

WIDEBAND DISPERSION ANALYSIS OF WAVEGUIDE GEOMETRIES USING FINITE SAMPLED DATA

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Abstract—Wideband analysis of frequency dispersive geometries is a challenge in inverse scattering problems. Waveguide duct is an important case in aerial targets with dominant returns. Its dispersive behavior affects the range profile analysis due to occurrence of unwanted range extension. A new high frequency analysis using model based parameter estimation (MBPE) approach is presented. A group delay criteria derived from the nonlinear scattering phase response represents the duct length. Wideband sparse measured frequency domain samples of various waveguides are used as inputs to the model. Comparison is made with joint time-frequency analysis (JTFA) and inverse fast Fourier transform (IFFT) results.

1. INTRODUCTION

FEM and FDTD are common approaches for forward scattering problems. However, they are computationally inefficient for large targets. Modeling cavities and ducts is a challenge in inverse problems. An effective approach that uses a large number of samples to study such frequency dispersive targets is the joint time-frequency technique. Its time domain resolution is limited by the width of the frequency window and vice versa. Another approach is using a scattering center model and super-resolution techniques. Zhang and Ge used FDTD to analyze dispersive objects [1]. Kusiek and Mazur implemented hybrid finite-difference mode-matching method for cylindrical objects [2]. A basis function for electrically large targets is reported by Nie et al. [3]. Raynal et al. used adaptive methods for feature extraction [4]. A maximum likelihood algorithm with a

GTD-based frequency dependence is also suggested by Potter et al. [5]. Here, we employ MBPE [6] as a flexible approach requiring only sparse sampled data for dispersion analysis.

2. THE NONLINEAR PHASE AND GROUP DELAY

The high frequency backscattered far field due to point scatterers is

$$E_S(k, r) = \sum_{m=1}^{\infty} B_m(k, r) e^{-j2kr_m} \quad (1)$$

where k is the free space wave number, m is the scattering center number and r is the line of sight (LOS) propagation path length or simply the radar range. r_m is the distance from the origin (phase reference) to the m th scatterer and B_m is the weighting coefficient. In waveguide geometries, the phase velocity is frequency dependent and can be used to model scattering centers [7]. In open-ended waveguides illuminated by a high frequency plane wave, the aperture field at each assumed sub-aperture is a discrete radial beam launched into the waveguide [8]. Here, such nonpoint dispersive scattering centers (DSCs) are considered as an ensemble of M equivalent point scatterers with an identical frequency dependent correction multiplicand $\xi_m(k)$ that is called dispersion factor (DF). Various geometries have different DFs. Thus:

$$B_m(k, r) = a_m(r) \xi_m(k) = a_m(r) \rho_m(k) e^{-j\phi_m(k)} \quad (2)$$

and the enhanced dispersive scattering center (EDSC) model is:

$$E_S(k) \approx \sum_{m=1}^M a_m \rho_m(k) e^{-j(2kr_m + \phi_m(k))} \quad (3)$$

The complex weighting coefficients consist of frequency dependent and frequency independent parts $\rho_m(k)$ and a_m respectively. $\phi_m(k)$ is the deviation from the linear phase encountered in dispersive media. Rewriting (3) in terms of the frequency dependent total nonlinear phase, $\Psi_m(k)$ and the total complex scattering amplitude, $A_m(k)$ yields:

$$\begin{aligned} E_S(k) &\approx \sum_{m=1}^M A_m(k) e^{-j\Psi_m(k)}; \\ \Psi_m(k) &= 2kr_m + \phi_m(k), \quad A_m(k) = a_m \rho_m(k) \end{aligned} \quad (4)$$

The dispersion is generally due to nonlinear frequency dependent amplitude and phase response. Here, the main idea is to characterize

the dispersion by an appropriate group delay quantity. For the m th equivalent point scattering center, a corresponding nonlinear group delay is defined as the derivative of the phase:

$$\tau_{dm}^{nl}(k) = -\frac{1}{c} (\partial \Psi_m(k) / \partial k) = \tau_{dm}^c + \tau_{dm}^{dev}(k) \quad (\text{sec}) \quad (5)$$

where $\tau_{dm}^c = -2r_m/c$ is a frequency independent term due to the radar range and c is the free space propagation velocity. The second term $\tau_{dm}^{dev}(k)$ is the deviation group delay due to the phase nonlinearity that contains dispersion information.

