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# IMPROVED SPURIOUS FREE PERFORMANCE OF MULTI-LAYER MULTI-PERMITTIVITY DIELECTRIC RESONATOR IN MIC ENVIRONMENT

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**Abstract**—In this paper, a novel approach has been suggested to obtain an improved spurious-free window for dielectric resonator in microwave integrated circuit environment. In microwave integrated circuit environment, the dielectric resonator placed on a thin dielectric substrate gets located asymmetrically with respect to its shielding enclosure. A reduced separation in frequencies (mode separation) is one of a consequence of this asymmetry that may become a cause of spurious modes. This adverse influence of asymmetry is sought to be compensated by proposing a multi-layer multi-permittivity dielectric resonator structure with several layers of differing permittivity. The suggested approach takes advantage of the fact that the mode separation of a dielectric resonator configuration can be correlated to relevant resonance mode fields. By perturbing the resonance mode fields through the suggested multi-layer multi-permittivity approach, the adverse influence of asymmetry is found to reduce considerably over a comparative Conventional Ring dielectric resonator in microwave integrated circuit configuration. Still more improvement in mode separation are shown when the shape of the multi-layer multi-permittivity ring dielectric resonator is further modified, suggesting a scope for optimization in present approach.

## 1. INTRODUCTION

The resonance mode spectrum of a dielectric resonator (DR) with several closely spaced frequencies is inherently dense. Because of this dense frequency spectrum it may become difficult to avoid interference

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between a desirable operating mode frequency and an undesirable one, leading to the well recognized problem of spurious modes [1]. Though the issue of improvements in spurious response of DRs has been studied for a long time, it still continues to attract attention [2, 3].

In the past, there have been several strategies to improve the mode spectrum of a DR either by suppressing an undesirable mode or by enhancing the separation between the desirable and undesirable frequencies [4–13]. In these approaches, usually the electric fields for the undesirable mode(s) are sought to be suppressed by an additional metal structure in close vicinity to the dielectric resonator or influenced by modifying the dielectric resonator shape and structure itself. Following the latter approach, the innovation of ring shaped DR compared to a conventional circular rod, is generally recognized to be the most successful approach in achieving an improved separation in frequency between the dominant resonance mode and the nearest higher mode. It has been found that for an ideal case of a ring DR located symmetrically in a shielding cavity the separation between the dominant $TE_{01\delta}$ mode frequency ($f_0$) and the nearest higher frequency is $0.58f_0$; much higher than $0.34f_0$ for a comparable conventional circular rod DR structure [6]. For convenience the mode separation as referred here has also been defined in Section 2 of this paper.

In microwave integrated circuit (MIC) configuration, certain structural deviation from the above idealized structure [6] (symmetrically placed DR in cavity) become inevitable. This causes reduction of mode separation from $0.58f_0$ to (0.35–0.38) $f_0$ [12]. This degradation of mode separation comes primarily due to influence of the properties of dielectric substrate, both, due to its dielectric constant ($\varepsilon_{rs}$) and the thickness ($H_S$). It may be easily visualized that for the usual thin dielectric substrate of an MIC, the location of DR deviates from the ideal symmetrical location; it becomes asymmetric with respect to shielding enclosure. Secondly, the dielectric constant of substrate $\varepsilon_{rs} > 1$, is a cause of undue perturbation to dielectric resonator fields. Recognizing these limitations of DR in MIC configuration, a more recent proposition of a modified ring DR has shown a significant improvement in mode separation by approximately $(0.06)f_0$ over a comparative, hitherto, state of the art MIC ring DR [13]. Evidently, this shown improvement in mode separation implies compensation, by some extent, of the degrading influence of the dielectric substrate mentioned above. On further exploration, to compensate the degrading influences on the mode separation in frequency of DR in MIC configuration, another approach of multilayer multi-permittivity (MLMP) ring DR has been suggested in this paper.

The proposed MLMP ring DR shown in Fig. 1 comprises several

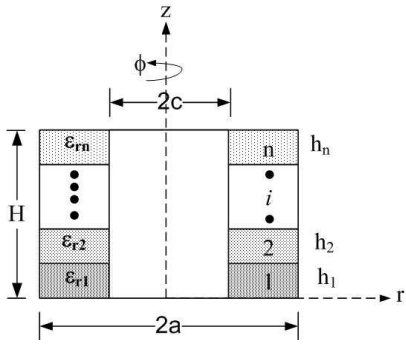



**Figure 1.** Structure of multilayer multi-permittivity ring dielectric resonator (MLMP Ring DR).

ring cross sectioned layers ($n$) of varying dielectric constant ($\varepsilon_{ri}$; $i = 1$ to $n$) and thickness ($h_i$) stacked together. The individual layers of dielectric rings are homogeneous though the layered dielectric ring resonator as a whole becomes an inhomogeneous artificially anisotropic medium. The concept of obtaining an artificial anisotropic dielectric medium in various microwave structures has been well known and applied since long [15–17]. However, for applying this concept to dielectric resonators, especially for a control of their resonance mode spectrum, there appear to be no studies. Again, in the past, the multi-layer anisotropic dielectric resonators have been analyzed though applied mostly for the single-crystal uniaxially anisotropic Sapphire dielectric resonators [17]. Also, to best of present authors' knowledge the utility of referred anisotropic dielectric resonators especially for an improvement in mode separation has not been studied so far except in [18].

The paper is organized as follows. The Section 2 describes the considerations for devising the multilayer multi-permittivity (MLMP) ring DR for improving its mode separation in MIC environment. The performance of MLMP ring DR has been determined by computer simulation using a known commercial AnSoft HFSS electromagnetic simulator as well by robust and accurate FDTD numerical software developed by the present authors [19]. Based on the data and results, a relatively optimized 4-layer ring DR in MIC configuration is suggested, fabricated and measured on a network analyzer (HP 8720B). Further, a modification of shape on the top layer of this 4-layer MLMP ring DR in a fashion similar to the modified ring DR structure [13] has also been incorporated to provide better performance over the 4-layer MLMP ring DR.

## 2. APPROACH AND ANALYSIS

### 2.1. Multilayer Multi-permittivity Ring DR (MLMP Ring DR)

*2.1.1. Preliminary Considerations*

The primary issue of this paper is to suggest a MLMP ring DR that can compensate for the usual degradations in mode separation of a conventional asymmetrically located ring DR in MIC configuration.

