COMPARISON OF SURFACE INTEGRAL EQUATIONS FOR LEFT-HANDED MATERIALS

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Abstract—A wide analysis of left-handed material (LHM) spheres with different constitutive parameters has been carried out employing different integral-equation formulations based on the Method of Moments. The study is focused on the accuracy assessment of formulations combining normal equations (combined normal formulation, CNF), tangential equations (combined tangential formulation, CTF, and Poggio-Miller-Chang-Harrington-Wu-Tsai formulation, PM-CHWT) and both of them (electric and magnetic current combined field integral equation, JMCFIE) when dealing with LHM's. Relevant and informative features as the condition number, the eigenvalues distribution and the iterative response are analyzed. The obtained results show up the suitability of the JMCFIE for this kind of analysis in contrast with the unreliable behavior of the other approaches.

1. INTRODUCTION

Left-handed materials (LHM's), also known as double negative (DNG) materials, were already theoretically proposed in 1968 [1], though their negative refraction was not experimentally demonstrated until 2000 [2]. Many theoretical and experimental studies focused on these artificial structures have been developed in last years due to their wide range of applications and potential capabilities [3, 4].

The electromagnetic numerical analysis of LHM has been usually tackled by means of volumetric differential-equation formulations [5–7], but the limited applicability of these techniques to realistic large

problems is making way for other alternatives. Surface integral equations (SIE) have been extensively used for solving scattering problems involving homogeneous or piecewise homogeneous dielectric objects [8–15]. Recent works have shown that their application may be extended to the homogeneous LHM analysis [16–19].

In this work, the well-known Method of Moments (MoM) SIE formulation [20] is applied to predict the electromagnetic scattering of homogeneous LHM. The most popular combined formulations that traditionally appear in the literature for analyzing dielectric objects are employed here for DNG materials with the aim of checking their accuracy when the matrix system is solved either in a direct form or by iterative techniques.

2. INTEGRAL EQUATION FORMULATIONS

When dealing with homogeneous dielectric materials, it is usual to consider the combination of normal and tangential equations derived from the boundary conditions imposed separately to the electric and magnetic fields, namely, the tangential electric field integral equation (T-EFIE), the tangential magnetic field integral equation (T-MFIE), the normal electric field integral equation (N-EFIE) and the normal magnetic field integral equation (N-MFIE). Among the multiple possibilities of combination of these equations, the following one has proven to be a stable proposal [10]:

$$\sum_{l=1}^{2} a_l \frac{1}{\eta_l} \text{T-EFIE}_l + \sum_{l=1}^{2} b_l \text{N-MFIE}_l \tag{1}$$

$$-\sum_{l=1}^{2} c_l \text{N-EFIE}_l + \sum_{l=1}^{2} d_l \eta_l \text{T-MFIE}_l \tag{2}$$

In Equations (1) and (2), η_l is the intrinsic impedance in medium R_l (R_1 and R_2 are the exterior and the interior regions of the material, respectively). Different known formulations can be obtained depending on the selection of the complex combination parameters a_l, b_l, c_l and d_l (see Table 1). The comparative study presented here involves the formulations known as Poggio-Miller-Chang-Harrington-Wu-Tsai (PMCHWT) [8], combined tangential formulation (CTF), combined normal formulation (CNF) [10] and electric and magnetic current combined field integral equation (JMCFIE) [9]. PMCHWT and CTF formulations combine only tangential equations, CNF combines only normal equations and JMCFIE combines both tangential and normal equations. The details concerning the extension of these formulations

Table 1. Parameters for combining equations corresponding to each formulation considered in the comparative study.

Formulation	a_l	b_l	c_l	d_l
JMCFIE	1	1	1	1
CNF	0	1	1	0
CTF	1	0	0	1
PMCHWT	η_l	0	0	$1/\eta_l$

to the analysis of LHM can be looked up in [18]. Their suitability for dealing with LHM will be tested next.

3. COMPARATIVE STUDY

A parametric analysis has been carried out using the MoM-based integral formulation in order to extract interesting conclusions about the accuracy of the combined formulations for analyzing metamaterials with different constitutive parameters (ϵ_r, μ_r). Two different sized spheres were chosen to accomplish the comparative study. The availability of the Mie's series reference result supports the selection of this target [21].

