*Progress In Electromagnetics Research C, Vol. 21, 217–228, 2011*

# ADAPTIVE ARRAY BEAMFORMING USING SIGNAL CYCLOSTATIONARITY AND FINITE DATA

**C.-C. Huang[1] and J.-H. Lee[2, *]**

[1]Graduate Institute of Communication Engineering, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan

[2]Department of Electrical Engineering, Graduate Institute of Communication Engineering, and Graduate Institute of Biomedical Electronics and Bioinformatics, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan

**Abstract**—This paper considers adaptive array beamforming using signal cyclostationarity. Due to the effect of using finite data samples, there exists an estimation error in computing the weight vector required by performing cyclic beamforming. To deal with this problem, we formulate a cost function consisting of a *posteriori* information of the received signal and a *priori* information regarding the probabilistic distribution of the error. By minimizing the cost function, we obtain a weight vector with a diagonal loading data covariance matrix under a white Gaussian estimation error. An analytical solution for determining the loading factor is further derived. Simulation results for showing the effectiveness of the proposed method are provided.

## 1. INTRODUCTION

For conventional array beamforming, the a *priori* information required for adapting the weights is either the direction vector or the waveform of the signal of interest (SOI) [1]. It will be too costly to provide the information in many applications such as the mobile radio system and the regenerative satellite communication system. A signal with cyclostationarity has the statistical property of correlating with either a frequency-shift or a complex-conjugate version of itself. By restoring this property at a known value of frequency separation, it is possible to extract the SOI and suppress the signals not of interest (SNOIs) and noise [2]. Adaptive beamforming utilizing signal cyclostationarity

*Received 17 April 2011, Accepted 18 May 2011, Scheduled 22 May 2011*
* Corresponding author: Ju-Hong Lee (juhong@cc.ee.ntu.edu.tw).

has been widely considered in [3–5]. These beamforming techniques do not require training signals and the knowledge of array manifold. For example, in [3], a class of spectral self-coherent restoral (SCORE) algorithms has been presented to deal with the problem of blind adaptive signal extraction by using signal cyclostationarity. As the number of data snapshots approaches infinity, it has been shown in [3] that the performance of the SCORE algorithms approaches that of the conventional beamforming methods developed by maximizing the output signal-to-interference plus noise ratio (SINR). The least-square SCORE (LS-SCORE) algorithm is the simplest among the SCORE algorithms. However, the LS-SCORE algorithm converges slowly due to finite sample effect. To overcome this drawback, the Cross-SCORE algorithm is proposed by [3]. Although the Cross-SCORE algorithm has the advantage of faster convergence rate over the LS-SCORE algorithm, it needs considerable computational complexity to solve the generalized eigenvalue problem required for finding the weights.

For many practical applications, such as satellite communications [6], an antenna array is required to possess the beamforming capability that receives more than one SOI with specified gain constraints while suppressing all SNOIs. This goal can be achieved by using an antenna array with a multiple-beam pattern [6, 7]. Recently, a novel cyclostationary beamforming method based on the LS-SCORE algorithm has been proposed by [8] for dealing with the situation of multiple SOIs. This method is referred to as the multiple LS-SCORE (MLS-SCORE) algorithm. It has been shown in [8] that the solution to the MLS-SCORE algorithm converges to the solution of the conventional linearly constrained minimum variance (LCMV) algorithm as the number of data snapshots approaches infinity. However, the MLS-SCORE algorithm converges slowly when the number of data samples available is finite. To overcome this drawback, an estimation error model was developed in [8] to represent the perturbation due to finite sample effect on the cyclic correlation vector. Then, a subspace projection approach was further presented to incorporate with the MLS-SCORE algorithm. The resulting method is called as the MLS-SCORE-SP algorithm. It has been shown that the MLS-SCORE-SP algorithm achieves better performance than the MLS-SCORE algorithm because the perturbation due to the estimation error is eliminated. Meanwhile, the MLS-SCORE-SP algorithm outperforms the Cross-SCORE algorithm in the case of multiple SOIs. This is because the required dominant eigenvector of the Cross-SCORE algorithm for computing the weight vector may no longer be able to converge to the desired solution in the presence of multiple SOIs [8]. Nevertheless, the essential drawbacks of the MLS-SCORE-SP algorithm are that it is efficient only if the number of signal sources is small and correctly known.