3. THE PARAMETRIC SCATTERING CENTER MODEL

Some direct scattering solutions for waveguide type geometries are addressed in [9, 10]. In our inverse scattering solution based on MBPE approach, a parametric model is used with less computational effort. A complex target consisting of at least one open-ended waveguide duct DSC is shown in Figure 1. The TE_x polarized illumination is from the open side of DSC at an aspect angle that permits penetration into the waveguide. For polarimetric representation, stokes vector may also be used [11]. The excitation at the aperture at a fixed aspect angle is

$$\bar{E}_i(\bar{r}) = E_0^+ e^{-j\bar{k} \cdot \bar{r}}, \quad \bar{k} = k\hat{k} \quad (6)$$

The radar range in non-dispersive medium (free space) and dispersive medium (waveguide) are denoted by r and r' respectively. Concentrating on the waveguide as an isolate DSC, the high frequency scattered field is composed of three terms [8]:

$$\bar{E}_S^{DSC} = \bar{E}_S^{wgd} + \bar{E}_S^{rim} + \bar{E}_S^{ext} \quad (7)$$

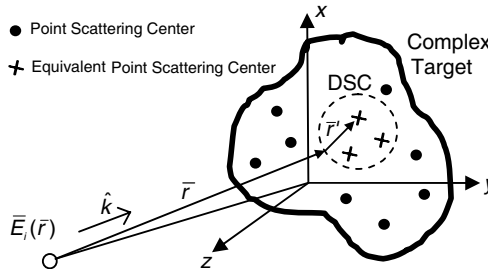


Figure 1. An arbitrary complex target consisting of at least one open ended waveguide DSC.

The backscattered field due to the penetrating wave into the waveguide, $\bar{E}_S^{wgd}$ is inherently highly dispersive and is the dominant contributor to the total field. The model for rim edge diffraction field, $\bar{E}_S^{rim}$ and waveguide surface current induced field, $\bar{E}_S^{ext}$ as non-dispersive scattering centers are given by various researchers [5, 12]. The backscattered field due to the round trip propagation inside the waveguide at the aperture is

$$\hat{E}_S^{wgd}(k, r') = \Gamma(r') E_0^+ e^{-2\gamma(k)r'} = \Gamma(r') E_0^+ e^{-2\alpha(k)r'} e^{-j2\beta(k)r'} \quad (8)$$

where $\Gamma(r')$ is the waveguide reflection coefficient and $\gamma(k)$ is the complex frequency dependent propagation constant. Assuming our man-made DSC has a semi-canonical geometry, the attenuation constant $\alpha(k)$ and the phase constants $\beta(k)$ could be approximated by

$$\alpha(k) \approx ((k^2 + 2k_c^2) q') / (k \sqrt{k^2 - k_c^2}), \quad \beta(k) \approx \sqrt{k^2 - k_c^2}; \quad k > k_c \quad (9)$$

where k_c is the cutoff frequency and q' is a waveguide structural constant. At frequencies below cutoff $k \leq k_c$, the backscattering is merely from the edges and the outer surface. The parametric form of (8) becomes:

$$\begin{aligned} \hat{E}_S^{wgd}(k, r') = & \hat{A}(r') \exp\left(-((k^2 + 2k_c^2) q) / k \sqrt{k^2 - k_c^2}\right) \\ & \exp\left(-j2\sqrt{k^2 - k_c^2} \cdot d\right) \end{aligned} \quad (10)$$

where $k > k_c$, $q = -2q'r'$ and $\hat{A}(r') = \Gamma(r') E_0^+$ is the weighting coefficient. d is called the “dispersion length” and is dependent on the waveguide physical length. The weighting coefficient and phase of (10) provide the required parametric form of DF. Inserting DF in the total scattered field of (3) results in the enhanced dispersive scattering center (EDSC) model:

$$\begin{aligned} E_S^{DSC}(k) \approx & \sum_{m=1}^M A_m \exp\left(-\frac{(k^2 + 2k_{cm}^2) q_m}{k \sqrt{k^2 - k_{cm}^2}}\right) \\ & \exp\left(-j\left(2kr_m + 2\sqrt{k^2 - k_{cm}^2} \cdot d_m\right)\right) \end{aligned} \quad (11)$$

