

## OSCILLATOR ACCURATE LINEAR ANALYSIS AND DESIGN. CLASSIC LINEAR METHODS REVIEW AND COMMENTS

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**Abstract**—This paper is a deep analysis of oscillator plane reference design methods. It defines applicable conditions and the expected accuracy that can be archived with these methods. Some examples will be shown to illustrate wrong solutions that the use of linear reference plane methods can produce. The wrong solutions will be justified by necessary conditions for proper use of these methods. The strengths and weaknesses of the, widely used, plane reference methods are described in this paper. Several classic topologies of microwave oscillators, as Grounded Collector Tuned Bases(GCTB) and Grounded Bases Tuned Oscillator (GBTO), are used to illustrate these results and the additional required conditions.

### 1. INTRODUCTION

The oscillators are fundamental elements for all RF and microwave systems, as Radar systems [1]. They are one of the most problematic circuits in design process. Nowadays, the linear simulation, as first approximation, is widely used for RF and microwave oscillator design [2–6]. Nonlinear simulation needs more computational resources, and the non-linear models for active devices must be available. These nonlinear models are not always available, and in some cases they do not have enough accuracy [7]. However, it is necessary, before starting a nonlinear simulation, to have a good approximation

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of the frequency and start-up conditions. To conclude, in nonlinear simulation it is desirable to have a good background in nonlinear simulation and nonlinear approximation of these circuits [8]. In some cases, knowledge in nonlinear solution stability is necessary [9, 10], specially if harmonic balance is used [11].

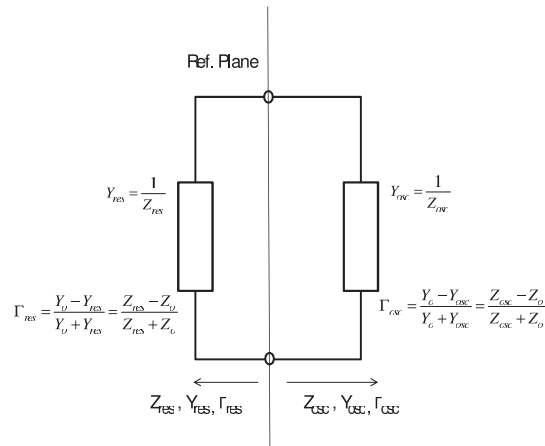
One of the most important reasons for using the linear simulation in oscillator design is that it is quicker and simpler than the nonlinear and that it is suitable for tuning the circuit [12, 13]. It is only necessary for these linear simulations to have the  $S$  parameters or the lineal model of the active device. These linear models are much easier to get than the non linear ones. A simple linear oscillator model and quick simulation give the chance of looking for new topologies. On the other hand, the linear simulation can only predict the oscillation frequency, gain margin and oscillator  $Q$  (quality factor), but not the output power, phase noise and harmonic levels. Therefore, nowadays oscillator design methodology consists in a first linear simulation step, followed by harmonic balance and transient simulations.

Linear oscillator analysis design techniques can be divided into two groups: Loop gain [3, 14] and reference plane [2, 15–17]. Negative resistance, negative conductance and reflection coefficient, for microwave circuits, are members of the second group. Each group has numerous advantages and disadvantages. This paper is focused on the reference plane methods. The problems of this group of methods, the possible solutions and the conditions of use will be described. At the first stage, the methods of the plane reference will be described. At the second stage, the use of these methods will be illustrated with the classical topologies as Grounded Collector Tuned Oscillator (GCTB) and Grounded Bases Tuned Oscillator (GBTC). Later, conclusions for the accurate and proper use of these design methods will be exposed.

## 2. REFERENCE PLANE METHODS

This section describes the principles of the linear oscillator analysis and the main strengths, weaknesses and limitations. Any oscillator may be analyzed using  $Z$ ,  $Y$  or  $\Gamma$  network functions. The network functions include all system poles, but general transfer function does not include them. The necessary condition for a circuit to be a proper oscillator is that it must have a pair of complex conjugated poles on the right half plane (RHP). Any pole factorized network function will have (1) time response, for the  $p$  order pole with  $k$  multiplicity.

$$L^{-1} \left[ \frac{a_k}{(s - s_p)^k} \right] = \frac{a_k \cdot t^{k-1} \cdot e^{s_p \cdot t}}{(k-1)!} \quad (1)$$



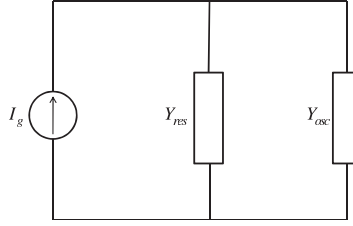
**Figure 1.** Oscillator as two subsystems.

It is also possible to demonstrate that if there is more than a pair of poles on the RHP the time solution will be a quasi-periodic solution, which is not desirable for an oscillator. If the pair of poles have a negative real part, the time solution will be a damped sine.

The traditional drawing way is conditioned in order to find the resonant structure as a dipole isolated from the negative  $Z/Y/\Gamma$  generator. The reference plane can be any (Fig. 1), without a real division between resonator and generator, as the denominator of any network function has all the information about system poles. But using one of the traditional divisions,  $Z/Y/\Gamma$ , simplifies the necessary conditions to assure a correct linear analysis, so these analyses will be used in the following sections. As we will see in the next sections, any of these traditional methods are really the application of the method of Nyquist for the detection of poles on the RHP of the used network functions.

## 2.1. Admittance Method (Impedance Network Function)

The negative conductance method divides the oscillator into two sub-circuits: negative conductance generator and resonator [15,18]. There is a left sub-circuit that is the resonator and a right sub-circuit that is the active device (with the required passive elements for its proper operation) in Fig. 2. This active device will work as negative conductance generator. With the first harmonic Kurokawa approximation [15], the parallel resonator fixes the oscillator frequency, and the negative generator offsets the resonator losses. This is the



**Figure 2.** Negative conductance method conceptual diagram.

classic view of negative  $Z/Y/\Gamma$  oscillators. This interpretation is intuitive and easy to understand, and it is widely used in literature. A more formal and powerful, but less intuitive, is the network function. The network function can be obtained by injecting current into the network with an ideal generator at the plane that divides the circuit. The obtained poles result from the addition of the two sub-circuits.

