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FRACTIONAL RECTANGULAR WAVEGUIDE INTERNALLY COATED WITH CHIRAL NIHILITY METAMATERIAL

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Abstract—The fields inside a rectangular waveguide with an internal coating of chiral nihility metamaterial are determined. These fields are then fractionalized utilizing the fractional curl operator to find the fields for the intermediate geometries which are also termed as the fractional order geometries. It is noted that no electric field exists inside the chiral nihility coating backed by perfect electric conductor (PEC) surface. The fractional order geometries are related through the principle of duality. The behavior of the fields with respect to the fractional parameter, α is analyzed.

1. INTRODUCTION

Chiral medium have attracted many researchers and scientists in the recent years [1–4]. Chiral nihility is a special kind of chiral medium, for which the permittivity and permeability are simultaneously zero and the chirality parameter is nonzero at certain frequency called the nihility frequency [5–7]. Wave propagation and energy transfer in chiral nihility have been discussed in [5]. Chiral media are characterized by two intrinsic eigenwaves with left-handed and right-handed circular polarizations [3–4], and both of them have different phase velocities and refraction indices. The two eigenwaves in chiral nihility are

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circularly polarized but one of them is a backward wave, whose phase velocity has antiparallel direction to that of the corresponding Poynting vector [6]. In order to overcome the strict conditions of permittivity and permeability equals to zero for the chiral nihility material, Qiu has also proposed the nonreciprocity route for the realization of backward waves and negative refractions [8], and that the backward waves can be produced using the gyrotropic parameters and permittivity and permeability very small [9]. On this topic a number of research articles have been contributed by different authors [10–16].

Fractional calculus is a branch of Mathematics that deals with the operators of general order that covers integer order, real non-integer order and complex order such as fractional derivatives and fractional integrals [17]. Fractional operator may be utilize to find the intermediate solutions between a given solution and dual of the given solution [18, 19]. The fractional duality in electromagnetics states that if $(\bar{E}, \eta\bar{H})$ is one set of solutions to the Maxwell's equations, then $(\eta\bar{H}, -\bar{E})$ is the dual solution of the same equations, where η is the impedance of medium. Engheta introduced the fractional curl operator in electromagnetics [20]. Naqvi extended the work to operators having higher and complex order [21]. Mathematical recipe to fractionalize the curl operator is discussed in [20]. The application of fractional curl operator to different problems are addressed in [22–27]. Naqvi modeled the transmission through chiral nihility slab in terms of fractional curl operator [28].

In this paper, the fractional dual fields inside a rectangular waveguide internally coated with chiral nihility material are obtained. It is shown that the electric field inside the chiral nihility coating is zero and therefor all the electromagnetic energy is propagated through the air region. The variation in the fields according to α is also discussed.

2. FIELDS INSIDE A CHIRAL NIHILITY COATED RECTANGULAR WAVEGUIDE

The problem geometry which is under consideration is shown in Figure 1. Where the rectangular waveguide, with internal coating of chiral nihility material is shown. This structure is assumed to be uniform and infinitely long in the x-direction. The time dependency is of the form $\exp(-j\omega t)$, which is suppressed through out the text. The waveguide is divided into two regions. Region 0 is between $(-d_1 < z < d_1)$ and $(-b_1 < y < b_1)$ which is occupied by air with the permittivity ϵ_0 and permeability μ_0. Region 1 $(-d_2 \le z < -d_1)$, $(d_1 < z \le d_2)$ and $(-b_2 \le y < -b_1)$, $(b_1 < y \le b_2)$ is covered with the chiral nihility material which is characterized by $(\epsilon = 0, \mu = 0, \kappa \neq 0)$.

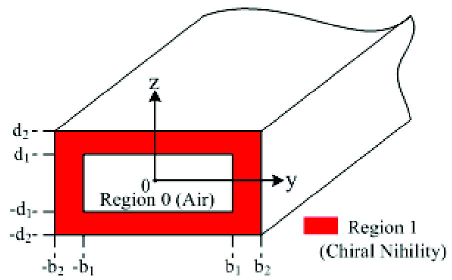

Figure 1. Rectangular waveguide internally coated with chiral nihility material.

The constitutive relations of an isotropic, reciprocal and lossless chiral medium are [3],

$$\bar{D} = \epsilon\bar{E} + j\kappa\sqrt{\epsilon_0\mu_0}\bar{H} \tag{1}$$

$$\bar{B} = \mu\bar{H} - j\kappa\sqrt{\epsilon_0\mu_0}\bar{E} \tag{2}$$

where, ϵ, μ and κ are permittivity, permeability and the chirality parameter of the chiral medium, respectively. As the chiral nihility is a special case of chiral medium for which ($\epsilon = 0,\ \mu = 0,\ \kappa \neq 0$), so the constitutive relations of the chiral nihility material becomes as,

$$\bar{D} = j\kappa\sqrt{\epsilon_0\mu_0}\bar{H} \tag{3}$$

$$\bar{B} = -j\kappa\sqrt{\epsilon_0\mu_0}\bar{E} \tag{4}$$

The PEC boundaries are located at $z = \pm d_2$ and $y = \pm b_2$.

The wave equation in free space inside the rectangular waveguide is,

$$\left(\nabla^2 + k_0^2\right)\bar{E} = 0 \tag{5}$$

$$\left(\nabla^2 + k_0^2\right)\bar{H} = 0 \tag{6}$$

where $k_0 = \omega\sqrt{\mu_0/\epsilon_0}$ and inside the chiral nihility material the wave equation becomes as:

$$\left(\nabla^2 + k^{\pm 2}\right)\bar{E} = 0 \tag{7}$$

$$\left(\nabla^2 + k^{\pm 2}\right)\bar{H} = 0 \tag{8}$$

where, $k^{\pm} = \pm\omega\kappa$ are the two wavenumbers for the left circularly polarized (LCP) and right circularly polarized (RCP) waves in the chiral nihility material at nihility frequency.