In this paper, we present an efficient method for dealing with the estimation error. It has been shown in [9] that the estimation error of a cyclic correlation function has a complex Gaussian distribution. Using this result, we first formulate a problem of finding an optimal cyclic correlation vector as an optimization problem. A cost function consisting of the first term related to the inverse of the array output power and a second term related to the likelihood function of the estimation error. The second term is employed to prevent that the obtained optimal solution becomes one of the interference direction vectors. Solving the optimization problem yields a weight vector with a generalized loading on the data covariance matrix. Under a white Gaussian estimation error, we obtain a weight vector with a diagonal loading data covariance matrix, where the loading factor is a key factor which should be appropriately determined. More recently, some related works on choosing the loading factor are presented by [10] and [11]. For instance, in [10], the optimal loading factor is found by the particle filters. The particle who has the highest posterior probability is chosen as the optimal loading factor. From [11], the loading factor is obtained by controlling the peak location of the main beam. However, the loading factors of those methods cannot be obtained directly and have to be solved numerically. Here, for determining the required loading factor, we derive an analytical solution based on the criterion of minimizing noise output power. The resulting loading factor can be computed easily from the received data vector based on the analytical formula. Several simulation examples are provided to confirm the validity of the proposed method and make comparison with the existing methods.

## 2. PRELIMINARIES

### 2.1. The MLS-SCORE Algorithm

It has been shown in [2] that a signal $s(t)$ is said to possess the signal cyclostationarity with cycle frequency $\alpha$ if and only if the cyclic or the cyclic conjugate autocorrelation function given by

$$r_{ss}(\alpha,\tau)=\left\langle s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)e^{-j2\pi\alpha t}\right\rangle_\infty$$

or

$$r_{ss^*}(\alpha,\tau)=\left\langle s\left(t+\frac{\tau}{2}\right)s\left(t-\frac{\tau}{2}\right)e^{-j2\pi\alpha t}\right\rangle_\infty \tag{1}$$

does not equal zero at cycle frequency $\alpha$ for some time delay $\tau$, where the superscript "$*$" denotes the complex conjugate and $\langle\cdot\rangle_\infty$ represents the infinite-time average operation.

Consider that there are total $K$ far-field narrowband signals including $d$ SOIs and $K$-$d$ SNOIs impinging on an $M$-element antenna array. Assume that the background noise is spatially white. The received data vector $\mathbf{x}(t)$ is given by [1]

$$\mathbf{x}(t) = \sum_{i=1}^{K} \mathbf{a}_i s_i(t) + \mathbf{n}(t) = \sum_{i=1}^{d} \mathbf{a}_i s_i(t) + \mathbf{v}(t) \tag{2}$$

where $s_i(t)$ and $\mathbf{a}_i$ represent the waveforms and direction vector of the $i$th signal, respectively; $\mathbf{v}(t)$ includes the $K$-$d$ SNOIs and noise.

Without loss of generality, we assume that the SOIs are cyclostationary and have the cycle frequencies $\underline{\alpha}$, where $\underline{\alpha} = \{\alpha_i | i = 1, 2, \ldots, d\}$ denotes the set of cycle frequencies of the SOIs. $\mathbf{v}(t)$ is not cyclostationary at $\alpha_i$, $i = 1, 2, \ldots, d$, and is temporally uncorrelated with the SOIs. Based on the MLS-SCORE algorithm [8], the optimal weight vector is given by

$$\hat{\mathbf{w}}_{mls} = \arg\min_{\mathbf{w}} \langle |y(t) - z(\underline{\alpha}, t)|^2 \rangle_T \tag{3}$$

where $y(t) = \mathbf{w}^H \mathbf{x}(t)$ denotes the array output with the superscript "$H$" the conjugate transpose and the reference signal $z(\underline{\alpha}, t)$ is given by $z(\underline{\alpha}, t) = \mathbf{c}^H \mathbf{x}^*(t-\tau) \sum_{i=1}^{d} e^{j2\pi\alpha_i t}$ with a fixed control vector $\mathbf{c}$. The solution of (3) is given by

$$\hat{\mathbf{w}}_{mls} = \hat{\mathbf{R}}_{xx}^{-1} \hat{\mathbf{r}}_{xz}(\underline{\alpha}) \tag{4}$$

where $\hat{\mathbf{R}}_{xx} = \langle \mathbf{x}(t)\mathbf{x}^H(t) \rangle_T$ and $\hat{\mathbf{r}}_{xz}(\underline{\alpha}) = \langle \mathbf{x}(t) z^*(\underline{\alpha}, t) \rangle_T$ denote the sample covariance matrix and the sample cyclic correlation vector, respectively.

