

## **A RIGOROUS TREATMENT OF VERTICAL DIPOLE IMPEDANCE LOCATED ABOVE LOSSY DPS, MNG, ENG, AND DNG HALF-SPACE**

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**Abstract**—In this paper, accurate analytical expressions for the impedance of vertical electric and magnetic dipoles which are located over the half-space materials of arbitrary permittivity and permeability are developed. In this regard, the impedance variations are expressed in integral forms. For metamaterial half-space, a proper expression for approximating the Fresnel reflection coefficient is proposed. Using this approximate expression, the impedance integrals are analytically solved, and exact formulas for impedance variations are obtained. The results for the metamaterial half-spaces are compared with the case of natural materials (positive permittivity and permeability), and key differences are explained. The influences of sign changing in permeability of the half-space material on the impedance of vertical dipole are studied, and the results are validated by comparison with those of numerical solution of integrals. It is shown that for elevated dipoles over materials with high and/or low conductivities, the results of both methods are in complete agreement. For vertical dipoles above low loss materials, the results are somewhat identical. However, a better agreement could be obtained using higher order approximations for the integrand.

## 1. INTRODUCTION

The metamaterials was introduced in 1968 as a novel class of electromagnetic materials [1, 2]. These materials retain attractive phenomena for antenna and microwave applications [3–7]. Various methods of fabricating materials with negative permittivity and/or permeability have already been proposed [8–11]. Therefore, nowadays, having desired metamaterials is presumable. Prior to extending the usage of metamaterials in microwave or photonic structures, thorough investigation on scattering from metamaterials is required. So far there has been few theoretical studies of wave scattering mechanism from metamaterials [12–16]. However, dipole scattering from the natural earth ( $\varepsilon$  and  $\mu$  positive) has been the research subject for many years. The scattering modelling is mainly based on the spherical source transformations via Fourier-Bessel transformations. Although many articles and books are devoted to this method, it is still an undergoing research issue. A good review of the Fourier-Bessel transformations method can be found in [17] and references therein.

When the environment is lossy material, an important aspect of a radiation system is the power efficiency. Here, we are interested in the amount of power that is dissipated in the lower lossy half-space in comparison with the amount of power that would be emitted by the same dipole when it is located in free space. As stated by [18], instead of using a tedious approach of integrating the real part of Poynting vector over a plane just above the lossy half-space, a much simpler method of direct calculation of the change in the self-impedance of dipole, as a quantitative criterion for loss, may be applied. As a result, both variations of the resistive and reactive parts are obtained. The simplest method to accomplish this investigation is to study the self-impedance variations of the dipole from its value,  $Z_0$ , in free space to its value,  $Z$ , in presence of the lossy material half-space. The impedance variations of short dipole over the lossy earth have been studied by many researchers. Integral expression for the impedance variations in terms of the media parameters and the elevation of dipole can be found in [18–21].

Metwally and Mahmoud have derived expressions for the impedance variations of VED (Vertical Electric Dipole) and VMD (Vertical Magnetic Dipole) over natural half-space with  $\mu = 1$  in terms of rapidly converging series [19]. They have also solved the integrals by the discrete and continuous image method [20, 21]. Analysis of microstrip dipole antennas on layered metamaterial substrate has been carried out in [22], where the method of moment and array scanning method were used for calculating the impedance of dipole

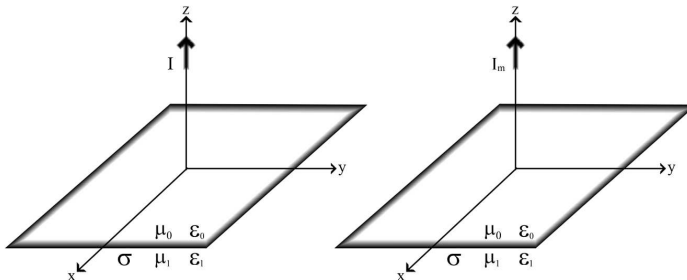
antennas in the vicinity of metamaterials. Recently, [23] developed exact expressions for EM field of VMD in the vicinity of the common lossy materials.

