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IMPROVEMENT OF THE PERFORMANCES OF 1D PHOTONIC CRYSTAL BY THE REDUCTION OF THE KIESSIG FRINGES

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Abstract—The goal of this work is to look for a technique of optimization making it possible to improve the optical performances of materials with photonic band gap by reducing of the Kiessig fringes. The techniques of apodization and smoothing were used. The combination of these two techniques made it possible to reduce the Kiessig fringes up to 95%.

1. INTRODUCTION

In signal processing, the presence of some kind of interference will corrupt the desired signal with noises also known under the name sidelobes. No matter how many names of these lobes we got, there is still no relevant information conceived in the profiles of the optical response. In some types of dielectric structures, whose dielectric constant varies periodically in space, the constructively interfering waves will have the strongest intensity during the propagation when the considered waves are in phase. This phenomenon gives a total inhibition (reflection) of the incident electromagnetic wave from which the notion of the photonic band gap (PBG) is derived. Significant interest has been attracted to research about this interesting type of structure based optical devices, such as waveguides [1], filters [2], photonic integrated circuits, optical communication network, and ultrahigh speed information processing. . . .

Optical spectral filtering with a several devices and designs gives rise to a very large technological field adapted to numerous applications in photonic domain. Whether being a monochromatic or polychromatic filter, the main function is to make a frequencies-locked against the electromagnetic wave propagation. Through the

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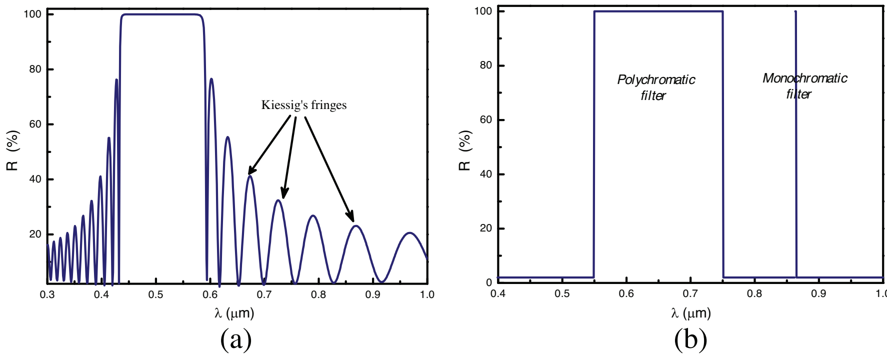


Figure 1. (a) Polychromatic filter with sidelobes. (b) Ideal poly and monochromatic filter (perfect reflection 100% and perfect transmission 100%).

PBG propriety the specific frequency restriction of wave propagation has been made. But in photonic crystal (PhC) the interferences of the amplitudes diffused by the first diopter (air-sample) and the last (last layer-substrate) have non desirable effects. These noises appear in the form of oscillations which are on both sides of the principal lobe (PBG). They are called "Kiessig fringes" revealed by Kiessig in 1931 [3].

It appears clearly that the analysis of data which have already been filtered becomes more significant if we eliminate parasite information and limit ourselves to study only the principal part of the signal. Also, when the intensity of these parasites, located near the main lobe, becomes equal to the intensity of the principal lobe, a phenomenon of convolution between them springs up and makes treatment of these data more difficult. For all these reasons and in object to provide a much higher degree of freedom in state-of-the-art filter design techniques, in this paper, we will describe a new method that will help us to have an ideal optical response without Kiessig fringes (Figure 1).

2. THEORY

To study the propagation of electromagnetic waves in one dimensional photonic crystal, we chose the transfer matrix method [4] and conveyable techniques to reduce the intensity of these side lobes. So, the fundamental idea is to break the intrinsic periodicity of the optical system [5]. As examples of these techniques we quote the chirp technique [6], cascade technique, apodization technique [7], and smoothing technique [8].

The aim of this work is to reduce the intensity of the slide lobes which appear in the reflectivity spectra's of periodical multilayer's systems. To reduce these slide lobes we increased the performance of the optical devices such as the omnidirectional mirror.

In our study, we selected the two later techniques (Apodization and Smoothing) for the reason that their principal goal is to reduce the amplitude of sidelobes and the irregularity produced in a signal.

2.1. Presentation of the Apodization Technique

The apodization is a mathematical technique which aims at reducing the phenomenon of Gibbs. It is a treatment that forces the amplitude of the signal to remain at zero level at the beginning and the end of the sidelobes, without influencing the intensity of the principal lobe.