It is recognized that for applying a comparative test between two DRs, on the basis of mode separation, a strict equivalence between the two structures need be maintained except of course the nature of their respective medium: homogeneous single dielectric constant for the Conventional Ring DR and inhomogeneous multi-layer permittivity for the MLMP ring DR. This necessity for equivalence, as can be expected, is imposed since the resonance-mode spectrum for a dielectric resonator configuration is highly dependent on their structural parameters. With this view, the considered equivalent dielectric resonator structures have been shown in Fig. 2, where the dimensional parameters of resonators ($H$, $2a$ and $2c$) and their shielding MIC enclosures parameters ($H_C$, $2b$, $H_S$ and $\varepsilon_{rs}$) for both the DRs are same. Along with the same dimensional parameters of two resonators, the resonant frequencies of $TE_{01\delta}$ mode of ring DR and MLMP ring DR are also maintained to be same for reasonable comparison of mode separation between two DRs. The reason for maintaining same resonant frequency is that

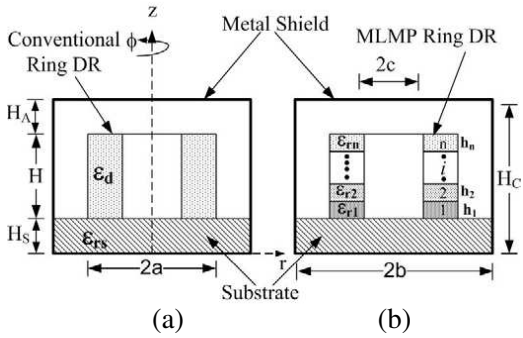



**Figure 2.** Configuration of circular cylindrical DR placed on a substrate shielded by a circular cylindrical metal enclosure. (a) Conventional Ring DR. (b) Multilayer multi-permittivity ring DR (MLMP ring DR). ($2b = 1.02$ in, $H_C = 0.6$ in, $2a = 0.68$ in, $H = 0.3$ in, $\varepsilon_{rs} = 2.2$, $\tan\delta_{sub} = 0.0009$, $\sigma_{metal} = 4.1 \times 10^7$ S/m).

enhancing permittivity in isotropic ring DR itself can enhance mode separation, but at the same time the resonant frequency of $TE_{01\delta}$ mode gets shifted. Hence to show that the improvement in mode separation in MLMP ring DR is not because of change in permittivity, instead arranging the different permittivity layers in some particular fashion for improving the mode separation and at the same time the resonant frequency of desired mode ($TE_{01\delta}$ mode) are same as Conventional Ring DR.

It is possible to maintain such equivalence by maintaining effective permittivity ($\varepsilon_{eff}$) of the MLMP ring DR equal to the permittivity ($\varepsilon_d$) for the conventional ring. The effective permittivity of MLMP ring DR has been evaluated here by the following well known rule of mixture for a composite dielectric medium. The permittivity of composite mixture can be defined as

$$\varepsilon_{com} = \sum_{i=1}^{n} \varepsilon_i v_i \,; \quad \text{with} \sum_{i=1}^{n} v_i \, = 1 \tag{1a}$$

where, $\varepsilon_{com}$ is the permittivity of the composite containing $i$ number of dielectric media of permittivity and their volume fractions, $\varepsilon_i$ and $v_i$, respectively.

Applying the above rule for MLMP ring DR, the effective permittivity ($\varepsilon_{eff}$) of MLMP DR is calculated using following expression

$$\varepsilon_{eff} = \frac{\varepsilon_{r1} \cdot h_1 + \varepsilon_{r2} \cdot h_2 + \ldots + \varepsilon_{ri} \cdot h_i + \ldots \varepsilon_{rn} \cdot h_n}{h_1 + h_2 + \ldots + h_i + \ldots h_n} \tag{1b}$$

where, $\varepsilon_{ri}$ and $h_i$ are the permittivity and thickness of $i$th layer. Here, all the layers of MLMP ring DR have equal cross-section, hence thickness of the layers are used in place of their respective volume. It is verified in Table 1 for MLMP ring DR structure that when the effective permittivity of MLMP ring DR and the Conventional Ring DR are arranged about the same ($\varepsilon_{eff} = \varepsilon_d$) within a close tolerance range, their resonant frequencies for dominant $TE_{01\delta}$ mode are also lie in similar range of tolerance. As noted in the Table 1, this has been validated by comparing the dominant frequencies of homogenous ring DR using various values of $\varepsilon_d$ and the equivalent 3-layer or 4-layer or 5-layer MLMP ring DR of corresponding $\varepsilon_{eff}$ value.

It is now necessary first to express the terms mode separation as well as the asymmetry that will be used in this paper subsequently. The mode separation for a DR considering the $f_{TE01\delta}$ as the resonant frequency mode of desired $TE_{01\delta}$ mode, is defined as

$$Mode\,Separation\ (in\,\%) = \left( \frac{f_{Nearest_Mode} - f_{TE01\delta}}{f_{TE01\delta}} \right) \times 100 \tag{2}$$

**Table 1.** Resonant frequency of $TE_{01\delta}$ mode of Conventional Ring DR and Multilayer Multi-permittivity (MLMP) Ring DR with 3, 4 and 5 layers. [$2b = 0.51\,\text{in}$, $H_C = 0.6\,\text{in}$, $2a = 0.68\,\text{in}$, $H = 0.3\,\text{in}$, $\varepsilon_{rs} = 2.2$, $H_S = 0.15\,\text{in}$, $c/a = 0.40$, **3-layer MLMP Ring DR**: $\varepsilon_{r1} = 41$, $\varepsilon_{r2} = 45$, $\varepsilon_{r3} = 33$, $h_1 = 0.20\,\text{H}$, $h_2 = 0.50\,\text{H}$, $h_3 = 0.30\,\text{H}$, **4-layer MLMP Ring DR**: $\varepsilon_{r1} = 43$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 43$, $\varepsilon_{r4} = 27$, $h_1 = 0.35\,\text{H}$, $h_2 = 0.1\,\text{H}$, $h_3 = 0.30\,\text{H}$, $h_4 = 0.25\,\text{H}$, **5-layer MLMP Ring DR**: $\varepsilon_{r1} = 41$, $\varepsilon_{r2} = 45$, $\varepsilon_{r3} = 10$, $\varepsilon_{r4} = 45$, $\varepsilon_{r5} = 33$, $h_1 = 0.10\,\text{H}$, $h_2 = 0.20\,\text{H}$, $h_3 = 0.10\,\text{H}$, $h_4 = 0.30\,\text{H}$, $h_5 = 0.30\,\text{H}$].

| MLMP Ring DR | | | Conventional Ring DR | |
|---|---|---|---|---|
| Number of Layers | $\varepsilon_{eff}$ | Frequency (in GHz) | $\varepsilon_d$ | Frequency (in GHz) |
| 3-layer | 40.6 | 3.2884 | 41.0 | 3.3178 |
| 4-layer | 35.7 | 3.5392 | 35.74 | 3.5557 |
| 5-layer | 37.5 | 3.4592 | 37.5 | 3.4894 |

where, and $f_{Nearest_Mode}$ is the resonant frequency of nearest higher mode that potentially can be a spurious mode. The resonant frequencies as above have been determined using commercially available software HFSS whose accuracy in predicting resonance frequencies to that of a rigorous analytical approach has already been established in [13]. Further, the simulation results obtained from HFSS have also been verified with FDTD program developed by authors [19].