The unknown parameters are: complex amplitude A_m , structural parameter q_m , down range r_m , the dispersion length d_m , and the cutoff frequency k_{cm} . Please note that r_m is related to free space non-dispersive range r and d_m is related to the waveguide dispersive range r' . The unknown parameters are found through an optimization

routine with the following objective function:

$$S(A, q, r, d, k_c) = \sum_{n=1}^L \left| X_n - \sum_{m=1}^M A_m \exp \left(-\frac{(k_n^2 + 2k_{cm}^2)q_m}{k_n \sqrt{k_n^2 - k_{cm}^2}} \right) \exp \left(-j \left(2k_n r_m + 2\sqrt{k_n^2 - k_{cm}^2} \cdot d_m \right) \right) \right|^2 \quad (12)$$

In the above, A , q , r , d , and k_c are vectors of length M and X_n s are the measured coherent stepped frequency backscattered samples with number L . Due to existence of many local minima, initialization of the parameters is very critical and super-resolution methods are used for this purpose. The algorithm steps are as follows:

- 1- Selection of order M is highly application dependent. The factors to be considered include complexity of the dispersive geometry, sampling conditions (bandwidth, signal-to-noise ratio, etc.) and range resolution [13]. For a model of order M over X_n samples in frequency domain, Prony method is used for initialization of $\{A_m\}_{m=1}^M$ and range $\{r_m\}_{m=1}^M$. Please note that M can't exceed $L/2$.
- 2- Considering the physical properties of the waveguide DSC, the initial guess for the remaining parameters $\{q_m, d_m, k_{cm}\}_{m=1}^M$ is selected from the rational range of values. The initial guess for the dispersion length parameter $\{d_m\}_{m=1}^M$ must be in order of the waveguide physical length. Similarly, the initial guess for the vector $\{q_m\}_{m=1}^M$ is selected considering the characteristics of the propagation medium. They are intrinsic impedance, aperture dimensions and walls surface resistivity. The initial waveguide cutoff frequency $\{k_{cm}\}_{m=1}^M$ is chosen to be the start of the frequency range.
- 3- The general-purpose genetic algorithm toolbox of MATLAB is applied to the objective function (12) to extract the parameters here, but any other optimization techniques could also be used [14].

Usually, a point scattering center is depicted as a vertical strip on the time axis of the JTFA spectrogram meaning a constant propagation time delay at all frequencies. A DSC with structural dispersive mechanism appears as a negative slope slanted curve [15]. A spectrogram with satisfactory spatial resolution often requires numerous data samples and a weak dispersion is hardly identified. According to (5) and (11), for waveguide type geometries, the parametric form of the nonlinear group delay is:

$$\tau_{dm}^{nl}(k) \approx -\frac{2}{c} \left(r_m + k d_m / \sqrt{k^2 - k_{cm}^2} \right) \quad (\text{sec}) \quad (13)$$

Please note the dynamic frequency dependent term that is added to r_m . This dynamic behavior is utilized to quantify the dispersion using the extracted parametric nonlinear phase. Curve fitting is applied to the unwrapped nonlinear phase data to create a deterministic function that describes the aforementioned dynamic behavior. Differentiating this function with respect to the frequency yields $\tau_{dm}^{nl}(k)$. The parametric nonlinear phase is useful in complex range profile analysis [16].

4. DISPERSION ANALYSIS RESULTS

The backscattered measured data of some metallic waveguides were used for analysis. A 57 cm long cylindrical duct with irregular semi-circular aperture is considered as a non-canonical target (Figure 2(a)).

