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H-INFINITY FILTER BASED PARTICLE FILTER FOR MANEUVERING TARGET TRACKING

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Abstract—In this paper, we propose a novel H-infinity filter based particle filter (H∞PF), which incorporates the H-infinity filter (H∞F) algorithm into the particle filter (PF). The basic idea of the H∞PF is that new particles are sampled by the H∞F algorithm. Since the H∞F algorithm can fully take into account the current measurements, when the new algorithm calculates the proposed probability density distribution, the sampling particles can take advantage of the system current measurements to predict the system state. The particles distribution we obtained approaches nearer to the state posterior probability distribution and the H∞PF alleviates the sample degeneracy problem which is common in the PF, especially when the maneuvers of the target tracking are large. Furthermore, the H∞F algorithm can adjust gain imbalance factor by adjusting disturbance attenuation factor, from that the new algorithm can get the compromise between the accuracy and robustness and we can obtain satisfied accuracy and robustness. Some simulations and experimental results show that the proposed particle filter performed better than the PF and the Kalman particle filter (KPF) in tracking maneuvering target.

1. INTRODUCTION

For linear or Gaussian problems of the tracking algorithm, the Kalman filter (KF) is widely used to get optimal solutions, and it can achieve good tracking performance [1, 2]. Unfortunately, many practical maneuvering target tracking problems are nonlinear or non-Gaussian. In this case, a variety of tracking algorithms have been proposed to

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evaluate the probability distribution, such as Extended Kalman filter (EKF) and Unscented Kalman filter (UKF) [3–5]. Since the PF can approach the Bayesian optimal estimate with infinite samples, it is more accurate than the EKF and UKF and is often chosen over the EKF or UKF. In recent years, the PF has attracted many researchers' attention, which are also known as sequential Monte Carlo (SMC) methods and can effectively deal with non-linear and non-Gaussian problems [6–12]. Meanwhile, a large number of tracking methods about the PF have been presented [13–15]. Although the PF has been proven successful in dealing with tracking maneuvering target, it has some disadvantages, and one of which is that employing uniform re-sampling leads to the particle impoverishment problem [16–18]. To deal with this problem in the filter, two key factors can be adopted, which are the proper selection of the proposed distribution and the re-sampling method mentioned in [19, 20]. In the light of the selection of proposed distribution, the PF algorithm uses system state transition probability as its importance density function. Since the density function, which the PF adopted, does not utilize the latest measurements to generate new particles, the result is that the produced particle samples focus on the last of the posterior probability distribution, which leads to a blind choice in the particles and makes the reduction of the filtering precision. So the PF filter usually has unsatisfactory performance, and sometimes the PF algorithm cannot be effectively utilized. For this problem, people try to find some other methods, which use latest measurements to enhance performance, to generate new particles, such as the KF, EKF and UKF [21–23]. For the KF, EKF and UKF, they cannot always get both high accuracy and robustness at the same time, but the H∞F can get the compromise between the accuracy and robustness by adjusting disturbance attenuation factor. In this paper, we incorporate the H∞F algorithm [24–28], which can fully take into account the current measurements, into the PF for maneuvering target tracking. The proposed H∞F based particle filter (H∞PF) both has the inherent advantages of the PF and the H∞F and shows a marked improvement in the maneuvering target tracking.

The layout of this paper is as follows: In Section 2, the PF algorithm and H∞ algorithm are formulated; meanwhile, the proposed H∞PF algorithm is presented in detail. Simulation results and discussions are given in Section 3, and we conclude this paper in Section 4.

2. H∞ FILTER BASED PARTICLE FILTER

2.1. Basic Theory of Particle Filter

Because the particle filter has good performance in tracking nonlinear and non-Gaussian problems, we employ it to solve the state estimation problem. The particle filter first starts with a number of particles which are initialized. After that, each particle is generated by the density function [29, 30].