The general solution of the reduced wave equation for the transverse magnetic (TM) mode in simple rectangular waveguide containing free space only, is given by [29].

$$\bar{E}_x = \frac{-j}{2}Ae^{jk_y y + jk_z z} + \frac{1}{2}Be^{jk_y y - jk_z z} + \frac{1}{2}Ce^{-jk_y y + jk_z z} + \frac{j}{2}De^{-jk_y y - jk_z z} \tag{9}$$

Now, exciting the rectangular waveguide internally coated with the chiral nihility material by the field expression (7), we work out the expressions for the electric and magnetic fields inside the Region 0 (free space) and Region 1 (the chiral nihility material) as [30, 31]

$$\begin{aligned}\bar{E}_0 =& -\frac{j}{2}e^{jk_{0y}y+jk_{0z}z}\hat{x} - \frac{j}{2}A^+\bar{N}_R^+e^{jk_{0y}y+jk_{0z}z} - \frac{j}{2}B^+\bar{N}_L^+e^{jk_{0y}y+jk_{0z}z}\\ &+\frac{1}{2}e^{jk_{0y}y-jk_{0z}z}\hat{x} + \frac{1}{2}A^-\bar{N}_R^-e^{jk_{0y}y-jk_{0z}z} + \frac{1}{2}B^-\bar{N}_L^-e^{jk_{0y}y-jk_{0z}z}\\ &+\frac{j}{2}e^{-jk_{0y}y-jk_{0z}z}\hat{x} + \frac{j}{2}C^+\bar{N}_R^+e^{-jk_{0y}y-jk_{0z}z} + \frac{j}{2}D^+\bar{N}_L^+e^{-jk_{0y}y-jk_{0z}z}\\ &+\frac{1}{2}e^{-jk_{0y}y+jk_{0z}z}\hat{x}+\frac{1}{2}C^-\bar{N}_R^-e^{-jk_{0y}y+jk_{0z}z}+\frac{1}{2}D^-\bar{N}_L^-e^{-jk_{0y}y+jk_{0z}z}\end{aligned} \tag{10}$$

$$\begin{aligned}\bar{H}_0 =& \\ &\frac{1}{2\eta_0}\left[\frac{k_{0z}}{k_0}\left(je^{jk_{0y}y+jk_{0z}z}+e^{jk_{0y}y-jk_{0z}z}-e^{-jk_{0y}y+jk_{0z}z}+je^{-jk_{0y}y-jk_{0z}z}\right)\hat{y}\right.\\ &+\frac{k_{0y}}{k_0}\left(-je^{jk_{0y}y+jk_{0z}z}+e^{jk_{0y}y-jk_{0z}z}-e^{-jk_{0y}y+jk_{0z}z}-je^{-jk_{0y}y-jk_{0z}z}\right)\hat{z}\\ &+A^+\bar{N}_R^+e^{jk_{0y}y+jk_{0z}z} - B^+\bar{N}_L^+e^{jk_{0y}y+jk_{0z}z} + jA^-\bar{N}_R^-e^{jk_{0y}y-jk_{0z}z}\\ &-jB^-\bar{N}_L^-e^{jk_{0y}y-jk_{0z}z} + C^+\bar{N}_R^+e^{-jk_{0y}y-jk_{0z}z} - D^+\bar{N}_L^+e^{-jk_{0y}y-jk_{0z}z}\\ &\left.-jC^-\bar{N}_R^-e^{-jk_{0y}y+jk_{0z}z} + jD^-\bar{N}_L^-e^{-jk_{0y}y+jk_{0z}z}\right]\end{aligned} \tag{11}$$

$$\begin{aligned}\bar{E}_1 =& -\frac{j}{2}E^+\bar{M}_R^+e^{jk_y^+y+jk_z^+z} - \frac{j}{2}F^+\bar{M}_L^+e^{jk_y^-y+jk_z^-z}\\ &+\frac{1}{2}E^-\bar{M}_R^-e^{jk_y^+y-jk_z^+z} + \frac{1}{2}F^-\bar{M}_L^-e^{jk_y^-y-jk_z^-z}\\ &+\frac{j}{2}G^+\bar{M}_R^+e^{-jk_y^+y-jk_z^+z} + \frac{j}{2}H^+\bar{M}_L^+e^{-jk_y^-y-jk_z^-z}\\ &+\frac{1}{2}G^-\bar{M}_R^-e^{-jk_y^+y+jk_z^+z} + \frac{1}{2}H^-\bar{M}_L^-e^{-jk_y^-y+jk_z^-z}\end{aligned} \tag{12}$$

$$\begin{aligned}\bar{H}_1 =& \frac{1}{2\eta}\left[E^+\bar{M}_R^+e^{jk_y^+y+jk_z^+z} - F^+\bar{M}_L^+e^{jk_y^-y+jk_z^-z} + jE^-\bar{M}_R^-e^{jk_y^+y-jk_z^+z}\right.\\ &-jF^-\bar{M}_L^-e^{jk_y^-y-jk_z^-z} + G^+\bar{M}_R^+e^{-jk_y^+y-jk_z^+z} - H^+\bar{M}_L^+e^{-jk_y^-y-jk_z^-z}\\ &\left.-jG^-\bar{M}_R^-e^{-jk_y^+y+jk_z^+z} + jH^-\bar{M}_L^-e^{-jk_y^-y+jk_z^-z}\right]\end{aligned} \tag{13}$$