The semi-circular aperture dimension is about 15 cm. The analysis is performed from 8 to 12 GHz with $L = 100$ and model order of $M = 50$. A linearly polarized plane wave at the aspect angle of $\theta \approx 15^\circ$ illuminates the target. Figure 2(b) compares the calculated range profile with IFFT results with good agreement. Figures 2(c)

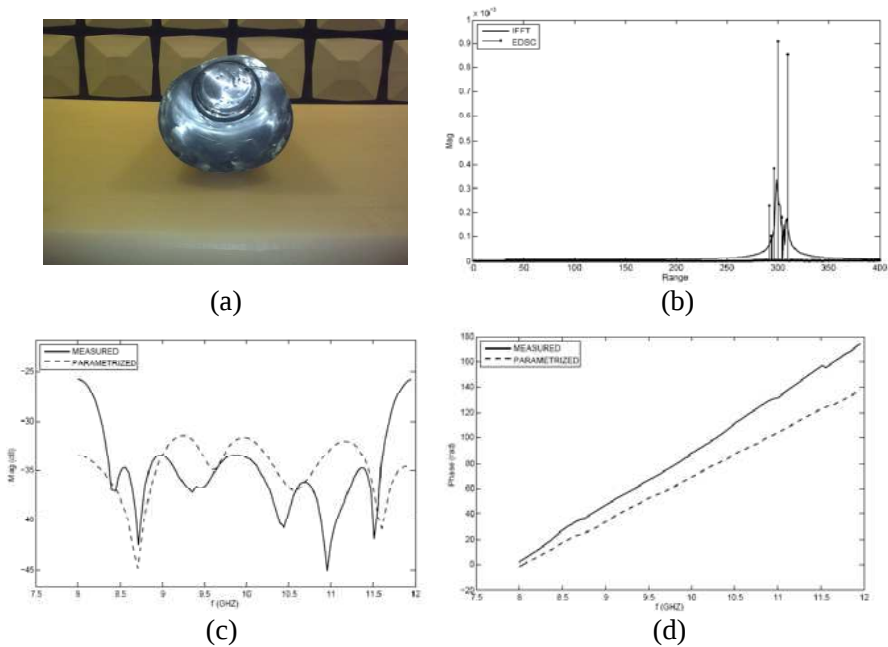
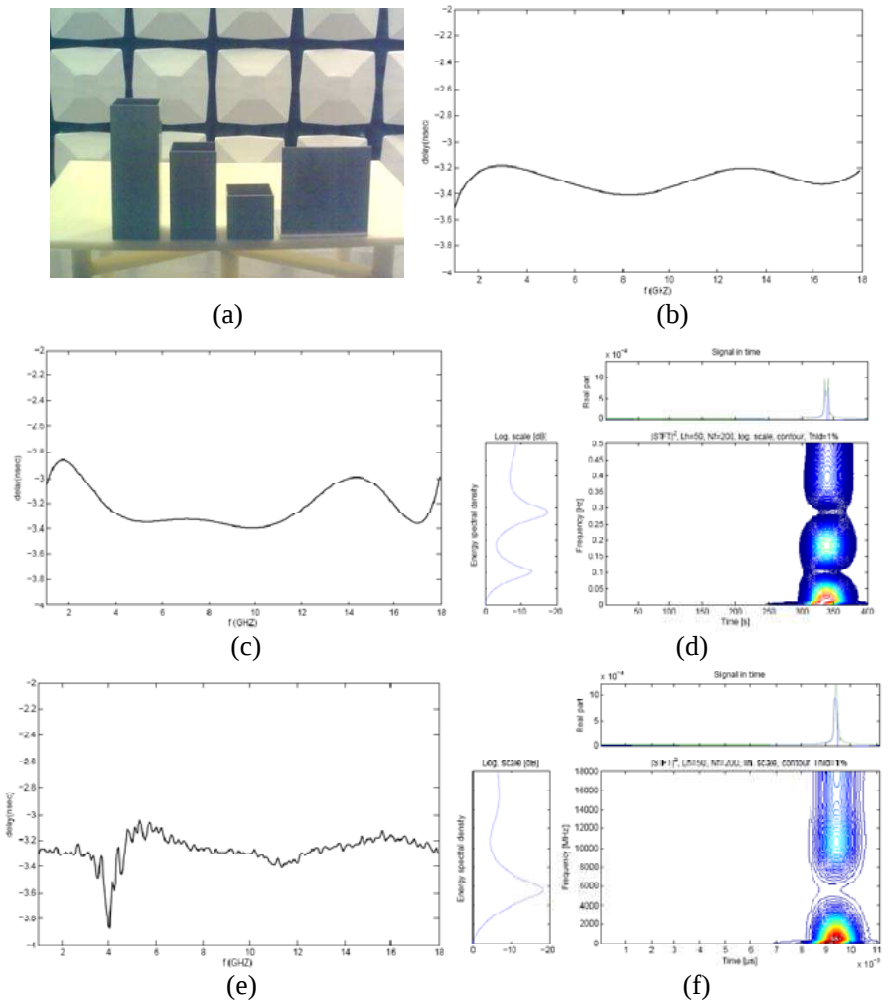


