NEGATIVE REFRACTIVE INDEX BEHAVIOR THROUGH MAGNETO-ELECTRIC COUPLING IN SPLIT RING RESONATORS

Z. Djeffal, H. Talleb, D. Lautru, and V. Fouad Hanna

UPMC Univ Paris 06
UR 2, L2E, 4 place Jussieu, F-75005, Paris, France

Abstract—An investigation on the possibility of obtaining a negative refractive index behavior in split ring resonators (SRRs) through magneto-electric coupling is presented. We have performed rigorous electromagnetic simulations using a full-wave 3D simulator, and the obtained results have been verified by our experimental realizations and measurements. The results confirm that the increase of magneto-electric gyrotropic activity inside a bi-anistropic medium can lead to the establishment of backward-wave propagation.

1. INTRODUCTION

The counterintuitive phenomena exhibited by metamaterials, such as the negative refraction and backward-wave propagation, attract many researchers all over the world, especially those who are working on realizing different structures presenting these phenomena in the microwave and optical domains such as mushrooms [1, 2] or photonic crystals [3]. So far, there have been very few successfully realized engineering applications making use of materials that can exhibit backward-wave propagation phenomenon. Most of them use planar microwave circuits [4] behaving as High-Impedance Surfaces (HIS). Nevertheless, experimental investigations on an edge of a prism structure composed of split ring resonators (SRRs) and metallic wires have demonstrated the presence of negative refraction phenomenon. However, these experimental investigations suffer from the presence of ohmic and magnetic losses due to electromagnetic interactions between both SRRs and metallic wires arrays [5].

In this paper, we confirm the role that the magneto-electric coupling plays in the left-handed behavior of bianisotropic materials as emphasized in previous works in other situations, e.g., it was explained that it is possible to establish of backward-wave propagation in Faraday chiral medium [6, 7] or in simply chiral medium [8–11]. This interpretation can be understood from the fact that a backward-wave can be produced when both of permeability, and permittivity is quite small at the working frequencies or smaller than the magneto-electrique parameter.

In this paper, a rigorous electromagnetic simulation of a periodic and bianisotropic SRRs medium [12, 13] illuminated by a plane-wave is performed. Our results demonstrate the presence of a backward wave propagation associated to gyrotropic phenomenon without a dichroism effect, results that confirm the existence of magnetoelectric coupling. This gyrotropic phenomenon, illustrated by the rotation of electric and magnetic fields for a plane wave propagating in the medium, is caused by a Faraday effect since a magnetic field is spontaneously induced along the direction of propagation of the plane-wave. To confirm this result, a measurement set-up is established using an edge of a prism formed of SRRs medium. The measured negative refraction behavior using the classic deviation angle of rotation shows a good concordance with the simulation results.

2. PROPAGATION IN BIANISOTROPIC MEDIUM

Consider a gyrotropic reciprocal bianisotropic medium whose response to an electromagnetic field is characterized by the following constitutive relations [12, 14, 15]:

$$D = \bar{\bar{\epsilon}}E + \bar{\bar{\chi}}H \tag{1}$$

$$B = -\bar{\bar{\chi}}^T E + \bar{\bar{\mu}}H \tag{2}$$

where D, B, E, H denote respectively, the electric displacement vector, the magnetic induction field, the electric field vector and the magnetic field. Theses relations can be written in the following harmonic standard form (the time-dependence is suppressed here):

$$E(r) = E_0 \exp(ik_0 \tilde{k}\hat{u}r) \tag{3}$$

$$H(r) = H_0 \exp(ik_0 \tilde{k}\hat{u}r) \tag{4}$$

$\hat{u}, \tilde{k}$ and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ denote the unit vector, the relative wave vector ($\tilde{k} \in \mathbb{C}$), and the free space wave number respectively. ϵ_0 and μ_0 are the permittivity and permeability in free space, respectively. Here, a uniaxial system is chosen considering that the optical axis is along the $\hat{z}$ axis propagation.

