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ARTIFICIAL MAGNETIC PROPERTIES OF DIELECTRIC METAMATERIALS IN TERMS OF EFFECTIVE CIRCUIT MODEL

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Abstract—An effective series RLC model for the electromagnetic response of weakly absorbing dielectric sphere near the first magnetic dipole resonance was developed, and the effective magnetic properties of Mie resonance-based dielectric metamaterials were obtained in terms of this model. In comparison with traditional effective medium theory such as extended Maxwell-Garnett (EMG) theory based on Mie model, this approach is more intuitive and can give an analytical dependence of the magnetic properties of the composite on the electromagnetic and geometric parameters of the constituting dielectric particles.

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1. INTRODUCTION

Left-handed metamaterials (LHM) are artificially structured media which exhibit many exotic electromagnetic behaviors not observed in nature [1, 2]. A big challenge for realization of LHM is to attain artificial magnetism, especially that of negative permeability. Since Pendry firstly proposed that metallic split ring resonator (SRR) can provide negative magnetic permeability [3], many researchers focused on the study of different metallic resonance structure to obtain the negative magnetic permeability, including the double SRR [3–5], single SRR [6–8], S-ring resonator [9, 10], fishnet structures [11, 12] and so on. In order to interpret the artificial magnetism of various metallic structures, researchers presented various models which are all based on the RLC-resonance of the metallic structure [3, 6, 13, 14]. Then the macroscopic electromagnetic parameters are obtained by use of the so-called effective medium theory. The RLC circuit model presented a clear and instructive physics paradigm for the metallic structured metamaterials, and became a powerful tool for the prediction and design of metamaterials.

As a new route to obtain LHM instead of metallic structure, metamaterials composed of dielectric particles in which the negative permeability is derived from the first Mie resonance (magnetic dipole resonance) of the particle attracted much attention for their simple structure and isotropy [15–20].

For the aforementioned studies on dielectric metamaterials, the analyses are based on the Extended Maxwell-Garnett (EMG) theory [21–23], which is essentially the dynamic generalization of Clausius-Mossotti relation. When the particles are arranged in cubic lattice, the effective magnetic permeability of the metamaterial is given by EMG as,

$$\mu_{\mathit{eff}} = \frac{x^3 + 3ifb_1}{x^3 - \frac{3}{2}ifb_1}, \tag{1}$$

where f is the volume fraction of the dielectric particles, and the magnetic dipole scattering coefficient is given by

$$b_1 = \frac{\psi_1(m_r x)\psi_1'(x) - m_r\psi_1(x)\psi_1'(m_r x)}{\psi_1(m_r x)\xi_1'(x) - m_r\xi_1(x)\psi_1'(m_r x)}, \tag{2}$$

where $m_r = m_p/m_h$ is the complex refractive index of the dielectric particle relative to the host, and m_p and m_h are respectively the refractive indexes of the particle and the host in vacuum. The size parameter of the dielectric particle is $x = m_h\omega r/c$, where ω is the angular frequency of incident wave, r is the radius of the dielectric particle, and c is the velocity of light in vacuum. ψ_1 and ξ_1 are the first order Riccati-Bessel functions.

The EMG approach can be used to numerically predict the effective permeability of the composite, whereas we cannot obtain analytically the dependence of the effective magnetic properties of the composite on the electromagnetic and geometrical parameters of the constituting particles due to the expression of b_1 being complicated and not intuitive.

In this work, we developed an effective series RLC loaded conductive loop model of the weakly absorbing dielectric particle metamaterials to overcome the deficiency of EMG. With this model, we can conveniently analyze the magnetic dispersion relation of dielectric metamaterials, and obtain the dependence of the important physical quantities such as resonance frequency and relative broadness of negative permeability on the parameters of constituting particles. This work provides us a new and intuitive insight into the physics of magnetic properties of dielectric metamaterials.