Give a state space model:

$$x_k = f(x_{k-1}, u_{k-1}) + w_k \tag{1}$$

$$z_k = h(x_k) + v_k \tag{2}$$

where, $f(.)$ is the system dynamic function, $h(.)$ is the system observation function, x_k and z_k are the target state vector and target observation vector at time k, respectively, w_k is the Gaussian system noise vector, v_k is the Gaussian observation noise vector, we take k as the time index. The detailed particle filter algorithm [5] is described as follows:

step 1: Initialization. Sample the initial particles $x_0^{(1)}, x_0^{(2)}, \ldots, x_0^{(N)}$, N is the number of particles and $\omega_0^i = 1/N$, $i = 1, 2, \ldots N$.

step 2: Importance Computation. We get the predicted particles at time k by bringing the particles at time $k-1$ into (1). The importance weight of each predicted particle is computed through (3) when we obtain the target observation at time k, and then, we normalize the particle weight by the Equation (4):

$$\omega_k^i = \omega_{k-1}^i \frac{p(z_k|x_k^i)p\left(x_k^i|x_{k-1}^i\right)}{q\left(x_k^i|x_{k-1}^i, z_k\right)} \tag{3}$$

$$\tilde{\omega}_k^i = \omega_k^i / \sum_{j=1}^{N} \omega_k^i \tag{4}$$

where, $q(x_k^i|x_{k-1}^i, z_k)$ is the importance density function, generally, $q(x_k^i|x_{k-1}^i, z_k) = p(x_k^i|x_{k-1}^i)$. After normalizing the weight, we can get the approximate posterior distribution $p(x_k|z_{1:k})$ by (5), $\delta(.)$ is the Dirac function.

$$p(x_k|z_{1:k}) = \sum_{j=1}^{N} \tilde{\omega}_k^i \delta(x_k - x_k^i) \tag{5}$$

step 3: Resample. Accept the particles that have higher importance weights, meanwhile, their cumulative probabilities are bigger than a given threshold. Eliminate those particles that have lower importance weights and their cumulative probabilities are smaller than the given threshold. Reset each particle weight $\omega_k^i = 1/N,\ i = 1, \ldots, N$.

step 4: Output calculation. The posterior probability estimation of the state is obtained approximately through (6):

$$\hat{x}_k \approx \sum_{i=1}^{N} \tilde{\omega}_k^i x_k^i \tag{6}$$

step 5: $k = k + 1$, and move to step 2.

In the particle filter, we can choose some different proposed distribution functions, and the most commonly used way is choosing the prior density as its proposed distribution function [11, 12] in the PF, that is, $q(x_k^i|x_{k-1}^i, z_k) = p(x_k^i|x_{k-1}^i)$. However, the shortcoming of this method is that it does not consider the system current measurements, which brings about the particle degeneracy problem. For now, two key factors in preventing particle degeneracy [10] are the proper selection of the proposed distribution and the re-sampling method. In the light of the selection of proposed distribution, we need an algorithm that can consider the system current measures. So we take advantage of the H∞ Filter and incorporate it into the particle filter.

2.2. Basic theory of H∞ Filter

Generally, we can consider a time-varying discrete state model in the Kerin space as following:

$$\begin{cases} X(k) = \Phi(k)X(k-1) + \Gamma(k)W(K) \\ Z(k) = H(k)X(k) + V(k) \\ \tilde{S}(k) = L(k)X(k), k = 1, 2, 3, \ldots \end{cases} \tag{7}$$

where, $X(k) \in R^n$ is the system state vector at time k, $Z(k) \in R^n$ is the observation vector at time k and $\tilde{S}(k)$ is the given state variable. The matrixes $\Phi(k)$, $\Gamma(k)$, $H(k)$ and $L(k)$ are preset to known matrixes in general. We suppose the system noise $W(k)$ and the observation noise $V(k)$ are energy bounded l_2 signals, that is, $\Sigma_{k=0}^{\infty}||W(k)||^2 < \infty$ and $\Sigma_{k=0}^{\infty}||V(k)||^2 < \infty$, where the sign $||.||^2$ denotes the l_2 norm. We have no hypothesis on their statistical properties. $X(0)$, $W(k)$ and $V(k)$ meet the following requirements:

$$\left[\begin{bmatrix} X(0) \\ W(j) \\ V(j) \end{bmatrix}, \begin{bmatrix} X(0) \\ W(k) \\ V(k) \end{bmatrix} \right] = \begin{bmatrix} \prod_0 & 0 & 0 \\ 0 & I/\delta_{jk} & 0 \\ 0 & 0 & R_\infty(k)\delta_{jk} \end{bmatrix} \tag{8}$$

where, $R_\infty(k) = \begin{bmatrix} I & 0 \\ 0 & -\gamma_f^2 I \end{bmatrix} \delta_{jk}$. For the given observations $Z(k)$, we denote $\hat{X}(k)$ as the estimate of $X(k)$, so we can obtain the state estimation error $e(k)$:

$$e(k) = \hat{X}(k) - X(k) \tag{9}$$

The H∞ filtering algorithm is a Kalman filter in Krein space actually, so we design H∞ filter based on (6) and (7) as follows:

$$X(k|k-1) = \Phi(k|k-1)X_\infty(k-1) \tag{10}$$

$$P(k|k-1) = \Phi(k|k-1)P(k-1)\Phi^T(k-1) + \Gamma(k-1)\Gamma^T(k-1) \tag{11}$$

$$X_\infty(k) = X(k|k-1) + K_\infty(k-1)\begin{bmatrix} Z(k) - H(k)X(k|k-1) \\ \tilde{S}(k) - L(k)X(k|k-1) \end{bmatrix} \tag{12}$$

$$K_\infty(k) = P(k|k-1)\begin{bmatrix} H^T(k) & L^T(k) \end{bmatrix} R_e^{-1}(k) \tag{13}$$

$$P(k) = (I - K_\infty(k)\begin{bmatrix} H(k) & L(k) \end{bmatrix})\, P(k|k-1) \tag{14}$$

$$R_e(k) = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} H(k) \\ L(k) \end{bmatrix} P(k|k-1) \begin{bmatrix} H^T(k) & L^T(k) \end{bmatrix} \tag{15}$$

The above formulas constitute the H∞ robust filtering algorithm. Because of using the different filtering gain algorithm, the H∞ filter is different from the standard kalman filter essentially. When the disturbance attenuation factor $\gamma \to \infty$, we can see that $M(k) \to 0$ through (16), which makes the H∞ filter recursion degenerate into the Kalman filter recursion. So the H∞ norm of the Kalman filter may be very large, which result in poor robustness performance. As the disturbance attenuation factor $\gamma \to \min$, we find $M(k) \to I$ through (16), where I is the identity matrix, though we can get good robustness, the estimation square error is quite large. So we can obtain satisfactory requirement by adjusting the parameter γ according to testing experiment in practice.

Under the Krein space conditions, suppose that $M(k)$ is the filtering gain imbalance factor, which describes the process that the disturbance attenuation factor γ adjusts filtering gain. They meet the following equation:

$$M(k) = \Lambda(k)(-\gamma^2 I + \Lambda(k))^{-1} \tag{16}$$

where, $\Lambda(k) = (I - K_s(k))H(k)P(k|k-1)$. From the analysis of the algorithms and the above formulas, we employ the decomposition of matrix and inverse theory in $R_e(k)$, and bring the results into the formula (13), assuming $L(k) = I$, we get:

$$K_\infty(k) = \begin{bmatrix} (I - M(K))K_s(k) & M(k) \end{bmatrix} \tag{17}$$

$$K_s(k) = P(k|k-1)H^T(k)(I + H(k)P(k|k-1)H^T(k))^{-1} \quad (18)$$

We can know by the above knowledge: the process that disturbance attenuation factor γ adjusts the robustness of the filter can be described as to adjust the gain of the filter process essentially.

$$K_s(k) \overset{\gamma \to M(k)}{\to} K_\infty(k) \overset{(14)}{\to} P(k) \overset{(11)}{\to} P(k+1|k) \overset{(18)}{\to} K_s(k+1) \quad (19)$$

2.3. Proposed H∞PF

The process of the H∞PF is shown in Table 1, where N is the total number of particles. x_k^j $(k > 0)$ is the new particle generated by H∞F. H∞_prediction(.) is the function that performs the H∞ filter algorithm using the formulas (10)–(15) described in Section 2.2. $Pr(.)$ is the probability that determines the resampling particle $\tilde{x}_k^j$ according to the discrete weight distribution $\tilde{\omega}_k^j$.