The first section presents the analysis of several discretizations of a sphere with radius $r = \lambda_0$, where λ_0 is the wavelength of the surrounding free-space medium. The main objective is to assess the precision of the radar cross section (RCS) prediction provided by each formulation for different interest cases. The second section deals with the study of the system matrix main features and each formulation response in the framework of an iterative solution scheme. A smaller sphere with $k_0 r = 3$, where $k_0 = 2\pi/\lambda_0$ is the free-space wave number, has been selected for this purpose in order to make lighter the computations involving the impedance matrix.

3.1. Accuracy Assessment

Initially, a sphere with $r = \lambda_0$, $\epsilon_r = -3$ and $\mu_r = -1$ was considered. The sphere has been modeled throughout 17 340 unknowns (including both electric and magnetic surface currents) employing the Rao-Wilton-Glisson (RWG) basis functions [22], resulting in a mean discretization size of approximately $\lambda_0/14$. An incident plane wave with horizontal $\hat{\phi}$ polarization and incidence angles $\phi = 0°$

and $\theta = 90°$ has been considered. The bistatic $\phi\phi$-RCS of the sphere has been obtained employing each one of the four formulations previously mentioned. The results are shown in Fig. 1 together with the reference solution provided by the Mie's series. The Christian Mätzler's implementation of the Mie's series for penetrable spheres was applied to compute the reference scattering parameters [23, 24]. It can be observed a nearly perfect agreement between the reference solution and the results of all the formulations analyzed. Since the RCS curves are almost totally overlapped, the root mean square (RMS) error of the RCS calculation has been evaluated for several sphere discretizations looking for an insight into the results. The error curves corresponding to this example and calculated for a mesh size varying from $\lambda_0/5.8$ to $\lambda_0/24$ are also shown in the bottom of Fig. 1, where the fourth abscissa sample corresponds to the $\lambda_0/14$ discretization used in the previous RCS representation. Though small error values and stable curves have been achieved, the error does not decrease further as the mesh is refined as could be expected, especially for tangential formulations.
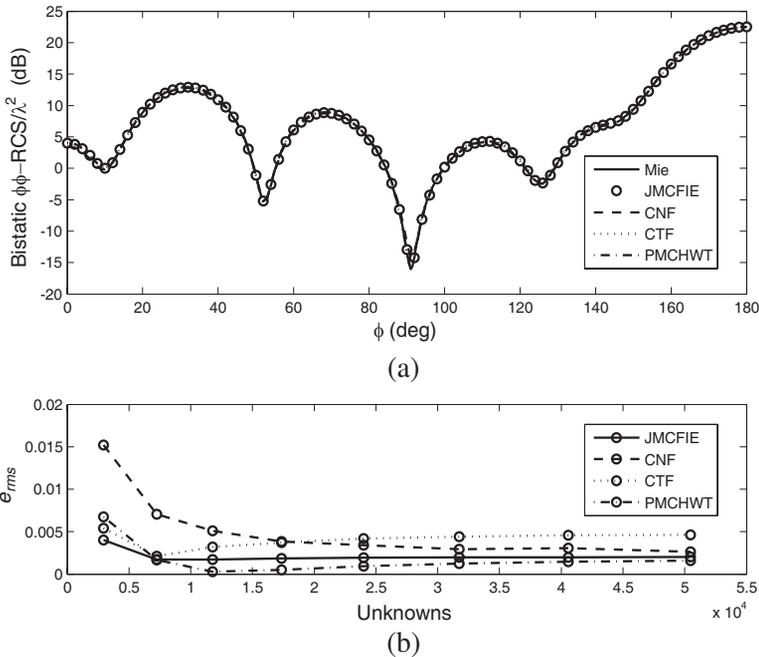