$\bar{\bar{\chi}}, \bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ denote, according to [15, 6], the coupling dyadic parameter expressing magnetoelectric effects, permittivity and permeability constitutive tensors, respectively:

$$\bar{\bar{\chi}} = [\chi_{xy}\,\hat{x}\hat{y} + \chi_{yx}\,\hat{y}\hat{x}] \tag{5}$$

$$\bar{\bar{\epsilon}} = \epsilon_0 \left[\epsilon_\perp \bar{\bar{I}} - i\epsilon_g \hat{z} \times \bar{\bar{I}} + (\epsilon_z - \epsilon_\perp)\hat{z}\hat{z} \right] \tag{6}$$

$$\bar{\bar{\mu}} = \mu_0 \left[\mu_\perp \bar{\bar{I}} - i\mu_g \hat{z} \times \bar{\bar{I}} + (\mu_z - \mu_\perp)\hat{z}\hat{z} \right] \tag{7}$$

$\chi_{xy}, \chi_{yx}, \epsilon_g$ and μ_g are the gyrotropic parameters due to the gyrotropic properties of a composite medium. $\epsilon_\perp$ and $\mu_\perp$ are the permittivity and the permeability in the perpendicular plan to the propagation axis and $\bar{\bar{I}}$ is the identity matrix.

The system can be reduced to a 2 dimensions one since the wave vector k is parallel to the $\hat{z}$ direction and both of Dz and Bz are zero. Using a regular procedure in the k-D-B vector coordinate system like that used in [16], we obtain the following quadratic polynomial equation:

$$u^4 - (2v + \chi_{xy}^2 + \chi_{yx}^2)u^2 + (v + \chi_{xy}^2)(v + \chi_{yx}^2) = b^2 \tag{8}$$

where $u = \frac{k}{\omega}$, $v = \epsilon_0\mu_0(\epsilon_\perp\mu_\perp + \epsilon_g\mu_g)$ and $b = \epsilon_0\mu_0(\epsilon_\perp\mu_g + \epsilon_g\mu_\perp)$.

In the case where $\chi_{xy} = -\chi_{yx} = -jK$, where K measures the magnetoelectric coupling [15], the solution can be written as:

$$u_{a\pm} = \pm\sqrt{\epsilon_0\mu_0(\epsilon_\perp + \epsilon_g)(\mu_\perp + \mu_g) - K^2} \tag{9}$$

$$u_{b\pm} = \pm\sqrt{\epsilon_0\mu_0(\epsilon_\perp - \epsilon_g)(\mu_\perp - \mu_g) - K^2} \tag{10}$$

by putting $\epsilon_\pm = (\epsilon_\perp \pm \epsilon_g)$ and $\mu_\pm = (\mu_\perp \pm \mu_g)$, the solutions yield:

$$k = \pm\omega\sqrt{\epsilon_0\mu_0\epsilon_\pm\mu_\pm - K^2} \tag{11}$$

This solution concords with that given in [15, 12] but it takes also into consideration the effect of gyrotropy. Let us now, study the case for which $k = \frac{\omega}{2c}(\epsilon_\pm + \mu_\pm)$, namely the case for which the material parameters satisfy the condition $K = \frac{1}{2}\sqrt{\epsilon_0\mu_0}(\mu_\pm - \epsilon_\pm)$. It can be seen that a backward-wave propagation becomes possible when $\mathrm{Re}(\epsilon_\pm + \mu_\pm) < 0$, i.e., when the following relation is satisfied:

$$-(\epsilon_g + \mu_g) > (\epsilon_\perp + \mu_\perp) \tag{12}$$

This last condition can be reached when the permeability and permittivity are quite small at the working frequencies. It is interesting to notice that this later condition is simpler to satisfy than the typical conditions: $\mathrm{Re}(\epsilon_\perp) < 0$ and $\mathrm{Re}(\mu_\perp) < 0$ for which the homogenization condition must be satisfied [17].

3. ELECTROMAGNETIC SIMULATION VALIDATION

Here, we examine the dense copper SRR medium represented in Fig. 1 for which the dimensions of a unit cell are given. The lattice periods along, x, y, and z axis are 18 mm, 16.5 mm, and 18 mm, respectively and the employed substrate is the Epoxy ($\epsilon_r = 4.4$) with a thickness of 1.5 mm. We restrict our study to the case of a plane wave propagating along the z axis when the electric field and magnetic field are applied along the x axis and the y axis, respectively.