This technique requires the use of a very specific function, and its choice is highly important and depends on the use made thereof. These functions are different by their spectral characteristics (width of the principal lobe, amplitude of the sidelobes). They generally have quite particular and known mathematical expressions under the names of their inventors. We can quote: The Flat top windows function [9], Hann window function [10], Kaiser-Bessel window function [11], Gaussian window function [12] and Tukey window function [13].

Hann window function:

$$\left\{W_n = \frac{1}{2}\left(1 - \cos\left(2\pi\frac{n}{L-1}\right)\right),\ 0 \leq n \leq L-1 \right. \tag{1}$$

Kaiser-Bessel window function:

$$W_n = \begin{cases} \frac{I_0\left[\beta\sqrt{1-(n-L)^2/L^2}\right]}{I_0(\beta)}, & 0 \leq n \leq L-1 \\ 0 \end{cases} \tag{2}$$

$$I_0(\beta) = \sum_{k=0}^{\infty}\left(\frac{0.5\beta^k}{k!}\right)^2 \tag{3}$$

$$\beta = \begin{cases} 0.1102(\alpha - 8.7) & \alpha > 50 \\ 0.5842(\alpha-21)^{0.4} + 0.07886(\alpha - 21) & 21 \leq \alpha \leq 50 \\ 0 & \alpha < 21 \end{cases} \tag{4}$$

Gaussian window function:

$$\mathrm{W}_n = e^{-\frac{1}{2}\left(\alpha\frac{2n}{L-1}\right)^2} \quad -\frac{L-1}{2} \leq n \leq \frac{L-1}{2}\ et\ \alpha \geq 2 \tag{5}$$

Tukey window function:

$$\mathrm{W}_n = \begin{cases} 1.0 & 0 \le |n| \le \alpha \frac{L-1}{2} \\ \frac{1}{2}\left(1+\cos\left(\pi \frac{n-\alpha\frac{L-1}{2}}{2(1-\alpha)\frac{L-1}{2}}\right)\right) & \alpha\frac{L-1}{2} \le |n| \le \frac{L-1}{2} \end{cases} \quad 0 \le \alpha \le 1 \tag{6}$$

In this work and after having tested these functions, we chose to study the apodization by using the Kaiser, Gaussian and Tukey functions because of their significant effect, and then we will choose the best of them.

2.2. Presentation of the Smoothing Technique

The difference between smoothing and apodization techniques seems to be negligible because their principles are very similar. However, the smoothing technique is generally applied to shape separating the studied system from the external medium (air and substrate). Therefore, the adjustment in the profile of the refractive index will just concern the two ends of the studied system. This adjustment consists in blunting the abrupt up variation of the refractive index at the time of the passage outside the photonic crystal up to get an eventual five order polynomial profile (quintic function) of the refractive index. This quintic variation constitutes the best function of smoothing to obtain an impressive reduction of the sidelobes [14, 15]. The expression of the refractive index in this case is given by:

$$n(t) = n_B + (n_H - n_B)\left(10t^3 - 15t^4 + 6t^5\right), \ \mathrm{t} \in [0,1] \tag{7}$$

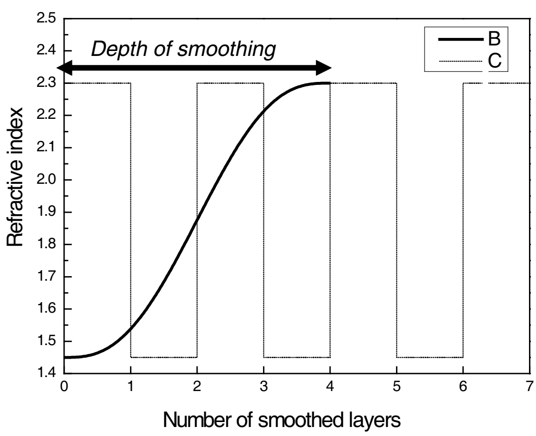


Figure 2. Quintic (B) and periodic (C) variation of the refractive index, for $n_H = 2.3$ and $n_L = 1.45$ versus the number of layers.

where t is a standardized parameter proportional to the depth of smoothing, n_H high refractive index, and n_L low refractive index.

Figure 2 shows the quintic and periodic variation of the refractive index.

3. RESULTS AND DISCUSSION

Let us consider a period structure of two alternating materials layers with different refractive indexes: The titanium TiO_2 ($n_H = 2.3$) and the dioxide of Silicon SiO_2 ($n_L = 1.45$). The distribution has the form: $H(LH)^j$ where H indicates the high refractive index layer, L the low refractive index layer and j the number of periods. The substrate on which the system is deposited is ordinary glass with a refractive index $n_S = 1.5$, and we suppose that the structure is immersed in air: $n_a = 1.0$. The optical thickness of each layer is equal to $\lambda_0/4$ where λ_0 is the reference wavelength which is equal to 0.5 µm.