Figure 2. (a) The non-canonical target. (b) Range profile. (c) (d) Measured and EDSC parameterized scattering responses.

and 2(d) compare the magnitude and phase of the original measured and model predicted data respectively with a RMS error of 0.31%.

$$RMS = \sqrt{\left(\sum_{n=1}^L \left| E_S^{original}(n) - E_S^{predicted}(n) \right|^2 \right) / L} \quad (14)$$

The canonical targets of Figure 3(a) are considered for dispersion analysis from 1 to 18 GHz with only $L = 400$ and model order of $M = 200$. The 30 cm square flat plate is the non-dispersive reference one. The group delay plot of the EDSC analysis for $\theta = 0^\circ$ is shown in Figure 3(b). As expected, a negligible group delay ripple of about



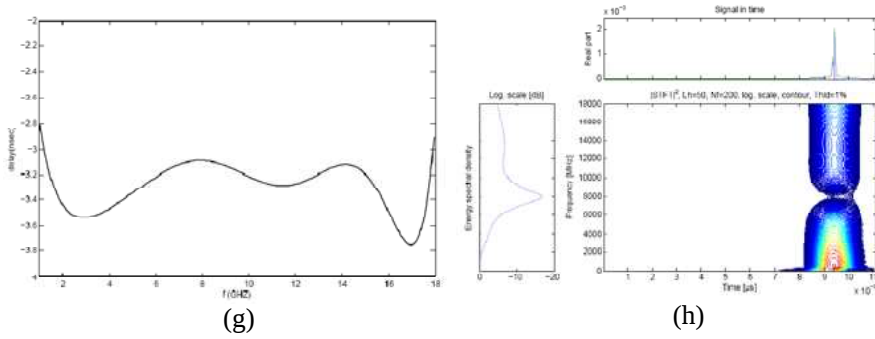


Figure 3. (a) The canonical targets; from right to left: Flat plate, waveguide 1, waveguide 2, waveguide 3. (b) Flat plate. (c), (d) Waveguide 1. (e) (f) Waveguide 2 and (g) (h) Waveguide 3.

0.3 nsec is observed that means no dispersion. In other words, all frequency components of a wide band radar pulse experience the same propagation time delay. It is expected that the group delay ripple approaches to a minimum possible level if the dimensions tend to a maximum possible size.

The open-ended waveguide1 is a 15 cm square aperture of 15 cm length. The results for $\theta = 15^\circ$ are shown in Figures 3(c) and 3(d). A minute group delay ripple of about 0.55 nsec shows a slight dispersion or range extension compared to the flat plate. However, a recognizable slanted curve is not observed in the corresponding JTFA spectrogram. The analysis is repeated for waveguides 2 and 3 with length of 30 cm and 45 cm respectively. The EDSC group delays (Figures 3(e) and 3(g)) demonstrate the ripple of about 0.75 nsec and 0.95 nsec for waveguides 2 and 3 respectively. Hence, the range extension is obviously increased. However, as observed in the corresponding JTFA spectrograms of Figures 3(f) and 3(h), no sensible slanted curve is detectable for the occurred range extension again. Comparison of the dispersive group delays of Figures 3(c), 3(e) and 3(g) with non-dispersive group delay of Figure 3(b) show a direct relation between the level of group delay ripple and the waveguide length at fixed aperture dimensions. In the spectrograms of Figures 3(d), 3(f) and 3(h) the brighter colors represent higher return magnitudes and the darker colors represent lower magnitudes. The constant measurement radar range shows itself as a level of about -3.2 to -3.3 nsec in group delay graphs too.

