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ROBUST ADAPTIVE BEAMFORMING BASED ON COVARIANCE MATRIX RECONSTRUCTION FOR LOOK DIRECTION MISMATCH

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Abstract—The performance degradation in traditional adaptive beamformers can be attributed to the imprecise knowledge of the array steering vector and inaccurate estimation of the covariance matrix. The inaccurate estimation of the covariance matrix is due to the limited data samples and presence of desired signal components in the training data. The mismatch between the actual and presumed steering vectors can be mainly due to the error in the look direction estimate. In this paper, we propose a novel algorithm to estimate the look direction and to reconstruct the covariance matrix so that near optimal performance without the effect of saturation can be achieved as the input SNR increases. Numerical results also show that all existing beamforming algorithms suffer from saturation effect as the input SNR increases.

1. INTRODUCTION

One of the most important challenges in adaptive beamforming design is to maintain its performance even in the presence of uncertainty due to the mismatch between the actual and the presumed steering vectors (SV). In traditional beamforming techniques, substantial degradation in performance can be observed due to this mismatch. The mismatch can be due to imprecise knowledge of one or any combination of look-direction, array geometry and array elements' gain-phase response.

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The errors other than the look-direction error can be overcome with reliable calibration technique.

During the past decade, several approaches, such as imposing multiple gain constraints in different directions in the vicinity of the presumed SV [1], diagonal loading [2], placing derivative constraints on the presumed SV [3] and eigenspace-based approaches [4], have been proposed to improve the robustness of adaptive beamformer design. The diagonal loading [2] has the advantage of being invariant to the type of mismatches but the choice of the loading factor is not obvious. The authors in [5–7] proposed robust capon beamformer (RCB) based on the idea that allows the presumed SV to be within a sphere whose radius determines the uncertainty level. These approaches efficiently calculate the loading factor. Other variable-loading based robust beamformers are also proposed in [8, 9]. Motivated by a similar idea, the authors in [10, 11] introduced different optimization formulation and proposed solutions that are based on semi-definite programming. Recently, iterative-based approaches are considered for adaptive robust beamforming problem. Hassaniean et al. [12] proposed a new optimization formulation to find the SV error that is solved iteratively using sequential quadratic programming (SQP). Later, Gu et. al. proposed to pre-estimate the covariance matrix prior to solving the optimization [13] where the pre-estimated covariance matrix has the diagonal loading form. In [14], the authors propose a robust adaptive beamforming based on the eigenstructure method to cancel the desired signal in a linearly constrained beamformer with imperfect arrays. The authors in [15] proposed an iterative RCB (IRCB) with adaptive uncertainty level, where in each iteration the estimated steering vector is updated based on the re-adjusted uncertainty level.

Theoretically, optimal beamformers' weight is a function of the interference-plus-noise covariance matrix. However, generally the beamformers' weight is formulated as a function of the covariance matrix estimate. Therefore, inaccurate estimation of the covariance matrix can also result in the performance degradation of the adaptive beamformer. The inaccurate estimation of the covariance matrix is mainly due to the limited data samples and the presence of the desired signal components in the training data.

In this paper, we aim to develop a new robust adaptive beamforming method that achieves near-optimal performance by addressing both the SV mismatch due to look-direction error as well as the inaccurate covariance matrix estimation problems. The essence of the idea is in finding the directions associated with the nulls of the standard capon beamformer's (SCB) beampattern. These directions are a good estimate of the interference directions and because of the signal self-nulling phenomenon, one of these directions is the good estimate of the look-direction. Therefore, the performance degradation due to the SV mismatch can be recovered by correcting the look-direction while the degradation due to the inaccurate covariance matrix can be recovered by replacing the matrix with a newly constructed matrix based on the nulls' directions associated with the interferences. Since the direction of the signal-of-interest (SOI) is nearer to the presumed look-direction, it is possible to distinguish the null's direction associated to the SOI from the interferences. Although numerous capon beamforming algorithms have been proposed [1–15], all these algorithms suffer from the problem of saturation in the output SINR as the input SNR increases.