The distribution of the refractive index and the spectrum of reflection of the chosen periodic system are shown respectively in Figures 3 and 4 for 31 layers. It is clear that the intensity of Kiessig fringes is elevated, and it can reach 76% in reflection.

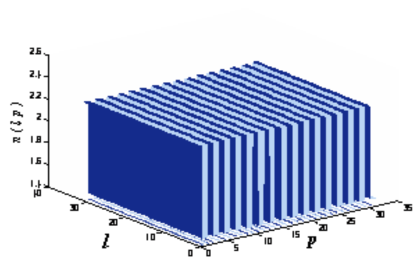


Figure 3. Profile of the refractive index for 31 layers. l represents the width of the crystal.

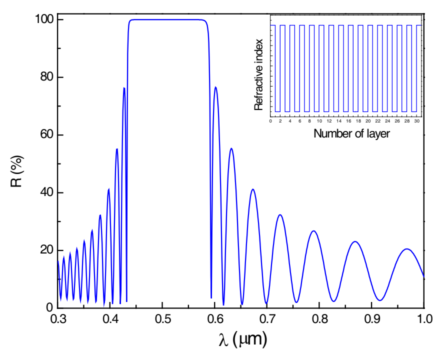


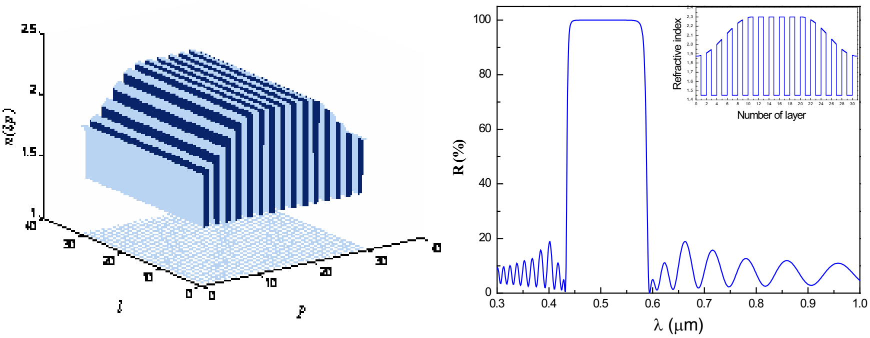
Figure 4. Spectrum of reflection according to the wavelength of a periodic system.

Thus, we will apply the two mentioned techniques by fixing the optimal parameters of each one, and then we will compare their various effects.

3.1. Application of the Apodization Technique

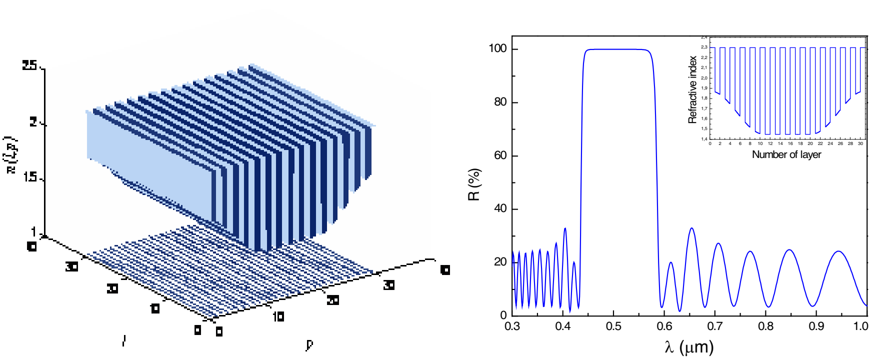
We will approach our study by the research of the optimal values of α and β for the functions of Tukeywin (L, α), Kaiser (L, β) and Gausswin (L, α). However, we were led to know, according to which criteria, the reducing effect of Kiessig fringes of this technique is maximum.

With this intention, we will present, respectively in Figures 5, 6 and 7 below, the effect of the apodization applied to the layers of high refractive index, layers of low refractive index and then both layers at



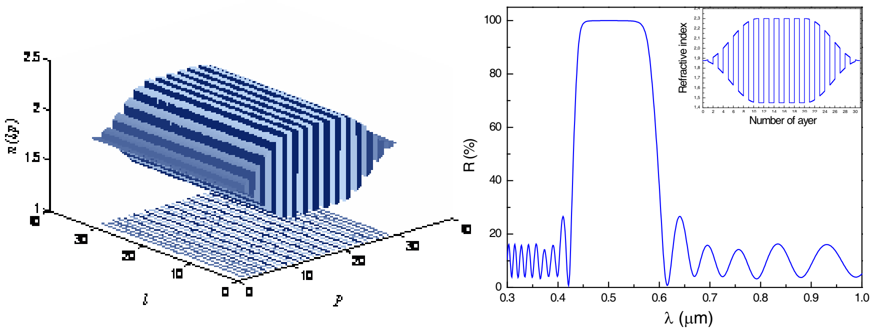
(a) (b)

Figure 5. High refractive index modulation in function of (a) the width of the crystal and the number of layers, (b) its optical response in the reflection mode.