The term asymmetry referred above is understood as:

$$Asymmetry\,(A) = \frac{H_A - H_S}{H_A + H_S} \tag{3}$$

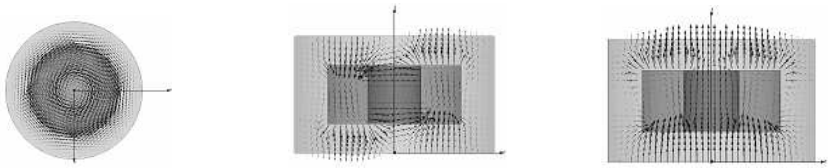
where $H_A$ and $H_S$ are shown in Fig. 2. For a symmetrically located DR in shielding enclosures, the value of asymmetry will be zero ($A = 0$) as $H_A = H_S$ and maximum value of asymmetry ($A = 1$) occurs when the DR is placed at the bottom metal plate of the shielding enclosure ($H_S = 0$). But DR placed at bottom metal plate becomes an image plate resonator with altogether different modal behaviour that does not concern our present discussions of MIC structure. Hence, value of asymmetry upto an extent of $A = 0.8$ has been considered here for evaluating the degradation in mode separation.

### *2.1.2. Strategy for Devising MLMP Ring DR*

A common approach for improvement in mode separation in a dielectric resonator is to influence the $E$-fields, selectively, of the undesirable mode(s) that is likely to interfere with the dominant or a desirable mode. Some attempts have been made in this regards for the ring

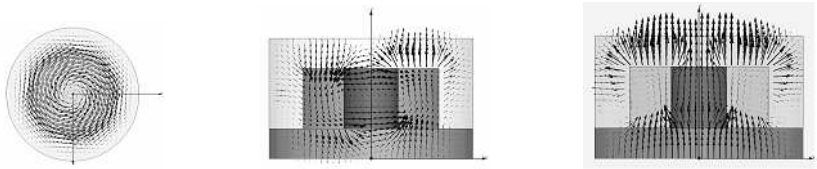
dielectric resonator in symmetrical configurations [6] and more recently for the modified ring resonator in a MIC configuration [13].

The degradation in mode separation can be correlated with its change in resonance mode field patterns of corresponding modes (desired and undesired modes). Here, as the objective paper, the degradation of mode separation due to structural change (substrate permittivity $\varepsilon_{rs}$ and thickness $H_S$) are considered and their respective resonance fields pattern ($E$-fields) for Conventional Ring DR are shown in Fig. 3 to Fig. 5. The fields in Fig. 3 belong to the ideal symmetrical case ($A = 0$, $\varepsilon_{rs} = 1$) that correspond to the non degraded mode separation. The fields in Fig. 4 are yet again for the symmetrical structure but with a degrading dielectric substrate ($A = 0$, $\varepsilon_{rs} = 2.2$)



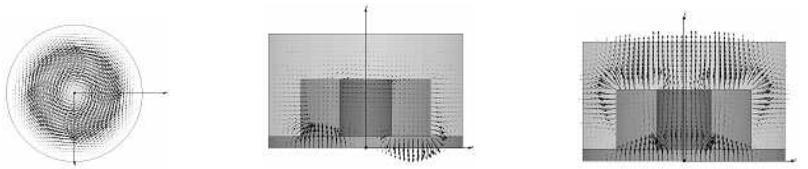
(a) E-field of $TE_{01\delta}$ at $z = H_s+H/2$ (b) E-field of nearest hybrid mode (c) E-field of $TM_{01\delta}$

**Figure 3.** Symmetrically loaded Conventional Ring DR in cylindrical metal cavity ($A = 0$, $\varepsilon_{rs} = 1.0$).



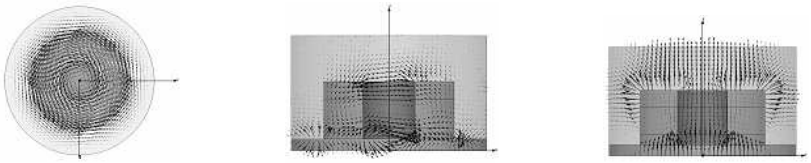
(a) E-field of $TE_{01\delta}$ at $z = H_s+H/2$ (b) E-field of nearest hybrid mode (c) E-field of $TM_{01\delta}$

**Figure 4.** Symmetrically placed Conventional Ring DR on substrate enclosed by cylindrical metal cavity ($A = 0$, $\varepsilon_{rs} = 2.2$).



(a) E-field of $TE_{01\delta}$ at $z = H_s+H/2$ (b) E-field of nearest hybrid mode (c) E-field of nearest $TM_{01\delta}$

**Figure 5.** Asymmetrically placed Conventional Ring DR on substrate enclosed by cylindrical metal cavity ($A = 0.6$, $\varepsilon_{rs} = 2.2$).



(a) E-field of $TE_{01\delta}$ at $z = H_s+H/2$ (b) E-field of nearest hybrid mode (c) E-field of $TM_{01\delta}$

**Figure 6.** Asymmetrically placed 3-layer MLMP Ring DR on substrate enclosed by cylindrical metal cavity ($A = 0.6$, $\varepsilon_{rs} = 2.2$).

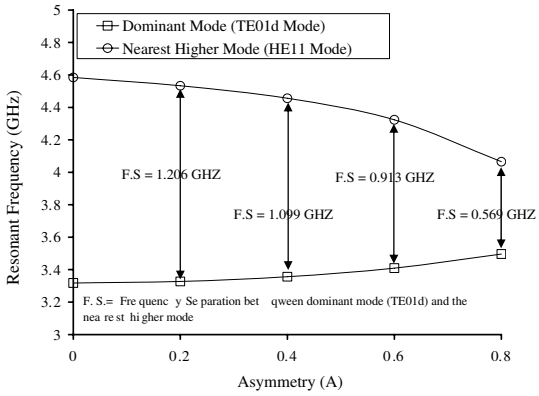



**Figure 7.** Resonant frequencies of dominant mode ($TE_{01\delta}$ mode) and nearest higher mode ($HE_{11}$ mode) of conventional ring DR (Fig. 2(a)) for various values of asymmetry. [$c/a = 0.4$, $\varepsilon_d = 41.0$, $\varepsilon_{rs} = 2.2$].

whereas the Fig. 5 shows for the asymmetrical MIC ring DR, $A = 0.6$, $\varepsilon_{rs} = 2.2$, that includes degradation in the mode separation due to asymmetry as well. The change in the field distribution can be observed prominently between the fields of the hybrid mode of symmetrical structures (Fig. 3(b) and Fig. 4(b)) and the asymmetrical structure (Fig. 5(b)). These differences in field distributions can be expected to contribute the degradation in mode separation. This degradation in mode separation, due to asymmetric position of DR, is also made clear in Fig. 7, by presenting the frequencies for the dominant ($TE_{01\delta}$) and the nearest higher mode ($HE_{11}$) of Conventional Ring DR. The reduction in frequency separation can be observed with increasing asymmetry, from 1.205 GHz at $A = 0.2$ to 0.569 GHz at $A = 0.8$.