where, $\bar{E}_0$ and $\bar{H}_0$ are the electric and magnetic fields in the free space and $\bar{E}_1$ and $\bar{H}_1$ are the electric and magnetic fields in the chiral nihility region respectively. $A^{\pm}$, $B^{\pm}$, $C^{\pm}$, $D^{\pm}$, $E^{\pm}$, $F^{\pm}$, $G^{\pm}$ and $H^{\pm}$ are the unknown coefficients for the RCP and LCP waves inside the air and chiral nihility regions of the rectangular waveguide and

$$\bar{N}_R^{\pm} = \hat{x} \pm j\frac{k_{0z}}{k_0}\hat{y} - j\frac{k_{0y}}{k_0}\hat{z} \tag{14a}$$

$$\bar{N}_L^{\pm} = \hat{x} \mp j\frac{k_{0z}}{k_0}\hat{y} + j\frac{k_{0y}}{k_0}\hat{z} \tag{14b}$$

$$\bar{M}_R^{\pm} = \hat{x} \pm j\frac{k_z^{\pm}}{k^{\pm}}\hat{y} - j\frac{k_y^{\pm}}{k^{\pm}}\hat{z} \tag{14c}$$

$$\bar{M}_L^{\pm} = \hat{x} \mp j\frac{k_z^{\pm}}{k^{\pm}}\hat{y} - j\frac{k_y^{\pm}}{k^{\pm}}\hat{z} \tag{14d}$$

Superscript $\pm$ in Equation (17) represents the eigenwaves propagating in the $\pm z$ and $\pm y$ directions. $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ and $\eta = \sqrt{\mu/\epsilon}$ is the impedance of free space and chiral nihility medium respectively. The subscript R and L refer to the RCP and LCP eigenwaves satisfying the dispersion relation

$$(k_y^{\pm})^2 + (k_z^{\pm})^2 = (k^{\pm})^2 \tag{15}$$

k_{0z} and k_{0y} satisfy the dispersion relation

$$k_{0y}^2 + k_{0z}^2 = k_0^2 \tag{16}$$

The relation between the normal components of wave vectors in chiral nihility medium is $k_z^+ = -k_z^-$ [32]. The unknown coefficients may be obtained by applying the boundary conditions. According to the boundary conditions at $z = \pm d_2$ and $y = \pm b_2$, i.e., at the PEC surfaces the tangential components of the electric field $\bar{E}_1$ must be zero and the tangential components of the electric and magnetic fields across the achiral-chiral interface located at $z = \pm d_1$ and $y = \pm b_1$ must be continuous. After solving the boundary problems, we obtain the coefficients as

$$A^+ = A^- = B^+ = B^- = C^+ = C^- = D^+ = D^- = \frac{-1}{2} \tag{17}$$

$$jE^+ = jF^+ = jG^+ = jH^+ = E^- = F^- = -G^- = -H^- \tag{18}$$

When we substitute Equations (20) and (21) in Equation (15), we get $\bar{E}_1 = 0$. The electric field inside the chiral nihility coating comes to be zero and all the electric field is localized in the air region. The reason for the disappearance of electric field inside the chiral nihility is discussed in [6].

It is obvious from above expressions that the $\bar{E}_1 = 0$, that is the electric field inside the Region 1 becomes zero. So the energy is only confined to the non-nihility regions of the waveguide as shown in Figure 2.

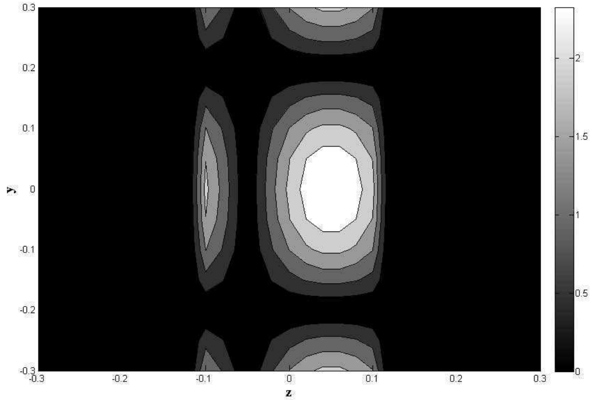

Figure 2. Amplitude of the electric field along the guide.

3. FRACTIONAL DUAL FIELDS INSIDE A CHIRAL NIHILITY COATED RECTANGULAR WAVEGUIDE

The fractional dual electric ($\bar{E}_{fd}$) and magnetic ($\bar{H}_{fd}$) fields may be obtained using the following relations [20]:

$$\bar{E}_{fd} = \frac{1}{(jk)^\alpha}(\nabla\times)^\alpha \bar{E} \tag{19}$$

$$\eta\bar{H}_{fd} = \frac{1}{(jk)^\alpha}(\nabla\times)^\alpha \eta\bar{H} \tag{20}$$

where $\frac{1}{jk}\nabla\times$ is equivalent to the cross product operator $(k_i\times)$. Using this concept of fractional curl operator $(k_i\times)^\alpha$ [20], the Maxwell equations for the $e^{j\omega t}$ time harmonic fields can be written as:

$$(\hat{k}\times)\bar{E}_{0fd} = \eta_0\bar{H}_{0fd} \tag{21a}$$

$$(\hat{k}\times)\eta_0\bar{H}_{0fd} = -\bar{E}_{0fd} \tag{21b}$$

$$(\hat{k}^\pm\times)\bar{E}_{1fd} = \eta\bar{H}_{1fd} \tag{21c}$$

$$(\hat{k}^\pm\times)\eta\bar{H}_{1fd} = -\bar{E}_{1fd} \tag{21d}$$

Fractional dual fields inside the air and chiral nihility coating may be obtained as [28]:

$$\bar{E}_{0fd} = (\hat{k}\times)^\alpha\eta_0\bar{H}_0 \tag{22a}$$

$$\eta_0\bar{H}_{0fd} = (\hat{k}\times)^\alpha\bar{E}_0 \tag{22b}$$

$$\bar{E}_{1fd} = (\hat{k}^{\pm}\times)^{\alpha}\eta\bar{H}_1 \tag{22c}$$

$$\eta\bar{H}_{1fd} = (\hat{k}^{\pm}\times)^{\alpha}\bar{E}_1 \tag{22d}$$

We may express the fields in the waveguide as a superposition of four waves, i.e.,

$$\bar{E}_i = \bar{E}_i^1 + \bar{E}_i^2 + \bar{E}_i^3 + \bar{E}_i^4$$

$$\eta\bar{H}_i = \eta\bar{H}_i^1 + \eta\bar{H}_i^2 + \eta\bar{H}_i^3 + \eta\bar{H}_i^4$$

where, $i = 0, 1$. The electric and magnetic fields associated with $i = 0$ waves i.e., fields inside the free space are written as:

$$\bar{E}_0^1 = -\frac{j}{2}e^{jk_{0y}y+jk_{0z}z}\hat{x} - \frac{j}{2}A^+\bar{N}_R^+e^{jk_{0y}y+jk_{0z}z} - \frac{j}{2}B^+\bar{N}_L^+e^{jk_{0y}y+jk_{0z}z} \tag{23}$$

$$\bar{E}_0^2 = \frac{1}{2}e^{jk_{0y}y-jk_{0z}z}\hat{x} + \frac{1}{2}A^-\bar{N}_R^-e^{jk_{0y}y-jk_{0z}z} + \frac{1}{2}B^-\bar{N}_L^-e^{jk_{0y}y-jk_{0z}z} \tag{24}$$

$$\bar{E}_0^3 = \frac{j}{2}e^{-jk_{0y}y-jk_{0z}z}\hat{x} + \frac{j}{2}C^+\bar{N}_R^+e^{-jk_{0y}y-jk_{0z}z} + \frac{j}{2}D^+\bar{N}_L^+e^{-jk_{0y}y-jk_{0z}z} \tag{25}$$

$$\bar{E}_0^4 = \frac{1}{2}e^{-jk_{0y}y+jk_{0z}z}\hat{x} + \frac{1}{2}C^-\bar{N}_R^-e^{-jk_{0y}y+jk_{0z}z} + \frac{1}{2}D^-\bar{N}_L^-e^{-jk_{0y}y+jk_{0z}z} \tag{26}$$

$$\eta_0\bar{H}_0^1 = \frac{1}{2}\left[\frac{k_{0z}}{k_0}je^{jk_{0y}y+jk_{0z}z}\hat{y} - \frac{k_{0y}}{k_0}je^{jk_{0y}y+jk_{0z}z}\hat{z} + A^+\bar{N}_R^+e^{jk_{0y}y+jk_{0z}z} - B^+\bar{N}_L^+e^{jk_{0y}y+jk_{0z}z}\right] \tag{27}$$

$$\eta_0\bar{H}_0^2 = \frac{1}{2}\left[\frac{k_{0z}}{k_0}e^{jk_{0y}y-jk_{0z}z}\hat{y} + \frac{k_{0y}}{k_0}e^{jk_{0y}y-jk_{0z}z}\hat{z} + jA^-\bar{N}_R^-e^{jk_{0y}y-jk_{0z}z} - jB^-\bar{N}_L^-e^{jk_{0y}y-jk_{0z}z}\right] \tag{28}$$

$$\eta_0\bar{H}_0^3 = \frac{1}{2}\left[\frac{k_{0z}}{k_0}je^{-jk_{0y}y-jk_{0z}z}\hat{y} - \frac{k_{0y}}{k_0}je^{-jk_{0y}y-jk_{0z}z}\hat{z} + C^+\bar{N}_R^+e^{-jk_{0y}y-jk_{0z}z} - D^+\bar{N}_L^+e^{-jk_{0y}y-jk_{0z}z}\right] \tag{29}$$

$$\eta_0\bar{H}_0^4 = \frac{1}{2}\left[-\frac{k_{0z}}{k_0}e^{-jk_{0y}y+jk_{0z}z}\hat{y} - \frac{k_{0y}}{k_0}e^{-jk_{0y}y+jk_{0z}z}\hat{z} - jC^-\bar{N}_R^-e^{-jk_{0y}y+jk_{0z}z} + jD^-\bar{N}_L^-e^{-jk_{0y}y+jk_{0z}z}\right] \tag{30}$$

For the above waves the $\hat{k}_i\times$ operators are defined

$$\begin{aligned}
\hat{k}_1\times &= -\frac{1}{k_0}\left(k_{0y}\hat{y}+k_{0z}\hat{z}\right)\times\\
\hat{k}_2\times &= -\frac{1}{k_0}\left(k_{0y}\hat{y}-k_{0z}\hat{z}\right)\times\\
\hat{k}_3\times &= -\frac{1}{k_0}\left(-k_{0y}\hat{y}-k_{0z}\hat{z}\right)\times\\
\hat{k}_4\times &= -\frac{1}{k_0}\left(-k_{0y}\hat{y}+k_{0z}\hat{z}\right)\times
\end{aligned}$$