2. DERIVATION OF THE MODEL

Consider a dielectric sphere embedded in a host with a refractive index m_h, being incident by a plane wave, as shown in Fig. 1(a). The radius of the dielectric sphere r is much smaller than the incident wavelength in the host, implying the particle size parameter $x \ll 1$. The complex refractive index of the dielectric particle relative to the

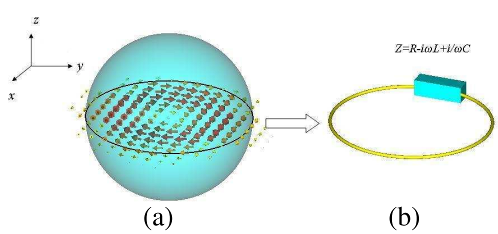


Figure 1. Schematic view of the structure under study. The left dielectric sphere is incident by a plane wave with the magnetic field directed along z axis and wave vector directed along y axis, and eddy currents are induced inside it. The eddy currents generate magnetic moment and provide the magnetic property. The arrows represent the eddy currents on the xy plane. The right conductive ring loaded by an impedance Z is the effective circuit model of the dielectric sphere. The radius of the ring is same as that of the sphere.

host is $m_r = m'_r + im''_r = (m'_p + im''_p)/m_h$, where m'_p and m''_p are the real part and imaginary part of particle refractive index in vacuum respectively. We assume that $m''_r \ll m'_r$, which is satisfied by weakly absorbing dielectrics including the $Ba_{0.5}Sr_{0.5}TiO_3$ (BST) cube used in the experiment [2, 20]. For small x, Videen ever gave $m_r x = N\pi$ where N is an integer, as the resonance condition of the scattering coefficient b_1. In order to meet the resonance condition and the effective medium theory which requires that the dimension of the particles must be much less than the wavelength in host, the real part of the particle refractive index have to be much larger than that of the host. When the dielectric sphere is excited into b_1 resonance, the eddy currents which consist of conductive and displacement currents, are generated around the electromagnetic wave magnetic field direction as shown in Fig. 1(a). The magnetic moment generated by the eddy currents interacts with the electromagnetic wave magnetic field, and provides the artificial magnetism. For the dielectric particle with weak absorption, b_1 can be expanded about the first resonance location and simplified as [24],

$$b_1 = \frac{x}{x + m_r m''_r} \frac{1}{1 + \frac{(\tilde{\Delta} + x/m_r)}{ix^2/m_r + im''_r x}}, \tag{3}$$

where $\tilde{\Delta} = m'_r x - \pi$. Considering that $m'_r \gg m''_r$ and $x \ll 1$, Eq. (3) can be simplified further,

$$b_1 \simeq \frac{x}{m'_r m''_r} \left[\frac{1}{1 + \left(\frac{m'_r}{m''_r} - \frac{\pi/m''_r}{x}\right)^2} + i \frac{\left(\frac{m'_r}{m''_r} - \frac{\pi/m''_r}{x}\right)}{1 + \left(\frac{m'_r}{m''_r} - \frac{\pi/m''_r}{x}\right)^2} \right]. \tag{4}$$

First of all, we derive the effective resistance R of the dielectric particle. We assume that the effective circuit is a circular conductive ring loaded by an effective impedance $Z = R - i\omega L + i/\omega C$ as shown in Fig. 1(b), where R, L, and C are the effective resistance, effective inductance, and effective capacitance respectively, which are in series connection. The radius of the ring is same as that of the sphere. Considering $x \ll 1$, the electromotive force (emf) induced on the ring is

$$\mathcal{E} = -\frac{\partial \left(B\pi r^2\right)}{\partial t} = i\omega\pi r^2 B, \tag{5}$$

where the harmonic factor $e^{-i\omega t}$ is assumed. The power dissipated in the resistor is equal to the power absorbed and scattered by the sphere. In the case of resonance, the power dissipated in the effective resistor of the effective RLC circuit is equal to $\mathcal{E}_0^2/2R$, where $\mathcal{E}_0$ is the amplitude of the emf. The power absorbed and scattered by the sphere is $cB_0^2 C_{ext}/2\mu_0 m_h$, where B_0 is the amplitude of the magnetic vector, and C_{ext} is the extinction cross section of the dielectric particle, which is equal to

$$C_{ext} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1)\mathrm{Re}(a_n + b_n). \tag{6}$$

The Mie scattering coefficients except for b_1 are negligible when the particle is excited into magnetic dipole resonance. So the extinction cross section is equal to $C_{ext} = 6\pi\mathrm{Re}(b_1)/k_0^2$, where k_0 denotes the wavenumber at resonance. Therefore we have

$$\frac{1}{2}\frac{\mathcal{E}_0^2}{R} = \frac{cB_0^2}{2\mu_0 m_h}\frac{6\pi}{k_0^2}\mathrm{Re}(b_1). \tag{7}$$