(a) (b)

Figure 6. Low refractive index modulation in function of (a) the width of the crystal and the number of layers, (b) its optical response in the reflection mode.



(a) (b)

Figure 7. High and low refractive index modulation in function of (a) the width of the crystal and the number of layers, (b) its optical response in the reflection mode.

the same time by using Tukeywin function with an arbitrary parameter α ($\alpha = 0.7$).

After scanning these results, we obtain 50% of reduction of the intensity of Kiessig frange for high refractive index modulation, 37.5% for low refractive index modulation and 12.5% for high and low refractive index modulation. It is clear that for these three cases the intensities of the Kiessig fringes are the lowest in the case of high refractive index modulation. However, for the last two cases (Figures 5 and 6), some anomalies appear in the stage of the PBG, and it is still noticed that the reducing effect of this technique is increasingly favorable when we have a small variation of the index, on the basis of the medium towards the two edges of the distribution. So, we can conclude that the application of this technique should be employed only for the high refractive indexes.

Let us now find out the optimal values of α and β for Tukeywin, Kaiser and Gausswin functions. The field of their variation depends on the used function; for Tukeywin function (L, α): $0 \leq \alpha \leq 1$, for Kaiser function (L, β): $0 \leq \beta \leq 40$ and for Gausswin function (L, α): $2 \leq \alpha \leq 20$. We present in Figure 8 the reflection response of the system according to these parameters.

We notice considerably that these functions act on the Kiessig fringes intensity and on the width of the PBG, but the effectiveness degree of this technique changes from one function to another.

The choice of the optimal values of these parameters must take into account of the minimal intensity of the Kiessig fringes as well as the width of the PBG. The founded optimal values are: for Tukeywin