To determine the fractional dual solutions $(\bar{E}_{0fd}, \eta\bar{H}_{0fd})$, the eigenvalues and the eigenvectors of the cross product operator $\hat{k}_i\times$ are required. Eigenvalues and the eigenvectors of the operator $(\hat{k}_1\times)$ are

$$A_{11}=\frac{1}{\sqrt{2}}\left(\hat{x}+j\frac{k_{0z}}{k_0}\hat{y}-j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_R^+, \qquad a_{11}=+j$$

$$A_{12}=\frac{1}{\sqrt{2}}\left(\hat{x}-j\frac{k_{0z}}{k_0}\hat{y}+j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_L^+, \qquad a_{12}=-j$$

$$A_{13}=-j\frac{k_{0y}}{k_0}\hat{y}-j\frac{k_{0z}}{k_0}\hat{z}, \qquad a_{13}=0$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_2\times)$ are

$$\begin{aligned}
A_{21} &= \frac{1}{\sqrt{2}}\left(\hat{x}-j\frac{k_{0z}}{k_0}\hat{y}-j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_R^-, &\qquad a_{21}&=+j\\
A_{22} &= \frac{1}{\sqrt{2}}\left(\hat{x}+j\frac{k_{0z}}{k_0}\hat{y}+j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_L^-, &\qquad a_{22}&=-j\\
A_{23} &= -j\frac{k_{0y}}{k_0}\hat{y}+j\frac{k_{0z}}{k_0}\hat{z}, &\qquad a_{23}&=0
\end{aligned}$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_3\times)$ are

$$\begin{aligned}
A_{31} &= \frac{1}{\sqrt{2}}\left(\hat{x}+j\frac{k_{0z}}{k_0}\hat{y}-j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_R^+, &\qquad a_{31}&=-j\\
A_{32} &= \frac{1}{\sqrt{2}}\left(\hat{x}-j\frac{k_{0z}}{k_0}\hat{y}+j\frac{k_{0y}}{k_0}\hat{z}\right)=\bar{N}_L^+, &\qquad a_{32}&=+j\\
A_{33} &= j\frac{k_{0y}}{k_0}\hat{y}+j\frac{k_{0z}}{k_0}\hat{z}, &\qquad a_{33}&=0
\end{aligned}$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_4\times)$ are

$$\begin{aligned} A_{41} &= \frac{1}{\sqrt{2}}\left(\hat{x} - j\frac{k_{0z}}{k_0}\hat{y} - j\frac{k_{0y}}{k_0}\hat{z}\right) = \bar{N}_R^-, & a_{41} &= -j \\ A_{42} &= \frac{1}{\sqrt{2}}\left(\hat{x} + j\frac{k_{0z}}{k_0}\hat{y} + j\frac{k_{0y}}{k_0}\hat{z}\right) = \bar{N}_L^-, & a_{42} &= +j \\ A_{43} &= j\frac{k_{0y}}{k_0}\hat{y} - j\frac{k_{0z}}{k_0}\hat{z}, & a_{43} &= 0 \end{aligned}$$

The fractional dual fields associated with the corresponding operators are

$$\bar{E}^1_{0fd} = -\frac{j}{2}\left[\left(\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y} + \frac{k_{0y}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{jk_{0y}y+jk_{0z}z} + (j)^\alpha A^+\bar{N}_R^+ e^{jk_{0y}y+jk_{0z}z} + (-j)^\alpha B^+\bar{N}_L^+ e^{jk_{0y}y+jk_{0z}z}\right] \quad (31)$$

$$\bar{E}^2_{0fd} = \frac{1}{2}\left[\left(\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y} + \frac{k_{0y}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{jk_{0y}y-jk_{0z}z} + (j)^\alpha A^-\bar{N}_R^- e^{jk_{0y}y-jk_{0z}z} + (-j)^\alpha B^-\bar{N}_L^- e^{jk_{0y}y-jk_{0z}z}\right] \quad (32)$$

$$\bar{E}^3_{0fd} = \frac{j}{2}\left[\left(\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_{0y}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{-jk_{0y}y-jk_{0z}z} + (-j)^\alpha C^+\bar{N}_R^+ e^{-jk_{0y}y-jk_{0z}z} + (j)^\alpha D^+\bar{N}_L^+ e^{-jk_{0y}y-jk_{0z}z}\right] \quad (33)$$

$$\bar{E}^4_{0fd} = \frac{1}{2}\left[\left(\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_{0y}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{-jk_{0y}y+jk_{0z}z} + (-j)^\alpha C^-\bar{N}_R^- e^{-jk_{0y}y+jk_{0z}z} + (j)^\alpha D^-\bar{N}_L^- e^{-jk_{0y}y+jk_{0z}z}\right] \quad (34)$$

$$\eta_0\bar{H}^1_{0fd} = \frac{1}{2}\left[j\left(\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{k_{0z}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_{0y}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{jk_{0y}y+jk_{0z}z} + (j)^\alpha A^+\bar{N}_R^+ e^{jk_{0y}y+jk_{0z}z} - (-j)^\alpha B^+\bar{N}_L^+ e^{jk_{0y}y+jk_{0z}z}\right] \quad (35)$$

$$\eta_0\bar{H}^2_{0fd} = \frac{1}{2}\left[\left(-\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{k_{0z}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y} + \frac{k_{0y}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{jk_{0y}y-jk_{0z}z} + j(j)^\alpha A^-\bar{N}_R^- e^{jk_{0y}y-jk_{0z}z} - j(-j)^\alpha B^-\bar{N}_L^- e^{jk_{0y}y-jk_{0z}z}\right] \quad (36)$$

$$\eta_0 \bar{H}^3_{0fd} = \frac{1}{2}\Bigg[j\left(-\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} + \frac{k_{0z}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_{0y}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{-jk_{0y}y-jk_{0z}z}$$
$$+(-j)^\alpha C^+\bar{N}^+_R e^{-jk_{0y}y-jk_{0z}z} - (j)^\alpha D^+\bar{N}^+_L e^{-jk_{0y}y-jk_{0z}z}\Bigg] \quad (37)$$

$$\eta_0 \bar{H}^4_{0fd} = \frac{1}{2}\Bigg[\left(-\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} - \frac{k_{0z}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_{0y}}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right)e^{-jk_{0y}y+jk_{0z}z}$$
$$-j(-j)^\alpha C^-\bar{N}^-_R e^{-jk_{0y}y+jk_{0z}z} + j\,(j)^\alpha D^-\bar{N}^-_L e^{-jk_{0y}y+jk_{0z}z}\Bigg] \quad (38)$$