When the particle is in magnetic dipole resonance, from Eq. (4) we have $\mathrm{Re}(b_1) = x/m'_r m''_r$. Together with the first b_1 resonance condition $m'_r x = \pi$, i.e., $\omega_0 = \pi c/m'_r m_h r = \pi c/m'_p r$, we derive the effective resistance R as follows,

$$R = \frac{\mu_0 \pi^4 c m''_r}{6 m'^2_r m_h} = \frac{\mu_0 \pi^4 c m''_p}{6 m'^2_p}. \tag{8}$$

Assuming that $m'_r/m''_r = A_1$, $\pi/m''_r = A_2$, and $X = A_2/x$, we can obtain real b_1 according to Eq. (4) as follows,

$$\mathrm{Re}(b_1) = \frac{x}{m'_r m''_r}\frac{1}{(X - A_1)^2 + 1} = \frac{x}{m'_r m''}y. \tag{9}$$

The right term of Eq. (9) $y = 1/[(X - A_1)^2 + 1]$ is a typical Lorentzian function, so it reaches its peak at $X = A_1$, and gets its half maximum at $X = A_1 \pm 1$, i.e., at $x = A_2/(A_1 \pm 1)$. Therefore the full width at half maximum (FWHM) of y is $\Delta x = 2A_2/(A_1^2 - 1)$. Seeing that $m''_r \ll m'_r$, which means that the dielectric particle is a high Q resonator, we can consider x as a constant in the FWHM. Therefore, we think that the FWHM of real b_1 is approximately equal to that of y as follows,

$$\Delta x = \frac{2\pi/m''_r}{(m'_r/m''_r)^2 - 1}. \tag{10}$$

So the relative broadness of b_1 resonance is

$$\frac{\Delta f}{f_0} = \frac{\Delta x}{x_0} = \frac{\Delta x}{\pi/m'_r} = \frac{2\pi/m''_r}{(m'_r/m''_r)^2 - 1}\frac{m'_r}{\pi} \simeq 2m''_r/m'_r, \tag{11}$$

which means that the quality factor of the first b_1 resonance is $Q = f_0/\Delta f = m'_r/2m''_r$. It is well-known that the quality factor of a series RLC circuit is $Q = (1/R)\sqrt{L/C}$. Therefore we have

$$(1/R)\sqrt{L/C} = m'_r/2m''_r. \tag{12}$$

On the other hand, the resonance frequency of RLC circuit is $\omega_0 = 1/\sqrt{LC}$. Together with $\omega_0 = \pi c/m'_p r$, Eqs. (8) and (12), we can derive the effective inductance and capacitance of the effective circuit as follows,

$$L = \frac{\mu_0 \pi^3 r}{12}, \quad C = \frac{12 r m_p'^2}{\mu_0 \pi^5 c^2}. \tag{13}$$

Now we can make use of previous theoretical research on SRR metamaterials to obtain the effective magnetic permeability of the dielectric particle array. The effective magnetic permeability of SRR array is given by [3, 6],

$$\mu_{zz}(\omega) = 1 - \frac{A\omega^2}{\omega^2 - \omega_r^2 + i\Gamma\omega}, \tag{14}$$

in which the parameters A, ω_r, Γ are related to the effective R, C, and L of the SRR element by [6]: $A = \pi^2 \mu_0 r^4 n L^{-1} \omega_r^2 \omega_0^{-2}$, $\omega_r = \omega_0 (1 + \mu_0 r \Sigma L^{-1} + \pi^2 \mu_0 r^4 n L^{-1}/3)^{-1/2}$, $\Gamma = R L^{-1} \omega_r^2 \omega_0^{-2}$, where $\omega_0 = 1/\sqrt{LC}$ is the resonance angular frequency of the SRR, r and n are the radius and number density of the SRR respectively. Σ is a dimensionless parameter which depends only on the lattice type and the values of the lattice constants, and in the case of cubic lattice it equals to zero due to symmetry. The relative broadness of negative magnetic permeability is given as follows [6],

$$\Delta = \frac{\tilde{\omega}^2 - \omega_r^2}{\omega_r^2} = \frac{\pi^2 \mu_0 r^4 n L^{-1}}{1 + \mu_0 r \Sigma L^{-1} - \frac{2}{3}\pi^2 \mu_0 r^4 n L^{-1}}, \tag{15}$$

where $\tilde{\omega}$ denotes the magnetic plasma frequency.

