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ANALYSIS OF HORN RADIATION PATTERN USING UTD EDGE AND CORNER DIFFRACTION

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Abstract—Finite edge geometrical theory of diffraction (FEGTD) approach is a new and latest improvement in GTD technique. This FEGTD technique is applied to the E-plane horn radiation problems with spherical source excitation. The radiation patterns obtained with the FEGTD approach are found to be in good agreement with measured results.

1. INTRODUCTION

Application of the geometrical theory of diffraction (GTD) to predict accurately the far side lobes and back lobes of an antenna has been the subject of interest for quite some time as it is a good canonical problem which has a broad range of low response in the backward (LRB) directions [1]. Keller's GTD [2] and later the uniform theory of diffraction (UTD) [3] and the uniform asymptotic theory (UAT) [4, 5] have been applied to horn antennas [6–12]. In the year 2010, finite edge GTD (FEGTD) technique is developed and applied to compute H-plane horn radiation pattern in [13]. The FEGTD approach in [13] shows improved agreement with measured results compared to earlier GTD formulations in [8, 9] for H-plane horn radiation pattern. This is a pure GTD formulation that gives good results in all regions an attractive proposition because it gives the physical inside of the

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problem and it would require insignificant computer time and resources (as compared to purely numerical technique).

The earliest work on E-plane horn radiation pattern using Keller's GTD was by Russo et al. [6]. They approximated the pyramidal horn by a dihedral corner reflector excited by a magnetic line source and then obtained its E-plane far field radiation pattern. A more comprehensive analysis of the E-plane radiation pattern was carried out by Yu et al. in [7]. Their analysis included the multiply reflected images and as a result, they obtained continuous far field E-plane patterns at all angle. The computed E-plane pattern in the later investigation [9–11] however, shows no improvement over that obtained as early as 1966 in [7]. A fundamental limitation in [7] was the approximation (up to $K^{-1/2}$ term) of the line source at the edge and the images to be isotropic. This limitation does not exist in UAT. UAT applied to determine near field [10] also gives term of order $K^{-1/2}$ which in general differs from that of UAT [14–16]. Further refinement in predicting the far out side lobes ripples, better than 1 dB at 35 dB below reference, in the E-plane radiation pattern was carried out by Sanyal and Bhattacharyya in [12].

From all this discussions, it is seen that GTD formulation have till now failed in the backward broad range of low response regions because the finiteness of the edges were not incorporated in the diffraction coefficient used. The problem of diffraction by the edge of an infinite half-plane/wedge is a canonical problem in GTD. GTD offers an asymptotic solution to this problem. Because, of this the GTD formulations having these canonical problem as their base have failed, wherever finiteness of the edges plays an important rule. In GTD formulation, finite edge would mean a corner discontinuity. In order to incorporate this finite effect in the horn problem, corner diffraction needs to be considered.

Recently, UTD technique has been applied to analysis of complex antenna around the scatterer in [17, 18]. An approximate, UTD based ray solutions are applied to analysis high frequency electromagnetic (EM) wave radiation/coupling mechanisms for antennas on or near a junction between two different thin planar slabs on ground plane in [19]. But corner diffraction has not been used explicitly in there computations.

In this paper, E-plane horn radiation patterns are computed using FEGTD technique. Firstly, a two dimensional (2D) model of the horn using spherical source at the apex of a dihedral corner reflector formed by inclined half-planes (see Fig. 1(b)) is considered and UTD of [20] is applied to compute the field. Then higher order fields are computed by taking the account of multiply diffracted rays from its edge (see

Fig. 5) and apex. The apex is considered as a wedge (see Fig. 6(b)) of wedge angle $2\theta_E$. Finally, corner diffractions arising in the case of three dimension (3D) horn model are added to take the accounts of finiteness of the horn edges. The computed patterns are compared with the measured patterns.