The total fractional dual electric and magnetic fields in the free space region may be written as:

$$\bar{E}_{0fd} = \bar{E}^1_{0fd} + \bar{E}^2_{0fd} + \bar{E}^3_{0fd} + \bar{E}^4_{0fd} \quad (39)$$
$$\eta_0\bar{H}_{0fd} = \eta_0\bar{H}^1_{0fd} + \eta_0\bar{H}^2_{0fd} + \eta_0\bar{H}^3_{0fd} + \eta_0\bar{H}^4_{0fd} \quad (40)$$

Similarly the electric and magnetic fields associated with $i = 1$, i.e., fields inside the chiral nihility coating may be written as as sum of four plan waves for convenience:

$$\bar{E}^1_1 = -\frac{j}{2}E^+\bar{M}^+_R e^{jk^+_y y+jk^+_z z} - \frac{j}{2}F^+\bar{M}^+_L e^{jk^-_y y+jk^-_z z} \quad (41)$$
$$\bar{E}^2_1 = \frac{1}{2}E^-\bar{M}^-_R e^{jk^+_y y-jk^+_z z} + \frac{1}{2}F^-\bar{M}^-_L e^{jk^-_y y-jk^-_z z} \quad (42)$$
$$\bar{E}^3_1 = \frac{j}{2}G^+\bar{M}^+_R e^{-jk^+_y y-jk^+_z z} + \frac{j}{2}H^+\bar{M}^+_L e^{-jk^-_y y-jk^-_z z} \quad (43)$$
$$\bar{E}^4_1 = \frac{1}{2}G^-\bar{M}^-_R e^{-jk^+_y y+jk^+_z z} + \frac{1}{2}H^-\bar{M}^-_L e^{-jk^-_y y+jk^-_z z} \quad (44)$$
$$\eta\bar{H}^1_1 = \frac{1}{2}E^+\bar{M}^+_R e^{jk^+_y y+jk^+_z z} - \frac{1}{2}F^+\bar{M}^+_L e^{jk^-_y y+jk^-_z z} \quad (45)$$
$$\eta\bar{H}^2_1 = \frac{j}{2}E^-\bar{M}^-_R e^{jk^+_y y-jk^+_z z} - \frac{j}{2}F^-\bar{M}^-_L e^{jk^-_y y-jk^-_z z} \quad (46)$$
$$\eta\bar{H}^3_1 = \frac{1}{2}G^+\bar{M}^+_R e^{-jk^+_y y-jk^+_z z} - \frac{1}{2}H^+\bar{M}^+_L e^{-jk^-_y y-jk^-_z z} \quad (47)$$
$$\eta\bar{H}^4_1 = -\frac{j}{2}G^-\bar{M}^-_R e^{-jk^+_y y+jk^+_z z} + \frac{j}{2}H^-\bar{M}^-_L e^{-jk^-_y y+jk^-_z z} \quad (48)$$

The $(\hat{k}_i\times)$ operators are defined for the fractionalization of fields in the chiral nihility regions:

$$\hat{k_1}^+\times = -\frac{1}{k^+}\left(k^+_y\hat{y} + k^+_z\hat{z}\right)$$
$$\hat{k_2}^+\times = -\frac{1}{k^+}\left(k^+_y\hat{y} - k^+_z\hat{z}\right)$$

$$\hat{k_3}^+ \times = -\frac{1}{k^+}\left(-k_y^+\hat{y} - k_z^+\hat{z}\right)$$
$$\hat{k_4}^+ \times = -\frac{1}{k^+}\left(-k_y^+\hat{y} + k_z^+\hat{z}\right)$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_1^+ \times)$ are

$$A_{11}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} + j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_R^+, \qquad a_{11}^+ = +j$$
$$A_{12}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} - j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_L^+, \qquad a_{12}^+ = -j$$
$$A_{13}^+ = -j\frac{k_y^+}{k^+}\hat{y} - j\frac{k_z^+}{k^+}\hat{z}, \qquad a_{13}^+ = 0$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_2^+ \times)$ are

$$A_{21}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} - j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_R^-, \qquad a_{21}^+ = +j$$
$$A_{22}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} + j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_L^-, \qquad a_{22}^+ = -j$$
$$A_{23}^+ = -j\frac{k_y^+}{k^+}\hat{y} + j\frac{k_z^+}{k^+}\hat{z}, \qquad a_{23}^+ = 0$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_3^+ \times)$ are

$$A_{31}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} + j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_R^+, \qquad a_{31}^+ = -j$$
$$A_{32}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} - j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_L^+, \qquad a_{32}^+ = j$$
$$A_{33}^+ = j\frac{k_y^+}{k^+}\hat{y} + j\frac{k_z^+}{k^+}\hat{z}, \qquad a_{33}^+ = 0$$