2. BACKGROUND

Consider K narrowband sources impinging on an L-element uniform linear array (ULA) ($L > K$). The array observation is given by

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} = [\mathbf{a}(\theta_s), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_{K-1})]$ comprises of all the impinging signal SVs, $\mathbf{s}(t)$ is a K-dimension vector containing the SOI and interferences and $\mathbf{n}(t)$ is the noise components. The beamformer's output tries to recover the SOI and is expressed as $y(t) = \mathbf{w}^H\mathbf{x}(t)$.

To have an optimal beamformer, its weight vector $\mathbf{w}$ has to be designed such that the interference-plus-noise output power is minimized while that of the desired signal is unchanged. This design criteria can be formulated mathematically as the following optimization to solve for the weight

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}_{in}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}(\theta_s) = 1 \tag{2}$$

where $\mathbf{w}$ is the complex vector of beamformer weights, $\mathbf{a}$ is the desired signal SV, θ_s is the direction-of-arrival (DOA) of the desired signal and $\mathbf{R}_{in}$ is the interference-plus-noise covariance matrix. Note that the superscript $(\cdot)^H$ denotes the Hermitian operations. The solution to this optimization will yield an optimum SINR output

$$\text{SINR}_{\text{opt}} = \sigma_s^2\mathbf{a}(\theta_s)\mathbf{R}_{in}^{-1}\mathbf{a}^H(\theta_s) \tag{3}$$

where σ_s^2 is the desired signal's power. In practice, the optimum SINR cannot be achieved because $\mathbf{R}_{in}$ is unavailable. It is then replaced with the estimate of the array covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{4}$$

where N is the number of snapshots and $\mathbf{x}(n)$ is the complex vector of array observations using M sensors.

The degradation due to replacing $\mathbf{R}_{in}$ with $\hat{\mathbf{R}}$ becomes significant as σ_s^2 increases. This can be shown from the analytical SINR expression derived by alienating the finite sample effect $(N \approx \infty)$

$$\begin{aligned}\text{SINR} &= \sigma_s^2 \mathbf{a}(\theta_s)\{\mathbf{R}_s + \mathbf{R}_{in}\}^{-1}\mathbf{a}^H(\theta_s) \\ &= \text{SINR}_{opt} - \sigma_s^2 \mathbf{a}(\theta_s)\left[\frac{\mathbf{R}_{in}^{-1}\mathbf{R}_s\mathbf{R}_{in}^{-1}}{1+\sigma_s^2\mathbf{a}(\theta_s)\mathbf{R}_{in}^{-1}\mathbf{a}^H(\theta_s)}\right]\mathbf{a}^H(\theta_s) \quad (5)\end{aligned}$$

Using the Sherman-Morrison formula

$$\begin{aligned}\{\mathbf{R_s} + \mathbf{R_{in}}\}^{-1} &= \left\{\mathbf{R}_{in} + \sigma_s^2\mathbf{a}(\theta_s)\mathbf{a}^H(\theta_s)\right\}^{-1} \\ &= \mathbf{R}_{in}^{-1} - \frac{\mathbf{R}_{in}^{-1}\mathbf{R}_s\mathbf{R}_{in}^{-1}}{1+\sigma_s^2\mathbf{a}(\theta_s)\mathbf{R}_{in}^{-1}\mathbf{a}^H(\theta_s)} \quad (6)\end{aligned}$$

From the expression in (5), it is clear that the degradation is unavoidable due to the presence of desired signal in the covariance matrix and the signal power σ^2 acts as a scaling factor that determines the amount of degradation suffered by the beamformer.

When there is a mismatch between the actual and the presumed SV due to the look-direction error, the beamformer's weight designed from solving (2) will fail to keep the desired signal unchanged. This is because the constraint is set inaccurately, thus resulting in signal self-nulling. Instead of maintaining the unity response of $\mathbf{w}^H\mathbf{a}(\theta_s)$, the solution to the optimization problem forms the null at θ_s and maintains the response at $\bar{\theta}_s$, the original DOA. This is the self-nulling effect that causes the break-down in adaptive beamformer design.