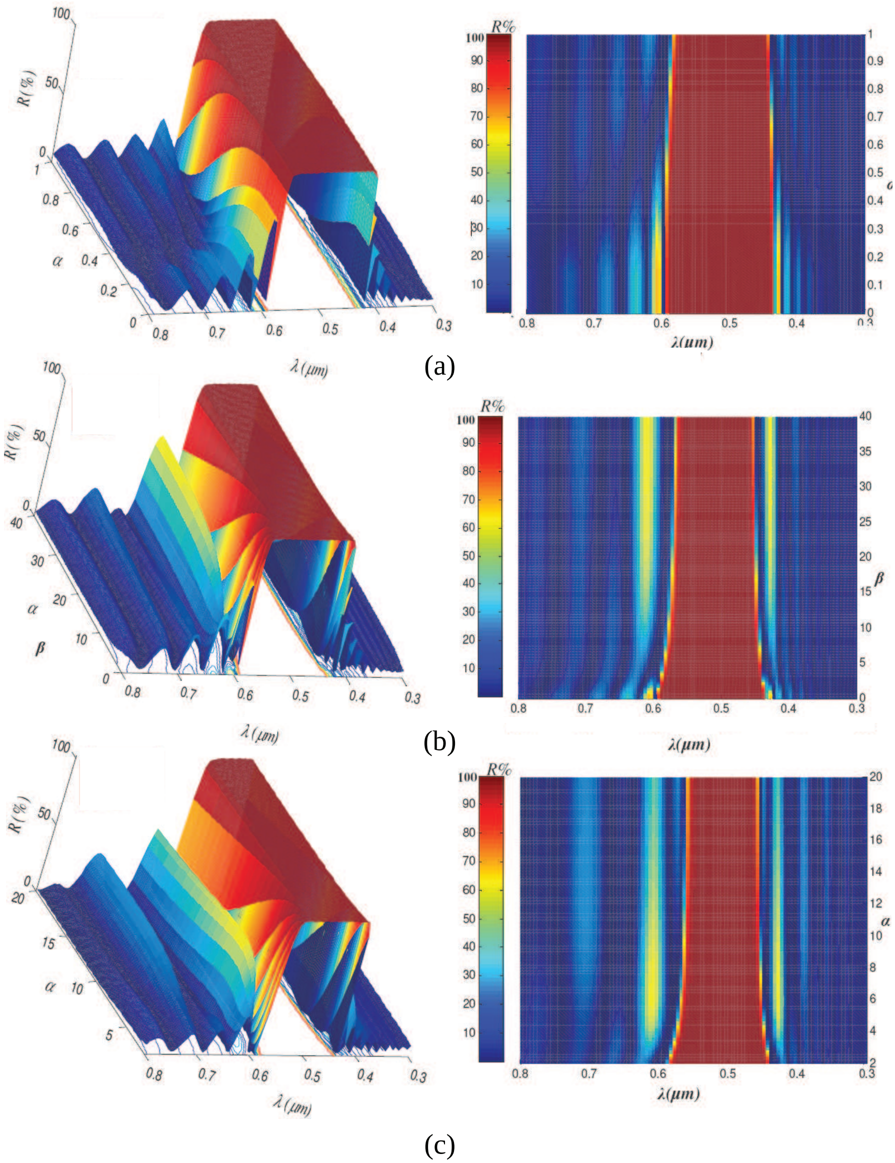
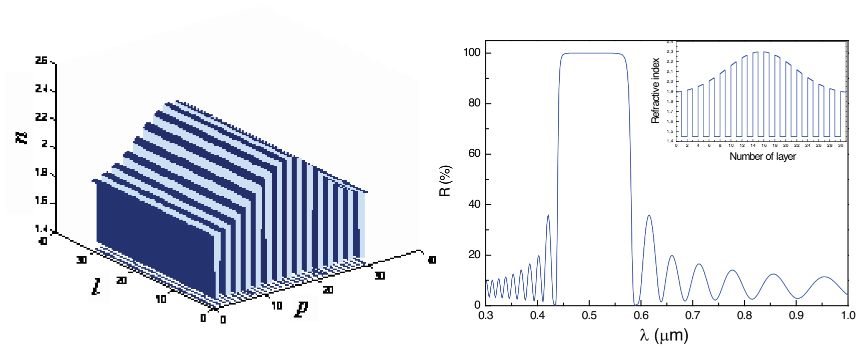
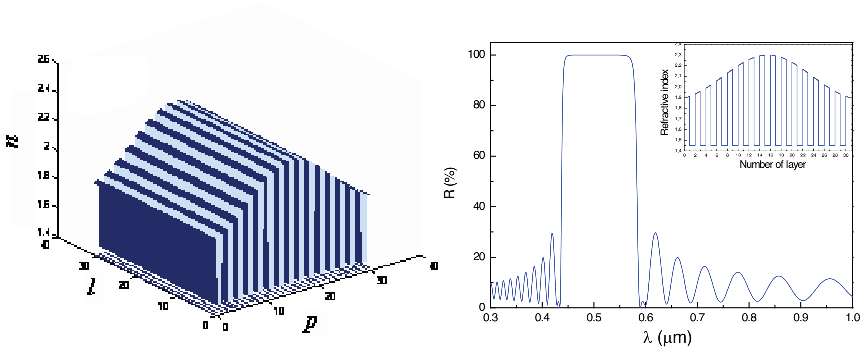


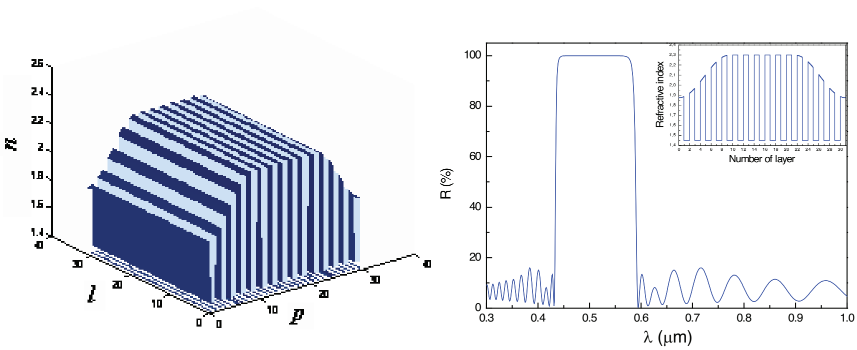
Figure 8. (a) Reflection according to α and β and the curves of variation of the PBG according to these parameters for (a) Tukeywin function, (b) Kaiser function, (c) Gausswin function.



(a)



(b)



(c)

Figure 9. Modulation of the refractive index according to several weight functions: (a) Gausswin, (b) Kaiser, (c) Tukeywin and their corresponding reflection spectra.

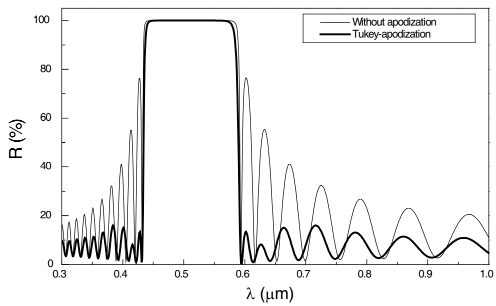


Figure 10. Simulation of the spectrum of reflectivity of the apodization Tukeywin functions with the spectrum not optimized.