## 2.2. The MLS-SCORE Algorithm with Subspace Projection

According to [8], an estimation error model has been presented to represent the perturbation due to the finite sample effect on the ideal cyclic correlation vector $\mathbf{r}_{xz}(\underline{\alpha})$. Let $\hat{\mathbf{r}}_{xz}(\underline{\alpha})$ be expressed by

$$\hat{\mathbf{r}}_{xz}(\underline{\alpha}) = \mathbf{r}_{xz}(\underline{\alpha}) + \Delta \tag{5}$$

where $\Delta$ represents the estimation error and vanishes asymptotically as $T$ approaches infinity. In practice, $\Delta \neq 0$ since the number of data snapshots is finite. This leads to performance degradation of the MLS-SCORE algorithm, especially when the number of data snapshots is small. To alleviate the performance degradation due to $\Delta$, a subspace projection in conjunction with the MLS-SCORE algorithm (termed as MLS-SCORE-SP algorithm) was proposed by [8]. The optimal weight vector of the MLS-SCORE-SP algorithm is given by

$$\hat{\mathbf{w}}_{sp} = \left(\mathbf{I} - \mathbf{E}_n \mathbf{E}_n^H\right) \hat{\mathbf{w}}_{mls} = \hat{\mathbf{w}}_{mls} - \mathbf{w}_{e_1} \tag{6}$$

where $\mathbf{w}_{e_1} = \mathbf{E}_n\mathbf{E}_n^H\hat{\mathbf{w}}_{mls}$ is the projection of $\hat{\mathbf{w}}_{mls}$ onto the noise subspace spanned by $\mathbf{E}_n$. $\mathbf{I}$ is the identity matrix with an appropriate size. The basis matrix $\mathbf{E}_n$ can be obtained as follows. Performing the eigenvalue decomposition (EVD) on $\hat{\mathbf{R}}_{xx}$ yields

$$\hat{\mathbf{R}}_{xx} = \sum_{m=1}^{M} \hat{\lambda}_m \mathbf{e}_m \mathbf{e}_m^H = \mathbf{E}_s\hat{\Lambda}_s\mathbf{E}_s^H + \mathbf{E}_n\hat{\Lambda}_n\mathbf{E}_n^H \tag{7}$$

where $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \ldots \geq \hat{\lambda}_M$ are the eigenvalues, $\{\mathbf{e}_m\}_{m=1}^M$ are the corresponding eigenvectors, $\hat{\Lambda}_s = \text{diag}\{\hat{\lambda}_1, \ldots, \hat{\lambda}_{K+d}\}$, $\hat{\Lambda}_n = \text{diag}\{\hat{\lambda}_{K+d+1}, \ldots, \hat{\lambda}_M\}$, $\mathbf{E}_s = [\mathbf{e}_1\ \mathbf{e}_2\ \ldots\ \mathbf{e}_{K+d}]^T$ and $\mathbf{E}_n = [\mathbf{e}_{K+d+1}\ \mathbf{e}_{K+2}\ \ldots\ \mathbf{e}_M]^T$ span the signal subspace and the noise subspace, respectively. It has been shown in [8] that the MLS-SCORE-SP algorithm achieves better performance than the MLS-SCORE algorithm because the perturbation due to the estimation error is alleviated. However, the drawbacks of this MLS-SCORE-SP algorithm are that it is efficient only when the dimension of the signal subspace is low and correctly known.