Much of the effort in the design of robust adaptive beamforming has been focused on addressing this problem while ignoring the inaccurate covariance matrix estimation issue. In the next Section, we describe our approach to design a new robust adaptive beamforming method that addresses both these issues in order to achieve near-optimal performance.

3. PROPOSED METHOD

The main idea is to remove the SOI from the covariance matrix estimate $\hat{\mathbf{R}}$, which suffers from the finite sample effect. Any attempt to remove the SOI from $\hat{\mathbf{R}}$ using eigen-decomposition or otherwise can only achieve partial removal of the SOI component, since the steering vectors are non-orthogonal and imprecise. Therefore to ensure that the SOI is completely removed from the covariance matrix, we propose to re-construct a new covariance matrix denoted as $\mathbf{R}_c$ with all the interfering signals except the SOI.

Recall that $\mathbf{R}_{in}$ is formulated from the interferences' SVs weighted by its power

$$\mathbf{R}_{in} = \sum_{k=1}^{K-1} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k) \tag{7}$$

Hence, we need to estimate the SVs as well as the interferences' power σ_k^2.

When the array gain/phase and geometry are perfectly calibrated, the array response function is given by $B(\theta) = \mathbf{w}^H\mathbf{a}(\theta)$. Without a loss of generality, we first discuss for the case where the array is strictly an uniform linear array (ULA) with half-wavelength inter-element spacing. In this case, the array SV expression is simplified to $\mathbf{a}(\theta) = [1, e^{j\pi\sin(\theta)}, \cdots, e^{j(L-1)\pi\sin(\theta)}]^T$ and $B(\theta)$ is now expressed as sum of $(L-1)$ weighted exponential terms

$$B(\theta) = \sum_{l=0}^{L-1} w_l^* e^{j\pi l \sin(\theta)} \tag{8}$$

where the superscript $*$ denotes the conjugate operation.

The DOAs of the SOI and interferences can be estimated as the beampattern of standard capon beamformer (SCB) is expected to form the nulls at both the interferences and SOI directions.

Based on the estimated interfernce's DOAs a new covariance matrix $\mathbf{R}_c$ can be re-constructed. Therefore, the proposed beamformer can be formulated as the following optimization problem

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}_c\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H\bar{\mathbf{a}}(\hat{\theta}_s) = 1 \tag{9}$$

It is similar to the expression in (2) except that $\mathbf{R}_{in}$ and $\mathbf{a}(\theta_s)$ are replaced with $\mathbf{R}_c$ and $\hat{\theta}_s$, respectively. $\hat{\theta}_s$ is a presumed SV defined as a function of SOI's estimated DOA.

We assume that the steering vector, $\mathbf{a}(\theta_s)$ corresponding to the SOI is known only approximately and the presumed angle, θ_s is bounded, $\theta_L < \theta_s < \theta_U$, where θ_L and θ_U are the lower and upper bounds respectively. In our formulation, we also require the knowledge of the presumed array geometry for the generation of the beampattern. We start off with the SCB solution given as

$$\mathbf{w}_{\mathrm{SCB}} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}^H(\theta_s)\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta_s)}. \tag{10}$$

Based on this solution, we generate the beam pattern using

$$B(\theta) = \mathbf{w}_{\mathrm{SCB}}^H\mathbf{a}(\theta) \qquad \text{for} \quad -90\deg \le \theta \le 90\deg. \tag{11}$$

By setting a suitable scanning resolution ϵ, we search for nulls outside the SOI region. The location of the nulls outside the SOI region would correspond to the interferers or grating nulls. We represent the set of all the angles corresponding to these nulls as $\mathcal{I} = \{\theta_1, \theta_2, \ldots, \theta_k\}$, where the maximum number of nulls found is $k < K$. This is done heuristically and can be written as,

$$\mathcal{I} = \{\mathcal{I} : \mathcal{I} = \theta \,|\{\theta < \theta_L\} \cup \{\theta > \theta_U\}, B(\theta - \epsilon) > B(\theta) < B(\theta + \epsilon)\} \tag{12}$$