Figure 1. (a) $\phi\phi$-RCS of a λ_0 radius sphere with $\epsilon_r = -3$ and $\mu_r = -1$ obtained with different formulations in contrast with the Mie's series reference solution; (b) RMS-error vs. number of unknowns.
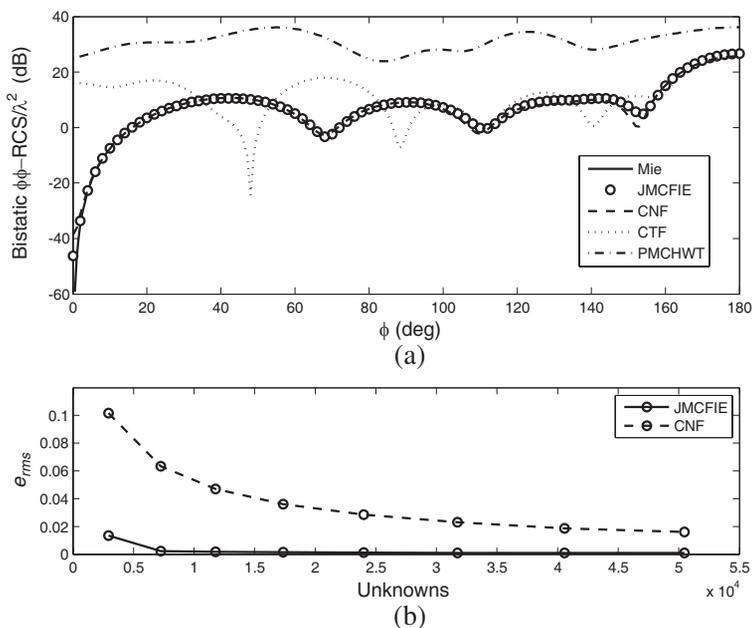
Figure 2. (a) $\phi\phi$-RCS of a λ_0 radius sphere with $\epsilon_r = -1$ and $\mu_r = -1$ obtained with different formulations in contrast with the Mie's series reference solution; (b) RMS-error vs. number of unknowns.

In the following example the sphere has been modeled as LHM matched to free-space, i.e., $\epsilon_r = -1$ and $\mu_r = -1$, so that the index of refraction is $n = -1$. This particular case has attained great relevance since Pendry [25] predicted that DNG metamaterial slabs with this refraction index could act as "perfect lens". Later works [26] have found that this concept does not exist in general for any realistic dispersive, lossy double negative medium, but it keeps an indisputable interest as a study case. The results of Fig. 2 show that, in this case, only the JMCFIE formulation fits properly the RCS prediction given by the Mie's series reference. A quite good agreement can be appreciated in the case of the CNF approach whereas significant discrepancies with the reference solution occur when using formulations combining only the tangential equations (PMCHWT and CTF). Since high and erratic RCS error values have been obtained for the latter cases, only the error curves corresponding to the JMCFIE and CNF formulations have been depicted in Fig. 2.

The erroneous results predicted by the tangential formulations in Fig. 2 are due to cancellations appearing in the formulations when dealing with lossless matched LHM objects. Under these circumstances, the inner and outer integral operators are complex

conjugate of each other, resulting in a zero imaginary part of the impedance matrix both in PMCHWT and CTF. This implies cancelation of main singularities in the operators and loss of information, which in turns results in extremely high condition numbers (see Table 2). Similarly, the main real part of the CNF matrix also becomes zero with the exception of the entries for the identity operator, thus giving a nearly diagonal matrix. The latter results in a slightly better behavior of this formulation in this case with respect to the tangential ones (a further explanation from the point of view of the eigenvalues distribution is given in the next section.) In contrast, the JMCFIE formulation retains both the real and the imaginary parts of the impedance matrix, resulting in lower condition numbers. Besides, this formulation is well tested with the RWG functions for both electric and magnetic currents even at resonance frequencies [27], which could also be determinant for its stability in problems involving lossless matched LHM objects.

A better understanding of each formulation behavior can be derived from Fig. 3, where the previous two examples have been analyzed involving a lossy term. Let us consider first the sphere with $\epsilon_r = -3 - 0.3i$ and $\mu_r = -1$. The RMS error representation at the top of the figure shows that all the formulations improve their results to a greater or lesser extent with regard to the lossless analogous
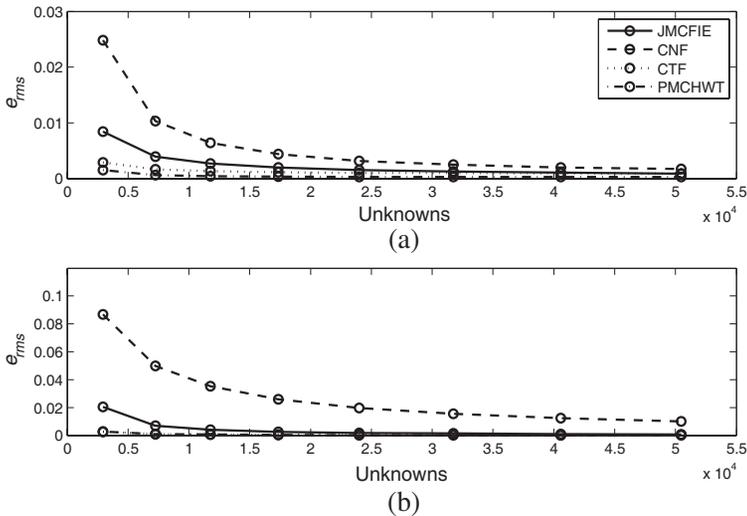