## 3. THE PROPOSED METHOD

It follows from [9] that the estimation error of a cyclic correlation function is asymptotically a complex Gaussian distribution. Hence, we let the estimation error $\Delta$ of (5) be a complex Gaussian random vector with zero mean and covariance matrix $\Sigma$. Its probability density function (pdf) can be written as

$$f(\Delta) = \frac{1}{\pi^M \det(\Sigma)} \exp\{-\Delta^H \Sigma^{-1} \Delta\}. \tag{8}$$

The log-likelihood function regarding the estimation error can be defined as

$$\mathcal{L}(\Delta) = -\Delta^H\Sigma^{-1}\Delta = -[\hat{\mathbf{r}}_{xz}(\underline{\alpha}) - \mathbf{r}_{xz}(\underline{\alpha})]^H \Sigma^{-1} [\hat{\mathbf{r}}_{xz}(\underline{\alpha}) - \mathbf{r}_{xz}(\underline{\alpha})]. \tag{9}$$

A cost function related to $\Delta$ is constructed as follows:

$$J(\mathbf{r}) = \mathbf{r}^H \mathbf{R}_{xx}^{-1} \mathbf{r} + \lambda[\mathbf{r} - \hat{\mathbf{r}}_{xz}(\underline{\alpha})]^H \Sigma^{-1} [\mathbf{r} - \hat{\mathbf{r}}_{xz}(\underline{\alpha})] \tag{10}$$

where the vector $\mathbf{r}$ is designated as the estimate of the ideal cyclic correlation vector $\mathbf{r}_{xz}(\underline{\alpha})$. The first term of (10) is related to the inverse of the array output power, while the second term represents a log-likelihood function related to the estimation error. $\lambda$ is a relative weight between the two terms. The optimal estimate of $\mathbf{r}_{xz}(\underline{\alpha})$ can be obtained by minimizing $J(\mathbf{r})$ as follows:

$$\mathbf{r}_o = \left(\mathbf{I} + \frac{1}{\lambda}\Sigma\hat{\mathbf{R}}_{xx}^{-1}\right)^{-1} \hat{\mathbf{r}}_{xz}(\underline{\alpha}). \tag{11}$$

Based on $\mathbf{r}_o$, the optimal weight vector is then obtained as

$$\mathbf{w}_{gl} = \left(\hat{\mathbf{R}}_{xx} + \frac{1}{\lambda}\Sigma\right)^{-1} \hat{\mathbf{r}}_{xz}(\underline{\alpha}). \tag{12}$$

Comparing (4) and (12), we note that the proposed method provides an appropriate weight vector which is the solution of the MLS-SCORE algorithm with a generalized loading on the data covariance matrix $\hat{\mathbf{R}}_{xx}$. From (12), there exists an issue that concerns the specification of the error covariance matrix $\Sigma$. In practice, the true $\Sigma$ seems to be unavailable. For simplicity and without loss of generality, we assume that the elements of $\Delta$ are independent complex Gaussian with zero mean and variance $\sigma^2$. Hence, $\Sigma$ is reduced to a scaled identity matrix, i.e., $\Sigma = \sigma^2\mathbf{I}$. Then, the optimal weight vector obtained in (12) becomes

$$\mathbf{w}_{dl} = \left(\hat{\mathbf{R}}_{xx} + \frac{\sigma^2}{\lambda}\mathbf{I}\right)^{-1} \hat{\mathbf{r}}_{xz}(\underline{\alpha}) = \left(\mathbf{I} + \frac{\sigma^2}{\lambda}\hat{\mathbf{R}}_{xx}^{-1}\right)^{-1} \hat{\mathbf{w}}_{mls}. \tag{13}$$

The loading factor $\lambda$ is a key factor which should be appropriately determined. For a large $\lambda$, the second term of (10) becomes dominant. The range for searching the minimum of $\mathbf{r}^H\mathbf{R}_{xx}^{-1}\mathbf{r}$ is constrained in the vicinity of $\hat{\mathbf{r}}_{xz}(\underline{\alpha})$. Consider the extreme situation of letting $\lambda$ be infinite, $\mathbf{w}_{dl}$ is exactly the same as $\hat{\mathbf{w}}_{mls}$ of (4). In contrast, using a smaller $\lambda$ allows a wider range for searching the minimum of $\mathbf{r}^H\mathbf{R}_{xx}^{-1}\mathbf{r}$. However, $\mathbf{w}_{dl}$ may approach a zero vector if $\lambda$ approaches zero. Thus, $\lambda$ must be appropriately chosen to achieve the best performance for the proposed method. More recently, some related works on choosing the loading factor are presented by [10] and [11]. For instance, in [10], the optimal loading factor is found by the particle filters. The particle who has the highest posterior probability is chosen as the optimal loading factor. From [11], the loading factor is obtained by controlling the peak location of the main beam. However, the loading factors of those methods cannot be obtained directly and have to be solved numerically.