In this paper we develop accurate analytical expressions for input impedance of vertical short dipole that is located above metamaterial half-space. In this regard, proper expressions for approximating the Fresnel reflection coefficient for VED and VMD radiation above DPS ( $\varepsilon$  and  $\mu$  positive), ENG ( $\varepsilon$  negative), MNG ( $\mu$  negative), and DNG ( $\varepsilon$  and  $\mu$  negative) materials of arbitrary permittivity and permeability are developed. Similar investigation for horizontal electric and magnetic dipoles over lossy half-space is under investigation by the authors.

The paper is organized in five sections as follows. In Section 2, the integral equation for a vertical dipole above lossy half-space is derived. Third Section is devoted to solution of the integral expressions by deriving proper approximation of the Fresnel reflection coefficient for VED and VMD radiation above the materials of arbitrary permittivity and permeability. In the fourth section, the solution of the derived expressions are numerically verified. Finally, in the last section, discussion of the results and conclusion is provided.

## 2. PROBLEM FORMULATION

A radiating electric and/or magnetic Hertzian dipole in free space is considered. The dipole is located at distance of  $z = h$  above a lossy material with arbitrary permittivity and permeability. The geometry of problem is shown in Fig. 1. In order to find the input impedance variations of the dipole, the near fields must be calculated. For VED, the electric field and for VMD the magnetic field calculations are required. The impedance variations are calculated in the following subsections for both dipoles.



**Figure 1.** The geometry of problem.

## 2.1. VED

The input impedance variation for VED in terms of the scattering field can be obtained from [18],

$$\Delta Z = \lim_{\substack{z \rightarrow z_0 \\ \rho \rightarrow 0}} \frac{-E_z^s dl}{I}. \quad (1)$$

The scattering field,  $E_z^s$ , can be calculated by solving the following wave equation for Hertzian potential in two mediums:

$$\nabla^2 \Pi - \gamma_i^2 \Pi = -\frac{Idl}{j2\pi\omega\epsilon_0} \delta(\rho) \delta(z-h), \quad (2)$$

where

$$\Pi = (0, 0, \Pi_z) \quad (3)$$

and  $\gamma_i, i = 0, 1$  are the complex propagation constants of mediums, given as,

$$\begin{aligned} \gamma_0^2 &= j\omega\mu_0(j\omega\epsilon_0) \quad z > 0 \\ \gamma_1^2 &= j\omega^2\mu_0\epsilon_0\mu_r(S|\epsilon_r| + j\epsilon_r) \quad z < 0 \\ S &= \frac{\sigma}{\omega|\epsilon_r|}. \end{aligned} \quad (4)$$

With reference to the cylindrical coordinate system, the solution for  $\Pi$  involves linear superposition of the eigenfunctions:

$$\Pi_z = \exp \left[ \pm (\lambda^2 + \gamma^2)^{1/2} z \right] \begin{cases} \cos(n\phi) J_n(\lambda\rho) \\ \sin(n\phi) Y_n(\lambda\rho) \end{cases} \quad (5)$$

where  $\lambda$  denotes the eigenvalue. Since the solutions should be finite at  $\rho = 0$  (except at the source) and the fields have to be symmetrical about the  $z$  axis, thus, the eigenfunctions would be of the following form with no  $\phi$  dependency:

$$J_0(\lambda\rho) \exp \left[ \pm (\lambda^2 + \gamma^2)^{1/2} z \right]. \quad (6)$$

Therefore,

$$\Pi_z = \frac{Idl}{j4\pi\omega\mu_0} \begin{cases} \int_0^\infty J_0(\lambda\rho) (e^{-u_0|z-h|} + R_e e^{-u_0(z+h)}) \frac{\lambda d\lambda}{u_0}, & z > 0 \\ \int_0^\infty T_e J_0(\lambda\rho) e^{u_1 z} d\lambda, & z < 0 \end{cases} \quad (7)$$

where  $R_e$  and  $T_e$  are the unknown reflection and transmission coefficients, respectively. The electromagnetic fields can be obtained as,

$$\begin{aligned} E &= \nabla(\nabla \cdot \Pi) - \gamma^2 \Pi \\ H &= \hat{y} \nabla \times \Pi \end{aligned} \quad (8)$$

where  $\hat{y} = \sigma + j\omega\varepsilon$ . Substituting for  $\Pi$  from (7) and applying the appropriate boundary conditions at  $z = 0$ , the following expressions for the unknown coefficients are obtained,

