

MODELLING RESONANCE FREQUENCIES OF A MULTI-TURN SPIRAL FOR METAMATERIAL APPLICATIONS

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Abstract—The planar metal particles, consisting of a multi-turn spirals, are studied with the aim of using them to realize high impedance surfaces or as an elementary cell to create an artificial material. These spirals present a resonant behaviour in a certain frequency band. To obtain miniature devices, a compromise between the surface and the efficiency of the resonance must be found. The compactness of the particles can be increased by using s spirals. However, the accuracy on resonant frequency of existing models is not sufficient for our applications. We present a simple analytical model that determines the resonant frequency from the geometric dimensions of the approximated model. This model is verified by electromagnetic simulations and by measurements.

1. INTRODUCTION

Artificial Metamaterials are usually realized with dielectrics and metal particles of different shapes: SRR, Omega, Jerusalem Cross, Spiral, ...). Periodic arrangement of these particles provides unusual properties that have been widely studied in literature. First studies have focused on free space applications [1,2] but planar circuits applications have also emerged rapidly [3]. Particles like SRR have also been used as resonators coupled with a propagation line [4] to perform

Received 12 January 2011, Accepted 17 February 2011, Scheduled 21 February 2011

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filtering functions. The disadvantage of this device comes from the size of particles. For applications in the 1–5 GHz band the SRR outer diameter must be varied from 1 to 5 cm. Thus the size of the device can not be greatly reduced. Our idea was to replace the SRR by multi-turn spiral particles to reduce the dimensions and to propose an analytical model that can accurately predict the resonant frequency of the spiral particle. The electrical modelling of spirals is difficult because the physical phenomenon are numerous and depend on many geometrical parameters as well as frequency bands. In [5], spirals with several turns are presented but the electrical size of the spiral is very small compared to the wavelength ($\lambda/80$, $\lambda/650$) and the studies are limited to radio frequencies. In [6], spirals of comparable size to the ones we want to use are investigated but simulations are only made using a commercial software. In [7], Baena has proposed a successful analytical modelling for spirals of 2 and 3 turns. Unfortunately, his model diverges quickly if the number of turns increases. In the present work, we propose an improved model that correctly predicts the resonant frequency. Modifications are made in the theory proposed by Baena and resonant frequency can be correctly obtained even for many turns. Results have been successfully compared to numerical simulations and to measurements.

2. MULTI-TURN SPIRAL

The multi-turn spiral that we propose consists of a metal strip wound in the form of spirals of different geometric shapes. The shape of each turn can be circular (Fig. 1(a)) or rectangular (Fig. 1(b)). Their manufacture can be made by a simple etching of a conductor deposited on a dielectric.

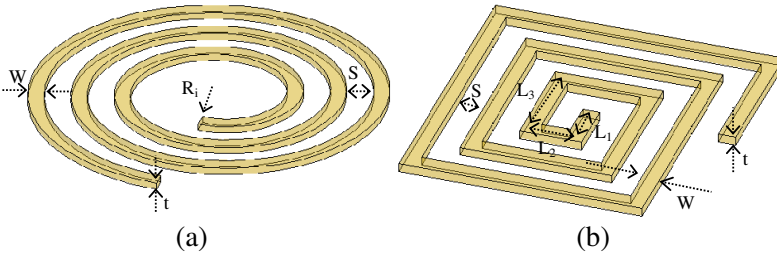


Figure 1. (a) Circular spiral structure. (b) Rectangular spiral structure. w : width of the line; S : space between the towers; t : thickness of the metallic strip. N : Number of turns. Case of circular spiral: R inner radius of 1st turn. Case of rectangular spiral: L_1 , L_2 and L_3 : length used for the 1st turn.

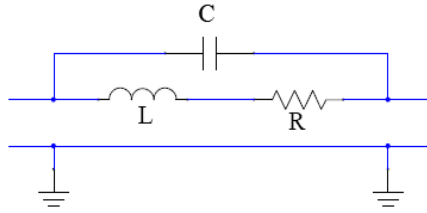


Figure 2. Equivalent electric circuit.

