness of the new technique was demonstrated by a microstructure design example that was optimized by GA. By using numerical simulations techniques and S-parameter

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retrieval method, we found that the GA-designed optimal solution can exhibit a wide LH frequency band with simultaneously negative values of effective permittivity and permeability. Therefore, the design methodology presented in this paper is a very convenient and efficient way to pursue a novel metamaterial microstructure of left-handed materials with desired electromagnetic characteristics.

1. INTRODUCTION

In recent years, metamaterials have been the subject of research interest for many investigators worldwide. Metamaterials are often characterized in terms of their effective material parameters, such as electric permittivity and magnetic permeability. These constituent parameters can either be both negative, or only one of them may be negative, while the other is positive. The former is often referred to as left-handed materials (LHM), double negative material (DNG), or negative index metamaterials (NIMs). The latter is called single negative material (SNG).

In 1968, Veselago first postulated the possibility of left-handed metamaterial with a simultaneously negative permeability and permittivity, along with several interesting properties associated with such a material, including backward-propagating waves, reversed Doppler shift and near-field focusing [1]. However, due to the absence of natural LHM, Veselago's research was not properly regarded over the last three decades. Until the late 1990s, Pendry et al. suggested a model of periodic array of metallic wires to get an effective negative permittivity [2, 3], and an array of split ring resonators (SRRs) to get an effective negative permeability [4].

The first artificial left-handed material was fabricated by Smith et al. through combining SRRs and continuous wires [5–7]. After that, all kinds of microstructure configurations of LHMs with different shapes were found in recent years, such as Omega-shaped [8], S-shaped [9], Double S-shaped [10], H-shaped [11], Fishnet Structure [12], and so on.

However, most of those microstructure were obtained by changing the shape and (or) the size of the components proposed by Pendry et al., which means those microstructures will have the similar properties with Pendry's. Furthermore, these works were mainly based on human intuition, experience or large numbers of simulation experiments which were time-consuming, ineffective or expensive.

Thus, an effective and systematic methodology for guiding and designing the microstructure of negative index metamaterials with novel properties is quite desirable and vital.

Genetic algorithms (GAs) have been successfully applied to many electromagnetic problems [13, 14] to pursue novel solutions which are difficult to obtain using the conventional design approaches. For instance, GAs have been used to optimize multi-band and broadband microstrip antennas [15], wire-based antennas [16], microwave absorbers [17], scannable circular antenna arrays [18], fractal antenna-array [19], and so on.

In this paper, genetic algorithm technique was introduced to design and optimize potentially microstructure for negative index metamaterials with simultaneously negative permeability and permittivity. Firstly, a novel genetic algorithm optimization model for wide frequency band of negative refraction was proposed. Then the effectiveness of the new technique was demonstrated by a microstructure design example that was optimized by GA. By using numerical simulations techniques and S-parameter retrieval method, we found that the GA-designed optimal solution can exhibit a wide LH frequency band with simultaneously negative values of effective permittivity and permeability.

2. GA-BASED DESIGN AND OPTIMIZATION MODEL

2.1. Design Concept

In this paper, the geometry and dimensions for the present example are chosen to be similar with the typical LHM by Smith [20]. The unit cell is cubic, with a cell dimension of $d = 2.5\,\mathrm{mm}$. A 0.25 mm thick substrate of FR4 ($\varepsilon = 4.4$, loss tangent of 0.02) is assumed. The copper thickness is 0.017 mm. The width of the wire is 0.14 mm, and it runs the length of the unit cell.

However, our work is different from Smith's because we assume that one of the sides of the substrate is a copper wire, but the other side is selected to be the design and optimization domain which is described in Figure 1.

For simplicity, we suppose there is a little gap equal to 0.05 mm between the edge of the design domain and the cell. The boundary conditions of periodic microstructure are also presented in Figure 1.

When using the concept of genetic algorithm optimization, the design domain is subdivided into discrete elementary pixels coded as 1 or 0, depending on whether they are covered by a printed copper or not. Figure 2 shows an example of the encoding of the design domain for the GA optimization. In Figure 2, the whole design domain is subdivided into a 6×6 grid, and the portion of the chromosome related to a 6×6 grid is 36 bits.