Figure 3. RMS-error vs. number of unknowns of a λ_0 radius sphere: (a) $\epsilon_r = -3 - 0.3i$ and $\mu_r = -1$; (b) $\epsilon_r = -1 - 0.3i$ and $\mu_r = -1$.
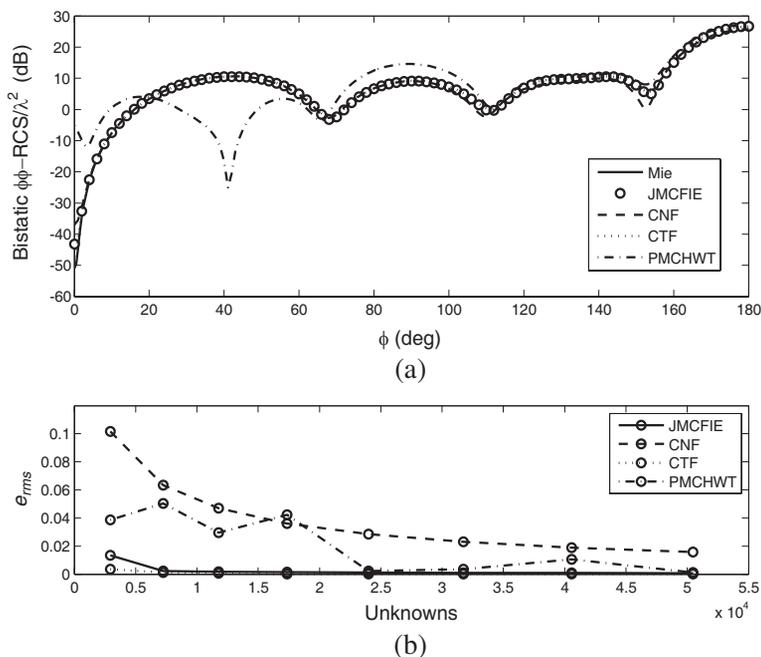
Figure 4. (a) $\phi\phi$-RCS of a λ_0 radius sphere with $\epsilon_r = -1 - 0.001i$ and $\mu_r = -1$ obtained with different formulations in contrast with the Mie's series reference solution; (b) RMS-error vs. number of unknowns.

case. Smooth error curves decreasing with the mesh refinement have been obtained. The improvement effect becomes magnified when comparing the error progress from constitutive parameters $\epsilon_r = -1$ and $\mu_r = -1$ (bottom of Fig. 2) to $\epsilon_r = -1 - 0.3i$ and $\mu_r = -1$ (bottom of Fig. 3). This is especially remarkable for CTF and PMCHWT formulations, for which stable error curves described by small error values have been obtained when losses are present, providing more accurate results than the CNF and JMCFIE formulations for these cases. This contrasts with the chaotic and out of scale error values achieved in the lossless assumption. What is more, the simulations have demonstrated that a noticeable improvement of the tangential formulations performance is achieved even with really a small amount of losses such as $\epsilon_r = -1 - 0.001i$. The results are shown in Fig. 4. On one hand, the comparison of the error values of Figs. 3 and 4 illustrates that extremely small losses seem to be enough to provide the same degree of improvement of the CTF results than that obtained with greater losses. On the other hand, Fig. 4 also shows that the

PMCHWT behavior starts to regularize after introducing a small amount of losses although the accuracy in the RCS prediction is still poor. This is explained by the fact that the condition numbers in the PMCHWT are always larger than the CTF, because the matrix blocks in the former are very unbalanced (in addition to being a first kind integral equation). This is improved in the CTF with the proper combination coefficients [10]. Regarding the normal formulation error level, it barely changes from $\epsilon_r = -1$ to $\epsilon_r = -1 - 0.001i$.

3.2. Iterative Performance

The iterative solution of the matrix system and the analysis of some relevant parameters of the impedance matrix can provide useful information in order to extract conclusions about the main features of the combined integral formulations. With this purpose, a restarted GMRES [28] has been selected as iterative solver with a restart value of 30 and a limit of 500 external iterations if the desired tolerance of 10^{-6} is not achieved. A sphere with $k_0 r = 3$, smaller than the previous one, has been employed to get a reasonable matrix size for carrying out an exhaustive analysis of the eigenvalues arrangement, the condition number, the convergence of the iterative scheme and the accuracy of the RCS prediction.