(L, α): $\alpha = 0.61$, for Kaiser (L, β): $\beta = 4.65$ and for Gausswin (L, α): $\alpha = 2.5$.

According to the results, it is shown in Figures 9, 10 that Tukeywin is the best function of apodization because the reduction in the intensity of a certain peak can reach 64% without influencing too much the width of the PBG (2.5% of reduction).

We conclude that the intensity of Kiessig fringes presents an average reduction (50%), and it is accompanied by a contraction of the PBG by using the apodization technique. In the next part of the paper and for the purpose of having a more optimized optical response, we will study the smoothing technique.

3.2. Application of the Smoothing Technique

First we will search for the best conditions to apply the smoothing technique. Thus, we will present, in Figure 11, the influence of a smoothed layers number on the smoothing effectiveness.

In these figures, it is noted that the intensity of Kiessig fringes falls when the number of smoothed layers increases and that we lose this property when this number exceeds four layers. So, the optimal number of layers is three.

Contrary to apodization, it is shown in Figure 12 that the effectiveness of this technique is related to the zone far from the PBG. The peaks near the PBG are less influenced than those which belong to the zone far from the PBG.

Despite the fact that these techniques of optimization and smoothing are effective in reducing the intensity of these lobes, they remain unable to remove completely the Kiessig fringes. But, by gathering the two optical responses in the same graph (Figure 12(b)), the idea comes, and the question arises: Do we manage to be inspired

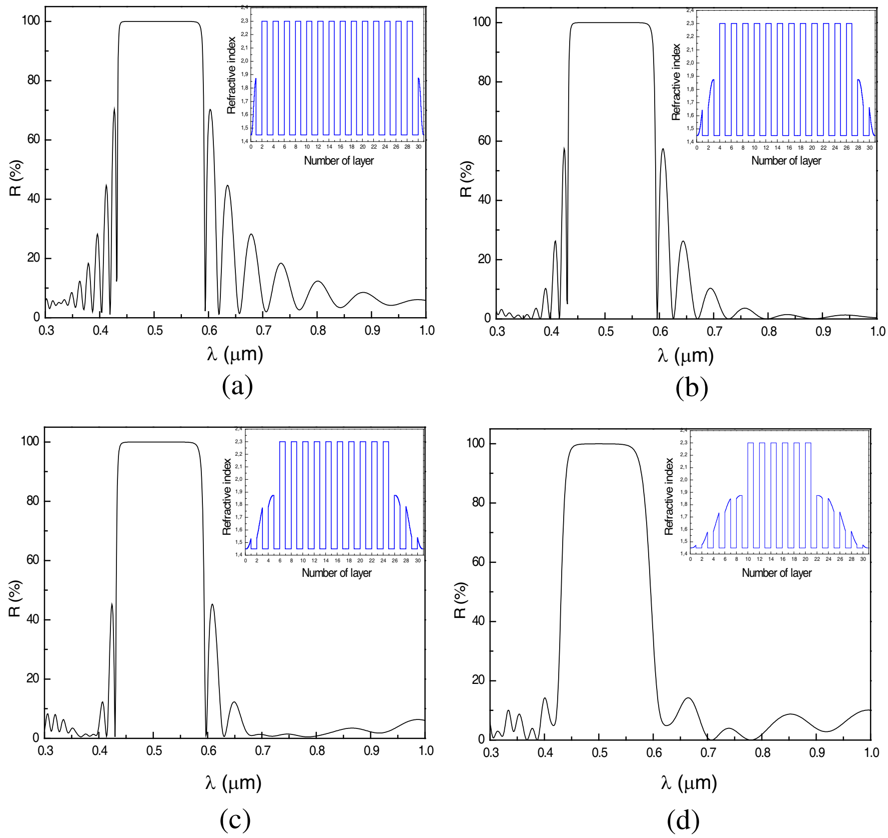


Figure 11. Reflection response for a smoothing of: (a) one, (b) two, (c) three and (d) five layers.

by these results to remove the Kiessig fringes completely by combining the two techniques instead?

3.3. Application of the Technique Tukeywin-apodization-smoothing

The new technique is inspired by the combination of the Tukeywin-apodization and smoothing techniques in order to have added effects and a reduction of all the Kiessig's fringes peaks.

The application of this new technique requires a parameter setting of the used functions. Moreover, it should be noted that for apodization, we will fix the number of smoothed layers and vary the coefficient α of Tukeywin function.