$$\begin{aligned} R_e &= \frac{\frac{\hat{y}_1}{\hat{y}_0}u_0 - u_1}{\frac{\hat{y}_1}{\hat{y}_0}u_0 + u_1} \\ T_e &= \frac{2\lambda}{\frac{\hat{y}_1}{\hat{y}_0}u_0 + u_1} e^{-u_0 h} \end{aligned} \quad (9)$$

where

$$u_i = (\gamma_i^2 + \lambda^2)^{1/2}, \quad i = 0, 1 \quad (10)$$

with  $\text{Re}(u_i > 0)$ . The scattering fields are, then, determined from (8). Now, after some mathematical manipulations of (1), the normalized impedance variation for VED can be found as,

$$\frac{\Delta Z}{R_{e0}} = \frac{3}{2\gamma_0^3} \int_0^\infty R_e e^{-2u_0 h} \frac{\lambda^3}{u_0} d\lambda \quad (11)$$

where  $R_e$  is given by (9) and  $R_{e0} = 20(k_0 dl)^2$  is radiation impedance of the electrical dipole in free space.

## 2.2. VMD

The input impedance variation for VMD can be calculated from [18],

$$\Delta Z = j\omega\mu_0 \lim_{\substack{z \rightarrow z_0 \\ \rho \rightarrow 0}} \frac{H_z^s dA}{I_m}. \quad (12)$$

In a same manner as the VED case, the scattering fields can be found by solving the following wave equation for Hertzian magnetic potential,

$$\nabla^2 \Pi^* - \gamma_i^2 \Pi^* = -\frac{I_m dl}{j2\pi\mu_0\omega} \delta(\rho) \delta(z-h), \quad (13)$$

with

$$\Pi^* = (0, 0, \Pi_z^*). \quad (14)$$

The solution is,

$$\Pi_z^* = \frac{IdA}{4\pi} \begin{cases} \int_0^\infty J_0(\lambda\rho) (e^{-u_0|z-h|} + R_m e^{-u_0(z+h)}) \frac{\lambda d\lambda}{u_0}, & z > 0 \\ \int_0^\infty T_m J_0(\lambda\rho) e^{u_1 z} d\lambda, & z < 0 \end{cases} \quad (15)$$

The electromagnetic fields are, then, found from the following equations:

$$\begin{aligned} E &= -\hat{z}\nabla \times \Pi^* \\ H &= \nabla(\nabla \cdot \Pi^*) - \gamma^2 \Pi^* \end{aligned} \quad (16)$$

where  $\hat{z} = j\omega\mu$ . Substituting from (15) and applying the appropriate boundary conditions at  $z = 0$ , the unknown coefficients are obtained as,

$$\begin{aligned} R_m &= \frac{\frac{\hat{z}_1}{\hat{z}_0}u_0 - u_1}{\frac{\hat{z}_1}{\hat{z}_0}u_0 + u_1}, \\ T_m &= \frac{2\lambda}{\frac{\hat{z}_1}{\hat{z}_0}u_0 + u_1} e^{-u_0 h}. \end{aligned} \quad (17)$$

Upon calculation of the scattering fields from (16), the impedance variation of VMD can be obtained as,

$$\frac{\Delta Z}{R_{m0}} = \frac{3}{2\gamma_0^3} \int_0^\infty R_m e^{-2u_0 h} \frac{\lambda^3}{u_0} d\lambda \quad (18)$$

where  $R_m$  is given by (17) and  $R_{m0} = 20(k_0^2 dA)^2$  is the free space radiation impedance of the magnetic dipole. This expression is similar to (11) for impedance variation of VED.

### 3. EXACT SOLUTION OF INTEGRAL WITH APPROXIMATED INTEGRAND

Regarding to the similarity of (11) and (18) for VED and VMD impedance variations, respectively, these integrals can be written in a general form,

$$\frac{\Delta Z}{R_0} = \frac{3}{2\gamma_0^3} \int_0^\infty R e^{-2u_0 h} \frac{\lambda^3}{u_0} d\lambda \quad (19)$$