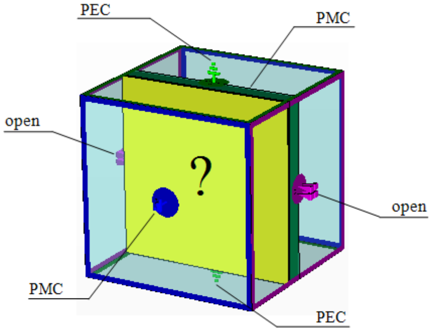
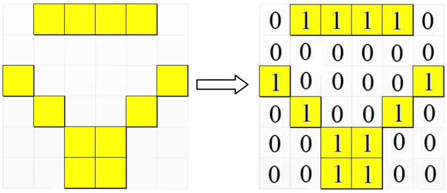


Figure 1. The design and optimization domain of negative index metamaterials.



0	1	1	1	1	0
0	0	0	0	0	0
1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
0	0	1	1	0	0

Figure 2. The encoding illustration of the design domain for the GA optimization.

In this paper, we assume that the distribution of the design area is symmetrical to the left and right sides of the unit cell. Therefore, the corresponding number of bits of the chromosome can be reduced to 18.

2.2. Genetic Algorithm

As a probabilistic global search strategy, GAs can converge to near-global optimality in the design of electromagnetic systems within a reasonable time. In this paper, a general GA scheme [21], proposed by Goldberg, is employed to perform the optimization.

The block diagram of the GA design for metamaterial microstructure optimization is illustrated in Figure 3. Firstly, an initial population is generated randomly. Then the chromosome of each individual is decoded to generate the microstructure of metamaterials for the finite-difference time-domain (FDTD) method [22, 23] to call. The fitness function is then calculated based on the S parameters of microstructure.

The GA judges if the termination criteria is met. If not, the

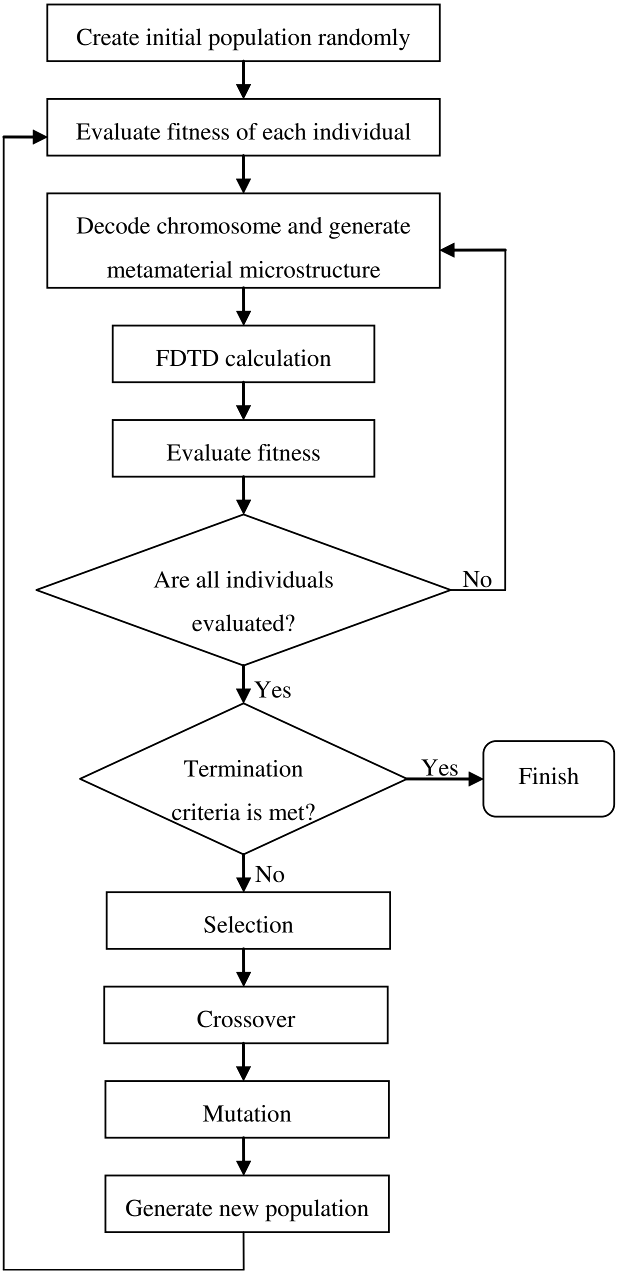


Figure 3. The block diagram of the GA design for metamaterial microstructure optimization.

population of chromosomes is regenerated through a GA process including selection, crossover and mutation. In this paper, the selection scheme is tournament selection with a shuffling technique for choosing random pairs for mating. The uniform crossover is used and the single-point mutation strategy is applied in the GA. The GA process is repeated until the maximum number of the generation is reached.