3. SIMULATION RESULTS

3.1. Case 1: One-dimension Target Tracking With Large Maneuvers

3.1.1. Target Scenario

For one-dimensional tracking problem, we adopt the following system state space model, and at the same time we compare the performance of the PF, KPF and H∞PF in terms of tracking accuracy.

$$\begin{cases} x_t = 0.5x_{t-1} + 25x_{t-1}/\left(1 + x_{t-1}^2\right) + 8\cos[1.2(t-1)] + w_t \\ y_t = x_t^2/20 + v_t \end{cases} \quad (20)$$

where, w_t and v_t are the vector input white noise with zero mean, $x_0 = 0.1$, and the particle number N is 50 and $\gamma = 0.5$. We have implemented the algorithms in MatlabR2009a.

3.1.2. Tracking Performance Comparison

Figure 1 shows the tracking results by the PF, KPF and the proposed H∞PF. We can see that the Kalman particle filter, and the proposed method can well estimate the motion state of target throughout the entire movement process.

Figure 2 shows the position error of estimated position corresponding to PF, KPF and H∞PF. It is obvious that when the target is during the maneuvering, the proposed H∞PF guarantees the tracking accuracy and performed better than the KPF, but the PF

Table 1. The simple description of the H∞PF algorithm.

Initialization: $k = 0$
for $j = 1, \ldots, N$
  Sample $\tilde{x}_0^j$ from $p(x_0)$; Calculate the weight $\omega_0^j = 1/N$
end
for $k = 1, 2, \ldots$ (main loop)
  H∞ Prediction
    for $j = 1, \ldots, N$
      x_k^j=H∞_prediction $(\tilde{x}_{k-1}^j)$
    end
  Importance Step
    for $j = 1 : N$, calculate the importance weights:
      $\omega_k^j = \omega_{k-1}^j \dfrac{p(z_k|x_k^j)p(x_k^j|x_{k-1}^j)}{q(x_k^j|x_{k-1}^j, z_k)}$,
      where, $q(x_k^j|x_{k-1}^j, z_k) = p(x_k^j|x_{k-1}^j)$.
    end
    Followed by normalization: $\tilde{\omega}_k^j = \omega_k^j / \sum_{j=1}^N \omega_k^j$
  Resampling Step
    for $j = 1, \ldots, N$
      Pr $(x_k^i = \tilde{x}_k^j)$=$\tilde{\omega}_k^j$,
      Reset the weights $\omega_k^i = 1/N$, $(i = 1, \ldots N)$.
    end
  State Estimation Step
    Calculate the desired estimate $\hat{x}_k$: $\hat{x}_k = \sum_{j=1}^N \tilde{\omega}_k^j x_k^j$
end

cannot ensure good tracking accuracy. The reason is that the PF does not think over the current measurements but the KPF and the H∞PF employ the current measurements. To the KPF and the H∞PF, H∞PF can get the compromise between the accuracy and robustness by adjusting disturbance attenuation factor, so, the proposed algorithm is the best among the three. From Fig. 3, we can see that the proposed algorithm is much more robust than the other two and that the H∞PF keeps the satisfactory results throughout the whole process.

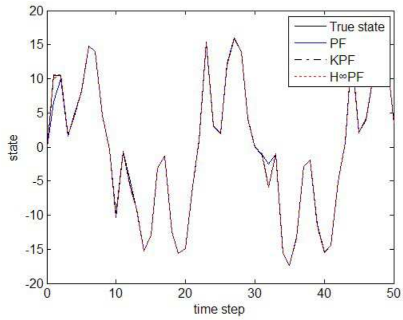

Figure 1. Estimated trajectory by PF, KPF and H∞PF.