2. HORN RADIATION PROBLEM

Geometry of a pyramidal horn along with its top (H-plane), side (E-plane) and front view are shown in Fig. 1. The TE_{10} mode at the waveguide horn junction gives rise to a tube of rays and the diverging wavefront shown emerging from the aperture in Fig. 2. The axial ray, which is surrounded by the tube of rays, coincides with the Z-axis. The horn E and H-planes intersect the principle direction of the wavefront surface. These planes intersect at a point z_0 on the wavefront. The principle radii of curvature of the diverging phase front are (ρ_E, ρ_H). Along the E and H-planes the phase variations of the diverging wavefront are coincident with those of spherical source excitations at S_H and S_E respectively.

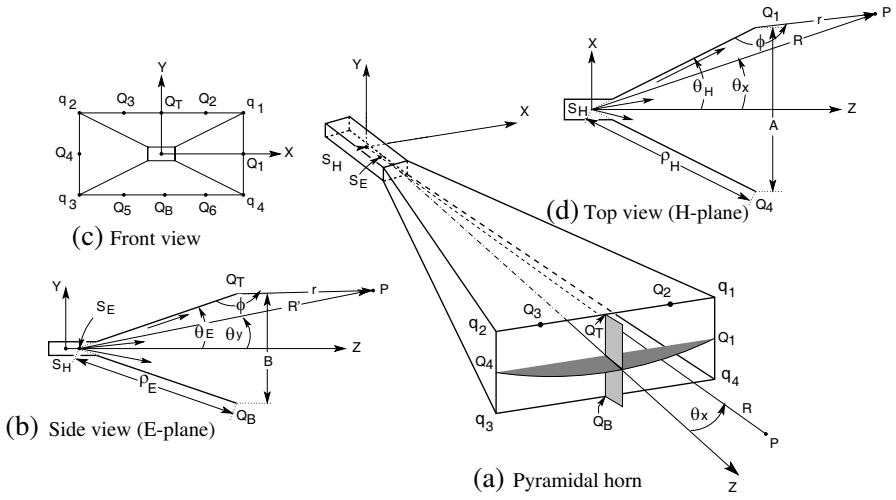

Figure 1. Geometry of a pyramidal horn antenna.

For the computation of E-plane radiation pattern, apex (S_E) is considered as the origin of the coordinate system. When the effects of top and bottom E-edges (edges perpendicular to the principal E-plane) are computed the corner reflector is assumed to be excited by an anisotropic spherical source at the vertex S_E and when the effects

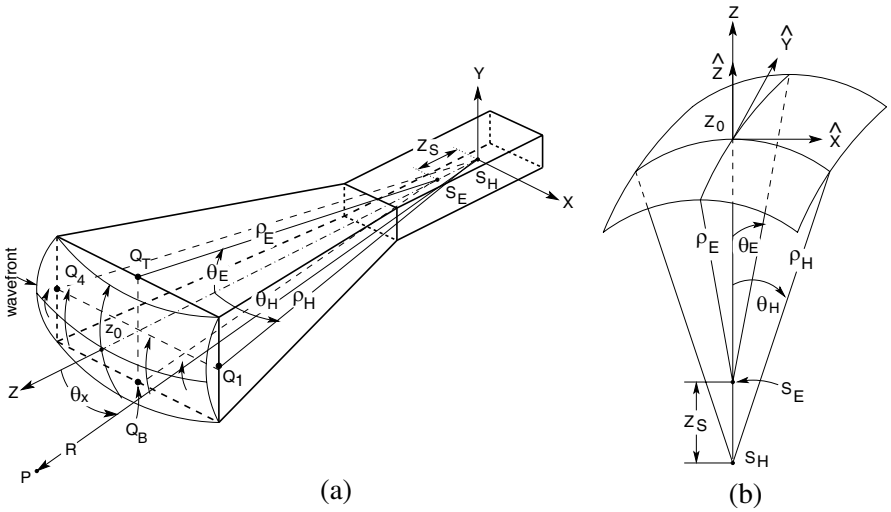

Figure 2. Diverging wavefront at the aperture of a pyramidal horn.