5. CONCLUSION

A new scattering center model, EDSC, based on MBPE approach is proposed for dispersion analysis of waveguide type geometries. A population-based algorithm is used for estimating the model unknown parameters. The total scattering nonlinear phase is considered as a combination of the linear free space propagation phase and an additional nonlinear frequency dependent term. The dynamic behavior of the latter represents the structural dispersion and leads to an extractable nonlinear group delay quantity. The group delay ripple is related to the waveguide length for a fixed aperture size. Hence, it is an effective quantity to represent dispersion and the radar range extension. Dispersion analysis using JTFA approach requires a huge amount of sampled data without providing scattering phase information. However, the EDSC can coherently analyze the dispersive targets using sparse data samples. The model is evaluated by various measured canonical and non-canonical target responses. The parametric EDSC model is flexible enough to be used in complex range profile and/or radar imaging studies that require phase information. In the latter, the range extension appears as the smudge effect.

REFERENCES

1. Zhang, Y.-Q. and D.-B. Ge, "A unified FDTD approach for electromagnetic analysis of dispersive objects," *Progress In Electromagnetics Research*, Vol. 96, 155–172, 2009.
2. Kusiek, A. and J. Mazur, "Analysis of scattering from arbitrary configuration of cylindrical objects using hybrid finite-difference mode-matchnig method," *Progress In Electromagnetics Research*, Vol. 97, 105–127, 2009.
3. Nie, Z.-P., S. Yan, S. He, and J. Hu, "On the basis functions with traveling wave phase factor for efficient analysis of scattering from electrically large targets," *Progress In Electromagnetics Research*, Vol. 85, 83–114, 2008.
4. Raynal, A. M., J. T. Moore, and H. Ling, "Broadband scattering data interpolation based on a relaxed adaptive feature extraction algorithm," *Progress In Electromagnetics Research*, Vol. 64, 99–116, 2006.
5. Potter, L. C., D.-M. Chiang, R. Carriere, and M. J. Gerry, "A GTD-based parametric model for radar scattering," *IEEE Trans. Antennas Propag.*, Vol. 43, 1058–1067, Oct. 1995.
6. Miller, E. K., "Model-based parameter estimation in electro-

- magnetics: Part I. Background and theoretical development," *IEEE Antennas and Propagation Magazine*, Vol. 40, No. 1, 42–52, Feb. 1998.
7. Censor, D., "Broadband spatiotemporal differential-operator representations for velocity-dependent scattering," *Progress In Electromagnetics Research*, Vol. 58, 51–70, 2006.
 8. Burkholder, R. J. and P. H. Pathak, "Analysis of EM penetration into and scattering by electrically large open waveguide cavities using Gaussian beam shooting," *Proc. IEEE*, Vol. 79, No. 10, Oct. 1991.
 9. Liao, S. and R. J. Vernon, "On the image approximation for electromagnetic wave propagation and PEC scattering in cylindrical harmonics," *Progress In Electromagnetics Research*, Vol. 66, 65–88, 2006.
 10. Wang, S., X. Guan, D.-W. Wang, X. Ma, and Y. Su, "Fast calculation of wide-band responses of complex radar targets," *Progress In Electromagnetics Research*, Vol. 68, 185–196, 2007.
 11. Sabry, R. and P. W. Vachon, "Advanced polarimetric synthetic aperture radar (SAR) and electro-optical (EO) data fusion through unified coherent formulation of the scattered EM field," *Progress In Electromagnetics Research*, Vol. 84, 189–203, 2008.
 12. Tong, M.-S. and W. C. Chew, "Nyström method with edge condition for electromagnetic scattering by 2D open structures," *Progress In Electromagnetics Research*, Vol. 62, 49–68, 2006.
 13. Fuller, D. F., A. J. Terzuoli, P. J. Collins, and R. Williams, "Approach to object classification using dispersive scattering centers," *IEE Proc. Radar Sonar Navig.*, Vol. 151, No. 2, Apr. 2004.
 14. Semnani, A. and M. Kamyab, "An enhanced method for inverse scattering problems using fourier series expansion in conjunction with FDTD and PSO," *Progress In Electromagnetics Research*, Vol. 76, 45–64, 2007.
 15. Chen, V. C. and H. Ling, *Time-frequency Transforms for Radar Imaging and Signal Analysis*, Artech House, 2002.
 16. Rihaczek, A. W. and S. J. Hershkowitz, *Theory and Practice of Radar Target Identification*, Artech House, 2000.