The same five mediums of Section 3.1 with a fixed permeability of $\mu_r = -1$ and different permittivity values have been considered in this analysis. Their results are gathered in the five separate blocks of Table 2 and they are discussed next.

A good agreement with Mie's reference has been achieved for the first assumption of $\epsilon_r = -3$ when checking JMCFIE, CNF and CTF formulations. However, the PMCHWT high error rate attained when using the iterative solver strongly differs with the 2% that would be obtained by means of direct solving. With regard to the convergence speed, CNF and PMCHWT behave worse and their condition numbers are higher. The slow convergence of the PMCHWT iterative solution observed in this case and in all the LHM proposals studied in this work was already reported in [29] for homogeneous dielectric objects. The improved results of the other tangential formulation, the CTF approach, point out again the importance of the adequate selection of the combination parameters (a_l and d_l in this case).

The second block of Table 2 collects the data obtained for the matched case. The JMCFIE formulation shows fast convergence, high accuracy in the RCS prediction and the lowest condition number with regard to the rest of combined formulations. Despite the CNF formulation seems to converge in terms of the iterative solver, a limited accuracy has been obtained. It is worth mentioning that the low

Table 2. 2-norm condition number (c) of the impedance matrix, number of GMRES iterations (i) for residue $< 10^{-6}$ and RMS error (e_{rms}) of the RCS calculation versus Mie's result for a $k_0 r = 3$ sphere with $\mu_r = -1$ and different ϵ_r values.

$\mathbf{\epsilon_r = -3}$	c	i	e_{rms}
JMCFIE	$2.0 \cdot 10^6$	7	2.1%
CNF	$3.8 \cdot 10^7$	$(7.5 \cdot 10^{-5})^*$	1.6%
CTF	$1.1 \cdot 10^6$	12	1.9%
PMCHWT	$1.0 \cdot 10^9$	$(1.8 \cdot 10^{-3})^*$	22.3%
$\mathbf{\epsilon_r = -1}$	c	i	e_{rms}
JMCFIE	$8.4 \cdot 10^6$	4	0.56%
CNF	$1.0 \cdot 10^7$	5	7.6%
CTF	$3.9 \cdot 10^{12}$	$(4.1 \cdot 10^{-3})^*$	2.3%
PMCHWT	$1.1 \cdot 10^{13}$	$(1.2 \cdot 10^{-3})^*$	20.7%
$\mathbf{\epsilon_r = -3 - 0.3i}$	c	i	e_{rms}
JMCFIE	$1.6 \cdot 10^6$	5	1.3%
CNF	$1.7 \cdot 10^7$	32	0.97%
CTF	$1.0 \cdot 10^6$	10	1.8%
PMCHWT	$2.5 \cdot 10^7$	$(1.4 \cdot 10^{-3})^*$	6.0%
$\mathbf{\epsilon_r = -1 - 0.3i}$	c	i	e_{rms}
JMCFIE	$2.7 \cdot 10^6$	3	0.98%
CNF	$8.3 \cdot 10^6$	5	6.6%
CTF	$7.1 \cdot 10^6$	15	0.37%
PMCHWT	$2.0 \cdot 10^8$	$(1.0 \cdot 10^{-4})^*$	0.68%
$\mathbf{\epsilon_r = -1 - 0.001i}$	c	i	e_{rms}
JMCFIE	$8.3 \cdot 10^6$	4	0.57%
CNF	$1.0 \cdot 10^7$	5	7.6%
CTF	$1.0 \cdot 10^9$	$(2.3 \cdot 10^{-4})^*$	0.43%
PMCHWT	$9.2 \cdot 10^{12}$	$(7.2 \cdot 10^{-5})^*$	0.51%
*Residue after 500 external iterations			

error value achieved with the CTF formulation employing an iterative scheme can be deceptive and could lead to a wrong interpretation of this formulation performance. In fact, the direct solution of the CTF formulation for this test case provides a poor result and a high error rate (around 25%) according to the behavior observed in the $1\lambda_0$ radius

sphere analysis of Fig. 2. There is a strong bond between this unreliable behavior of the CTF formulation and the high condition number of its system matrix shown in Table 2. Also the PMCHWT error shoots up absolutely in this particular case when direct solving is applied.