of side H-edges (edges parallel to the principal E-plane) are computed the corner reflector is assumed to be excited by an anisotropic spherical source at the vertex S_H. Rays from this source are assumed to have electric vector parallel to the corner reflector H-edges. The ray-optical expansion of this source field (GO field) in the H-plane is assumed to be given by

$$E^i = E_0 \cos\left(\frac{\pi}{A}\rho_{_H} \cos\theta_H \tan\theta_x\right) \frac{\exp\left(-jkR\right)}{R} \tag{1}$$

which satisfies the boundary condition for the horn, i.e., the field E^i at ($x = \pm A/2$, $-B/2 \leq y \leq B/2$, $z = \rho_{_H} \cos\theta_H$) is zero and the polarization is along Y-axis. This direct GO field ($E^i(P)$) contributes in the H-plane and E-plane horn radiation patterns within angle $|\theta_x| \leq \theta_H$ and $|\theta_y| \leq \theta_E$ respectively. The angles are shown in Fig. 1.

2.1. Measurement Setup and Process

To validate the computation, computed results are compared with measurement. The schematic of measurement setup is shown in Fig. 3. It is a 4-axis fully automated measurement system in an anechoic chamber. The specifications of the anechoic chamber are given below

- Size: $10\,\mathrm{m}(L) \times 6\,\mathrm{m}(W) \times 5\,\mathrm{m}(H)$
- Frequency Rang: $0.8 \sim 40\,\mathrm{GHz}$

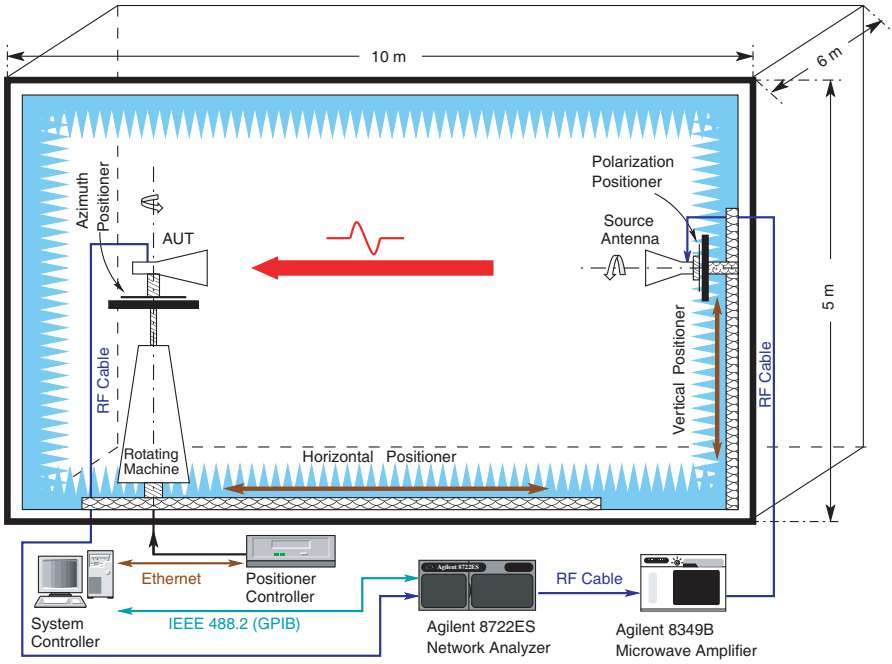

Figure 3. A schematic of measurement setup.

- Type: Rectangular
- Quiet Zone: 1 m @ 800 MHz
- Temperature: $21 \pm 2°C$
- Humidity: $45 \pm 8\%$

A broadband double-ridged horn antenna is used in transmitting mode and the antenna under test (AUT) on a rotating machine is used in receiving mode as shown in the Fig. 3. The rotating machine rotes in one degree step and the fields capture by the AUT are recorded with respect to the angular position of the AUT. Graphical representation of the field with respect to angular position of the AUT is known as radiation pattern. These patterns of a horn antenna are measured for different frequency.