Our objective is to calculate the resonant frequency, which can be determined from the equivalent circuit. Each winding of the spiral acts as a closed loop. An induced current runs through this loop when a magnetic flux passes through it. This gives an inductive effect (L). Thus, these structures are similar to flat coils connected in series between the input and the output of the equivalent circuit. All conductors have a certain resistivity; hence, an internal resistance (R) of the circuit connected in series with the inductor must be added. Another important effect is the electrical coupling between the turns of the spiral. This gives rise to a capacitive effect (C) connected in parallel with the inductor (L) and with its internal resistance (R).

The equivalent circuit obtained is very classical [8]. The real challenge is now to propose a model (Fig. 2) for these elements which allows us to calculate an accurate resonant frequency value.

3. MODELLING OF THE INDUCTIVE AND RESISTIVE EFFECTS OF A MULTI-TURN SPIRAL

In microwave range (HF), the current is confined within the area of skin effect (Kelvin effect). Our components are made of a metal strip of rectangular section width (W), thickness (t), length (l) and resistivity (ρ). To determine the resistive effect of a multi turn spiral, we can make an approximation that considers the spiral as a straight metal strip [9]. Hence the resistance (R) modelled by the following equation were t_{eff} is the effective thickness and δ is the skin depth.

$$R = \rho \frac{l}{W \cdot t_{eff}} \quad (1)$$

$$\text{With } t_{eff} = \delta (1 - e^{-t/\delta}) \text{ and } \delta = \sqrt{\frac{\rho}{\pi \cdot \mu \cdot f}}.$$

The inductive effect can be determined using the inductive model proposed by Grover [9, Chp. 3 & 8].

4. MODELLING THE CAPACITIVE EFFECT IN A MULTI-TURN SPIRAL

The modelling of the capacitive effect of a spiral with two and three turns has been studied by Baena [7]. He has determined the capacitance (C_0) between two adjacent turns.

$$C_0 = 2\pi r_0 \cdot C_{pul} \quad (2)$$

where C_{pul} is the capacitance per unit length (determined with a conformal mapping technique [10]) and r_0 is the spiral average radius.

According to [7], when the spiral has three turns, the coupled capacitance is equal to $2C_0$. We propose to extend this assumption to N -turns spiral with:

$$C_{tot} = (N - 1) \cdot C_0 \quad (3)$$

This approximation is effective for a small number of turns, but diverges rapidly when the number of turns increases as it can be seen in Table 1.

Table 1. Extension of the Baena's analytical model for N turns comparison with simulations in the case of a circular spiral. With $W = 450 \mu\text{m}$, $S = 150 \mu\text{m}$ and $R_i = 600 \mu\text{m}$.

N	2	3	4	5
L (nH)	6566	6672	7008	7409
$C_{tot} \times 10^{-6}$ (pF)	25.34	61.36	108.05	165.42
Analytical frequency (GHz)	12.33	7.86	5.78	4.54
Simulated frequency (GHz)	11.70	6.99	4.90	3.70
$\Delta f\%$	-5.4	-12.3	-18.0	-22.8

This is mainly due to the rough approximation made on the current distribution in the spiral. To improve the calculation of the capacitance, we have performed electromagnetic simulations based on the finite element method. We have observed the induced current in the frequency band of the resonance of the spiral. The maximum current is located close to the center of the spiral. As a consequence, the current mainly decreases from the center to the external turns (Fig. 3).

Taking into consideration these observations in the simulation in 3D finite element software and to keep a simple model (without an exact calculation of the current distribution), we consider that the capacitance must be calculated at the external turn. So we choose to calculate the capacitive effect on the last half turn using the formula:

$$C'_0 = l' \cdot C_{pul} \quad (4)$$

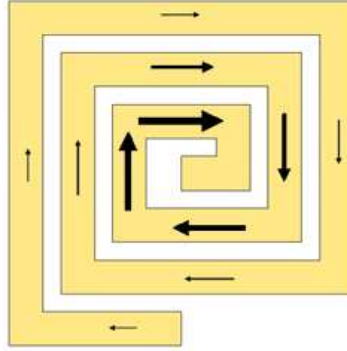


Figure 3. Schema of the current density distribution (full width of the strip).