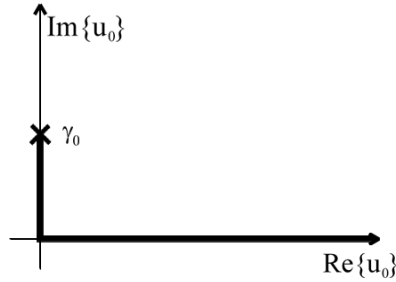
defining  $R$  as,

$$R = \frac{eu_0 - u_1}{eu_0 + u_1} \quad (20)$$

where  $e$  is equal to  $\hat{y}_1/\hat{y}_0$  and  $\hat{z}_1/\hat{z}_0$  for VED and VMD cases, respectively, and

$$R_0 = \begin{cases} R_{e0} = 20(k_0 dl)^2, & \text{VED} \\ R_{m0} = 20(k_0^2 dA)^2, & \text{VMD} \end{cases} \quad (21)$$

Now, changing the variable of integration to  $u_0$ , the contour of integration runs along the imaginary axis from  $\gamma_0$  to origin and then



**Figure 2.** The integration path in  $u_0$  plane.

along the real axis to  $\infty$ , as shown in Fig. 2. There exists another integration path which starts from  $-\gamma_0$  to 0 on the imaginary axis and proceeds to  $-\infty$  on the real axis. However, this is an improper path, because it results in exponentially growing results. Therefore, (19) becomes,

$$\frac{\Delta Z}{R_0} = \frac{3}{2\gamma_0^3} \int_{\gamma_0}^{\infty} R(u_0^2 - \gamma_0^2) e^{-2u_0 h} du_0. \quad (22)$$

Defining,

$$\alpha_0 \triangleq (u_1^2 - u_0^2)^{\frac{1}{2}} = (\gamma_1^2 - \gamma_0^2)^{\frac{1}{2}}, \quad (23)$$

so that,

$$\alpha_0 = [\omega^2 \mu_0 \varepsilon_0 (1 - \mu_r \varepsilon_r) + j\omega^2 \varepsilon_0 (\mu_r |\varepsilon_r| S)]^{\frac{1}{2}}, \quad (24)$$

we may rewrite (20)) as,

$$R = \frac{1}{e^2 - 1} \left[ \frac{(e^2 + 1) u_0^2 + \alpha_0^2}{u_0^2 - g^2} - 2e \frac{u_0 u_1}{u_0^2 - g^2} \right], \quad (25)$$

where  $g^2 = \alpha_0^2/(e^2 - 1)$ . To approximate the Fresnel reflection coefficient, we use the first order binomial expansion for  $u_1$  in (25) as,

$$u_1 = (u_0^2 + \alpha_0^2)^{\frac{1}{2}} = (u_0 + \alpha_0) \left( 1 - \frac{2u_0 \alpha_0}{(u_0 + \alpha_0)^2} \right)^{\frac{1}{2}}, \quad (26)$$

$$u_1 \simeq (u_0 + \alpha_0) \left( 1 - \frac{u_0 \alpha_0}{(u_0 + \alpha_0)^2} \right), \quad (27)$$

This is the same approximation as in [19] for DPS half-space. The convergence condition for binomial expansion in (25) is,

$$F_1 \triangleq \left| \frac{2u_0 \alpha_0}{(u_0 + \alpha_0)^2} \right| < 1. \quad (28)$$

When both  $\varepsilon_r$  and  $\mu_r$  are positive (DPS), according to (10) and (24),  $\alpha_0$  is in the first quadrant. Therefore, the real and imaginary parts of  $u_0$  and  $\alpha_0$  are positive and the condition of (28) is always satisfied. However, for the metamaterial half-space, this condition may be violated. In this case the real and imaginary parts of  $\alpha_0^2$  may become negative, resulting in  $F_1 > 1$ .

Next, we investigate the proper expressions for  $u_1$  for the ENG, MNG, and DNG metamaterials, individually. For ease of explanation, we separate the path of integration into two parts. The first path runs along the imaginary axis from  $\gamma_0$  to the origin and the second path goes along the real axis from origin to  $\infty$ .

- ENG half-space ( $\varepsilon_r < 0$  and  $\mu_r > 0$ ): In this case, according to (24), both real and imaginary parts of  $\alpha_0^2$  are positive. Therefore, the expression (27) for  $u_1$  is still valid.
- MNG half-space ( $\varepsilon_r > 0$  and  $\mu_r < 0$ ): According to (24),  $\alpha_0^2$  is in the forth quadrant and, thus,  $\alpha_0$  would be in the second and/or forth quadrant. If  $\alpha_0$  is chosen to be in the second quadrant, although, on the first path  $F_1 < 1$ , but along the second path  $F_1 > 1$ . Therefore, in this case, the proposed expansion of (27) diverges and would be an improper choice.