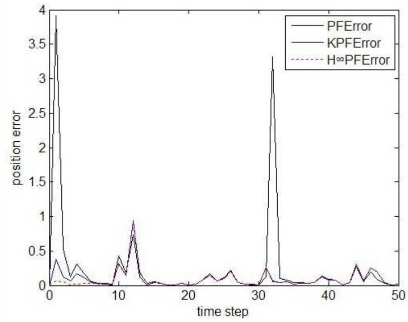

Figure 2. Position Error by PF, KPF and H∞PF.

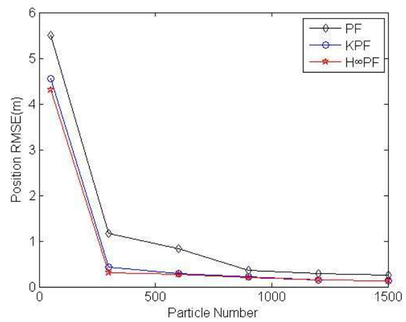

Figure 3. Position RMSE by PF, KPF and H∞PF with different particle number.

3.2. Case 2: Two-dimension Target Tracking With Large Maneuvers

3.2.1. Target Scenario

For two-dimensional tracking problem, we consider a relatively complicated scenario. There are seven target motion patterns as shown in Table 2 for our experiment in which we track a target with large maneuvers and the period is longer. We set the initial motion pattern as ($u^x = 28\,\text{m/s}$, $u^y = 28\,\text{m/s}$, $\theta = 0°/\text{s}$). The initial position is (1000 m, 1000 m) and $\gamma = 100$. The target motion is constant velocity or constant turn in different time interval. Here, the particle is described by a vector containing the position on X, the velocity on X, the position on Y, the velocity on Y. The target is modeled by the following model:

$$x_t = F(\theta, T)x_{t-1} + B(T)w_t \tag{21}$$

where, θ is the turn rate, $F(\theta,T)=\begin{bmatrix}1 & \sin(\theta T)/\theta & 0 & -(1-\cos(\theta T))/\theta \\ 0 & \cos(\theta T) & 0 & -\sin(\theta T) \\ 0 & -(1-\cos(\theta T))/\theta & 1 & \sin(\theta T)/\theta \\ 0 & \sin(\theta T) & 0 & \cos(\theta T)\end{bmatrix}$ is the state transition matrix, $B=\begin{bmatrix}T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T\end{bmatrix}$ is the input matrix, w_t is the matrix of input white noise with zero mean.

Table 2 lists the detailed description of the target motion. The target starts a constant velocity motion from position (1000 m, 1000 m) with initial speed ($u^x = 28\,\mathrm{m/s},\ u^y = 28\,\mathrm{m/s},\ \theta = 0°/\mathrm{s}$) and $\gamma = 100$.

3.2.2. Tracking Performance Comparison

Figure 4 shows the tracking results by the PF, KPF and the proposed H∞PF. Fig. 5 and Fig. 6 show the tracking results by the PF, KPF and the proposed H∞PF on X and Y, respectively. It is clear that the proposed method can well estimate the motion state of target.

Figure 7 shows the position error of estimated position corresponding to the PF, KPF and H∞PF. We can see that though the maneuvers are large, good performance can be obtained by H∞PF.

Figure 8 shows the position RMSE by the three filters with different particle number. Form this figure, it is obvious that the proposed algorithm is more precise and robust than the PF and KPF even when the particle number is small.

Table 2. The process of target motion.

Time interval	The target motion patterns
0–20	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = 0°/\mathrm{s}$
21–100	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = 6°/\mathrm{s}$
101–200	$u^x = 28\,\mathrm{m/s}$, $u^y = 28m/s$, $\theta = 0°/\mathrm{s}$
201–240	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = 9°/\mathrm{s}$
241–300	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = 0°/\mathrm{s}$
301–400	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = -7°/\mathrm{s}$
401–440	$u^x = 28\,\mathrm{m/s}$, $u^y = 28\,\mathrm{m/s}$, $\theta = 0°/\mathrm{s}$

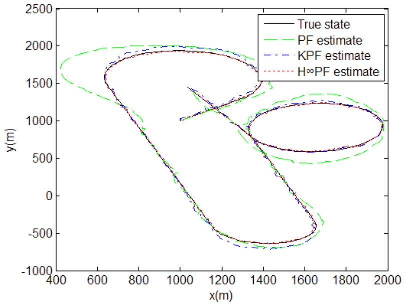

Figure 4. Estimated trajectory by the PF, KPF and H∞PF.