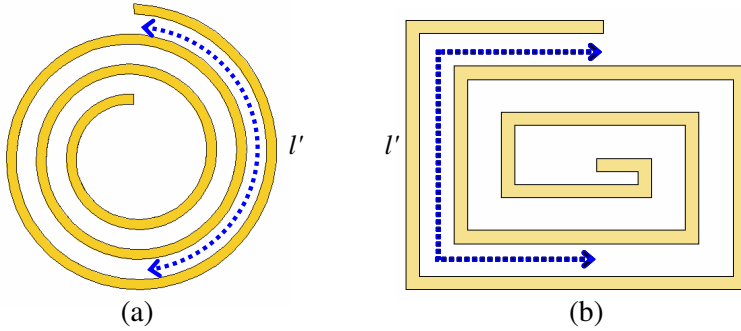


Figure 4. Average physical length (l') of the last two half-turns. (a) Case of a circular spiral. (b) Case of a rectangular spiral.

Were l' is the average length described in (Figs. 4(a), (b)).

To obtain the total capacitor value, we must add the capacitance of the two half spirals and take into account the number of turns. This leads to the approximate formula:

$$C'_{tot} = 2(N - 1)C'_0 \quad (5)$$

5. PARAMETRIC STUDY

We present a parametric study to validate our analytical model of the capacitive effect of multi-turn spiral. The equivalent circuit of the

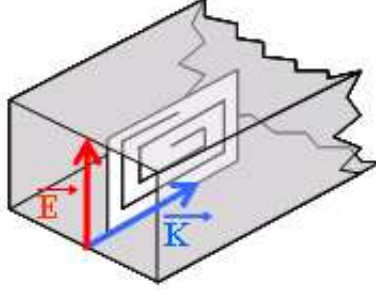


Figure 5. The spiral position in the waveguide.

Table 2. Comparison between the analytical model and simulation based on the number of turns N in the case of a circular spiral.

N	2	3	4	5
L (nH)	6566	6672	7008	7409
$C'_{tot} \times 10^{-6}$ (pF)	28.01	77.37	148.07	240.12
Analytical frequency (GHz)	11.73	7.00	4.94	3.77
Simulated frequency (GHz)	11.70	6.99	4.90	3.70
$\Delta f\%$	0.2	0.1	0.8	1.9

spiral (Fig. 2) has a resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (6)$$

Numerical simulations are performed using a finite element method. We tried to put the spiral in an empty suitable rectangular waveguide. The spiral is placed in the centre of the waveguide (Fig. 5). The electromagnetic coupling between the propagated wave and the spiral is strong, and is at the same time isolated from the external environment. We took into account the substrate (Epoxy) in the simulations. These simulations allow us to clearly observe the resonance frequency (Eq. (7)) of a multi-turn spiral from the transmission scattering parameter (S_{21}). We present some simulation results described by Fig. 6 and Tables 2 and 3.

5.1. Variation of Number of Turns

The spiral structure is circular, with an internal radius $R_i = 600 \mu\text{m}$ (Fig. 1). The width of strip is $W = 450 \mu\text{m}$. The distance between

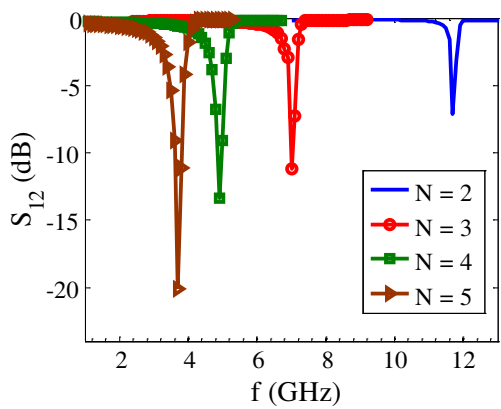


Figure 6. Insertion loss (S_{21}) of circular spiral structure for different numbers of turn $N = 2, 3, 4$ and 5 (turns).