To resolve this divergence problem, we rewrite the first order binomial expansion of  $u_1$  in the following form,

$$u_1 = (u_0^2 + \alpha_0^2)^{\frac{1}{2}} = (u_0 - \alpha_0) \left( 1 + \frac{2u_0\alpha_0}{(u_0 - \alpha_0)^2} \right)^{\frac{1}{2}}, \quad (29)$$

$$u_1 \simeq (u_0 - \alpha_0) \left( 1 + \frac{u_0\alpha_0}{(u_0 - \alpha_0)^2} \right). \quad (30)$$

Therefore, the convergence condition for the binomial expansion is,

$$F_2 \triangleq \left| \frac{2u_0\alpha_0}{(u_0 - \alpha_0)^2} \right| < 1. \quad (31)$$

Figure 3 shows the values of proper and improper expansions on the second path at two different frequencies. It is obvious from this figure that in this case  $F_2 < 1$  and so the binomial expansion converges. The other way to resolve the problem is choosing  $\alpha_0$  in the forth quarter and using the same Equation (27) that was used for ENG and DPS cases.

- DNG half-space ( $\varepsilon_r < 0$  and  $\mu_r < 0$ ): For both negative  $\mu_r$  and  $\varepsilon_r$ ,  $\alpha_0^2$  is located in the third quadrant. Therefore,  $\alpha_0$  is located in the second and/or forth quadrant and the same procedure as MNG can be used for DNG.

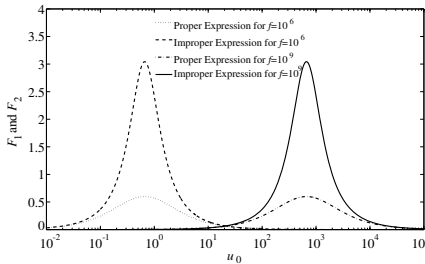


Substituting (30) in (25) and the result in (22), and after some mathematical manipulations, based on the desired values of  $e$ , the solution of the integral for VED and VMD is obtained as,

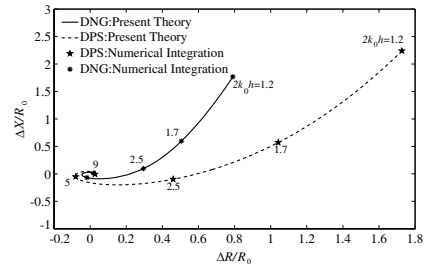
$$\frac{\Delta Z}{R_0} = \frac{3}{2\gamma_0^3} \left[ A \frac{\partial^2 F}{\partial x^2} + BF + CI_1 + DI_2 + EI_3 \right] \quad (32)$$

where

$$\begin{aligned} A &= \frac{e-1}{e+1} \\ B &= \frac{(g^2 - \gamma_0^2)(e-1)^2 + \alpha_0^2(1-2e)}{e^2 - 1} \\ C &= \frac{2e}{e^2 - 1} \frac{\alpha_0^3(\alpha_0^2 - \gamma_0^2)}{(\alpha_0^2 - g^2)} \\ D &= \frac{-\alpha_0^2\gamma_0^2 + g^2[\alpha_0^2 + g^2 - \gamma_0^2(e^2 + 1)]}{2g(e^2 - 1)} \\ &\quad - \frac{2e}{e^2 - 1} \frac{(g^2 - \gamma_0^2)[\alpha_0^2 + g(\alpha_0 + g)]}{2(g + \alpha_0)} \\ E &= \frac{\alpha_0^2\gamma_0^2 - g^2[\alpha_0^2 + g^2 - \gamma_0^2(e^2 + 1)]}{2g(e^2 - 1)} \\ &\quad - \frac{2e}{e^2 - 1} \frac{(g^2 - \gamma_0^2)[- \alpha_0^2 + g(\alpha_0 - g)]}{2(g - \alpha_0)}, \end{aligned} \quad (33)$$



**Figure 3.** Convergence condition.



**Figure 4.** Verification of proposed expressions for VED over the half-space with  $|\epsilon_r| = 10$ ,  $|\mu_r| = 10$  and  $S = 10$ .