In this paper, the goal of GA is to pursue a novel metamaterial microstructure for maximization of the LH bands within a specified frequency range. Therefore, the fitness function of GA can be defined as follows:

$$\text{Maximize}: \ F = \int_{f_{\min}}^{f_{\max}} H(f)df \tag{1}$$

$$H(f) = \begin{cases} 1 & \text{if } \varepsilon_{real} < 0 \text{ and } \mu_{real} < 0 \\ 0 & \text{others} \end{cases} \tag{2}$$

In the formula (1), the $f_{\min}$, $f_{\max}$ denote the lower and the upper limits of the frequency range respectively. In the formula (2), the ε_{real} represents the real part of permittivity, and the μ_{real} represents the real part of permeability.

Furthermore, when the S parameters of microstructure are obtained by a FDTD solver, we can determine the wave impedance Z, refractive index n, permittivity ε and permeability μ of this metamaterial microstructure by the S-parameter retrieval methods [20, 24, 25] which can be described as following:

$$Z = \pm\sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}, \quad Z' \geq 0 \tag{3}$$

$$e^{ink_0 d} = \frac{S_{21}}{1 - S_{11}\frac{Z-1}{Z+1}} \tag{4}$$

$$n = \frac{1}{k_0 d}\left\{\left[\left[\ln\left(e^{ink_0 d}\right)\right]'' + 2m\pi\right] - i\left[\ln\left(e^{ink_0 d}\right)\right]'\right\} \tag{5}$$

$$\varepsilon = n/Z, \quad \mu = nZ \tag{6}$$

where $(\cdot)'$ and $(\cdot)''$ denote the real part and imaginary part operators, respectively. Z' denotes the real part of the impedance, m is an integer related to the branch index of n', k_0 is the wave number of the incident waves, d is the length of the unit cell, and S_{ij} is the scattering parameters.

3. DESIGN RESULTS AND DISCUSSION

3.1. Parameters for the GA Optimization

In this paper, the metamaterial microstructure design was optimized to operate at 0–20 GHz, and the parameters for GA optimization are listed in Table 1.

Table 1. Parameters for the GA optimization.

Microstructure Grids	6×12
Chromosome Length	72 bits
Population Size	80
Crossover Probability	0.8
Mutation Probability	0.08
Maximum Generations	50

The GA optimization experiments were executed under Microsoft Windows Server 2003 with 3.00 GHz of Intel(R) Pentium(R) 4 CPU and 2 GB of RAM. In our simulation experiments, the CPU calculation time for fitness evaluation of one individual is about 2.5 minutes or so. Therefore, the whole CPU calculation time for 50 generations with 80 individuals is about one week.

The convergence curves of GA for microstructure optimization are presented in Figure 4. From Figure 4 it can be seen that the genetic algorithm almost converges before the maximum number of the generation is reached.

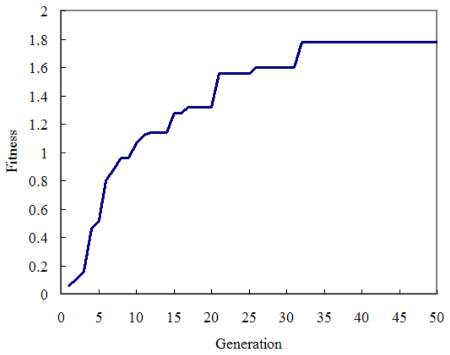


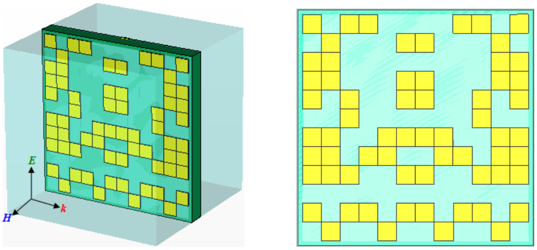
Figure 4. The convergence curves of GA for microstructure optimization.

3.2. Experimental Results and Discussion

The three suboptimal solutions and the optimal solution for the metamaterial microstructure design are shown in Figure 5, Figure 6, Figure 7 and Figure 8 respectively.