Table 3. Comparison between the analytical model and simulation based on the width of the strip (w), in the case of a circular spiral.

w (μm)	100	200	300	400
L (nH)	5304	5811	6131	6423
$C'_{tot} \times 10^{-6}$ (pF)	15.42	19.12	22.71	26.25
Analytical frequency (GHz)	17.59	15.09	13.48	12.25
Simulated frequency (GHz)	17.50	15.00	13.40	12.20
$\Delta f\%$	0.5	0.6	0.6	0.4

turns is $S = 150 \mu\text{m}$ and the number of turns increased from 2 to 5. The obtained results are presented in (Table 2) and (Fig. 6). The results show the accuracy of our model that takes more precisely the capacitive effect in a multi-turn spiral. Note that the relative offset between the analytical resonances frequencies versus the simulated ones (Δf) is less than 2%.

5.2. Variation of the Width of the Metal Strip and Its Dimensions

The spiral shape remains the same, but the width of the strip varies. The number of turns is fixed to two. Again our analytical model is in complete agreement with the numerical simulation.

5.3. Other Parametric Variations

We have varied other parameters R and (L_1, L_2, L_3) of circular and rectangular spiral structures respectively. We have noticed that the relative difference with the numerical simulation is still less than 3%. Hence, the proposed model is a good improvement of Baena's model which can give relative errors of 22% for the same spirals.

6. EXPERIMENTAL VALIDATION OF THE MODEL

To validate our model, we have done some measurements. The experiment is the same as in the numerical simulation (Fig. 7). We present experimental results for particles with geometric dimensions of the same order as those presented in the parametric study (Figs. 8 and 9). These particles consist of an Epoxy-FR4 substrate ($\varepsilon_r = 4.4$) with a metallic layer of copper ($\rho = 17 \cdot 10^{-9} \Omega \cdot \text{m}$, thickness = $35 \mu\text{m}$). The spirals are placed into a rectangular waveguide. In order to increase the response, the measurement is done on an array of spirals. There, we measure an array of structures (2×3 spiral) in order to increase the signal attenuation.

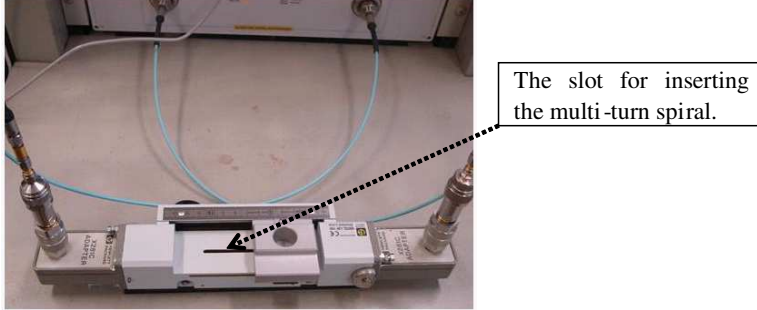


Figure 7. The array of multi-turn spirals under test.

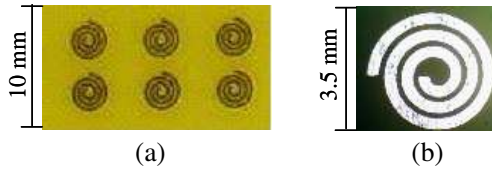


Figure 8. (a) The array of multi-turn circular structure spiral measurement. (b) The dimensions of one spiral: $N = 3$, $W = 355 \mu\text{m}$, $S = 100 \mu\text{m}$, $Ri = 150 \mu\text{m}$.

6.1. Circular Spiral

The test is realized with 3 turns circular spirals. The used dimensions are: $W = 355\text{ }\mu\text{m}$, $S = 100\text{ }\mu\text{m}$, $R_i = 150\text{ }\mu\text{m}$. The transmission coefficient is presented on Fig. 9. Where the spiral is in resonance, the power is absorbed, and as a result, the transmission is very low at this frequency.

We can extract the measured resonance frequency at $f_{\text{measurement}} = 9.73\text{ GHz}$, according to figure Fig. 9. The distance between the spirals in the array of spirals is sufficient enough to avoid coupling between them. The global effect is thus an increase of the resonant peak, without any frequency shift.

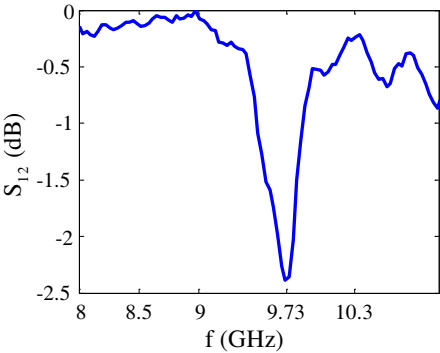


Figure 9. The transmission response S_{21} (dB) for the circular spiral structure.