Eigenvalues and the eigenvectors of the operator $(\hat{k}_4^+ \times)$ are

$$A_{41}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} - j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_R^-, \qquad a_{41}^+ = -j$$
$$A_{42}^+ = \frac{1}{\sqrt{2}}\left(\hat{x} + j\frac{k_z^+}{k^+}\hat{y} - j\frac{k_y^+}{k^+}\hat{z}\right) = \bar{M}_L^-, \qquad a_{42}^+ = j$$
$$A_{43}^+ = = j\frac{k_y^+}{k^+}\hat{y} - j\frac{k_z^+}{k^+}\hat{z}, \qquad a_{43}^+ = 0$$

The fractional dual fields to the corresponding operators are

$$\bar{E}^1_{1fd} = -\frac{j}{2}(j)^\alpha E^+ \bar{M}^+_R e^{jk_y^+ y + jk_z^+ z} - \frac{j}{2}(-j)^\alpha F^+ \bar{M}^+_L e^{jk_y^- y + jk_z^- z} \quad (49)$$

$$\bar{E}^2_{1fd} = \frac{1}{2}(j)^\alpha E^- \bar{M}^-_R e^{jk_y^+ y - jk_z^+ z} + \frac{1}{2}(-j)^\alpha F^- \bar{M}^-_L e^{jk_y^- y - jk_z^- z} \quad (50)$$

$$\bar{E}^3_{1fd} = \frac{j}{2}(-j)^\alpha G^+ \bar{M}^+_R e^{-jk_y^+ y - jk_z^+ z} + \frac{j}{2}(j)^\alpha H^+ \bar{M}^+_L e^{-jk_y^- y - jk_z^- z} \quad (51)$$

$$\bar{E}^4_{1fd} = \frac{1}{2}(-j)^\alpha G^- \bar{M}^-_R e^{-jk_y^+ y + jk_z^+ z} + \frac{1}{2}(j)^\alpha H^- \bar{M}^-_L e^{-jk_y^- y + jk_z^- z} \quad (52)$$

$$\eta\bar{H}^1_{1fd} = \frac{1}{2}\left[(j)^\alpha E^+ \bar{M}^+_R e^{jk_y^+ y + jk_z^+ z} - (-j)^\alpha F^+ \bar{M}^+_L e^{jk_y^- y + jk_z^- z}\right] \quad (53)$$

$$\eta\bar{H}^2_{1fd} = \frac{j}{2}\left[(j)^\alpha E^- \bar{M}^-_R e^{jk_y^+ y - jk_z^+ z} - (-j)^\alpha F^- \bar{M}^-_L e^{jk_y^- y - jk_z^- z}\right] \quad (54)$$

$$\eta\bar{H}^3_{1fd} = \frac{1}{2}\left[(-j)^\alpha G^+ \bar{M}^+_R e^{-jk_y^+ y - jk_z^+ z} - (j)^\alpha H^+ \bar{M}^+_L e^{-jk_y^- y - jk_z^- z}\right] \quad (55)$$

$$\eta\bar{H}^4_{1fd} = \frac{j}{2}\left[-(-j)^\alpha G^- \bar{M}^-_R e^{-jk_y^+ y + jk_z^+ z} + (j)^\alpha H^- \bar{M}^-_L e^{-jk_y^- y + jk_z^- z}\right] \quad (56)$$

The total electric and magnetic fields inside the chiral nihility coating can be written as sum of the four fractionalized fields obtained in the chiral nihility:

$$\bar{E}_{1fd} = \bar{E}^1_{1fd} + \bar{E}^2_{1fd} + \bar{E}^3_{1fd} + \bar{E}^4_{1fd} \quad (57)$$

$$\eta\bar{H}_{1fd} = \eta\bar{H}^1_{1fd} + \eta\bar{H}^2_{1fd} + \eta\bar{H}^3_{1fd} + \eta\bar{H}^4_{1fd} \quad (58)$$

Changing the values of α between 0 and 1, the field behavior inside intermediate geometries is obtained. For $\alpha = 0$

$$\bar{E}_{ifd} = \bar{E}_i$$
$$\eta\bar{H}_{ifd} = \eta\bar{H}_i$$

yields the original field solution in the PEC waveguide. For $\alpha = 1$

$$\bar{E}_{ifd} = \eta\bar{H}_i$$
$$\eta\bar{H}_{ifd} = -\bar{E}_i$$

The field for $\alpha = 1$ is dual to the field for $\alpha = 0$. The PEC waveguide for $\alpha = 1$ reduces to the PMC waveguide and for $0 < \alpha < 1$ the fields may be regarded as intermediate between the original and dual to the original solutions, and may be called as fractional dual fields. It shows that for $0 < \alpha < 1$, behavior changes from PEC to PMC and TM mode changes to TE mode. For $\alpha = 0, 1, 2, 3, 4$ it may be shown that fractional dual fields are periodic with respect to fractional parameter α with period 4.

4. CONCLUSION

In this paper, fractional dual fields for a rectangular waveguide internally coated with chiral nihility material are obtained. It is shown that the electric field inside the chiral nihility coating is zero. For $\alpha = 0$, the original fields $(\bar{E}, \eta\bar{H})$ are obtained and for $\alpha = 1$ dual to the original fields $(\eta\bar{H}, -\bar{E})$ are obtained. For $0 < \alpha < 1$ intermediate fields called the fractional fields are obtained where the PEC waveguide changes to PMC and TM mode changes to TE mode. It is noted that original and dual to the original solutions are periodic with respect to fractional parameter α with period 4.