With Fig. 2 as reference, the impedance network function is defined by (2), where  $I_g$  is the external current;  $V$  is the circuit response; and  $Z$  is the inverse of the admittances of Fig. 2. The circuit is a proper oscillator if the network function has only a pair of conjugated complex poles on the RHP.

$$V = Z \cdot I_g$$

$$Z = \frac{1}{Y_{res} + Y_{osc}} \quad (2)$$

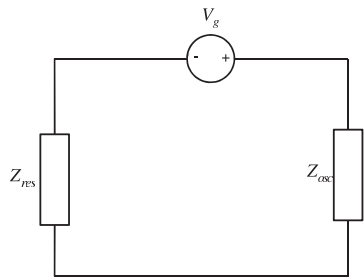
The poles of the network function are defined by the zeros of (3), and it is the characteristic function of the circuit.

$$Y_T = Y_{res} + Y_{osc} = 0 \quad (3)$$

The classical oscillator start-up condition,  $\Im(Y_T) = \Im(Y_{res} + Y_{osc}) = 0$  and  $\Re(Y_T) = \Re(Y_{res} + Y_{osc}) < 0$ , is a first harmonic approximation of the descriptive function as defined by Kurokawa [15], and this will be shown in the next paragraphs. It is not sufficient condition to guarantee the start-up. Once the start-up condition is satisfied, the oscillation stability condition and the minimum noise are defined in Table 1. The variable  $V$  is the voltage at the plane that separates the active from the passive sub-circuit. This approximation considers that the voltage is only from the fundamental frequency and that  $\omega$  is the frequency, then  $V_0$  and  $\omega_0$  are the voltage and frequency at the oscillation condition. But this formal representation is only an approximation that is more accurate when  $V$  has less harmonic tones, and it is more a pure tone signal. The definition of the division plane between the active sub-circuit and the resonator is not arbitrary. The

**Table 1.** Admittance oscillation conditions.

Parameter	Definition
Characteristic Equation	$Y_T(V, \omega) = Y_{osc}(V) + Y_{res}(\omega) = 0$
Oscillation Condition	$Y_T(V_0, \omega_0) = Y_{osc}(V_0) + Y_{res}(\omega_0) = 0$
Stability	$-Y_{osc}(V)$ with $Y_{res}(\omega)$ cross into a clockwise angle from 0 to $\pi$
Minimum noise	$-Y_{osc}(V)$ with $Y_{res}(\omega)$ cross into a $\frac{\pi}{2}$ clockwise angle



**Figure 3.** Negative impedance method conceptual diagram.

proper point makes the real spectrum of  $V$  to be near the predicted by linear analysis. The equations in Table 1 also consider that the variation of the negative impedance circuit is small with the frequency; this variation is smaller if the resonant circuit on the left side of the division plane has a high  $Q$ . The conditions of Table 1 are automatically fulfilled if the resonator is a parallel resonator. The use of parallel resonators has been traditionally recommended for generator of negative conductance.

**2.2. Impedance Method (Admittance Network Function)**

As in Section 2.1, the circuit is divided into two sub-circuits, a resonator on the left and an active device (with the associated passive devices) on the right (see Fig. 3). The active device works as a negative resistance generator when the first harmonic premise is considered [15].

The network function for this configuration is (4).

$$I = Y \cdot V_g$$
$$Y = \frac{1}{Z_{res} + Z_{osc}}$$

(4)

In (4),  $V_g$  is the external voltage;  $I$  is the circuit response; and  $Y$

**Table 2.** Impedance oscillation conditions.

Parameter	Definition
Characteristic Equation	$Z_T(I, \omega) = Z_{osc}(I) + Z_{res}(\omega) = 0$
Oscillation Condition	$Z_T(I_0, \omega_0) = Z_{osc}(I_0) + Z_{res}(\omega_0) = 0$
Stability	$-Z_{osc}(I)$ with $Z_{res}(\omega)$ cross into a clockwise angle from 0 to $\pi$
Minimum noise	$-Z_{osc}(I)$ with $Z_{res}(\omega)$ cross into a $\frac{\pi}{2}$ clockwise angle

is the inverse of the sum of the impedances of Fig. 3.

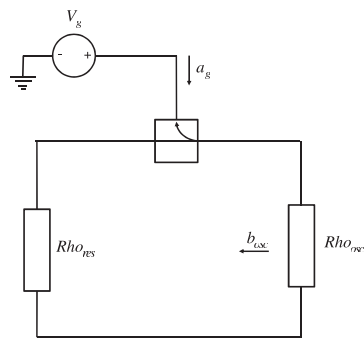
The poles of the network function are defined by the zeros of (5), and it is the characteristic function of the circuit. The circuit is a proper oscillator if the network function has only a pair of conjugated complex poles on the RHP, and they are the zeros of the characteristic function.

$$Z_T = Z_{res} + Z_{osc} = 0 \quad (5)$$

The widely used start-up condition  $\Im(Z_T) = \Im(Z_{res} + Z_{osc}) = 0$  and  $\Re(Z_T) = \Re(Z_{res} + Z_{osc}) < 0$  shall be complemented with the equations in Table 2 for the first harmonic approximation (this is not sufficient condition to guarantee the start-up). The extended operation considerations are described in Table 2. The variable  $I$  is the current between the two sub-circuits at the separation plane. Considering only the first harmonic of the signal,  $\omega$  is the frequency, and  $I_0$  and  $\omega_0$  are the current and oscillation frequency at state oscillation condition. The considerations for the division planes made in 2.1 for admittances are now applicable to impedances, as the previous oscillator. The conditions of Table 2 are automatically fulfilled if the resonator is a serial resonator. The use of serial resonators has been traditionally recommended for generators of negative impedance.

### 2.3. Reflection Coefficient Method (Reflection Coefficient Network Function)

The last case of division plane is the reflection coefficients. A plane is defined; the resonant circuit is placed on the left and the active device (with its associated passive components) on the right. The active device acts as a reflection generator with a  $\Gamma$  that varies with  $a_g$  as shown in Fig. 4. The approximation at the first harmonic defined by Kurokawa [15] is necessary for the following considerations as in previous cases.



**Figure 4.** Reflection method conceptual diagram.

**Table 3.** Reflection coefficient oscillation conditions.

Parameter	Definition
Characteristic Equation	$\Gamma_T(A, \omega) = -\frac{1}{\Gamma_{osc}(A)} + \Gamma_{res}(\omega) = 0$
Oscillation Condition	$\Gamma_T(A_0, \omega_0) = -\frac{1}{\Gamma_{osc}(A_0)} + \Gamma_{res}(\omega_0) = 0$
Stability	$\frac{1}{\Gamma_{osc}(A)}$ with $\Gamma_{res}(\omega)$ cross into a clockwise angle from 0 to $\pi$
Minimum noise	$\frac{1}{\Gamma_{osc}(A)}$ with $\Gamma_{res}(\omega)$ cross into a $\frac{\pi}{2}$ clockwise angle

The network function for reflection coefficient is (6).

$$b_{osc} = \frac{\Gamma_{osc}}{1 - \Gamma_{osc} \cdot \Gamma_{res}} \cdot a_g = \Gamma \cdot a_g$$

(6)

In (6),  $\Gamma$  is a function of the reflection coefficients of the two sub-circuits [17];  $a_g$  is the incident wave of the generator; and  $b_{osc}$  is the reflected wave of the active device. The condition for an oscillation is satisfied if  $\Gamma$  has a pair of conjugated complex poles on the RHP. The poles of  $\Gamma$  are defined by the zeros of the characteristic Equation (7).

$$\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res} = 0$$

(7)

It must be guaranteed that  $\Gamma_{osc}$  does not have any pole. The extended operation considerations are described in Table 3. The variable  $A$  is the incident wave at the reference plane. Considering only the first harmonic of the signal,  $\omega$  is the frequency, and  $A_0$  and  $\omega_0$  are the incident wave and oscillation frequency at state oscillation condition. The considerations for the division planes proposed for admittances and impedances, in the previous sections, are applicable to this case.

It is important to point out that it seems to be more appropriate to use the  $Z$  or  $Y$  network methods than the reflection coefficient network function, even though the reflection coefficient network function provides the oscillator poles and zeros position. The use of the reflection coefficient network function is less intuitive, and its compression behavior is more complex than that for other cases. This method has important practical issues; the reflection coefficient was easily measured in the pass when good simulation tools were not available.