While applying the new technique, we realized that the reduction of the number of smoothed layers influences the effectiveness of apodization. In other words, the latter does not manage sufficiently to reduce the Kiessig fringes intensity which has already been slightly reduced by smoothing. However, if we extend more smoothing in the system, the intensity and width of the PBG decrease remarkably.

The found results show that by fixing the number of smoothed layer to three and Tukeywin parameter to ($\alpha = 0.82$), it is possible to considerably reduce the sidelobes in the spectrum of reflectivity with a weak loss in the PBG (Figure 13).

With such a result, the idea to have secondary lobes in the

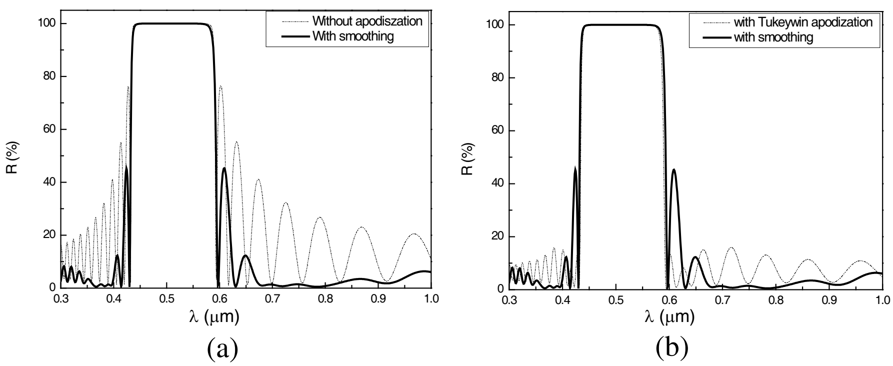


Figure 12. Comparison of the optical responses, (a) case of three smoothed layers and without Tukeywin-apodization, (b) between Tukeywin apodization and with three smoothed layers.

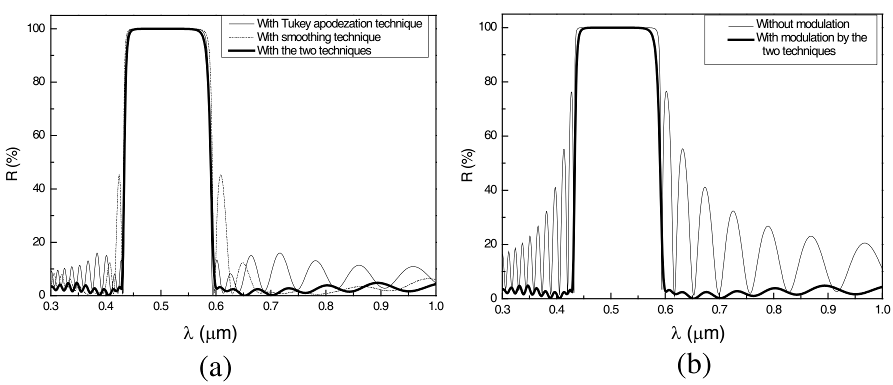


Figure 13. (a) Validation of the effectiveness of the association of two techniques, (b) illustration of the reduction of Kiessig fringes.

spectrum of reflectivity is absolutely dissimulated with the least loss in the width of the PBG (Figure 13(b)). Moreover, the validation of our choice for this method is shown in Figure 13(a) by a comparison of the spectrum of reflectivity of each technique (the apodization, smoothing and both associated at the same time).

In this stage, we could considerably optimize the optical response in reflection (or transmission) of the 1D photonic crystals, and the reduction of the intensity of certain peaks exceeds 85%.

3.4. Deformed Periodic System

We will study in this part the effect of the Tukeywin-apodization-smoothing technique on the Kiessig fringes intensity for a deformed periodic system. The studied deformation follows the law [16]: $y = x^{1+h}$. Where, h is the degree of the law, and y and x are, respectively, the deformed and the initial systems.

The initial optical phase thickness is: $\varphi = \frac{2\pi}{\lambda} x_0 \cos\theta$, and when we apply the y function, the initial optical phase thickness takes the following form: $\varphi_j = \frac{2\pi}{\lambda} x_0 [j^{h+1} - (j-1)^{h+1}] \cos\theta_j$. j represents the jth layer; θ_j is the complex refractive angle; and $x_0 = \frac{\lambda_0}{4}$ is the optical thickness of each layer of the multilayer structure with the reference wavelength λ_0.

We must, firstly, seek the optimum deformation degree "h" which enables us to have the broadest PBG without being out of our studied field: $\lambda \in [0.3,\ 1.0]$ µm. Thus, we present in Figure 14 the variation of the reflection according to this coefficient, where $\lambda \in [0.1, 1.5]$ µm.