Rigorous electromagnetic simulations performed using the full-wave 3D CST Microwave Studio software have shown a backward-wave phenomenon behavior between 9.3–10 GHz (see Fig. 2). To demonstrate this phenomenon, a prism, composed of the same SRRs shown above and having an apex angle A of 45°, has been illuminated by a horn antenna working at 10 GHz. Fig. 3 shows a cross sectional view of the near field patterns while Fig. 4 shows a maximum deviation
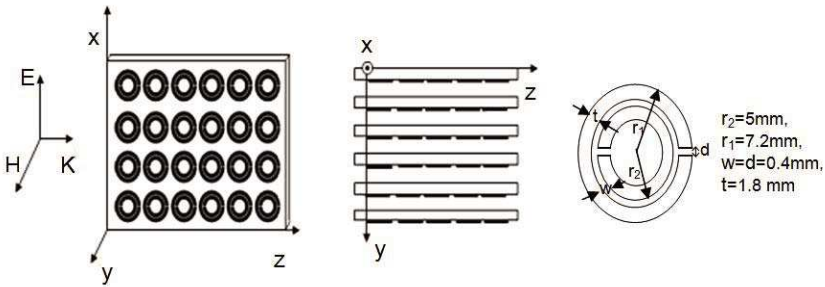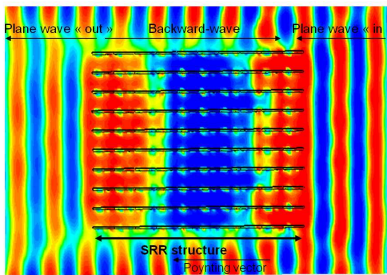

Figure 1. The studied SRRs structure.

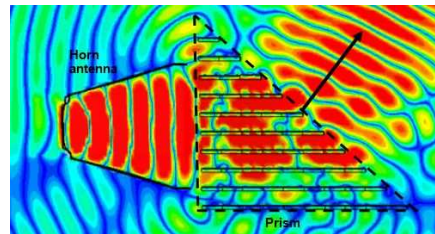**Figure 2.** Top view of the SRRs structure excited by a plane wave at 9.8 GHz.

Figure 3. Near zone electric field pattern at 10 GHz.

angle D of the steering beam equals $-55°$ in the far field zone. The refractive index can be calculated from Equation (13) according the formulation given in [18]. Thus, we obtain an effective index of -0.23.

$$n = \frac{\sin\left(\left(D+A\right)/2\right)}{\sin\left(A/2\right)} \qquad (13)$$

To better understand the causality of this phenomenon, some magnetic and electric probes have been placed inside and along the studied structure. Fig. 5 shows a cross sectional view inside the medium in which we can notice the presence of a gyrotropic effect involving two elliptically polarized electric and magnetic fields.

This gyotropic effect, expressed by the gyrotropic parameters K, ϵ_g and μ_g, is caused by a Faraday effect since a quasi-static effective magnetic field Hz is found to oscillate around a mean value of $1.1\,\text{mA/V}$. In fact, each rings cell behaves as magnetic and electric
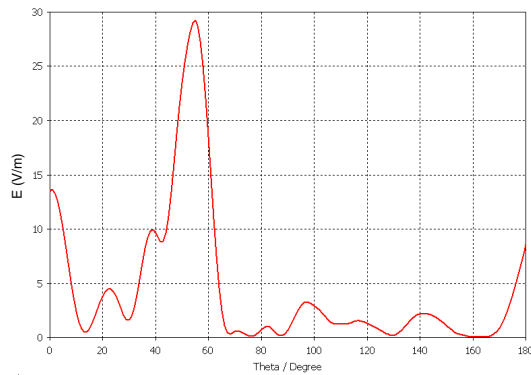

Figure 4. Far field radiation pattern in cartesian coordinates at $10\,\text{GHz}$.
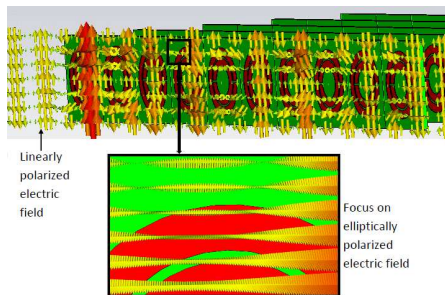

Figure 5. Elliptical polarization of electrical filed inside a SRR structure at $9.5\,\text{GHz}$.

dipole particles where the permeability and permittivity can be quite small in the working frequency band around resonance. It is important to emphasize that normally the Faraday effect is a nonreciprocal effect since it is due to the application of an external magnetic field in the direction in propagation of the system, i.e., involving a variance of the results when the transmitter and receiver are interchanged. However, in our case, this effect is reciprocal since the same Hz field is spontaneously induced according to the employed direction of propagation.