### 3. CONDITIONS FOR PROPER USE OF REFERENCE PLANE METHODS

The unconditional validity of the reference plane methods is widely accepted [6, 19, 20]. But these methods cannot be unconditionally used. Their formal validity to predict the presence of a pair of poles on the RHP must be defined. The presence of these two poles is the condition for the proper start-up of an oscillator. This section covers the additional conditions for guaranteeing that it is appropriate to use the linear approximation.

The classical admittance, impedance and reflection coefficient oscillator start-up analysis conditions previously described are summarized in Table 4.

**Table 4.** Start-up conditions for reference plane oscillator analysis method.

Impedance	Admittance	Reflection Coefficient
$R_{osc}(\omega) + R_{res}(\omega) < 0$	$G_{osc}(\omega) + G_{res}(\omega) < 0$	$ \Gamma_{osc}(\omega)  \cdot  \Gamma_{res}(\omega)  > 1$
$X_{osc}(\omega) + X_{res}(\omega) = 0$	$B_{osc}(\omega) + B_{res}(\omega) = 0$	$\Phi_{osc}(\omega) + \Phi_{res}(\omega) = 0$

In a general analysis, Table 4 conditions are neither necessary nor sufficient to guarantee the oscillation start-up. They have been defined at oscillation frequency and are particularizations of the Nyquist analysis. The necessary and sufficient condition for the oscillator start-up is the existence of one unique pair of conjugated complex poles on the RHP. As the access to the explicit  $S$  domain (Laplace's domain) functions is difficult and sometimes impossible for MW and RF circuits, the analysis can be performed by means of Nyquist using the function frequency response. This analysis principle has been taken as the base for the extended practice of using a series resonant circuit in series with a negative impedance generator, and a parallel resonant circuit in parallel with a negative admittance generator.



The previous considerations are valid if they are combined with some additional conditions as the zero cross only occurs at one frequency, and the imaginary part of the characteristic function changes from negative to positive on its crossing. This is true for simple circuits as the serial resonator in series with a negative resistance and the parallel resonator in parallel with a negative admittance. This belief has been fed with the use of simple examples to assure the validity of Tables 1, 2 and 3.

Talking about these concepts, the work of Jackson [17] is very interesting. Jackson concludes that the classical reflection coefficient conditions for oscillation are not sufficient neither necessary for the oscillator start-up. The sufficient and necessary condition for oscillator start-up is the Nyquist criteria verification, assuring that the right sub-circuit (active part) does not have any poles on the RHP. These can be verified for (8) and (9).

$$\begin{aligned} |\Gamma_{osc}(\omega)| \cdot |\Gamma_{res}(\omega)| &> 1 \\ \Phi_{osc}(\omega) + \Phi_{res}(\omega) &= 0 \end{aligned} \quad (8)$$

$$\begin{aligned} |\Gamma_{osc}(\omega)| \cdot |\Gamma_{res}(\omega)| &< 1 \\ \Phi_{osc}(\omega) + \Phi_{res}(\omega) &= 0 \end{aligned} \quad (9)$$

It is also possible to expand the zeros of characteristic function (reflection coefficient, impedance or admittance); all their zeros are the same. In this way, it is demonstrated that all the methods are equivalent and that the poles of the system are unique. The reflection coefficient is (10).

$$F(s) = 1 - \Gamma_{osc}(s) \cdot \Gamma_{res}(s) \quad (10)$$

If the impedances and admittances are defined as (11) and (12).

$$Z_{res}(s) = \frac{N_L(s)}{D_L(s)} \quad \text{and} \quad Z_{osc}(s) = \frac{N_d(s)}{D_d(s)} \quad (11)$$

$$Y_{res}(s) = \frac{D_L(s)}{N_L(s)} \quad \text{and} \quad Y_{osc}(s) = \frac{D_d(s)}{N_d(s)} \quad (12)$$

The expansion of admittances or impedances, as functions of variable  $s$ , is (13).

$$\begin{aligned} F(s) &= 1 - \frac{Z_{res} - Z_0}{Z_{res} + Z_0} \cdot \frac{Z_{osc} - Z_0}{Z_{osc} + Z_0} \\ &\quad \frac{(N_L + Z_0 \cdot D_L) \cdot (N_d + Z_0 \cdot D_d)}{-(N_L - Z_0 \cdot D_L) \cdot (N_d - Z_0 \cdot D_d)} \\ F(s) &= \frac{-(N_L - Z_0 \cdot D_L) \cdot (N_d - Z_0 \cdot D_d)}{(N_L + Z_0 \cdot D_L) \cdot (N_d - Z_0 \cdot D_d)} \end{aligned} \quad (13)$$

When  $Z_0$  is a high impedance then (13) is simplified to (14).

$$F(s) \approx \frac{2}{Z_0} \cdot (Z_{res} + Z_{osc}) \quad (14)$$

As  $\frac{2}{Z_0}$  is a constant, it does not affect the position (semi-plane) of the poles and zeros of the function  $F(s)$ . As this function is studied by means of Nyquist there is no any difference between  $F(s) = \frac{2}{Z_0} (Z_{res} + Z_{osc})$  and  $F(s) = Z_{res} + Z_{osc}$ , and the  $F(s)$  function is simplified as (15).

$$F(s) \approx \frac{N_d \cdot D_L + N_L \cdot D_d}{D_L \cdot D_s} \quad (15)$$

When  $Z_0$  is a small impedance then (13) is simplified to (16).

$$F(s) \approx 2 \cdot Z_0 \cdot \frac{Z_{res} + Z_{osc}}{Z_{res} \cdot Z_{osc}} \quad (16)$$

By means of a similar process, with the exclusion of the constant  $2 \cdot Z_0$ , (16) is expanded as (17).

$$F(s) \approx Y_{res} + Y_{osc} \quad (17)$$

$$F(s) \approx \frac{N_d \cdot D_L + N_L \cdot D_d}{N_L \cdot N_s}$$

The equivalence of the presented plane reference methods, as in Jackson papers [17], seems that the use of these methods is valid for all oscillator circuits, but it is not enough to guarantee the start-up and stability in some cases. The necessary and sufficient condition is the presence of a pair of conjugated complex poles on the RHP. As the analytical network functions in  $s$  domain are not practical to get, the Nyquist analysis for the network functions is used. But the Nyquist analysis only provides information of the difference between poles and zeros, so it is necessary to define the conditions to assure the proper use of Nyquist analysis in the network functions.

It is necessary to define the condition or conditions which assure the proper use of these methods. Based on the Jackson [21], Ohtomo [22] and Platzer [23] papers, it is possible to conclude the additional conditions for each sub-circuit (divided by the reference plane) to assure that the Nyquist analysis of the characteristic equations ( $Z_T$ ,  $Y_T$ ,  $\Gamma_T$ ) provides, without error possibility, the existence of one pair of conjugated complex poles on the RHP.

### 3.1. Negative Conductance Additional Conditions (Impedance Network Function)

The response of negative conductance circuits, which is easily simulated, is the frequency response of  $Y_T$ . The frequency response

of  $Y_T$  is suitable for Nyquist criteria application. The Nyquist criteria informs of the  $N_z - N_p$  value.  $N_p$  is the number of poles on the RHP, and  $N_z$  is the number of zeros on the RHP. When  $N_z - N_p$  is positive, the turning circle around the origin is clockwise.