REFERENCES

1. Zouhdi, S., A. Sihvola, and A. P. Vinogradov, *Metamaterials and Plasmonics: Fundamentals, Modelling, Applications*, 2008.
2. Jaggard, D. L., A. R. Mickelson, and C. H. Papas, "On electromagnetic waves in chiral media," *Appl. Phys.*, Vol. 18, 211–216, 1979.
3. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media Norwood*, Artech House, MA, 1994, ISBN 0-89006-684-1.
4. Mackay, T. G. and A. Lakhtakia, "Simultaneously negative and positive phase velocity propagation in an isotropic chiral medium," *Microwave and Optical Technology Letters,* Vol. 49, 1245–1246, 2007.
5. Tretyakov, S., I. Nefedov, A. Sihvola, S. Maslovski, and C. Simovski, "Waves and energy in chiral nihility," *Journal of Electromagnetic Waves and Applications*, Vol. 17, No. 5, 695–706, 2003.
6. Zhang, C. and T. J. Cui, "Negative reflections of electromagnetic waves in strong chiral media," *App. Phys. Lett.*, Vol. 91, 194101, 2007.
7. Cheng, Q., T. J. Cui, and C. Zhang, "Waves in planar waveguide containing chiral nihility metamaterial," *Optics Communications*, Vol. 276, 317–321, 2007.
8. Qiu, C. W., H. Y. Yao, L. W. Li, S. Zouhdi, and T. S. Yeo, "Routes to left-handed materials by magnetoelectric couplings," *Physical Review B*, Vol. 75, 245214, 2007.
9. Qiu, C. W., H. Y. Yao, L. W. Li, S. Zouhdi, and T. S. Yeo, "Backward waves in magnetoelectrically chiral media:

Propagation, impedance, and negative refraction," *Physical Review B*, Vol. 75, 155120, 2007.
10. Dong, J. F. and C. Xu, "Characteristics of guided modes in planar chiral nihility meta-material waveguides," *Progress In Electromagnetics Research B*, Vol. 14, 107–126, 2009.
11. Naqvi, Q. A., "Fractional dual solutions to the Maxwell equations in chiral nihility medium," *Optics Communications*, Vol. 282, 2016–2018, 2009.
12. Naqvi, Q. A., "Fractional dual interface in chiral nihility medium," *Progress In Electromagnetics Research Letters*, Vol. 8, 135–142, 2009.
13. Dong, J. F., "Surface wave modes in chiral negative refraction grounded slab waveguides," *Progress In Electromagnetics Research,* Vol. 95, 153–166, 2009.
14. Naqvi, A., A. Hussain, and Q. A. Naqvi, "Waves in fractional dual planar waveguides containing chiral nihility metamaterial," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 11–12, 1575–1586, 2010.
15. Naqvi, A., S. Ahmed, and Q. A. Naqvi, "Perfect electromagnetic conductor and fractional dual interface placed in a chiral nihility medium," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 14–15, 1991–1999, 2010.
16. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from a chiral-coated nihility cylinder," *Progress In Electromagnetics Research Letters*, Vol. 18, 41–50, 2010.
17. Oldham, K. B. and J. Spanier, *The Fractional Calculus*, Academic Press, New York, 1974.
18. Engheta, N., "On fractional paradigm and intermediate zones in electromagnetism: I. Planar observation," *Microwave and Optical Technology Letters,* Vol. 22, No. 4, 236–241, Aug. 20, 1999.
19. Engheta, N., "On fractional paradigm and intermediate zones in electromagnetism: II. Cylindrical and spherical observations," *Microwave and Optical Technology Letters,* Vol. 23, No. 2, 100–103, Oct. 20, 1999.
20. Engheta, N., "Fractional curl operator in electromagnetics," *Microwave and Optical Technology Letters,* Vol. 17, 86–91, 1998.
21. Naqvi, Q. A. and M. Abbas, "Complex and higher order fractional curl operator in electromagnetics," *Optics Communications*, Vol. 241, 349–355, 2004.
22. Hussain, A., S. Ishfaq, and Q. A. Naqvi, "Fractional curl oerator and fractional waveguides," *Progress in Electromagnetics*

Research, Vol. 63, 319–355, 2006.

23. Hussain, A., Q. A. Naqvi, and M. Abbas, "Fractional duality and perfect electromagnetic conductor (PEMC)," *Progress In Electromagnetics Research,* Vol. 71, 85–94, 2007.
24. Hussain, A., M. Faryad, and Q. A. Naqvi, "Fractional curl oerator and fractional chiro-waveguides," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1119–1129, 2007.
25. Hussain, A. and Q. A. Naqvi, "Perfect electromagnetic conductor (PEMC) and fractional waveguide," *Progress In Electromagnetics Research,* Vol. 73, 61–69, 2007.
26. Hussain, A. and Q. A. Naqvi, "Fractional curl operator in chiral medium and fractional non-symmetric transmission line," *Progress In Electromagnetics Research,* Vol. 59, 199–213, 2006.
27. Faryad, M. and Q. A. Naqvi, "Fractional rectangular waveguide," *Progress In Electromagnetics Research*, Vol. 75, 383–396, 2007.
28. Naqvi, Q. A., "Planar slab of chiral nihility metamaterial backed by fractional DUAL/PEMC interface," *Progress In Electromagnetics Research,* Vol. 85, 381–391, 2008.
29. Pozar, D. M., *Microwave Engineering*, 2nd edition, 170176, John Wiley and Sons, 1998.
30. Lakhtakia, A., *Beltrami Fields in Chiral Media*, Contemporary Chemical Physics, World Scientific Series, 1994.
31. Harrington, R. F., *Time-harmonic Electromagnetic Fields*, McGraw-Hill Inc., New York, 1961.
32. Qiu, C. W., N. Burokur, S. Zouhdi, and L. W. Li, "Chiral nihility effects on energy flow in chiral materials," *J. Opt. Soc. Am. A*, Vol. 25, No. 1, 55–63, 2008.