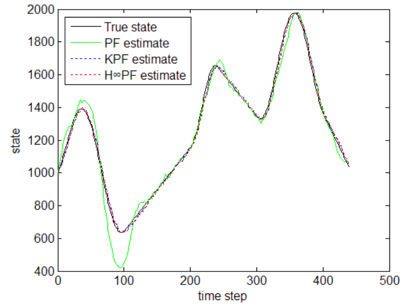

Figure 5. Estimated trajectory on X by the PF, KPF and H∞PF.

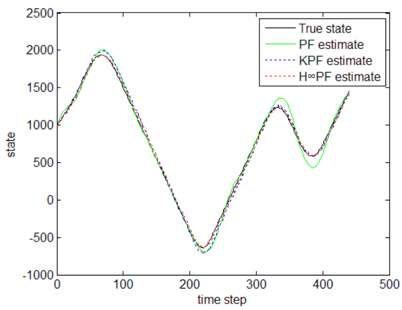

Figure 6. Estimated trajectory on Y by the PF, KPF and H∞PF.

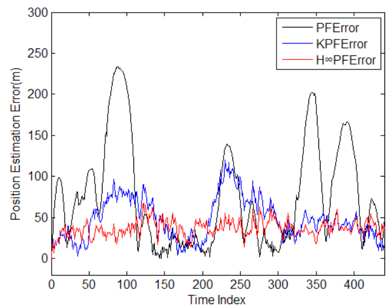

Figure 7. Position error of estimated position to the PF, KPF and H∞PF.

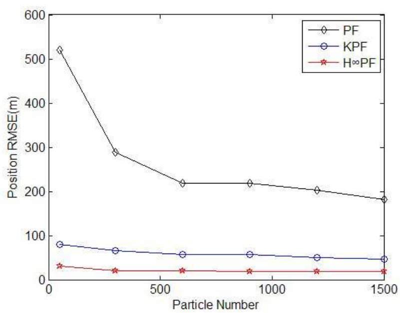

Figure 8. Position RMSE by the PF, KPF and H∞PF with different particle number.

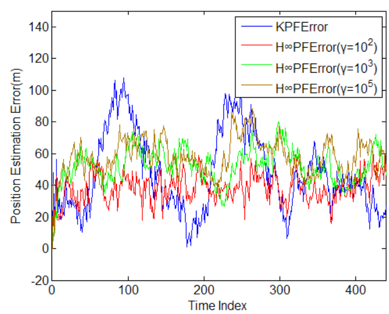

Figure 9. Position error of estimated position to the KPF and the H∞PF with different γ.

Figure 9 shows the position error of estimated position to the KPF and the H∞PF with different values of γ, where the values of γ are 10^2, 10^3, and 10^5, respectively. When $\gamma = 10^2$, we can get good robustness and accuracy. But when $\gamma = 10^3$ or $\gamma = 10^5$ or larger, the filter recursion may approach nearer to the KPF recursion which leads to poor robustness properties.

4. CONCLUSION

In this paper, a novel H∞PF for maneuvering target tracking has been proposed, and the simulation results demonstrate that the new algorithm has better accuracy and robustness in tracking maneuvering target. The proposed H∞PF algorithm incorporates the H∞F algorithm into the standard particle filter, so the new algorithm can fully take into account the current measures and make the particles distribution more approach to the station posterior distribution. Furthermore, the proposed algorithm can get the compromise between the accuracy and robustness by adjusting disturbance attenuation factor. Finally, simulation results demonstrate that the proposed algorithm can achieve higher prediction precision and better robustness, and meanwhile, the H∞PF is proved to be effective and practicable in tracking the maneuvering target with large maneuvers.

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