For the proper use of Nyquist criteria, it is necessary that  $Y_T$  does not have any poles on the RHP. This additional condition must be satisfied by  $Y_{osc}$ , which must not have any poles on the RHP (visible or hidden poles). It is important to remember that  $Y_{osc}$  is a reduced network function of a bigger and complex network, so it is possible that some of the poles of the original network are not visible [23]. These non-visible poles are not detectable in  $Y_{osc}$  but may produce a bad oscillation operation. As  $Y_{res}$  is a passive network, it cannot have poles.

The only way to assure that  $Y_{osc}$  does not have any pole (visible or hidden) on the RHP is the use of the Normalized Determinant Function (NDF) [23] for a network built with the active subnetwork terminated with a short circuit, as described by Jackson based on Platzker and Ohtomo papers [21–24, 26]. The possible existence of hidden poles obliges to use the NDF to  $Y_{osc}$ , for the proper use of Nyquist criteria to  $Y_T$  and for the negative conductance oscillator analysis ( $Z$  network function).

NDF is the quotient of the network determinant and normalized network determinant (18). The normalized network determinant is the result of the cancelation of all active devices of the network. The Nyquist analysis of this function, described by Platzker [23], provides the information about the number of poles on the RHP. Each clockwise turning circle around the origin, for positive frequencies, confirms the existence of a pair of conjugated poles. As NDF has an asymptotic response with frequency to 1, the upper analysis frequency is easily determined.

$$NDF = \frac{\Delta(s)}{\Delta_0(s)} \quad (18)$$

The NDF is easily calculated by the return relation functions ( $RR$ ), defined by Bode [25], as Platzker exposed in [24]. The NDF as a function of  $RR$  is given by equation (19).  $RR_i$  is the return response when the  $(i - 1)$  previous dependent generators have been disabled.

$$NDF = \prod_{i=0}^n (RR_i + 1) \quad (19)$$

It is important to remark that the Nyquist analysis for the  $Y_T$  (NDF test to  $Y_{osc}$ ) does not predict the oscillator frequency. It only provides the number of poles in  $Z$ . The cross over zero of  $\Im(Y_{res} + Y_{osc})$  is near to the poles frequency. This cross is nearest

to the oscillation frequency with the highest poles  $Q$ . So, the cross over zero is a function of the poles  $Q$  and the chosen  $Y_{res}$  and  $Y_{osc}$  division plane.

For one active device oscillators, the gain compression ( $g_m$  compression) modifies the  $Y_T$  response until it crosses over zero. This cross verifies  $\Im(Y_{res} + Y_{osc}) = 0$  and  $\Re(Y_{res} + Y_{osc}) = 0$ . The crossing point is the oscillation frequency as first harmonic approximation, which is better for FET devices than for BJT devices.

### 3.2. Negative Resistance Additional Conditions (Admittance Network Function)

The negative resistance oscillator is analyzed by means of the admittance network function. Using the same procedure as in Section 3.1, the  $Z_T$  function can be analyzed. To assure the proper start-up and oscillation stability at a unique frequency, the  $Y$  function must have a pair of conjugated poles on the RHP. For a proper use of the Nyquist criteria for  $Z_T$  zeros analysis, it is necessary to assure that  $Z_{osc}$  does not have any pole, visible or hidden, on the RHP. As  $Z_{res}$  is a passive network, it cannot have any poles on the RHP.

The only way to assure that  $Z_{osc}$  does not have any visible or hidden poles on the RHP is to calculate the NDF of a network formed by  $Z_{osc}$  terminated with an open circuit. So it is necessary to analyze the NDF of  $Z_{osc}$  to assure that the  $Z_T$  Nyquist analysis provides the correct information about the  $Y$ , admittance network function, poles.

In the same way as in the previous case, the Nyquist analysis of  $Z_T$  (after assured by means of NDF that  $Z_{osc}$  does not have any poles on the RHP) provides the information about the  $Y$  poles on the RHP, but not the oscillation frequency. The  $\Im(Z_{res} + Z_{osc})$  cross over zero will be closer to the oscillation frequency when the poles  $Q$  are the highest. The oscillation frequency is also modified by the poles of  $Z_T$ . As in the previous cases, if the transistor  $g_m$  is compressed the frequency response of  $Z_T = Z_{res} + Z_{osc}$  will cross over zero, then it will be the oscillation frequency as first harmonic approximation. This approximation is better for FET than for BJT, because FET input capacity has a minor modification with compression.

### 3.3. Reflection Coefficient Additional Conditions (Reflection Coefficient Network Function)

The last studied plane reference method, the reflection coefficient method, is analyzed by means of simulation software solving the frequency response of  $\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res}$ , but it is more common to analyze  $\Gamma_{osc} \cdot \Gamma_{res}$  changing the encircling point from 0 to +1. The

frequency analysis of  $\Gamma_{osc} \cdot \Gamma_{res}$  is the application of the Nyquist criteria, which provides information about the  $N_z - N_p$  (zeros minus poles) on the RHP.

As in Sections 3.1 and 3.2, the Nyquist criteria is used to determine the existence of a pair of conjugated complex zeros on the RHP, and it is the condition of a unique stable oscillation. To assure the correct use of the Nyquist to analyze  $\Gamma_T$ , it is necessary that  $\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res}$  does not have any poles on the RHP. Then, it is necessary that  $\Gamma_{osc}$  does not have any visible or hidden poles on the RHP.  $\Gamma_{res}$  does not have any poles on the RHP because it is a passive network.

The way to assure that  $\Gamma_{osc}$  does not have any poles on the RHP is by analyzing a network formed by  $\Gamma_{osc}$  and terminated with  $Z_0$  by means of NDF. Then it is necessary to perform a NDF analysis of  $\Gamma_{osc}$  to assure that it does not have any pole on the RHP before analyzing  $\Gamma_T$  with the Nyquist criteria. After verifying that  $\Gamma_{osc}$  does not have any poles on the RHP, the Nyquist analysis of  $\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res}$  is suitable for determining the existence of the necessary conjugated complex pair of poles for proper oscillation start-up.

As in the previous cases, the Nyquist cross over zeros of  $\Im(\Gamma_{res} \cdot \Gamma_{osc})$  will be the nearest to the oscillation frequency if the poles  $Q$  is higher. This cross is modified by the presence of the poles of  $\Gamma_T$ .