4. EXPERIMENTAL CONFIRMATION

To verify these principal simulation results, an experimental setup operating in the X band was established like that described in [19]. It enables to determine the angular position of a microwave beam generated from a network analyzer and passes across a SRR prism medium having an apex angle of $45°$. The SRR prism has the same dimensions of the above simulated prism. As illustrated in Fig. 6, the microwave beam was generated by a horn antenna placed at a short distance from the prism. This setup enables to reduce the angular spread of the incident beam caused by diffraction and also to maximize the received power. A mobile horn antenna controlled by a stepping motor was used to capture the microwave beam and to determine the refractive angle corresponding to its angular position. Fig. 7 shows the experimental results for the electrical field intensity at $10\,\mathrm{GHz}$ normalized with respect to the maximum intensity received without the prism at $\theta = 0°$ (i.e., when the transmitting antenna is placed directly in front of the receiving antenna without any obstruction). It can be seen that there are different levels for the received field intensity according to the position of the receiving antenna. The deviation angle
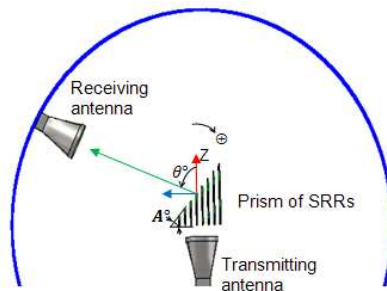

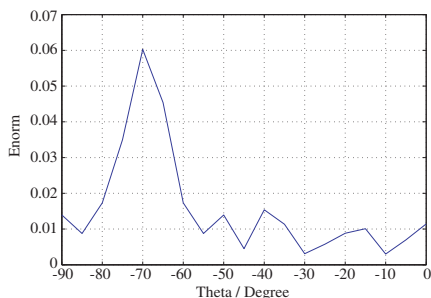**Figure 6.** Experimental setup for measuring the refractive angle.

Figure 7. Variation of the normalized received electrical field intensity at 10 GHz as a function of the angular displacement of the mobile antenna.
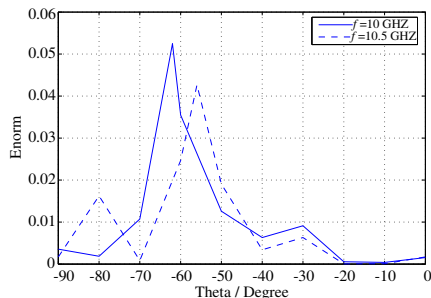
Figure 8. Variation of the normalized received electrical field intensity at 10 GHz and 10.5 GHz as a function of the angular displacement of the mobile antenna when an array of ten units of SRRs has been added.

corresponding to the maximum measured electrical field intensity is $D = -\theta = -70°$, which means that the value of the refractive index n equals -0.56 according (13).

In order to confirm these experimental results and also to reduce the influence of diffraction phenomenon, an array of ten units of SRRs was added to the studied prism. The experimental results show a negative refraction between 9.5 GHz to 10.5 GHz. Fig. 8 shows the variation of the electrical field intensity at 10 GHz and 10.5 GHz as a function of the angular displacement of the mobile antenna. One can notice that the deviation angle at both 10 GHz and 10.5 GHz, corresponds to a negative refraction, since the angle of the maximum measured electrical field intensity at both 10 GHz and 10.5 GHz is $-62°$ and $-56°$ respectively with a corresponding refractive index (according to (13)) $n = -0.4$ and $n = -0.25$ respectively. One can also notice a decrease of the value of the measured maximum received electric field intensity at 10 GHz due to additional losses introduced by the added SRR structure.

5. CONCLUSION

In this paper, it is shown that magneto-electric coupling, which induces the Faraday Effect inside a bianisotropic medium composed only of SRR cells, can lead to a Left-Handed Material (LHM) behavior in a specific frequency band. This result leads to a simplification in

the realization of LHMs with negligible ohmic and magnetic losses since it is not recessionary to add metallic wires to obtain a backward propagation. The measurement setup, using an edge of a prism formed of SRRs medium, has enabled us to measure a negative refraction and validated the simulation results. The addition of an array of SRRs structure has confirmed the measured negative refractions. This study has been performed in the microwave domain, and it is evident that a transposition can be extended to other frequency ranges since the scaling principle can be applied.

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