2.2. Analysis of *E*-plane Horn Radiation Pattern by GTD (UTD) Method

The Y-Z plane is the E-plane, and it is explicitly shown in Fig. 1(b). The horn antenna is approximated by a dihedral comer reflector formed

by two infinite perfectly conducting, inclined half-planes and excited by a spherical source at the vertex S_E (Fig. 1). Edge diffraction from the mid point (Q_T) of E-edge q_1q_2 and edge diffraction from the mid point (Q_B) of E-edge q_3q_4 are shown in Fig. 4(a). Detailed geometry of E-edge diffraction for the calculation of E-plane horn radiation pattern is also shown in Fig. 4(b). The edge diffracted fields from the E-edges q_1q_2 ($E^d_{Q_T}$) and q_3q_4 ($E^d_{Q_B}$) are calculated using UTD of [3]. The spreading factor as well as phase function for the edge or wedge diffracted field are according to [3].

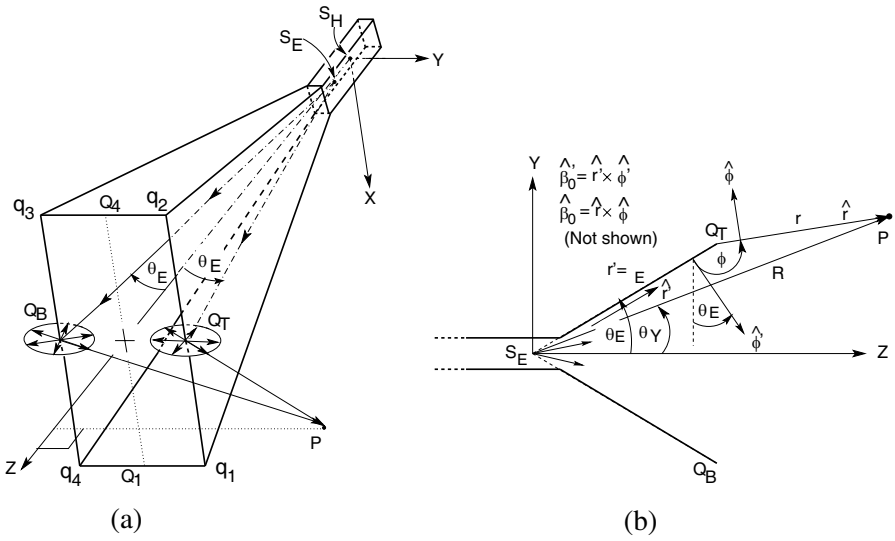

Figure 4. E-plane horn radiation pattern (a) E-edge diffractions, (b) Geometry of E-edge diffraction.

The multiply diffractions from E-edges are shown in Fig. 5. First order diffracted field ($E^d_{Q_B}$) from the midpoint Q_B of edge q_3q_4 incident on the midpoint Q_T of edge q_1q_2 creates second order edge diffraction ($E^{2d}_{Q_T}$). Similarly, first order edge diffracted field ($E^d_{Q_T}$) from Q_T incident on Q_B creates second order edge diffraction ($E^{2d}_{Q_B}$). Again, second order diffracted field ($E^{2d}_{Q_B}$) from the midpoint Q_B of edge q_3q_4 incident on the midpoint Q_T of edge q_1q_2 creates third order edge diffraction ($E^{3d}_{Q_T}$). Similarly, second order edge diffracted field ($E^{2d}_{Q_T}$) from Q_T incident on Q_B creates third order edge diffraction ($E^{3d}_{Q_B}$). These multiple edge diffracted fields are calculated using UTD technique of [3].