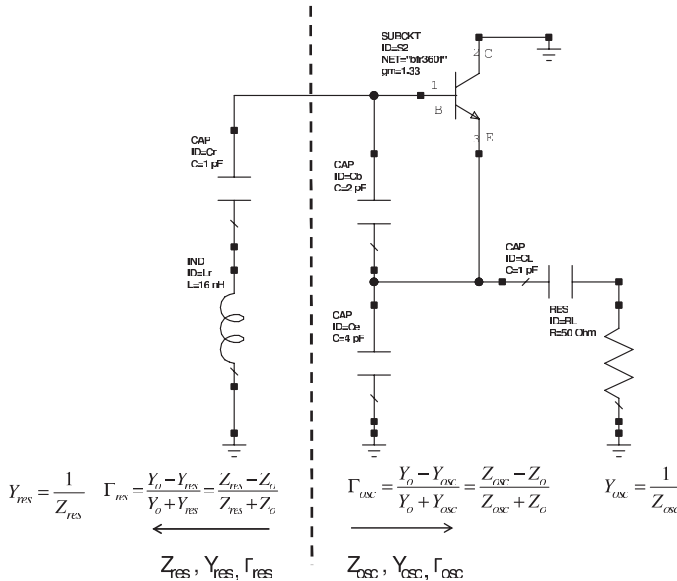
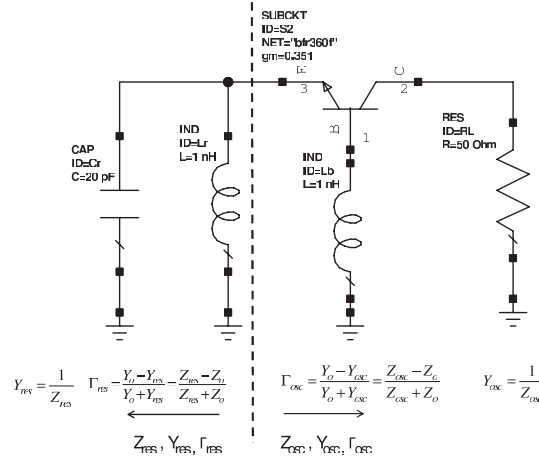


Figure 5. Common collector oscillator.



**Figure 6.** Common base oscillator.

The compression of the transistor  $g_m$  makes the  $1 - \Gamma_{osc} \cdot \Gamma_{res}$  cross occur over the zero. In this situation, the cross frequency is the oscillation frequency as Kurokawa defines the first harmonic approximation. In the same way as in the previous cases, this approximation is better for FET than for BJT.

#### 4. PRACTICAL EXAMPLES. CANONICAL TOPOLOGIES

In this section, the exposed analysis conditions are used with two classical topologies, common base and common collector oscillator. These two topologies are usually analyzed with negative resistance (see Fig. 5) and negative admittance (see Fig. 6).

The values illustrated in Fig. 5 and Fig. 6 are not optimized and just illustrative for the L band, used in these examples. The used transistor is BFR380F with a polarization collector current of 40 mA and a collector to emitter voltage of 5 V. The transistor package parasite elements are considered in the used model. The simulation software used for these examples is AWR Microwave Office.

##### 4.1. Common Collector Oscillator

The common collector oscillators are usually analyzed by negative resistance. Its first harmonic approximation response behaves as a negative impedance generator. The Nyquist representations of

impedance, admittance and reflection are: Fig. 7, Fig. 8 and Fig. 9(a). The Nyquist analysis predicts an oscillation frequency of 1245MHz for the  $Z_T$  analysis, but it predicts no oscillation for  $Y_T$  and  $\Gamma_{res} \cdot \Gamma_{osc}$  ( $\Gamma_T$ ) analysis. It can be explained as that  $Y_{osc}$  and  $\Gamma_{osc}$  have poles that hide the Nyquist analysis for  $Y_T$  and  $\Gamma_{res} \cdot \Gamma_{osc}$  ( $\Gamma_T$ ). The Cartesian Bode representation for  $Z_{osc}$  is shown in Fig. 10(a).  $Z_{osc}$  analysis with a short-circuit termination predicts an oscillation frequency at 1234MHz. At this point, the NDF analysis of Fig. 11 circuit, as Platzer describes [24], is used using the RR. The circuit is analyzed for open-

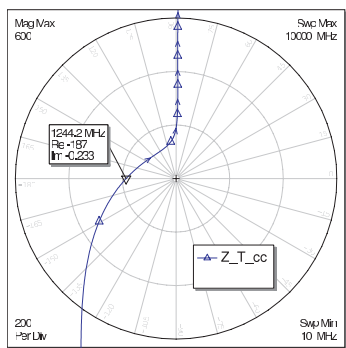


Figure 7. Nyquist impedance representation.

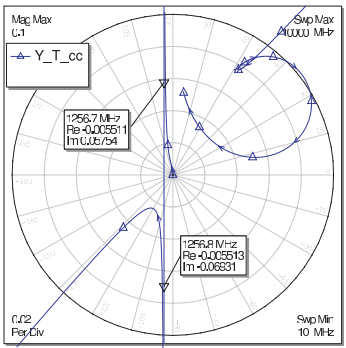
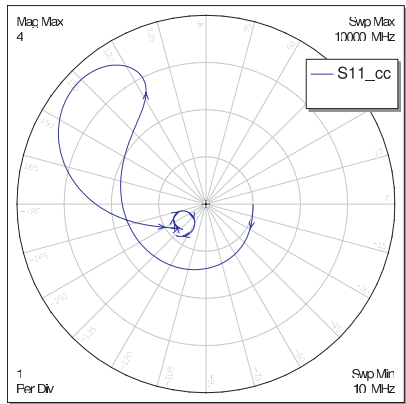
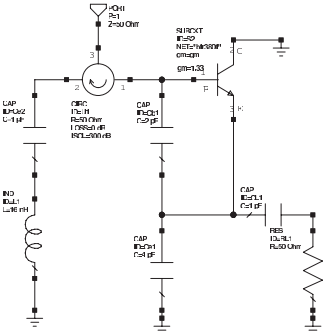


Figure 8. Nyquist admittance representation.

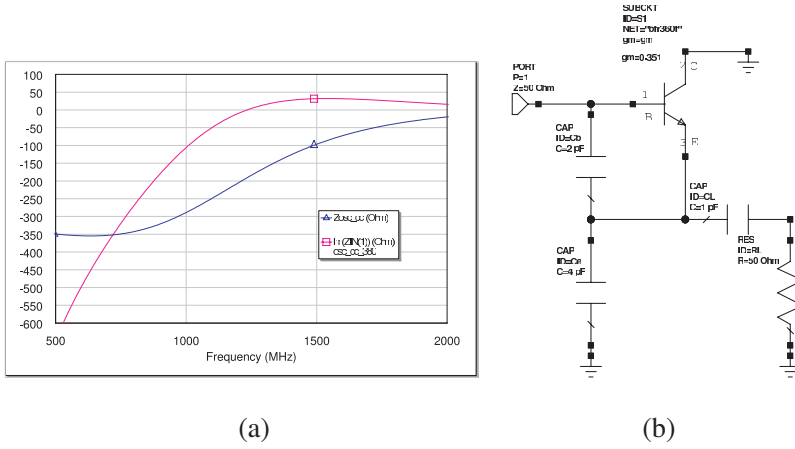


(a)

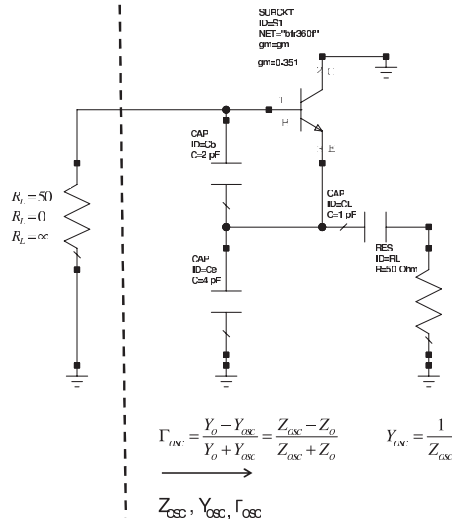


(b)

Figure 9. (a) Nyquist reflection coefficient representation, (b) measurement model.