Now reverting back to resonance mode fields, it is asserted that if by *some means* the fields for the *asymmetrical* structure in Fig. 5 can be influenced such that these are *restored back* as of the symmetrical structures in Fig. 4 (or Fig. 3), its mode separation will

also be restored (improved) back to that of the symmetrical structure. This modification in modal pattern contributes to change in resonant frequency and correspondingly the mode separation. This restoration of fields is the key basis for devising the MLMP ring DR. Following the above assertion, in the asymmetrical MIC structure, when the conventional ring DR may be replaced with a suitably equivalent MLMP ring DR, the mode separation for the structure should improve. To test the above assertions, at the first instance, a tentative three-layer (3-layer) MLMP ring DR is conceived and its field patterns are shown in Fig. 6 for purpose of comparison with the asymmetric Conventional Ring DR structure.

For implementing the concept of restoration of fields through the MLMP ring DR, the resonance mode fields shown in Fig. 3 to Fig. 5 are need to be studied in details. A comparison of $TE_{01\delta}$ mode $E$-fields ($E_\phi$ only, $E_r = 0$ and $E_z = 0$) in Fig. 3 to Fig. 5 easily reveals that there is only a small noticeable difference even due to enhanced asymmetry in Fig. 5 and this is corroborated with a small change in frequency as noticed in Fig. 7. Hence, it can be assumed that asymmetry has small influence on the TE mode frequency and mode separation degradation is mostly because of hybrid mode ($HE_{11}$ mode) fields.

It may be observed, that in case of ideal symmetric structure (Fig. 3(b) $A = 0$, $\varepsilon_{rs} = 1$), the field distributions for the $E$-fields of the HE modes within and outside the DR are symmetric with respect to the geometrical centre of DR, and as may be expected for the HE modes that contain the transverse as well as the axial components. This distribution of fields is of course due to similar boundary conditions at the two flat surfaces of the DR and also due to their equal separations with metal walls ($H_S = H_A$). When the DR still remains symmetric ((Fig. 4(b), $A = 0$) and substrate ($\varepsilon_{rs} = 2.2$, $H_A = H_S$) is introduced in between the bottom metal plate and DR, the variance in the fields are noticed at the top and the bottom flat surface of the DR. In case the dielectric constant of the substrate is varied, say enhanced from 2.2 to 10, the effect can be much more pronounced, though not shown here. However, more significant influence on the fields can be observed in asymmetric case (Fig. 5(b), $A = 0.6$, $\varepsilon_{rs} = 2.2$), where the variance in the fields at the two dielectric interfaces (top and the bottom flat surface of the DR) are more pronounced and effect of closer metal wall are clearly visible. The absence of transverse components of $E$-fields at the bottom half of DR are easily noticed, otherwise are present in the comparative fields in the symmetric case in Fig. 4(b). The task of devising the MLMP dielectric resonator now gets more specific to restoring back the HE mode fields in Fig. 5(b) to that of in Fig. 4(b) (or possibly even to Fig. 3(b)). The following considerations

are need to be maintained while devising the MLMP DR for improving its performance over the Conventional Ring DR in MIC configuration.

*2.1.3. General Considerations*

(i) To get equal concentration of fields of the hybrid mode at both the flat surfaces of DR in asymmetric case, the permittivity needs to be increased for the bottom layer and decreased for the top layer depending on their respective separation from metal plates ($H_S$ and $H_A$).

(ii) For the less pronounced influence of the closer bottom plate, a relatively higher confinement of HE mode fields within the DR is desirable. Enhancing the permittivity of the bottom layer help to confine the fields within the DR and in direction of approach proposed in (i).

(iii) Since the lower value of permittivity in top layer and higher value of permittivity in bottom layer shift the centre of effective permittivity of DR to the lower half of DR (between 0 to $H/2$), resulting shift in the concentration of resonance mode fields of $TE_{01\delta}$ mode to lower half and may get influenced by the closer metal wall. Hence the permittivity and thickness of middle layers are chosen such that the centre of effective permittivity and geometrical centre should coincide.

(iv) Simultaneous to above considerations, the equivalence conditions (same resonant frequency of $TE_{01\delta}$ mode) between the Conventional Ring and MLMP ring DR are also to be met.

The center of effective permittivity, as discussed in 3rd consideration, is being defined here as a fictitious point of intersection of $z$-axis and a transverse plane ($r$-$\phi$), where the contributions to effective permittivity in (1b) from the regions below and above are equal. Based on above considerations a tentative 3-layer MLMP ring DR is devised and analyzed in Fig. 6 and its evaluated mode separation is shown in Fig. 8. It is corroborated from Fig. 6(b) that by the implementation of above discussed strategies in MLMP ring DR one can restore the fields of symmetrically placed Conventional Ring DR (Fig. 3(b)) in MLMP ring DR and can enhance its mode separation.

## 2.2. Optimizing MLMP Ring DR

An attempt here is made for improving the performance MLMP ring DR by optimizing the number of layers, their permittivity and thickness. Rather than claiming to be the most optimized, the suggestions here are meant to emphasize as to how a correct choice

of number of layers, their thickness and their respective permittivity can lead to improvements in mode separation.

In 3-layer MLMP ring DR, implementing the 3rd consideration gets limited while simultaneously meeting the 1st and 2nd considerations. Because of the higher and lower permittivity of the bottom and top layer respectively, the effective permittivity centre falls lower to the geometrical centre of DR, a situation contrary to the 3rd consideration. The effective permittivity centre can be brought closer to its geometrical centre when a redistribution of permittivity can be affected by adjusting the number of layers, their permittivity and thickness. Fig. 9 shows the 5-layer MLMP ring DR where the effective permittivity at the upper half is 18.9 $[(33 * 0.3H + 0.2 * 45H)/0.5H = 37.8]$, which is almost equal to the effective permittivity at lower half

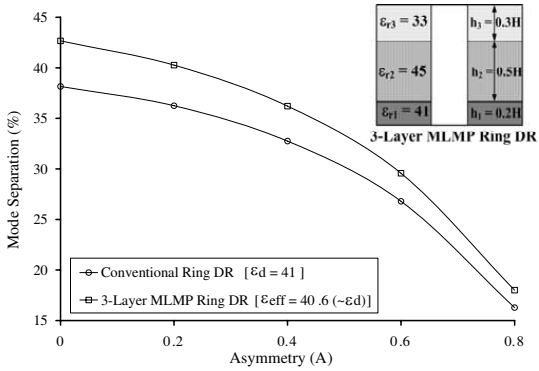



**Figure 8.** Comparison of mode separation of $TE_{01\delta}$ mode with the nearest mode for configurations shown in Fig. 2, with asymmetry. (Conventional Ring DR: $c/a = 0.4$, $\varepsilon_d = 41.0$, 3-layer MLMP Ring DR: $c/a = 0.4$, $\varepsilon_{r1} = 41$, $\varepsilon_{r2} = 45$, $\varepsilon_{r3} = 33$, $h_1 = 0.20\,\mathrm{H}$, $h_2 = 0.50\,\mathrm{H}$, $h_3 = 0.30\,\mathrm{H}$).