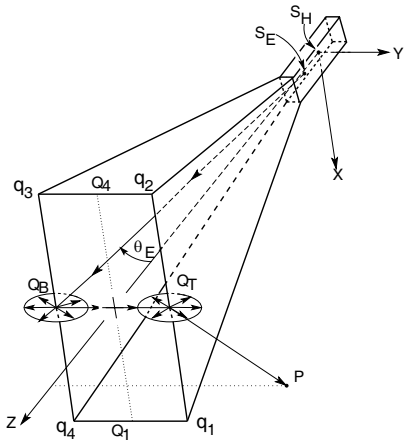

Figure 5. Multiple diffractions from E-edges.

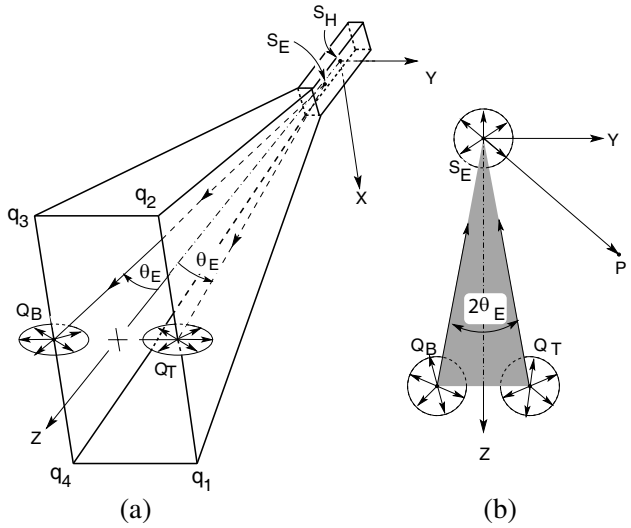

Figure 6. Second order wedge diffracted from apex S_E. (a) First order edge-diffractions due to the spherical source at S_E (b) Second order wedge diffractions due to the incidence of E-edge diffracted field.

First order diffracted field ($E^d_{Q_T}$) from the midpoint Q_T of edge q_1q_2 (see Fig. 6(a)) incident on the apex of the horn at S_E (see Fig. 6(b)) creates second order wedge diffraction ($E^{2dW}_{Q_T}$). The apex of the horn is considered as a wedge of wedge angle $2\theta_E$. Similarly, first order diffracted field ($E^d_{Q_B}$) from the point Q_B incident on the apex of

the horn at S_E creates second order wedge diffraction ($E^{2dW}_{Q_B}$). These edge and wedge diffracted fields are calculated using UTD technique of [3].

Adding the above diffracted fields and using symmetry the E-plane radiation pattern is

$$\begin{aligned} E(P) = {} & E^i(P)\big|_{for\ |\theta_y|\le\theta_E} \\ & + \Big[E^d_{Q_T}(P) + E^{2d}_{Q_T}(P) + E^{3d}_{Q_T}(P)\Big]\Big|_{for\ 0^\circ\le\theta_y\le 180^\circ\ and\ 270^\circ<\theta_y\le 360^\circ} \\ & + \Big[E^d_{Q_B}(P) + E^{2d}_{Q_B}(P) + E^{3d}_{Q_B}(P)\Big]\Big|_{for\ 0^\circ\le\theta_y<90^\circ\ and\ 180^\circ\le\theta_y\le 360^\circ} \\ & + \Big[E^{2dW}_{Q_T}(P) + E^{2dW}_{Q_B}(P)\Big]\Big|_{for\ \theta_E\le\theta_y<360^\circ-\theta_E} \end{aligned} \tag{2}$$

The computed radiation pattern using (2) is compared with the measurement shown in Fig. 7.