and

$$\begin{aligned}
 F &= \frac{e^{-\gamma_0 x}}{x} \\
 I_1 &= \exp(\alpha_0 x) E_1((\gamma_0 + \alpha_0) x) \\
 I_2 &= \exp(-gx) E_1((\gamma_0 - g) x) \\
 I_3 &= \exp(gx) E_1((\gamma_0 + g) x)
 \end{aligned} \tag{34}$$

where  $E_1$  is the exponential integral. In derivation of (33), we have used the following integral identities [24],

$$E_1(\gamma_0) = \int_{\gamma_0}^{\infty} \frac{e^{-x}}{x} dx \tag{35}$$

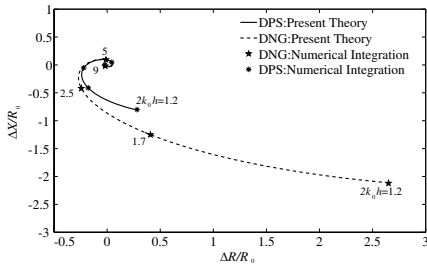
$$\int_{\gamma_0}^{\infty} u^n e^{-u_0 x} du_0 = (-1)^n \frac{\partial^n}{\partial x^n} \left( \frac{e^{-u_0 x}}{x} \right). \tag{36}$$

#### 4. NUMERICAL RESULTS AND DISCUSSION

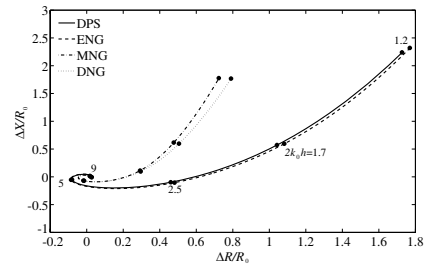
In order to verify the validity of our proposed approximate method, we solve (11) and (18) numerically and compare the results with those of (32) for VED and VMD. The impedance variation is expressed in terms of a resistive variation,  $\Delta R$ , and a reactance variation,  $\Delta X$ , in the usual manner as:  $\Delta Z = \Delta R + j\Delta X$ . These parameters are normalized with respect to  $R_0$ , which is the radiation resistance of the dipole when it is located in the free space. In what it follows, the results for the normalized values of  $\Delta R$  and  $\Delta X$  are obtained for various values of parameter  $2k_0 h$  for  $1.2 < 2k_0 h < 9$ . Argand plots are used to show the impedance variations of VED and VMD located over DSP, ENG, MNG, and DNG half-spaces.

Figures 4 and 5 compare the results for VED and VMD over DPS and DNG materials with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 10$  and  $S = 10$ , respectively. Although, as it is seen that the results are in complete agreement, the proposed method is much faster than the numerical integration. As an example, we mention that the computation time for 80 impedances, on a computer with Intel(R) Core(TM) 2 Duo CPU E7500 @ 2.93 GHz 2.95 GHz, for numerical integrations was 91.8 seconds. However, for the proposed algorithm this computation time was decreased to 0.12 seconds, a reduction of about 765 times in CPU time!

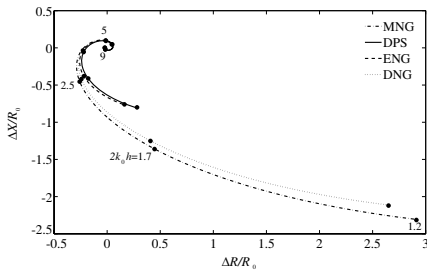
Figures 6 and 7 show the impedance variations for VED and VMD located above DPS, DNG, MNG and ENG materials for the same elevation range of dipoles and absolute values of permittivity and permeability. It is seen that the variation ranges of impedances are highly dependent on the sign of permeability.



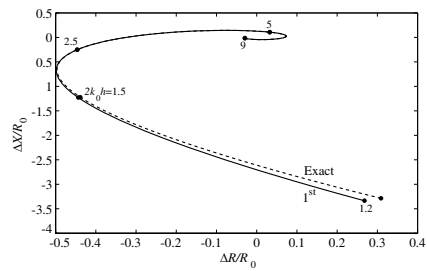
**Figure 5.** Verification of proposed expressions for VMD over half-space with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 10$  and  $S = 10$ .



**Figure 6.** Impedance variations of VED above various materials with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 10$  and  $S = 10$ .