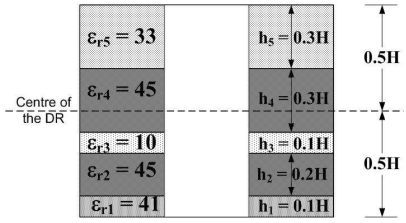



**Figure 9.** Permittivity and thickness distribution of 5-layer MLMP ring DR.

$[(45*0.1H+10*0.1H+45*0.2H+41*0.1H)/0.5H = 37.2]$ of DR. The improvement in mode separation in 5-layer MLMP ring DR is shown in Fig. 10 and it remains constant over a range of asymmetry, unlike the 3-layer MLMP ring DR where it starts reducing for higher value of asymmetry ($A = 0.4$ to 0.8 in Fig. 8).

Implementing the all four considerations in above 5-layer MLMP ring DR is just one of a possibly optimized structure. In fact a simpler 4-layer MLMP ring DR suggested in the following can also achieve an equivalent improvement. The 4-layer structure (as may be seen in the inset of Fig. 11) has not only lesser number of layers, it also requires only three types of dielectric materials of permittivity ($\varepsilon_r = 43$, 27, 10) compared to four materials for the 5-layer structure. The results of the mode separation with respect to asymmetry for 4-layer MLMP ring DR compared to an equivalent Conventional Ring DR are shown in Fig. 11, which are self explanatory. Finally the optimization discussed here is meant merely to show the scope for improving MLMP approach rather than claiming the best mode separation.

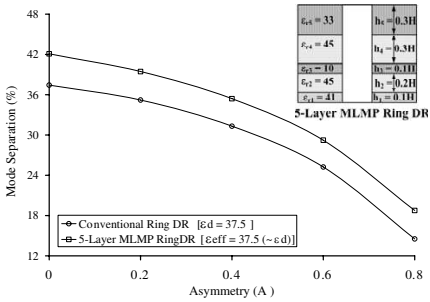



**Figure 10.** Comparison of mode separation of $TE_{01\delta}$ mode with the nearest mode for Conventional Ring DR and 5-layer MLMP Ring DR with asymmetry (A). (Conventional Ring: $c/a = 0.4$, $\varepsilon_d = 37.5$, $\tan\delta_{DR} = 0.00012$, 5-layer MLMP Ring DR: $c/a = 0.4$, $\varepsilon_{r1} = 41$, $\varepsilon_{r2} = 45$, $\varepsilon_{r3} = 10$, $\varepsilon_{r4} = 45$, $\varepsilon_{r5} = 33$, $\tan\delta_{DR} = 0.00012$, $h_1 = 0.10\,\mathrm{H}$, $h_2 = 0.20\,\mathrm{H}$, $h_3 = 0.10\,\mathrm{H}$, $h_4 = 0.30\,\mathrm{H}$, $h_5 = 0.30\,\mathrm{H}$).

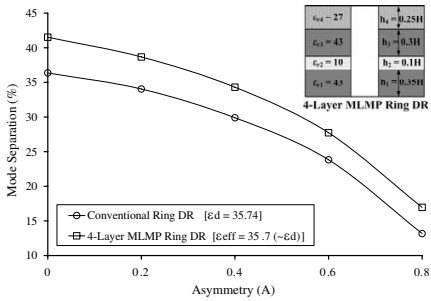



**Figure 11.** Comparison of mode separation of $TE_{01\delta}$ mode with the nearest mode for Conventional Ring DR and 4-layer MLMP Ring DR with asymmetry (A). (Conventional Ring DR: $c/a = 0.4$, $\varepsilon_r = 35.74$, 4-layer MLMP Ring DR: $c/a = 0.4$, $\varepsilon_{r1} = 43$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 43$, $\varepsilon_{r4} = 27$, $h_1 = 0.35\,\mathrm{H}$, $h_2 = 0.1\,\mathrm{H}$, $h_3 = 0.30\,\mathrm{H}$, $h_4 = 0.25\,\mathrm{H}$).

### 2.3. Modified MLMP Ring DR

In an earlier study by the authors, a modified ring DR structure was proposed that provides significant improvement over the conventional ring DR in MIC environment [13]. The suggested modified ring DR structure is reproduced here in Fig. 12(a). The suggested modification in modified ring DR was to influence the HE mode fields (also the objective of this study) by removing the axial plug from the top and bottom region. It is found interesting to combine this strategy with MLMP ring DR approach and accordingly a modified 4-layer MLMP ring DR structure is suggested in Fig. 12(b). It may be seen that the modification has been carried out only on the top layer of the 4-layer MLMP ring DR being in direction of present approach, however similar modification at the bottom side can be the violation of 1st and 2nd considerations of MLMP ring DR, hence not been incorporated here.

It can be understood that the principle of equivalence (same resonant frequency of $TE_{01\delta}$ mode) will not be feasible between the MLMP ring DR and modified MLMP ring DR as removal of dielectric plug from the top layer of MLMP ring DR. Fig. 13 gives the variation in mode separation over the range of asymmetry for all four configurations shown in Fig. 2 and Fig. 12. The MLMP ring DR and modified ring DR individually gives the improvement in mode separation over Conventional Ring DR and their combined improvement can also been seen in modified MLMP ring DR.

The comprehensive comparisons of all four DR structures have also been made in Table 2 for different ring radius with symmetric ($A = 0$) and asymmetric ($A = 0.6$) position of DRs. The improvement in symmetric ($A = 0$) position of MLMP DR can be interpreted as compensation of degradation due to dielectric constant

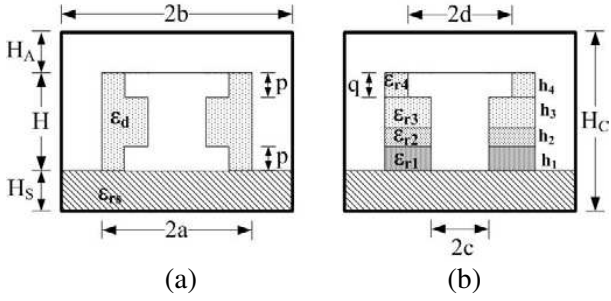



**Figure 12.** Cross-sectional view of (a) Modified Ring DR. (b) Modified Multilayer Multi-permittivity Ring DR. [Structural dimension of DR and shielding MIC dimensions are same as in Fig. 2].