The experimental result is in agreement with the numerical model. The comparison with our analytical model is presented in (Table 4). The relative error between the analytical and the measured results is 1%.

Table 4. Comparison between the analytical model and measurement of a 3-turns circular spiral.

$R\text{ (n}\Omega\text{)}$	627.4
$L\text{ (nH)}$	3988.3
$C'_{tot} \times 10^{-6}\text{ (pF)}$	68
Analytical frequency (GHz)	9.6
Measurement frequency (GHz)	9.73
$\Delta f\%$	1

6.2. Rectangular Spiral

Another test is done on a rectangular spiral structure with 4 turns. Referring to (Fig. 1(b)), its size is defined by: $W = 400 \mu\text{m}$, $S = 60 \mu\text{m}$, $L_1 = 60 \mu\text{m}$, $L_2 = 550 \mu\text{m}$ and $L_3 = 650 \mu\text{m}$. This structure is presented in (Fig. 10).

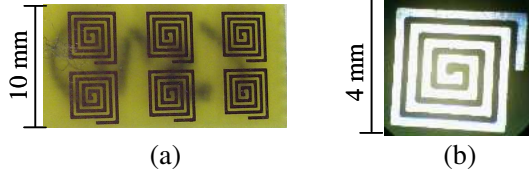


Figure 10. (a) The array of multi-turn rectangular spiral structure. (b) The dimensions of one spiral: $N = 4$, $W = 400 \mu\text{m}$, $S = 60 \mu\text{m}$, $L_1 = 60 \mu\text{m}$, $L_2 = 550 \mu\text{m}$ and $L_3 = 650 \mu\text{m}$.

The measured transmission is presented in Fig. 11.

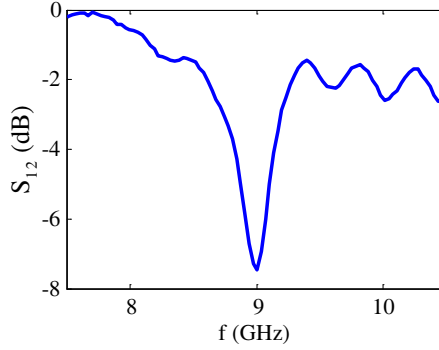


Figure 11. The transmission response S_{21} (dB) for the rectangular spiral structure.

According to the response, the measured resonant frequency is $f_{\text{measurement}} = 9 \text{ GHz}$. Table 5 represents the comparison between the measurement and the analytical model. A very good agreement is observed in view of the fact that the relative error is only 1.2%.

These experimental results lead to the conclusion that our modelling of the capacitance of a multi-turn spiral studied is robust. Note that the relative offset between the resonance frequencies of measured analytical and results (Δf) is smaller than 2% when we vary the number of turns and the width of the strip.

Table 5. Comparison between the analytical model and the measurement for 4 turns rectangular spiral.

R (n Ω)	963.5
L (nH)	2791.9
$C'_{tot} \times 10^{-6}$ (pF)	112
Analytical frequency (GHz)	8.9
Measurement frequency GHz)	9
$\Delta f\%$	1.2

7. CONCLUSION

A spiral with multi-turns has a great advantage compared to other resonant particles like SRR, BC SRR For the same resonance frequency, the use of multi-turn spirals reduces the surface area occupied by the resonant particle. But the existing analytical models that describe the behavior of these resonant particles diverge quickly especially when the number of turns increases. To model the resonant frequency of a multi-turn spiral, we started from the model proposed by Baena and took into account, in a different way, the non-uniformity of the current on the spiral. Our model was tested on a rectangular and a circular spiral with several turns (from 1 to 5). The results were compared with the measurements. In all cases, our basic analytic model gives precisely the value of the resonant frequency. In almost all tests, the relative error was less than 2%. The model can be used as a precision tool and can be reliably used for the design of multi-turn spirals.

The work to be done is now to propose a complete model that can analytically predict also the quality factor of the resonance.

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