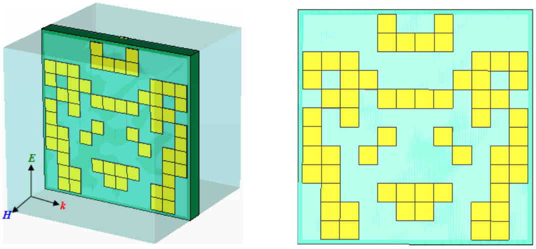
From the optimization results of Figures 5, 6, 7 and Figure 8, it can be seen that the fitness of GA or the LH bands of metamaterial microstructure becomes more and more wider which increases from 1.32 GHz to 1.78 GHz. Furthermore, the GA-designed suboptimal solutions and optimal solution all indicate that the split ring resonator (SRR) structure may be the fundamental component for negative index metamaterial microstructure design.

Finally, the electromagnetic characteristic of GA-designed optimal solution is analyzed in detail. The S parameters and the retrieved material parameters of optimal solution are presented in Figure 9.



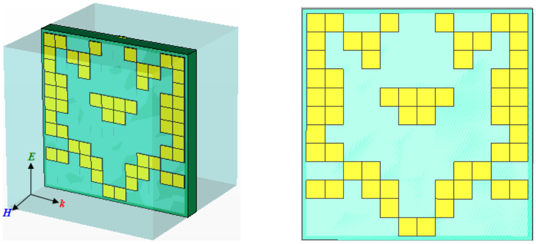
(a) (b)

Figure 5. (a) the GA-designed suboptimal solution for metamaterials microstructure design with the LH bands of 1.32 GHz, (b) planar view of the unit cell.



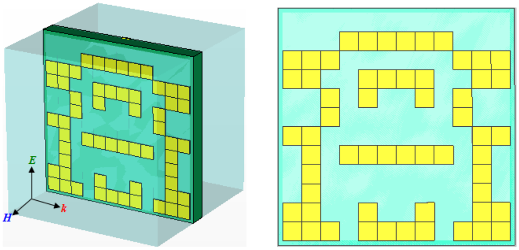
(a) (b)

Figure 6. (a) the GA-designed suboptimal solution for metamaterials microstructure design with the LH bands of 1.56 GHz, (b) planar view of the unit cell.



(a) (b)

Figure 7. (a) the GA-designed suboptimal solution for metamaterials microstructure design with the LH bands of 1.6 GHz, (b) planar view of the unit cell.



(a) (b)

Figure 8. (a) the GA-designed optimal solution for metamaterials microstructure design with the LH bands of 1.78 GHz, (b) planar view of the unit cell.

From Figure 9(a), it can be seen that the transmission coefficient S_{21} is quite small on the band lower than 12.66 GHz because the copper wire exhibit a resonant electric response ($\varepsilon < 0$) which is shown in Figure 9(e). But the effect permeability μ is still greater than zero which is seen in Figure 9(f). However, when the frequency increases, both an electric and a magnetic resonance are exhibited, associated with a negative permittivity and negative permeability regime, respectively. Therefore a LH transmission regime can be achieved in the frequency band from 12.66 GHz to 14.44 GHz which is described in Figure 9(c).

Moreover, in order to further verify the electromagnetic characteristic of GA-designed optimal solution, we analyzed the surface current and electric field of the optimal microstructure in the LH frequency band of 12.82 GHz which are shown in Figure 10 and Figure 11 respectively.