**Figure 10.** (a) Bode  $Z_{osc}$  analysis, (b) active sub-circuit for stability analysis.

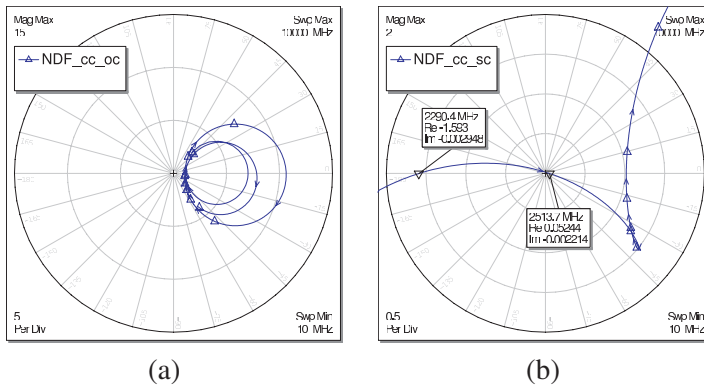


**Figure 11.** Circuit model for NDF analysis.

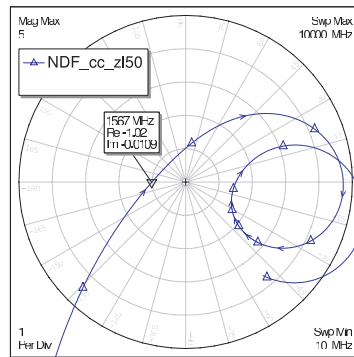
circuit, short-circuit and  $R_L = 50$ , that is for the necessary conditions for  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ) analysis.

The NDF Nyquist analysis for open-circuit (see Fig. 12(a)) predicts stability, which means that  $Z_{osc}$  is stable for open-circuit condition, so  $Z_T$  can be analyzed by Nyquist to predict the  $Y$  poles that guarantee the proper oscillation operation.





**Figure 12.** NDF analysis for (a) open-circuit and (b) short-circuit.



**Figure 13.** NDF analysis for  $Z_L = 50$ .

The NDF Nyquist analysis for short-circuit (see Fig. 12(b)) predicts instability. There are two poles on the RHP for  $Y_{osc}$ . As  $Y_{osc}$  is unstable for short-circuit condition,  $Y_T$  cannot be analyzed by Nyquist to predict the  $Z$  poles. As the existence of poles in  $Y_{osc}$  masks the zeros in an Nyquist analysis, the Nyquist analysis does not detect the zeros, and the circuit seems to be stable (but it is unstable).

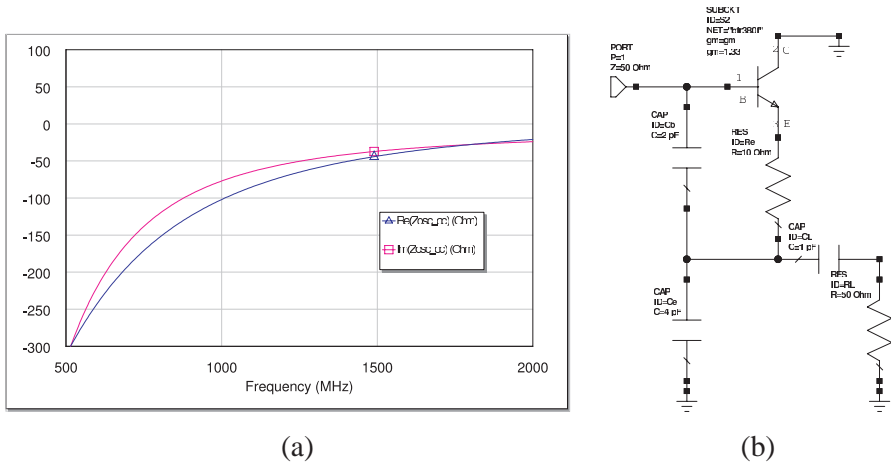
The NDF Nyquist analysis for  $Z_L = 50$  (see Fig. 13) predicts instability. There are two poles on the RHP for  $\Gamma_{osc}$ . As  $\Gamma_{osc}$  is unstable for  $Z_L = 50$  condition,  $\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res}$  cannot be analyzed by Nyquist to predict the  $\Gamma$  poles. As the existence of poles in  $\Gamma_{osc}$  masks the zeros in a Nyquist analysis, the Nyquist analysis does not detect the zeros, and the circuit seems to be stable (but it is unstable).

As the circuit in Fig. 11 is not stable for short-circuit and  $Z_L =$

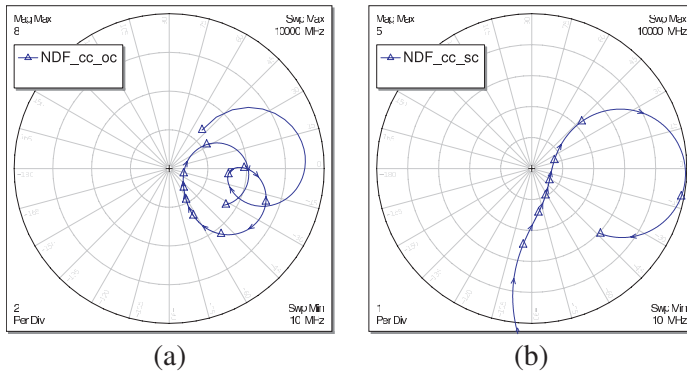
50, it is redesigned to make it stable for the three conditions (see Fig. 14(b)). In Fig. 14(a) the Bode response shows stability for this circuit.

The NDF Nyquist analysis of the oscillator active sub-circuit (see Fig. 14(b)) for the three load conditions (OC, SC and  $Z_0$ ), assures that the analysis of  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ) provide correct solutions.

Figs. 15 and 16 show the NDF analysis of the active sub-circuit for the three load conditions. The NDF analysis probes that the active sub-circuit does not have any poles on the RHP, so the  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ) Nyquist analysis can be performed without the risk of wrong solution (see Fig. 17). Now the three Nyquist analyses predict



**Figure 14.** (a) Bode  $Z_{osc}$  analysis and (b) stabilized active sub-circuit.



**Figure 15.** (a) NDF analysis for open-circuit and (b) short-circuit.

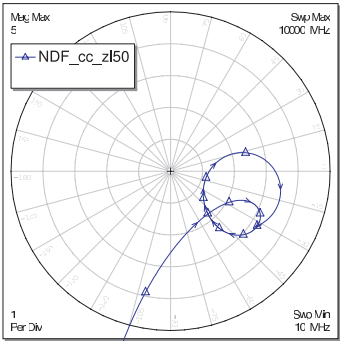


Figure 16. NDF analysis for  $Z_L = 50$ .