After found the best value of the deformation coefficient ($h = 0.1$),

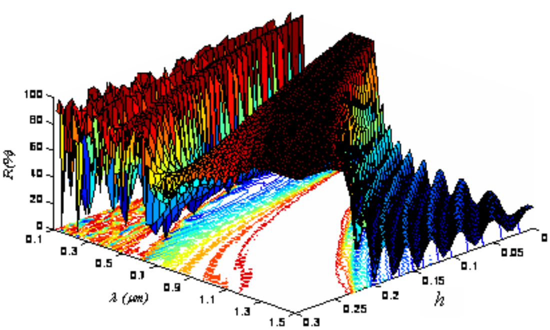


Figure 14. Simulation of spectrum of reflectivity according to the coefficient of deformation h and the wavelength λ.

we can apply the new technique adopted to the system, and due to the modification of its total thickness, the parameters used in the initial periodic system change. But, since the difference between these thicknesses is weak, we can keep the same value of α ($\alpha = 0.82$), and the number of smoothed layers becomes equal to 4.

In Figure 15, it is noted that the intensity of Kiessig fringes presents a great reduction (83%), particularly, in the high frequency zone. Then, it should be noted that the combination of the smoothing and Tukeywin-apodization techniques is validated in both deformed and non-deformed systems (Figure 15(d)). These results give a good reduction of Kiessig fringes compared to published results by other researchers where the maximum of reduction is 33% [17].

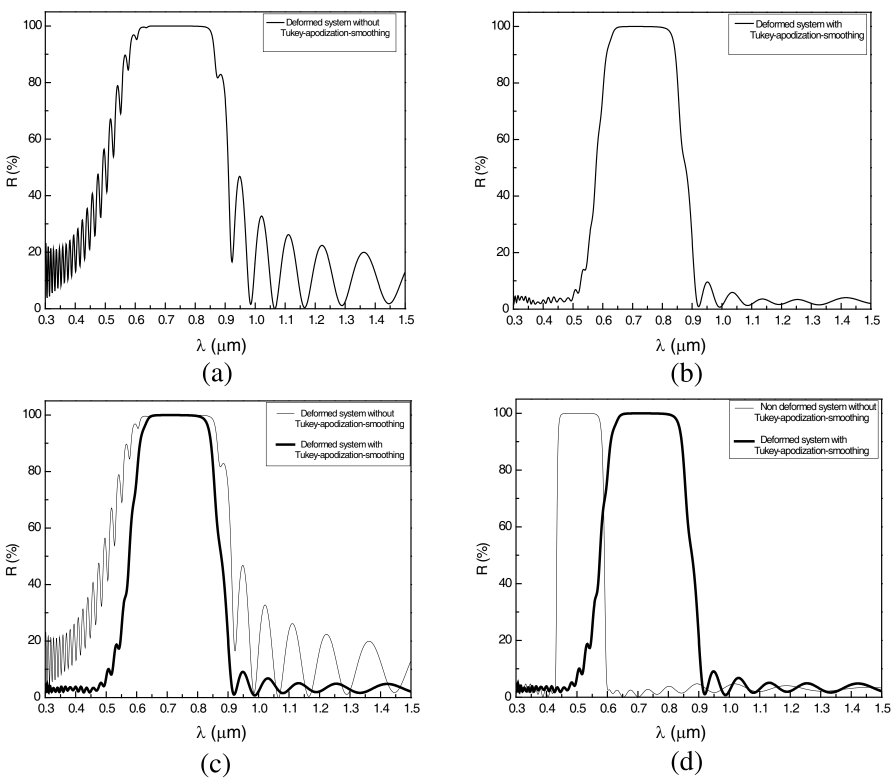


Figure 15. (a) Reflectivity of the deformed system, (b) verification of the reducing of Kiessig fringes, (c) comparison of the reflectivity of the system deformed before and after suppression of Kiessig fringes, (d) comparison of the reflectivity of the deformed system and not deformed (after suppression of Kiessig fringes).

4. CONCLUSION

There are various techniques suggested to reduce the intensity of Kiessig fringes. In this work, we adopt two techniques similar to the technique of optimization: smoothing and apodization. In the first part, we were able to reduce the intensity of sidelobe through the technique of apodization, to minimize the side lobes intensity. And in the second part, we could apply the technique of smoothing which allowed the reduction in these intensities.

In spite of the effectiveness of these two techniques of optimization in reducing the intensity of these lobes, they remain unable to entirely pack these lobes. Thus, we are inspired by these results, a new technique derived from a combination of the two previous techniques. This method almost completely reduces the lobes.

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