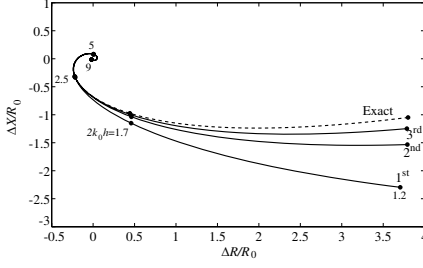


**Figure 7.** Impedance variations of VMD above various materials with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 10$  and  $S = 10$ .

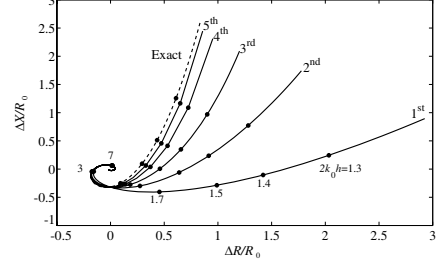


**Figure 8.** Comparison of exact and 1st order binomial expansion results for half-space with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 1$  and  $S = 10$ .

The accuracy of algorithm is highly influenced by the material loss, since the convergence rate of the series expansions are directly dependent on the amount of loss. Therefore, as the loss decreases, inclusion of higher order terms are required. For VMD over DNG metamaterial with  $|\varepsilon_r| = 10$ ,  $|\mu_r| = 1$  and higher loss ( $S = 10$ ), the first order approximation of (30) is adequate for accurate calculation of (19), as shown in Fig. 8. For low loss cases of  $S = 1$  and  $S = 0.01$ , as shown in Figs. 9 and 10, respectively, it is seen that the first order approximation for the binomial expansion is not sufficient for accurate solution of integrals, although the results may agree. It is seen that for  $S = 1$ , three terms expansion is sufficient, while for  $S = 0.01$ , at least 5 terms is required. Obviously, using more terms of series for integrand approximations increases the computation time. For example, for 5 terms approximation, the computation times increases to 11.07 seconds.



**Figure 9.** Comparison of exact results with those of binomial expansion for half-space with  $\varepsilon_r = 10$ ,  $|\mu_r| = 1$  and  $S = 1$ .



**Figure 10.** Comparison of exact results with those of binomial expansion for half-space with  $\varepsilon_r = 10$ ,  $|\mu_r| = 1$  and  $S = 0.01$ .

Based on scrutinized and presented diagrams, it is obvious that locating a dipole above half-space of arbitrary characteristics would cause some variations in its input impedance. The additional impedance, regardless of being real or imaginary, is illustrative of losses. However, it is important to investigate that which one of the half-space materials introduces lower amount of losses. We perform this study for VED and VMD cases, separately.

Referring to Fig. 6, it can be observed that there is not much variation in input impedance of VED, when it is located above DPS and ENG half-spaces. However, if for the lower half-space a material with negative permeability is used, the diagram of impedance would be contracted. This means that lower amount of losses would be introduced to the structure for the cases of MNG and DNG.

As it can be seen from Fig. 7, when VMD is situated above the half-space of DPS or ENG, as for the VED case, the difference in input impedances is negligible. However, if the lower half-space is made of a material with negative permeability, the diagram of impedance, in contrast to the VED case, would be expanded. This means that if MNG or DNG metamaterial half-spaces are used, the amount of losses would be much more than the DPS or ENG cases.

This discussion reveals the importance of sign changing in permeability of the half-space material for vertical dipoles.

## 5. CONCLUSION

In this paper accurate analytical expressions for the input impedance variations of electric and magnetic dipoles, located over the half-space material with arbitrary permittivity and permeability, were developed.

The input impedance variations of VED and VMD were found in integral forms. These integrals were solved by introducing appropriate approximation for the Fresnel reflection coefficient. Accuracy of the analytical expressions was verified by numerical solution of the integrals. A complete agreement between the results was observed for the materials with any amount conductivity, regardless of the operating frequency and elevation height of dipoles. However, for very low loss half-space materials, higher order approximations were required for likeness of the results.

Finally, the relationship between the type of half-space materials and introduced losses to the structure were carefully investigated. It was found out that for VED over MNG and DNG half-spaces, in comparison with DPS case, the amount of the introduced losses to the structure were lower, while it was higher for the case of VMD.

The proposed method of analysis would found its applications in future works on microstrip structures involving metamaterial substrates.

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