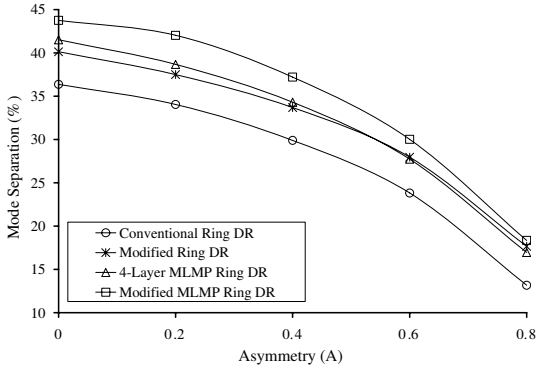



**Figure 13.** Comparison of mode separation of $TE_{01\delta}$ mode with the nearest mode for various DR structures shown in Fig. 2 and Fig. 12, with asymmetry (A). ((i) Conventional Ring DR: $c/a = 0.4$, $\varepsilon_r = 35.74$. (ii) Modified Ring DR: $c/a = 0.4$, $c/d = 0.6$, $p/H = 0.25$. (iii) MLMP Ring DR (4-layers): $c/a = 0.4$, $\varepsilon_{r1} = 43$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 43$, $\varepsilon_{r4} = 27$, $h_1 = 0.35\,\mathrm{H}$, $h_2 = 0.1\,\mathrm{H}$, $h_3 = 0.30\,\mathrm{H}$, $h_4 = 0.25\,\mathrm{H}$. (iv) Modified MLMP Ring DR: $c/a = 0.4$, $c/d = 0.6$, $q/H = 0.25$).

**Table 2.** A comparison of mode separation for $TE_{01\delta}$ mode with nearest mode for all four configurations shown in Fig. 2 and Fig. 12 ($\varepsilon_{rs} = 2.2$. (i) Conventional Ring DR: $\varepsilon_d = 35.74$. (ii) Modified Ring DR: $p/H = 0.25$. (iii) MLMP Ring DR (4-layers): $\varepsilon_{r1} = 43$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 43$, $\varepsilon_{r4} = 27$, $h_1 = 0.35\,\mathrm{H}$, $h_2 = 0.1\,\mathrm{H}$, $h_3 = 0.30\,\mathrm{H}$, $h_4 = 0.25\,\mathrm{H}$. (iv) Modified MLMP Ring DR: $q/H = 0.25$).

| Types of DR Structures | Parameters | Mode Separation | | Parameters | Mode Separation | | Parameters | Mode Separation | |
|---|---|---|---|---|---|---|---|---|---|
| | | A = 0 | A = 0.6 | | A = 0 | A = 0.6 | | A = 0 | A = 0.6 |
| Conventional Ring DR | c/a = 0.3 | 32.96 | 21.88 | c/a = 0.4 | 36.37 | 23.82 | c/a = 0.5 | 38.18 | 25.24 |
| MLMP Ring DR | c/a = 0.3 | 38.72 | 25.41 | c/a = 0.4 | 41.54 | 27.68 | c/a = 0.5 | 42.39 | 28.50 |
| Modified Ring DR | c/a = 0.3, c/d =0.57 | 38.96 | 26.65 | c/a = 0.4, c/d = 0.6 | 40.13 | 27.96 | c/a = 0.5, c/d =0.67 | 40.12 | 28.61 |
| Modified MLMP Ring DR | c/a = 0.3, c/d =0.57 | **41.14** | 28.31 | c/a = 0.4, c/d = 0.6 | **43.76** | 30.02 | c/a = 0.5, c/d =0.67 | **43.98** | 29.91 |

of substrate ($\varepsilon_{rs}$) alone whereas the asymmetric results include the effect of substrate permittivity along with its thickness. The maximum improvement in mode separation in 4-layer MLMP ring DR is 6% for $c/a = 0.3$, which has been enhanced further in modified MLMP ring DR to 8%. The maximum obtained mode separation in 4-layer MLMP ring DR is 42.39% which is more than the authors' earlier reported best mode separation [13] of modified ring DR of 40.12%. The best mode separation of 44% obtained for the modified MLMP ring DR in this

paper, though may still be optimized further, is highest ever reported in MIC environment.

### 2.4. *Q*-factor

Ensuring a high $Q$-factor is an important issue in the study of dielectric resonator. So, it needs to be shown here that the improvement of mode separation is not at the cost of $Q$-factor. The 5-layer MLMP ring DR, discussed earlier that has shown the improvement in mode separation over a wide range of asymmetry (Fig. 10), has been considered here for comparing its unloaded $Q$-factor with its equivalent ring DR. The values of unloaded $Q$-factors of the dominant $TE_{01\delta}$ mode and nearest mode for MLMP ring DR and the Conventional Ring DR are shown Fig. 14. For the calculation of unloaded $Q$-factor, the loss due to substrate, dielectric resonator, side metal wall, top and bottom metal plate, are taken into account.

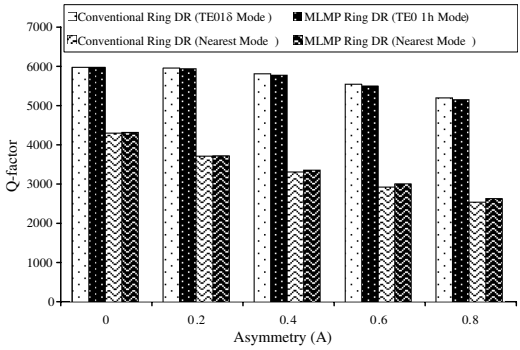


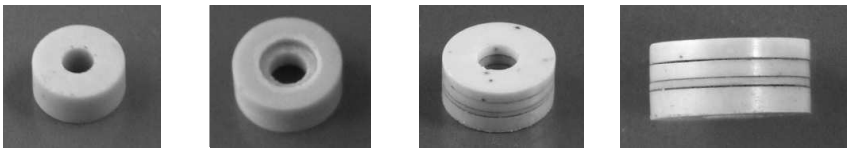

**Figure 14.** Comparison of $Q$-factor of $TE_{01\delta}$ mode with the nearest mode for Conventional Ring DR and 5-layer MLMP Ring DR with asymmetry (A). (All the parameters are same as given in Fig. 10).