Looking at third and fourth blocks of Table 2 it is clear that the condition number, the convergence rate and the precision become better when significant losses are involved. The iterative convergence of PMCHWT is still bad but, as occurred in the previous unmatched lossless case ($\epsilon_r = -3$, $\mu_r = -1$), direct solving by means of this formulation in these lossy cases gives error rates comparable to CTF data. Also, it is worth pointing out that, in these cases, CTF and PMCHWT tangential formulations provide better accuracy than CNF and JMCFIE.

Last block of Table 2 shows the results obtained for $\epsilon_r = -1 - 0.001i$. While JMCFIE and CNF formulations maintain almost invariable the response given for the ideal matched case, an evident improvement of the CTF and PMCHWT iterative parameters can be observed. The direct solution of the problem considering both formulations has been obtained and similar conclusions concerning the accuracy of the tangential formulations as those extracted in Section 3.1 for this epsilon value can be derived now, that is: the accuracy level (which is the same for both iterative and direct solutions) provided by the CTF formulation improves drastically as soon as a small amount of losses is considered; greater losses are required to achieve a comparable degree of improvement with the PMCHWT formulation, since in this example a slight reduction of the condition number is observed and the obtained accuracy is around 12.17%.

At this point, all this information given by the condition number and the iteration convergence will be complemented by the observation of the eigenvalues distributions. The representations of Figs. 5 and 6 together with the data of Table 2 confirm that the convergence of the iterative solutions of the proposed formulations improves as the eigenvalues move away from the origin. Paying special attention to the particular case ($\epsilon_r = -1$, $\mu_r = -1$), it can be appreciated in Fig. 6 how the eigenvalues of CTF and PMCHWT formulations are clustered near the origin according to the ill-conditioning of their system matrix indicated in Table 2. It can be also derived from the subplot displayed inside both figures that the bad iterative performance of the PMCHWT solution is connected with the expanded arrangement of its eigenvalues along the plane with respect to any other eigenvalues distribution. All these key points are along the lines of those reported in [17] and [30].

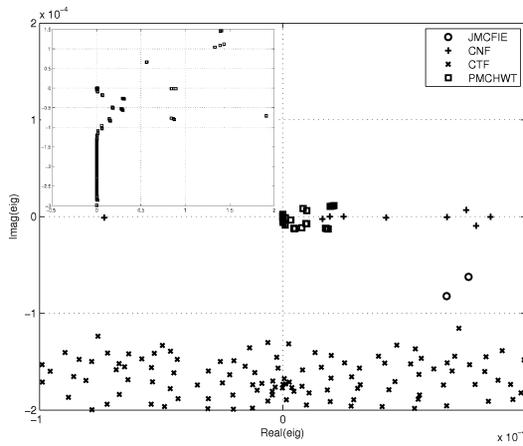To further support the conclusions drawn for the moment, a

Figure 5. Eigenvalues distributions for a $k_0 r = 3$ sphere with $\epsilon_r = -3$ and $\mu_r = -1$.
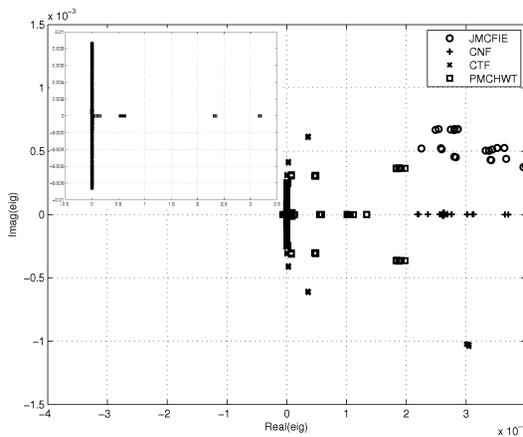

Figure 6. Eigenvalues distributions for a $k_0 r = 3$ sphere with $\epsilon_r = -1$ and $\mu_r = -1$.