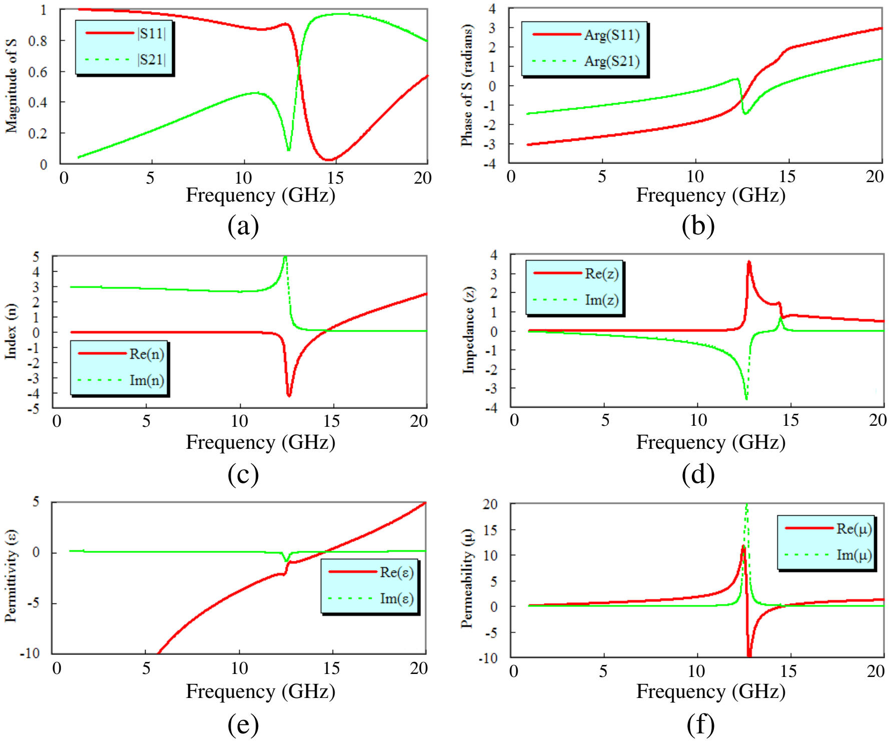


Figure 9. (a) Magnitude and (b) phase of the simulated S parameters for the GA-designed optimal solution. (c) Retrieved index, (d) impedance, (e) permittivity and (f) permeability are also shown.

From Figure 10, it can be found that the surface current direction of outer ring is opposite to that of inner two rings and short rod. The electric field of the optimal microstructure in Figure 11 shows that the electric field in split place of outer ring is strongest, and the electric field of inner rings and short rod is mainly distributed at both ends. The GA-designed optimal solution shown in Figure 8 is indeed a LHM.

To sum up, the design methodology presented in this paper that uses a FDTD solver optimized by genetic algorithm technique is a more convenient means to pursue a novel metamaterial microstructure of LHMs with desired electromagnetic characteristics. Experimental results demonstrate that the bandwidth of negative index metamaterials can be maximized by optimizing using genetic algorithm technique.

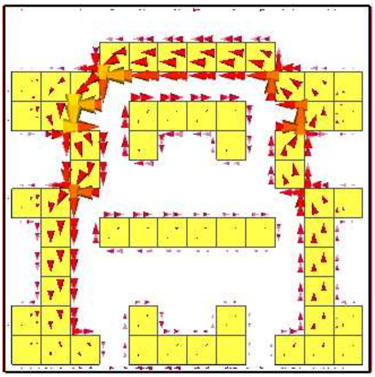


Figure 10. The surface current of the optimal microstructure.

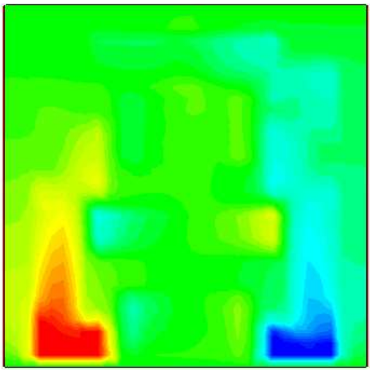


Figure 11. The electric field of the optimal microstructure.

4. CONCLUSION

In this paper, we propose a novel negative index metamaterial microstructure design methodology that uses a FDTD solver optimized by genetic algorithm technique in order to achieve a simultaneously negative permeability and permittivity. Firstly, an novel genetic algorithm optimization model for wide frequency band of negative refraction was proposed. Then the effectiveness of the new technique was demonstrated by a microstructure design example that was optimized by GA. By using numerical simulations techniques and S-parameter retrieval method, we found that the GA-designed optimal solution can exhibit a wide LH frequency band with simultaneously negative values of effective permittivity and permeability. Therefore, the design methodology presented in this paper is a very convenient and efficient way to pursue a novel metamaterial microstructure of LHMs with desired electromagnetic characteristics.

Future research directions are as follows: 1) Further extend the design methodology presented in this paper to near-IR and optical wavelengths. 2) Conduct the GA optimization on computer clusters with parallelized fitness evaluation in order to enhance the convergence speed of algorithm. 3) The Micro-Genetic Algorithm (MGA) has the advantages of requiring only a small population and achieving a near-optimal solution with a limited generation, therefore, the metamaterials design optimization using MGA is another potential area of research.

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