In the case of cubic lattice and the volume fraction of the dielectric particles being f, by substituting the formulae of R, L, C, and ω_0 into the expressions of A, ω_r, and Γ, we derive the parameters as follows,

$$A = \frac{9f}{3f + \pi^2}, \quad \Gamma = \frac{2\pi^3 c m''_p}{m_p'^2 r (3f + \pi^2)}, \quad \omega_r = \frac{\pi^2 c}{m'_p r \sqrt{3f + \pi^2}}. \tag{16}$$

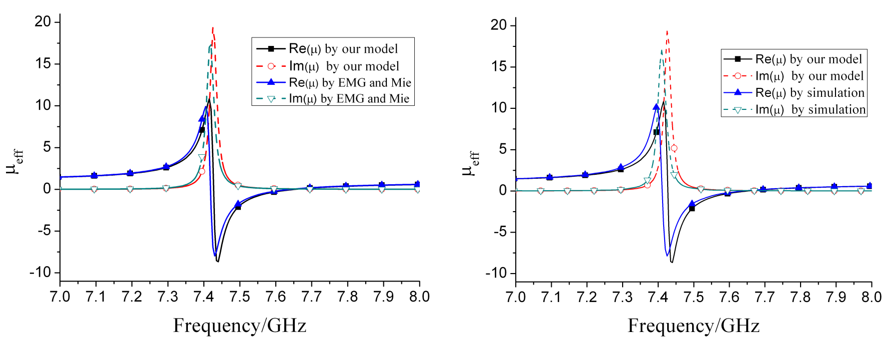
Therefore the effective magnetic permeability of the dielectric composite can be obtained by substituting these parameters into Eq. (14). On the other hand we can calculate the effective magnetic polarizability of the dielectric particle by using the expressions of the emf, R, L, and C, and then substitute the effective magnetic polarizability into Clausius-Mossotti relation to obtain the same result as well.

The relative broadness of negative magnetic permeability is also derived as follows,

$$\Delta = \frac{9f}{\pi^2 - 6f}. \tag{17}$$

3. NUMERICAL VERIFICATION AND DISCUSSION

In order to verify our derivation, we compare the calculation results using our model with those provided by EMG based on accurate Mie theory and numerical simulations respectively. The S parameters are computed by numerical simulation software CST Microwave Studio, and the boundary conditions are set as paired perfect conductor conditions and paired perfect magnetic conditions perpendicular to the propagation direction. Thus we used the calculated S parameters to retrieve the effective permeability by using the methods presented by Refs. [25, 26]. We choose the BST sphere as the constituting dielectric particle. The radius of the BST sphere is 0.5 mm and the lattice constant is 2 mm, and the dielectric permittivity is 1600+4.8i [20]. The permittivity of the host is set to 2. The BST spheres are arranged in cubic lattice. The calculation results using the model are shown in Fig. 2, and are both in good agreement with the results provided respectively by EMG and simulation. As shown in Fig. 2(a), compared with the results given by EMG, the maximums of the real and imaginary parts given by our model are a little higher, and the resonance frequency shifts a little to high frequency. The little discrepancy is due to the approximation we adopt in Eq. (4). There



(a) (b)

Figure 2. Comparisons of the effective magnetic permeability of the composite consisting of BST sphere obtained by three different methods. The permittivity of the BST sphere and the host are 1600 + 4.8i and 2 respectively. The radius of the BST sphere is 0.5 mm, and the lattice constant is 2 mm. (a) Permeability obtained by use of our model and the permeability obtained by EMG. (b) Permeability obtained by our model and the permeability retrieved from the S parameters calculated by CST Microwave Studio.

exists a similar discrepancy in Fig. 2(b), but a little bigger than that in Fig. 2(a). The reason is that the accuracy of the numerical simulation software is limited, and we can decrease the difference between Figs. 1(a) and (b) by increasing the mesh density.