different example is considered next. The interaction of a diverging Gaussian beam with the theoretical perfect lens first proposed by Pendry in [25] is analyzed. The lens consists of a 3D planar slab made of impedance matched LHM with refraction index $n = -1$. The dimensions of the slab are $7\lambda_0$ high, $7\lambda_0$ wide and $2\lambda_0$ deep. The Gaussian beam has a total angular spread far from the waist of $70°$ and is normally impinging onto the first wall of the slab from the bottom,
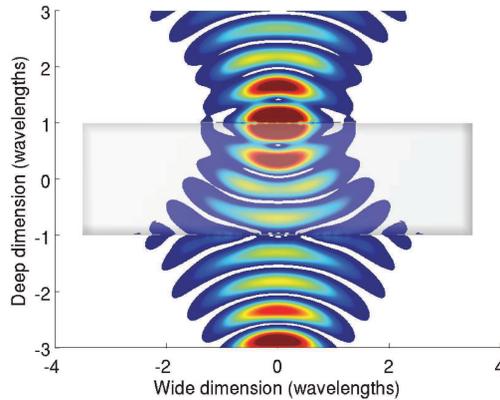
Figure 7. Horizontal cut of the electric field magnitude computed with the JMCFIE formulation for an incident diverging Gaussian beam focused by an impedance matched 3D perfect lens with $\epsilon_r = -1$ and $\mu_r = -1$.

as illustrated in Fig. 7. The incident field is horizontally polarized and the waist is at a distance of $3\lambda_0$ from the slab.

The example was solved with 23 100 unknowns for the electric and magnetic currents induced on the slab surfaces using the different considered formulations. Fig. 7 shows the magnitude of the total electric field computed with the JMCFIE in the horizontal plane. The computed field shows how the beam is interacting with the lens, where the negative angle of refraction is clearly observed: the lens bends the wave towards the beam axis with an angle that is equal (but opposite) to the incident angle at each point of the slab wall, thus focusing the divergent beam just as expected from the theoretical prediction [25]. Besides, no reflection effects are observed, which was also expected since the slab is impedance matched to the exterior medium (vacuum). The condition number of the JMCFIE matrix for this case is $7.5 \cdot 10^6$, and it took 5 extern GMRES iterations with restart 30 to reach a residue below 10^{-6}.

Regarding the other formulations, the CNF provided poorer accuracy, showing unexpected field hot spots and a noisier field distribution (the results are not shown for conciseness). The condition number for this formulation is $3 \cdot 10^7$. The tangential formulations did not converge at all for this case, showing extremely high condition numbers of $1 \cdot 10^{23}$ and $7.1 \cdot 10^{24}$ for the CTF and PMCHWT, respectively.

The remarkable issues extracted from the previous parametric

studies and analysis can be summarized as follows:

- The JMCFIE results show that the studied parameters (error level, condition number, number of required iterations) keep similar in all the examples. This fact is indicative of the JMCFIE stability, whereas great variability can be appreciated in the other formulations results.

- Only the results obtained by means of JMCFIE formulation are satisfactory in the matched case. The CNF formulation shows fast iterative convergence but it lacks accuracy despite iterative or direct solving is applied.

- In general terms, the iterative convergence of SIE gets worse as the distance from the origin to the eigenvalues decreases. We have checked that the eigenvalues move away from the origin when adding losses, so the results get better under these conditions.

- The unweighted magnitude of the combination parameters in PMCHWT formulation leads to high condition numbers and disperse eigenvalues distribution, resulting in slow convergence and inaccurate results when solving iteratively.

- The tangential formulations, PMCHWT and CTF, provide more accurate results in those cases where they are applicable. However, the JMCFIE is more reliable, in the sense that it is able to provide good results for all cases.

It must be taken into account that the results included here constitute only a representative part of an extensive set of tests that have allowed us to extract general conclusions. In addition to those listed above, we have appreciated that the formulations that have shown an unstable behavior are also more sensitive to the quality of the mesh used to model the geometry, with quality meaning a good and uniform aspect ratio of the involved triangular facets.

4. SUMMARY

The analysis of LHM's with different constitutive parameters has been carried out by means of four MoM-based integral-equation formulations. The reliability and stability of JMCFIE formulation has been demonstrated in all the test cases. Though CNF, CTF and PMCHWT formulations can provide satisfactory results in some particular cases such as those involving losses, the inaccuracy of the formulations employing only tangential or only normal equations in the matched case points out that the consideration of both types of equations is required. Nevertheless, it must be noted that the

PMCHWT and the CTF still provide more accurate results than CNF and JMCFIE formulations in the cases they can be successfully applied.

The iterative performance of the formulations has also been studied. Again, the JMCFIE proposal has shown both reliability and good convergence in contrast with the irregular behavior of the rest of formulations.

ACKNOWLEDGMENT

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