where ϵ is the scanning resolution used. The angles in $\mathcal{I}$ can then be used to generate the steering vectors of all the interferer signals. We generate a new covariance matrix,

$$\bar{\mathbf{R}} = \gamma_{\max} \sum_{i=1}^{k} \mathbf{a}(\theta_i)\mathbf{a}^H(\theta_i) \tag{13}$$

where $k < K - 1$ and $\gamma_{\max}$ is the largest eigenvalue of $\hat{\mathbf{R}}$. The reason for doing this is that it is difficult to assign the eigenvalues to the right steering vectors and whenever errors occur in the assignment, the performance deteriorates sharply. We find that by assigning the energy of the largest eigenvalue to all the interferences simply nulls all of the interferences equally regardless of their true energy. The rank of $\hat{\mathbf{R}}$ is not full when the number of interferences is less than the number of antennas. This is resolved by simply adding an identity matrix weighted by the smallest eigenvalue of $\hat{\mathbf{R}}$

$$\tilde{\mathbf{R}} = \gamma_{\min}^2 \mathbf{I} \tag{14}$$

Therefore the final reconstructed covariance matrix is simply

$$\mathbf{R}_c = \bar{\mathbf{R}} + \tilde{\mathbf{R}} \tag{15}$$

The reconstructed covariance matrix $\mathbf{R}_c$ does not contain the SOI and as such the SCB formulation applied to this matrix gives a weight vector that gives better results. The new weight vector, $\mathbf{w}_{\text{rec}}$ is

$$\mathbf{w}_{\text{rec}} = \frac{\mathbf{R}_c{}^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}^H(\theta_s)\mathbf{R}_c{}^{-1}\mathbf{a}(\theta_s)}. \tag{16}$$

4. SIMULATION RESULTS

Consider 10-element ULA with half-wavelength spacing receiving three i.i.d signals (SOI and two interferences of equal power (20 dB)), which are arbitrary and well separated from one another. A white Gaussian distributed random variable (0 dB) is considered as the additive noise. Also, $\hat{\mathbf{R}}$ calculated from 100 snapshots is used to implement all the beamformers discussed here. The simulations are carried out on a personal computer with Intel, Core i5 CPU, 3.25 GHz, 4 GB RAM. We carried out two different sets of simulations to show the robustness of the proposed approach compared to the existing beam forming techniques.

First, we only consider the mismatch due to the look-direction. Second,to show the robustness of the proposed algorithm, along with look direction error, we introduce array geometry error which is modeled as uniform random variable according to $\mathcal{U}(-0.15\lambda, 0.15\lambda)$, where λ is the signal wavelength. For both the cases, we evaluate the SINR performance of the proposed approach calculated from 100 Monte Carlo realizations and compare with the existing approaches as well as the theoretically optimal SINR. The algorithm is run for different SNR inputs ranging from −30 dB to +30 dB.

Figures 1 and 2 show the output SINR as a function of input SNR, while Table 1 details the output SINR obtained for the proposed approach for cases 1 & 2. Table 1 lists the mean, standard deviation, best and worst output SINR obtained from the 100 Monte Carlo realizations for each case.

From the results in Table 1, it can be observed that the best results of the proposed algorithm is approximately equal to the optimal values across all the simulated input SNR. From Fig. 1 and Fig. 2, as the input SNR increases the difference in performance of the proposed algorithm and the existing robust beam forming algorithms can be clearly observed. With the increase in the input SNR, saturation can be observed in the performance of SCB, RCB, iterative robust capon

Table 1. SNR vs SINR.