The resonance frequency is an important parameter of Mie resonance-based dielectric metamaterials. We calculate the dependence of the resonance frequency on the volume fraction f according to Eq. (16), as shown in Fig. 3. The maximum filling ratio of the composite with cubic lattice is $4\pi \times 0.5^3/3 = \pi/6$. With the increasing of the volume fraction of particles, the interactions among particles get strong [27], so the EMG is not applicable any more [13]. Therefore we set the maximum f to be 0.425. We change the lattice constant to obtain different volume fraction, and carry out the numerical simulations. The simulation and EMG results are also shown in Fig. 3. In the case of low volume fraction f, the three results are in good agreement. It is evident that the resonance frequency shifts to lower frequencies with the increase of volume fraction. According to Ref. [6], we obtained the expression of resonance frequency in the case of cubic lattice: $\omega_r = \omega_0(1 + \pi^2\mu_0 r^4 nL^{-1}/3)^{-1/2}$. We can see that ω_0 and L are all constants for a dielectric particle with given permittivity and radius from Eq. (13). So ω_r is the decreasing function with respect to the number density n. Therefore the resonance frequency ω_r decreases with the particle volume fraction f increasing.

With the further increase of f, the simulated resonance frequencies get higher than those given by the model and EMG. The reason is that when the particles get denser the interactions among them play a more

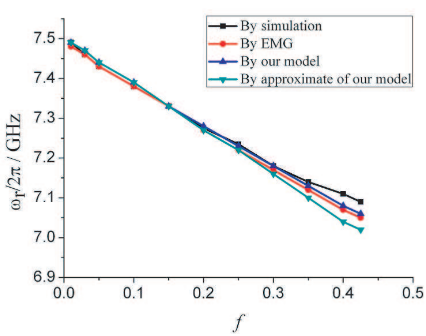


Figure 3. Resonance frequency against volume fraction of dielectric particles.

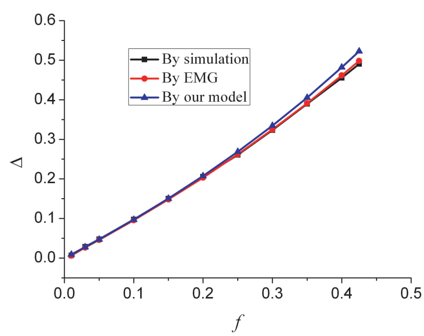


Figure 4. Relative broadness of negative magnetic permeability against volume fraction of dielectric particles.

important role, and the model is not applicable as well as the EMG. We note that at low f the dependence of the resonance frequency on f takes on a kind of linear relation with a negative slope. Considering $3f \ll \pi^2$ at low f, we expand the expression of resonance frequency as follows,

$$\omega_r = \frac{\pi^2 c}{m_p' r\sqrt{3f+\pi^2}} \simeq \frac{\pi c}{m_p' r}\left(1-\frac{3}{2\pi^2}f\right). \tag{18}$$

The approximate expression Eq. (18) is also depicted in Fig. 3. The four curves are in good agreement at the low volume fraction as expected.

Another important parameter of the dielectric metamaterials is the relative broadness of negative permeability Δ. The dependence of Δ on the volume fraction f, obtained by three methods, are all shown in Fig. 4. The three curves are in good agreement at low volume fraction, while the relative broadness Δ given by Eq. (17) is a little wider at high volume fraction than those provided by EMG and numerical simulation. Despite the little discrepancy, Eq. (17) gives a direct depiction with regard to the dependence of the relative broadness of negative permeability on the volume fraction of the dielectric particles. Eq. (17) can be changed into $1/\Delta = (\pi^2/9)/f - 2/3$, which means that Δ is an increasing function with respect to volume fraction f.

4. CONCLUSION

In summary, we developed an effective series RLC circuit model for weakly absorbing dielectric sphere to approximate and simulate its electromagnetic response near the first magnetic dipole resonance. In terms of the model we can give analytically the relation between the effective magnetic properties of weakly absorbing dielectric metamaterials and the physical parameters of the dielectric particles. Although this approach is derived in the case of spherical particles, it is also valid to analyze approximately the magnetic properties of metamaterials consisting of dielectric cube [20], or finite dielectric cylinder. The reason is that the dielectric cube and finite dielectric cylinder can be considered approximately as sphere when the wavelengths in host are much larger than the dimension of particles. This approach cannot be applied to interpret the effective magnetic properties of dielectric metamaterials consisting of infinite dielectric cylinder, such as in Ref. [28] The simple effective RLC circuit model analysis provides us an intuitive and deep understanding on the Mie resonance-based dielectric metamaterials, and will play an important role in the future design of dielectric metamaterials.

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