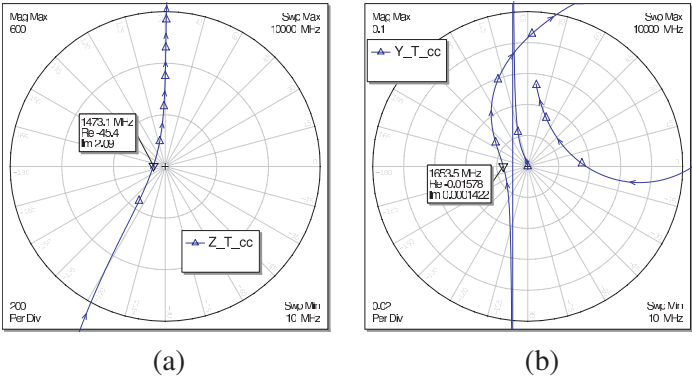


Figure 17. Nyquist representation for (a)  $Z_T$  and (b)  $Y_T$ .

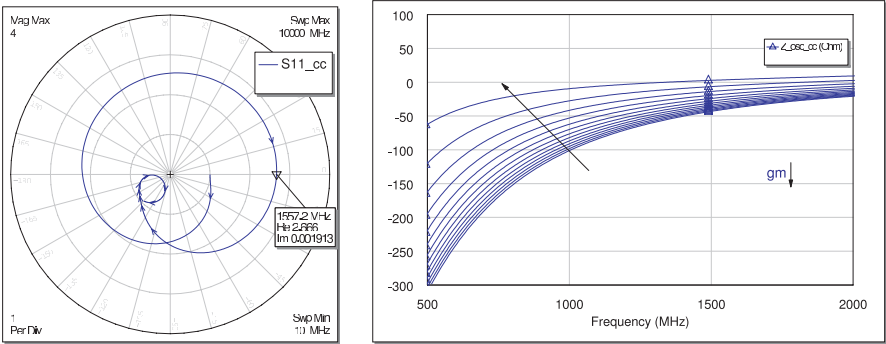


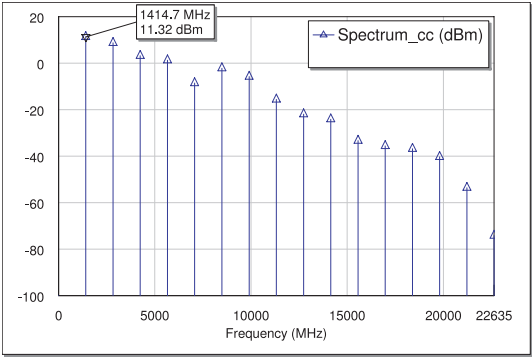
Figure 18. Nyquist representation for  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ).

Figure 19.  $Z_{osc}$  real part evolution with  $g_m$  variation.

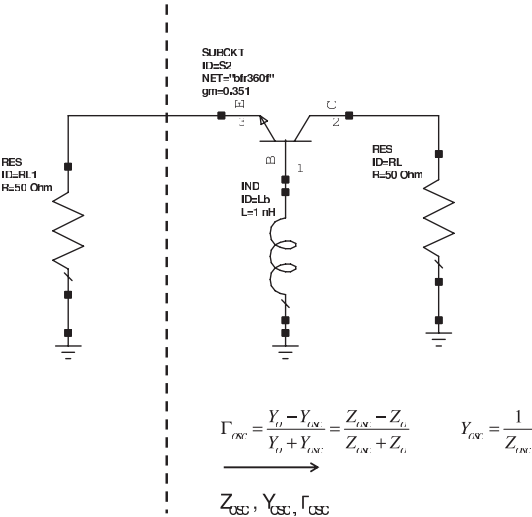


circuit and resonator, is calculated (see Fig. 20).

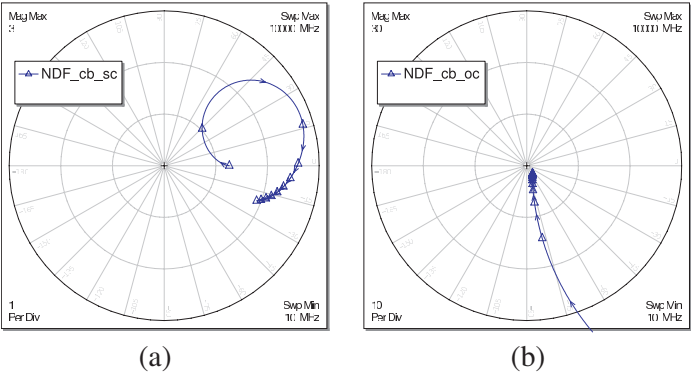
Fig. 20(a) represents the Oscillator Nyquist NDF without any resistance in the emitter of the transistor. The NDF analysis without emitter resistance (see Fig. 20(a)) predicts a pair of poles on the RHP. The NDF analysis with an emitter resistance also predicts a pair of poles on the RHP. So, with the NDF analysis it is possible to certify the  $Z_T$  analysis for the first case and the  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ) for the second case.



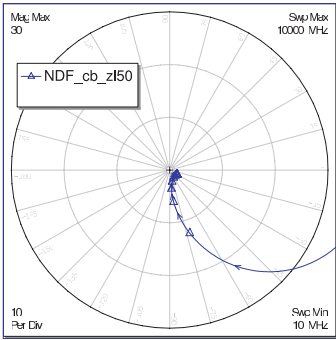
**Figure 22.** Spectrum of harmonic balance solution for oscillator with emitter resistance.



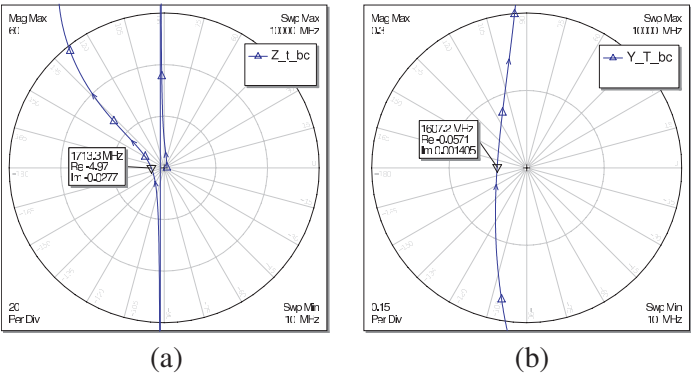
**Figure 23.** Common base oscillator circuit for NDF analysis.



**Figure 24.** Common base oscillator NDF analysis for (a) short-circuit and (b) open-circuit.



**Figure 25.** Common base oscillator NDF analysis for  $Z_L = 50$ .

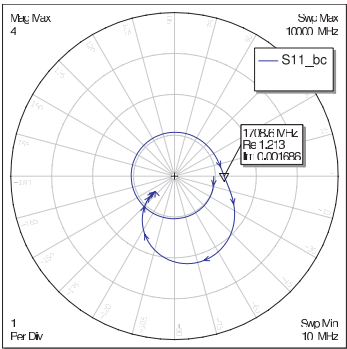


**Figure 26.** Common base oscillator Nyquist representation for (a)  $Z_T$  and (b)  $Y_T$ .

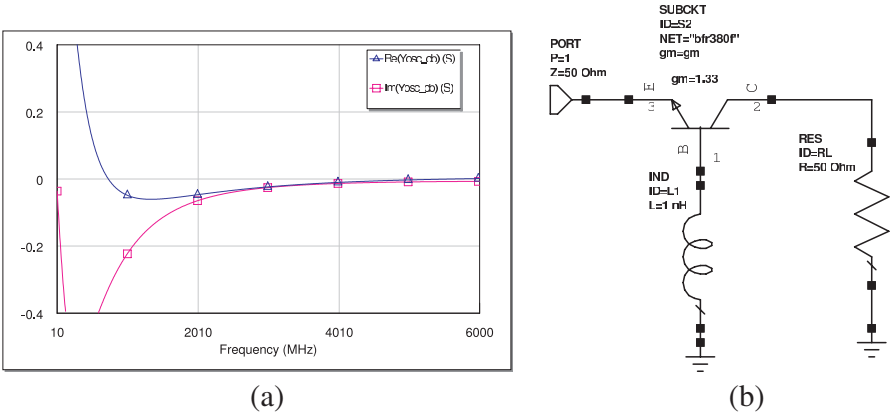
To finalize the verification of the oscillator, the harmonic balance analysis is applied to the circuit with emitter resistance (see Fig. 21). The solution predicts an oscillation at 1414 MHz (see Fig. 22) which is in accordance to the  $Z_T$  and NDF predicted frequencies.