	SINR [dB]							
SNR [dB]	Look Direction Error Only				Look Direction & 15% Geometry Error			
	Mean	STD	Best	Worst	Mean	STD	Best	Worst
−30	−21.0150	0.5355	−20.3101	−23.0707	−23.8917	1.8497	−20.9435	−29.6051
−25	−16.0368	0.5495	−15.3101	−18.1252	−18.9198	1.8579	−15.9200	−24.6336
−20	−11.0961	0.5926	−10.3162	−13.3190	−13.9845	1.8709	−10.8681	−19.7217
−15	−6.2937	0.7437	−5.3706	−8.9879	−9.1927	1.9551	−5.8558	−14.9950
−10	−1.8672	1.2047	−0.4687	−6.0816	−4.7724	2.2230	−0.9704	−11.0030
−5	1.3534	2.1756	4.3753	−3.4040	−1.6023	2.7912	3.0135	−8.7213
0	6.6894	1.2561	9.2436	3.4130	3.9769	2.2677	8.1945	−2.1245
5	13.8021	0.5573	14.5695	12.1945	13.1632	0.8178	14.3740	10.5613
10	18.8207	0.5436	19.5731	17.2003	18.1750	0.8687	19.4377	14.3065
15	23.8324	0.5398	24.5047	22.1892	23.2052	0.7976	24.4381	20.7942
20	28.8310	0.5370	29.5030	27.1901	28.2080	0.7831	29.4390	25.8203
25	33.8305	0.5366	34.5055	32.1903	33.2039	0.7817	34.4402	30.8536
30	38.8290	0.5351	39.4848	37.1906	38.2063	0.7776	39.4410	35.8515

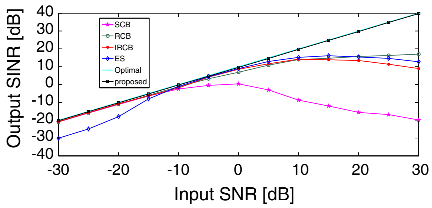

Figure 1. SINR (Median) vs SNR for look direction error only.

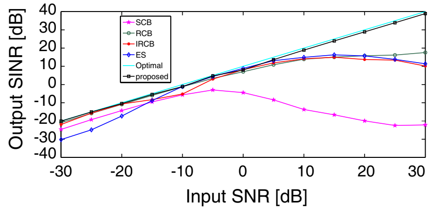

Figure 2. SINR (Median) vs SNR for look direction & 15% geometry error.

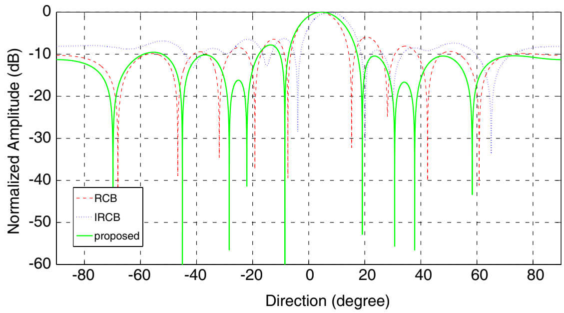

Figure 3. Beampattern comparison of the best results of RCB, IRCB and the proposed beamformer for look direction & 15% geometry error.

beamformer (IRCB) [15] and eigenstructure (ES) method, while the proposed algorithm could achieve near optimal performance. From Figs. 1 & 2, it can be observed that the performance of the proposed algorithm is consistent even when a considerable amount of geometry error is introduced. Among the existing robust beamformers, the performance of RCB, IRCB and ES is almost similar and better than SCB. SCB, RCB, IRCB and ES all saturate as the input SNR increases.

The time taken per one realization by each of the algorithms, SCB, RCB, IRCB, ES and proposed, are $2.04 * 10^{-2}, 2.16 * 10^{-2}, 3.02 * 10^{-2}$, $1.32 * 10^{-2}$ and $2.33 * 10^{-2}$ seconds respectively.

Figure 3 shows the beampattern plot comparison of the proposed approach against the RCB and the IRCB approaches. These are obtained from one of the realization in the simulation at 10 dB SNR. The proposed approach provides deeper null in the directions of interferences as well as lower side lobe level.

5. CONCLUSION

In this paper, we proposed a new robust adaptive beamforming mehtod that achieves near optimal performance by addressing both the SV mismatch due to look-direction error and the inaccurate covariance matrix estimation problems. We evaluated the performance of the proposed algorithm over a range of different input SNR values and compared with some of the existing adaptive beamforming techniques. From the simulation results we observed that the proposed algorithm is able to obtain near optimal performance and does not exhibit any saturation even when the input SNR is increased unlike the existing beamforming techniques.

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