2.3. Analysis of *E*-plane Horn Radiation Pattern by FEGTD Method

From (1), at $\theta_x = \theta_H$, incident field E^i is zero. So, the corner diffracted fields from end of the E-edges and H-edges are also zero. However, the slope of the incident field is nonzero. For grazing incidence, the edge slope diffraction coefficient in this case of hard boundary condition (BC) is also identically zero [21]. However, the edge slope diffraction coefficient for the soft BC case is non zero. The edge slope diffracted

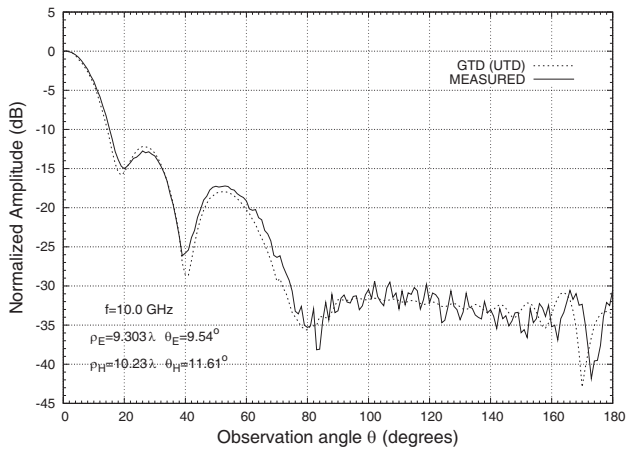

Figure 7. E-plane radiation pattern of a pyramidal horn antenna.

field for the soft BC case is much less than the edge diffracted field for the hard BC case at LRB directions. So the effects of corner slope diffracted field for the soft BC case are negligible in the LRB directions.

In order to incorporate the contributions of corner diffraction in the E-plane horn radiation pattern, we need to consider skewed rays incident on the E-edges (see Fig. 8). These rays appear to emanate from a point source located at S_E. The field diffracted from the E-edge at Q_2 gives rise to the ray incident on the corner q_4 at the end of adjacent H-edge q_1q_4 and simultaneously at the end of the opposite E-edge r_3q_4. The corner q_4 is first considered to be at one end of the half-plane $q_3q_4S_H$ (see [13, Fig. 4]). The Y-component of the corner diffracted field $E^{dH}_{Q_2q_4}(P)$ is obtained using the heuristic diffraction coefficient [22, Eq. (20)]. The spreading factor as well as phase function for the corner diffracted field is according to [22]. Next, the corner q_4 is considered to be at the end of the half-plane $q_1q_4S_E$ (see [13, Fig. 5]) and Y-component of the corner diffracted field $E^{dE}_{Q_2q_4}(P)$ is obtained using the heuristic diffraction coefficient [22, Eq. (20)] Let, $E^{d}_{Q_2q_4}(P)$ be the sum of fields $E^{dE}_{Q_2q_4}(P)$ and $E^{dH}_{Q_2q_4}(P)$ which is the contribution of corner q_4 in E-plane radiation pattern. Similarly, the contributions of the others corners q_1, q_2 & q_3 are also obtained.

On adding the corner diffracted field to the right hand side of (2)

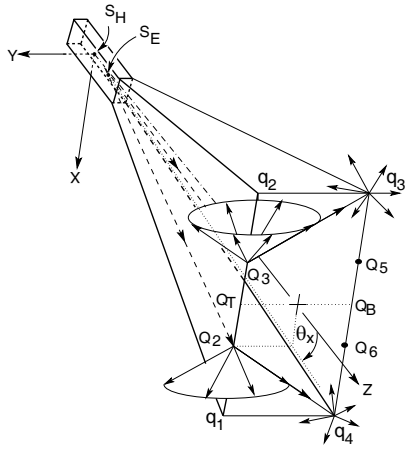

Figure 8. E-edge diffracted corner incident rays from a spherical source at S_E.

the E-plane radiation pattern using symmetry is

$$
\begin{aligned}
E2(P) = {} & E^{i}(P)\big|_{for\ |\theta_y|\le\theta_E} \\
& + \Big[E^{d}_{Q_T}(P) + E^{2d}_{Q_T}(P) + E^{3d}_{Q_T}(P)\Big]\Big|_{for\ 0^\circ\le\theta_y\le 180^\circ\ and\ 270^\circ<\theta_y\le 360^\circ} \\
& + \Big[E^{d}_{Q_B}(P) + E^{2d}_{Q_B}(P) + E^{3d}_{Q_B}(P)\Big]\Big|_{for\ 0^\circ\le\theta_y<90^\circ\ and\ 180^\circ\le\theta_y\le 360^\circ} \\
& + \Big[E^{2dW}_{Q_T}(P) + E^{2dW}_{Q_B}(P)\Big]\Big|_{for\ \theta_E\le\theta_y<360^\circ-\theta_E} \\
& + 2\Big[E^{d}_{Q_2q_4}(P) + E^{d}_{Q_6q_1}(P)\Big]
\end{aligned}
\tag{3}
$$