4.2. Common Base Oscillator.

This section shows the study of the Grounded Base Tuned Oscillator (GBTO)(see Fig. 23). As a first step, the NDF of the active sub-circuit is calculated for open-circuit, short-circuit and  $Z_0 = 50$ . With these NDF analyses the necessary conditions for  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ )



**Figure 27.** Common base oscillator Nyquist representation for  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ).



**Figure 28.** (a)  $Y_{osc}$  bode diagram and (b) active sub-circuit for stability analysis.

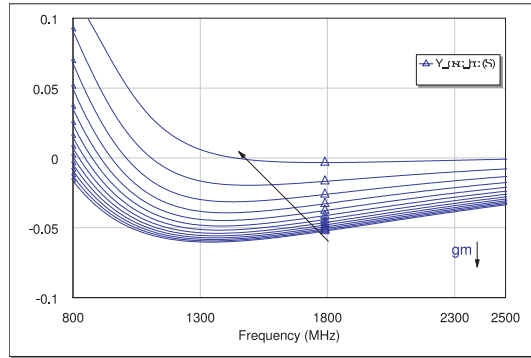
are checked.

As in Figs. 24 and 25, none of the NDF analyses show any poles on the RHP, and the oscillator can be analyzed by  $Z_T$ ,  $Y_T$  and  $\Gamma_{osc} \cdot \Gamma_{res}$  ( $\Gamma_T$ ) Nyquist analysis; these analyses are shown in Figs. 26 and 27.

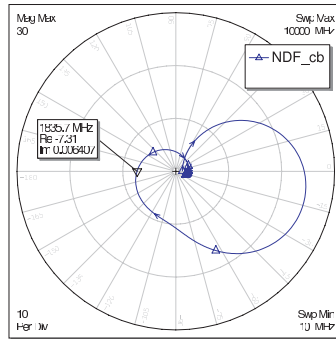
The three solutions in Figs. 26 and 27 predict an oscillation, and the predicted frequencies are similar. It is possible to check the Bode diagram of  $Y_{osc}$  (see Fig. 28(a)). The  $Y_{osc}$  of the active sub-circuit Bode plot does not have any  $\Im(Y_{osc})$  cross over zero while  $\Re(Y_{osc}) < 0$ .

In Fig. 29, the negative conductance of the active sub-circuit is verified. The  $\Re(Y_{osc})$  is increased with the  $g_m$  compression. This operation is the complementary of the negative resistance circuits. The circuits with negative conductance operation (as this one) should be analyzed by means of  $Y_T$  for the first harmonic approximation.

The NDF Nyquist analysis of the oscillator (see Fig. 30) detects

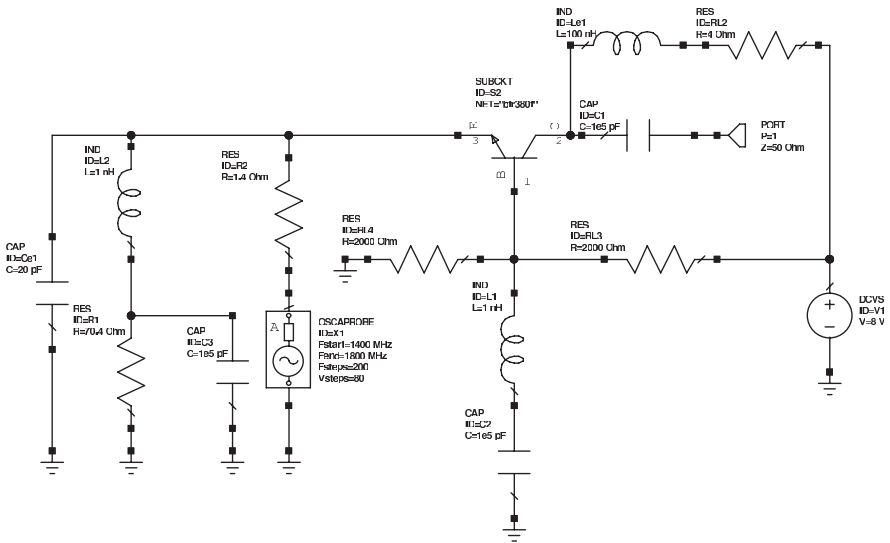


**Figure 29.**  $Y_{osc}$  real part evolution with  $g_m$  variation.

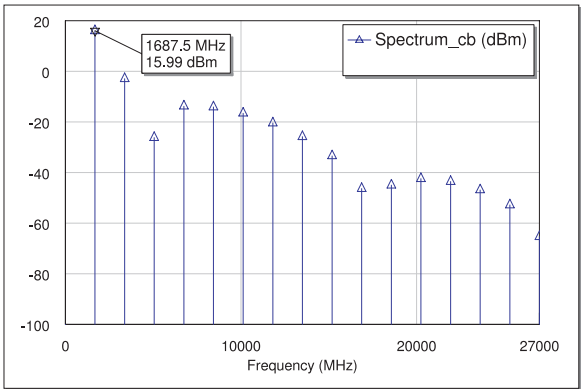


**Figure 30.** GBTO oscillator NDF Nyquist plot.





**Figure 31.** Harmonic balance solution for GBTO oscillator circuit.



**Figure 32.** Spectrum of harmonic balance solution for GBTO oscillator.

a pair of conjugated complex poles on the RHP.

To finalize the verification of the oscillator, as in Section 4.1, the harmonic balance analysis is applied to the complete oscillator circuit (see Fig. 31). The solution predicts an oscillation at 1687 MHz (see Fig. 32) which is in accordance with the  $Y_T$  and NDF predicted frequencies.

## 5. CONCLUSION

This paper has reviewed the classic reference plane linear methods: admittance, impedance and reflection coefficient. It has been demonstrated that the classic conditions are not “sufficient”, and in fact they are a partial use of Nyquist’s criteria. The complete use of Nyquist criteria with these methods has been analyzed, and an additional condition has been defined to assure the correct use of these classic methods for oscillation analysis. This condition is usually avoided by the oscillator designers, and it causes some erroneous predictions of oscillators start-up. These additional conditions are necessary to assure that none of the sub-circuits, divided by the reference plane, have any visible or hidden poles on the RHP. The unique way to assure the non-existence of these poles is the use of Nyquist analysis for the NDF of the sub-circuits. If the sub-circuits fulfill the Nyquist NDF analysis, it is possible to properly use impedance, admittance or reflection coefficient to analyze the oscillator.

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