The only approximation is that the dielectric loss tangent for each layer of MLMP ring DR and Conventional Ring have been considered the same as $\tan\delta_{DR} = 0.00012$. Practically, this assumption may not always hold but for the purpose here to show a comparative improvement in mode separation in MLMP ring DR over a ring DR, it is a reasonable assumption and also in line of principle of equivalence adopted in present study. Results show that the unloaded $Q$-factor of dominant mode for Conventional Ring DR and MLMP ring DR are almost same implying no additional losses on account of MLMP approach.

## 3. EXPERIMENTAL RESULTS

In this section, the improvements in mode separation observed by simulations have been verified experimentally. The required MLMP ring DR and its equivalent Conventional Ring DR have been fabricated by the ceramic processing technology commonly employed for dielectric resonators. However, MLMP ring DR not being a monolith has been assembled by stacking the ring layers of desired permittivity cut to desired depth from the ceramic tubes possessing their respective permittivity. Since the microwave dielectric resonator materials of required dielectric constant, of values $\varepsilon_{r1} = \varepsilon_{r3} = 42$, $\varepsilon_{r2} = 10$ and $\varepsilon_{r4} = 27.5$ needed for various layers of 4 layer MLMP ring were not available readily, these were prepared in Laboratory. The Niobate material systems $(1-x)\mathrm{ZnNb_2O_{6-x}}$. $\mathrm{TiO_2}$ reported in [20] have been considered suitable for the needed dielectric constants, respectively, for $\varepsilon_{r1} = \varepsilon_{r3} = 42$ and $\varepsilon_{r4} = 27.5$ with values for fractions $x$ adjusted in laboratory as desired. The starting (precursor) oxide material of high purity (>99.5%) have been used. For the remaining layer of $\varepsilon_{r2} = 10$, a high purity alumina ceramic (AKP-4, Sumitomo, Japan) has been used. For providing rigidity to stacked rings, an adhesive has been used to make it monolithic. Prior to using the adhesive, it has been verified separately that the required thin layers of the adhesive have negligibly small influence on the resonance characterization.

The dimension and other properties of the above fabricated ring DRs are provided in Fig. 15. As may be seen, this figure containing the photographs of the fabricated DRs also includes a modified ring DR, reported earlier by the present authors and referred in this paper while suggesting the modified MLMP ring DR structure. The resonant



(a) Conventional Ring DR (b) Modified Ring DR (c) MLMP Ring DR (d) Side View of MLMP Ring DR

**Figure 15.** Fabricated sample of Conventional Ring, Modified Ring and Multilayer Multi-permittivity (MLMP) Ring DR. (a) $2a = 9.96\,\mathrm{mm}$, $H = 4.26\,\mathrm{mm}$, $2c = 3.4\,\mathrm{mm}$, $\varepsilon_d = 36.0$. (b) $2a = 9.93\,\mathrm{mm}$, $H = 4.14\,\mathrm{mm}$, $2c = 3.36\,\mathrm{mm}$, $2d = 5.48\,\mathrm{mm}$, $p = 1\,\mathrm{mm}$, $\varepsilon_d = 36.0$. (c) & (d) $2a = 9.94\,\mathrm{mm}$, $2c = 3.47\,\mathrm{mm}$, $\varepsilon_{r1} = 42$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 42$, $\varepsilon_{r4} = 27.5$, $h_1 = 1.51\,\mathrm{mm}$, $h_2 = 0.44\,\mathrm{mm}$, $h_3 = 1.36\,\mathrm{mm}$, $h_4 = 1.08\,\mathrm{mm}$.

**Figure 16.** Fabricated cylindrical metal cavity.

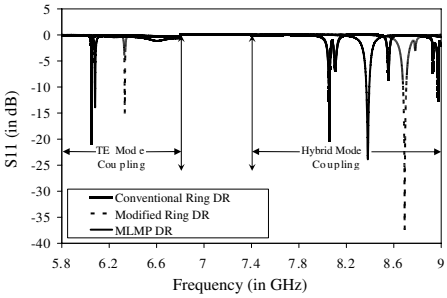



**Figure 17.** Measured resonant frequency of three fabricated dielectric resonators shown in Fig. 15. [$2b = 15\,\text{mm}$, $H_C = 9.68\,\text{mm}$, $H_S = 1.6\,\text{mm}$, $\varepsilon_{rs} = 2.2$]. Note that the first resonant frequency of dominant $\text{TE}_{01\delta}$ mode is measured from TE coupling, whereas the higher order modes are measured using hybrid coupling.

frequencies of all fabricated ring DRs are measured in single cavity, as shown in Fig. 16, to provide the same shielding enclosures to all the fabricated DRs. The measured resonant frequencies of first two modes of all the fabricated DRs are shown in Fig. 17, where the first modes, $\text{TE}_{01\delta}$ modes, are measured with TE coupling whereas to measure the nearest higher mode (HE mode), hybrid coupling is used. The measured and simulated frequencies for all three fabricated DRs are compared in Table 3.

The evaluated mode-separation from the measured frequencies of fabricated ring DRs for two different substrate thickness are compared in Table 4. As may be seen in the Table 4, the experimental results also show the improvement of mode separation in MLMP ring DR over the Conventional Ring DR, validating the simulated results.

The difference in simulated and measured frequencies both for the Conventional Ring and modified ring DR are within 0.9%, but it becomes higher to 1.4% for MLMP ring DR as noted in Table 3. The possible cause of observed differences between the simulated and measured frequencies in MLMP DR can be attributed mostly

**Table 3.** Comparison of simulated (In HFSS) and measured resonant frequencies of all three fabricated DR's placed on substrate enclosed by circular metal cavity [$2b = 15\,\text{mm}$, $H_C = 9.68\,\text{mm}$, $H_S = 1.6\,\text{mm}$, $\varepsilon_{rs} = 2.2$, fabricated DRs dimensions are given in Fig. 15].

| Types of structures | Mode | Simulated | Measured | Error (in %) |
|---|---|---|---|---|
| Conventional Ring DR | $TE_{01\delta}$ Mode | 6.0772 | 6.0525 | −0.406 |
| | Nearest Mode | 8.0039 | 8.0560 | 0.651 |
| Modified Ring DR | $TE_{01\delta}$ Mode | 6.3427 | 6.3313 | −0.179 |
| | Nearest Mode | 8.6132 | 8.6927 | 0.923 |
| MLMP Ring DR | $TE_{01\delta}$ Mode | 6.1285 | 6.0812 | −0.771 |
| | Nearest Mode | 8.211 | 8.3270 | 1.413 |

**Table 4.** Evaluated mode separation (in %) from measured resonant frequencies of fabricated ring DR's for different cavity dimensions and substrate thicknesses. [$H_C = 8.48\,\text{mm}$, $2b = 15\,\text{mm}$, $\varepsilon_{rs} = 2.2$, Fabricated DRs dimensions are given in Fig. 15].

| Substrate Thickness(mm) | $Hs = 0.8$ | $Hs = 1.6$ |
|---|---|---|
| Conventional Ring DR | 24.76 | 35.01 |
| MLMP Ring DR | 30.68 | 40.13 |
| Modified Ring DR | 31.13 | 40.24 |

to fabrication tolerances of the individual ring layers and also in accurately assembling and gluing them together for a monolith shape. For an accurate assembly of MLMP ring, the individual ring layers of the same outer and inner diameters ($2a$, $2c$ respectively) are needed. Since the desired rings of differing permittivity have been obtained by cutting from the ceramic tubes of their corresponding permittivity, the above dimensional equality in diameters can be maintained only within the obtainable tolerance of ceramic fabrication technology, particularly when the ceramic tubes using differing materials need be manufactured individually. Further, the accuracy in thickness of the rings gets limited by the resolution in cutting by a diamond circular saw employed here and this typically can cause an error in thickness up to $25\,\mu\text{m}$. Furthermore, the adhesive layer though minimized in the experimental samples, is still finite compared to its assumed zero thickness in simulation. All these factors can contribute to the observed differences in simulated and the measured resonant frequencies.