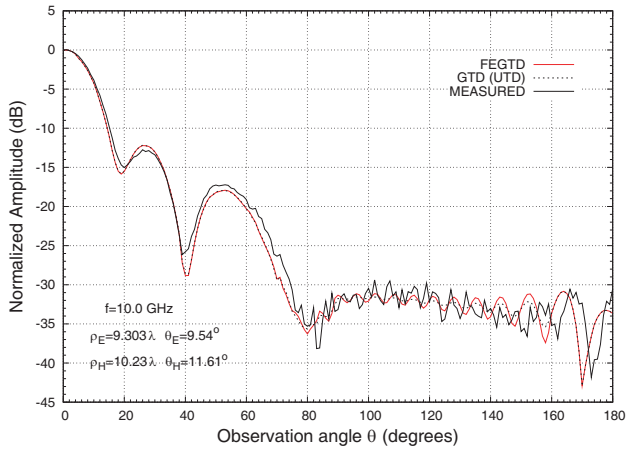

Figure 9. E-plane radiation pattern of a pyramidal horn antenna.

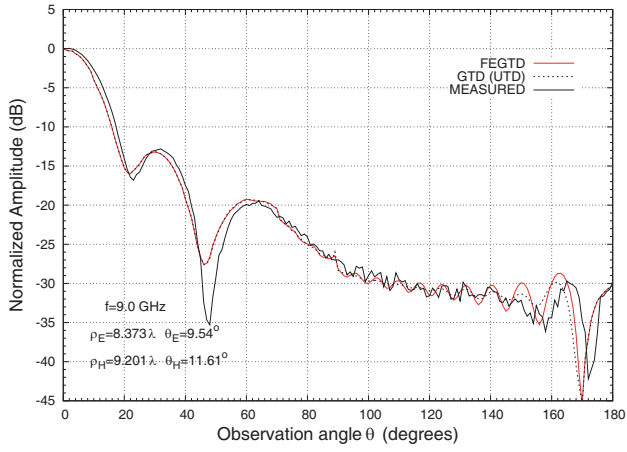

Figure 10. E-plane radiation pattern of a pyramidal horn antenna.

The computed radiation patterns using (3) are compared with the measurement and computed radiation patterns using GTD (UTD) technique (using (2)) shown in Figs. 9 & 10.

3. RESULTS AND DISCUSSIONS

E-plane horn radiation pattern is computed using UTD with spherical source excitations. Firstly, diffracted fields from the E-edges due to the spherical source at the apex S_E are computed. Then multiple (double and triple) diffraction from the E-edges and double diffraction from the apex S_E due the E-edge diffracted field incidence on the apex are added. The computed pattern is compared with the measurement shown in Fig. 7. The computed pattern is well matched with the measurement in overall manner. The presence of ripples in the field pattern is predicted like in [12].

Skewed ray that result in corner diffraction is introduced and augmented E-plane horn radiation pattern is also computed. The results are compared with the radiation pattern using GTD (UTD) technique and measurements shown in Figs. 9 & 10. Corner diffraction was found to have significant influence on the E-plane horn radiation pattern in the LRB directions. The ripples in the measurement between the angle 90° and 160° are well predicted in compared with GTD (UTD) technique. As the far out (90° to 160°) side lobe levels in the E-plane vary around 25 to 40 dB the effect of corner diffraction is less compared to the H-plane case in [13], where the far out sidelobe levels were around 40 to 55 dB.

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