The $Q$ factors for the MLMP ring-DR both with and without the adhesive between the layers have been measured by the well known 2-port transmission mode method [21, 22]. A copper shielding cavity

of relatively larger dimensions than the one used for measuring the frequencies as above has been used with expectedly a higher accuracy for measuring the loaded $Q$ factor and also to achieve a lighter coupling said as a desirable condition for a simplified approach for transmission mode method for measuring the $Q$ factors [21]. Then, for equally light couplings for both the ports the unloaded $Q$ factor is simply determined as $Q_0 = Q_L/(1 - |S_{21}|$. The loaded $Q$ factor ($Q_L$) can be easily determined by the 3 dB frequency method, $Q_L = f_0/BW$; where BW-the bandwidth, as usual is the difference between the frequencies at $-3$ dB amplitudes of the resonator transmission response.

Since the couplings at the two ports ($K_1$, $K_2$) are difficult to be arranged to be equal, these are evaluated here as following, by measuring $|S_{110}|$ and $|S_{220}|$ at the resonant frequency.

$$K_1 = \frac{1 - |S_{110}|}{|S_{110}| + |S_{220}|} \; ; \; K_2 = \frac{1 - |S_{220}|}{|S_{110}| + |S_{220}|} \; ; \tag{4}$$

and subsequently the $Q_0 =, Q_L(1 + K_1 + K_2)$.

Following the above method, the $Q_L$ and $K_1$, $K_2$ have been determined for the $TE_{01\delta}$ mode of the MLMP ring-DR as a monolith (with adhesive) and as stacked ring layer DR after the adhesive was removed (without adhesive). Expectedly, a comparison for the two cases will reveal the degradations in $Q$-factor, if any, on account of the used adhesive.

The measured $Q$ factors as above are presented in Table 5. The 4-layer MLMP Ring DR with adhesive and without adhesive have

**Table 5.** Measured $Q$-factors of $TE_{01\delta}$ mode 4 layer multilayer multi-permittivity (MLMP) ring DR using $S$-parameters of 2-port transmission mode method [**Cavity**: $2b = 1.01$ in, $H_C = 0.6$ in, $2a = 0.68$ in, $H = 0.3$ in, $\varepsilon_{rs} = 2.2$, $H_S = 0.15$ in, $c/a = 0.40$, **4-layer MLMP Ring DR**: $2a = 9..94$ mm, $2c = 3.47$ mm $\varepsilon_{r1} = 42$, $\varepsilon_{r2} = 10$, $\varepsilon_{r3} = 42$, $\varepsilon_{r4} = 27.5$, $h_1 = 1.51$ mm, $h_2 = 0.44$ mm, $h_3 = 1.36$ mm, $h_4 = 1.08$ mm].

| Sample | Loaded $Q$-factor ($Q_L$) | Coupling coefficients ($K1 + K2$) | Unloaded $Q$-factor ($Q_0$) |
|---|---|---|---|
| 4 Layer MLMP Ring-DR (With adhesive) | 1393 | 1.172 | 3026 |
| | | 1.124 | 2959 |
| | | 1.146 | 2989 |
| 4 Layer MLMP Ring-DR (Without adhesive) | 1564 | 0.987 | 3109 |
| | | 0.979 | 3096 |
| | | 0.972 | 3084 |

been measured thrice for different coupling coefficients to get the approximate value of DR's quality factor. The measured values for $Q_0$ with adhesive are approximately $2990 \pm 30$ compared to $3097 \pm 13$ for the case of without adhesive. The observed difference between the $Q$ values between the MLMP rings with and without adhesive is less than (3%) that can be considered well within the measurable errors. It is evident that the adhesive used for binding various rings to obtain a monolith 4 layer MLMP ring-DR causes a negligibly small degradation in $Q$ factors. It need be mentioned that the most of the material (except the thin layer 2 contributing $< 2.77\%$ to the $\varepsilon_{eff}$) used for fabricating the MLMP rings is reported [20] to posses the product $(Q \times f) = 20{,}000$–$25{,}000$. As such, the dielectric resonators made from this ceramic can offer unloaded $Q$ factors 3500 to 4000 at 6 GHz frequency when measured in an ideal cavity of large dimensions and of sufficiently high conductor quality factor so that to assume that $Q_0 = (1/\tan\delta)$; $\tan\delta$ being the dielectric tangent loss. The values of $Q_0$ as measured here are somewhat lower since the used shielding cavity is not an ideal one and different dielectric layers having different dielectric loss tangent. Nevertheless, the comparative view of $Q$-factor measured here is to assess the influence of the adhesive only not the individual $Q$-factor of MLMP Ring DR.

## 4. CONCLUSION

It has been demonstrated that the mode separation in a MIC structure can be improved by the MLMP DR approach. Implying that, the inevitable degradation in mode separation due to substrate properties of DR in MIC configuration can be compensated effectively. A significant feature of the MLMP approach lies in implementing the suggested scheme of restoration of field patterns of spurious mode(s). Such restoration of fields in asymmetric DR in MIC configuration can be anticipated as an improvement in mode separation. The anticipated improvements have been validated theoretically as well as experimentally by adopting the MLMP approach. Some of the results allowing such conclusion are as follows:

1. The improvement of mode separation in 4-layer MLMP DR over Conventional Ring DR is 4 to 6% for a wide range of asymmetry in MIC structure.
2. This mode separation is further enhanced in modified MLMP DR with a maximum improvement by 8%.
3. From the optimization of MLMP ring DR structures conducted in this study, it is expected that further improvement may be

possible in MLMP DR by optimizing the number of layers, their permittivity and thickness

Further, it appears that the approach opens up new possibilities in DR structure, in respect to control and tailor-making of their other important characteristics. For example, improving the $Q$-factors and in temperature compensation of DR structures simultaneous to an improvement in mode separation.

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