Next, we present an analytical solution for determining the loading factor optimal in some sense. First, we rewrite $\mathbf{w}_{dl}$ of (13) as follows:

$$\mathbf{w}_{dl} = \left(\mathbf{I} + \kappa\mathbf{R}_a^{-1}\right)^{-1} \hat{\mathbf{w}}_{mls} \tag{14}$$

where $\kappa = 1/\lambda$ and $\mathbf{R}_a = \sigma^{-2}\hat{\mathbf{R}}_{xx}$. Consider the term $h(\kappa) = (\mathbf{I}+\kappa\mathbf{R}_a^{-1})^{-1}$ and a small $\kappa$. We can approximate $h(\kappa) = (\mathbf{I}+\kappa\mathbf{R}_a^{-1})^{-1}$ by taking the first two terms of its Taylor series expansion about $\kappa = 0$ as follows. The differentiation of $h(\kappa)$ with respect to $\kappa$ is given by

$$h'(\kappa) = -\left(\mathbf{I} + \kappa\mathbf{R}_a^{-1}\right)^{-1} \mathbf{R}_a^{-1} \left(\mathbf{I} + \kappa\mathbf{R}_a^{-1}\right)^{-1}. \tag{15}$$

Clearly, we have

$$h(0) = \mathbf{I} \tag{16}$$
$$h'(0) = -\mathbf{R}_a^{-1}. \tag{17}$$

Hence, we obtain the following approximation

$$h(\kappa) = \left(\mathbf{I} + \kappa\mathbf{R}_a^{-1}\right)^{-1} \approx \mathbf{I} - \kappa\mathbf{R}_a^{-1}. \tag{18}$$

Substituting (18) into (14) yields

$$\mathbf{w}_{dl} \approx \hat{\mathbf{w}}_{mls} - \underbrace{\kappa\mathbf{R}_a^{-1}\hat{\mathbf{w}}_{mls}}_{\mathbf{w}_{e_2}}. \tag{19}$$

Since the noise power at the beamformer's output is proportional to the squared norm of $\mathbf{w}_{dl}$, we then derive a solution for finding the optimal $\kappa$ so that the noise power at the beamformer's output is minimized. The squared norm of $\mathbf{w}_{dl}$ of (19) is given by

$$\|\mathbf{w}_{dl}\|^2 = \gamma_1\kappa^2 - 2\gamma_2\kappa + \gamma_3 \tag{20}$$

where $\gamma_1 = \hat{\mathbf{w}}_{mls}^H\mathbf{R}_a^{-2}\hat{\mathbf{w}}_{mls}$, $\gamma_2 = \Re\{\hat{\mathbf{w}}_{mls}^H\mathbf{R}_a^{-1}\hat{\mathbf{w}}_{mls}\}$, and $\gamma_3 = \|\hat{\mathbf{w}}_{mls}\|^2$. Since (20) is a quadratic function of $\kappa$, the optimal solution of $\kappa$ for minimizing (20) is clearly given by

$$\kappa_o = \frac{\gamma_2}{\gamma_1}. \tag{21}$$

Substituting $\kappa_o$ into (19) yields

$$\mathbf{w}_{dl}|_{\kappa_o} \approx \hat{\mathbf{w}}_{mls} - \frac{\Re\{\hat{\mathbf{w}}_{mls}^H\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{w}}_{mls}\}}{\hat{\mathbf{w}}_{mls}^H\hat{\mathbf{R}}_{xx}^{-2}\hat{\mathbf{w}}_{mls}}\hat{\mathbf{R}}_{xx}^{-1}\hat{\mathbf{w}}_{mls}. \tag{22}$$

Equation (22) reveals that optimal weight of the proposed method can be computed easily from the received data vector $\mathbf{x}(t)$. It means that the proposed method can avoid the computational burden for solving and evaluating the eigenvalue problem. Accordingly, the proposed method dose not have the drawbacks of the MLS-SCORE-SP algorithm and has ability to cope with the difficulty due to the estimation error as compared with the MLS-SCORE algorithm.

To analyze the performance of the proposed method, we note from (6) and (19) that the difference between the weight vectors of the MLS-SCORE-SP algorithm and the proposed method is the difference between $\mathbf{w}_{e_1}$ and $\mathbf{w}_{e_2}$. The squared norms of $\mathbf{w}_{e_1}$ and $\mathbf{w}_{e_2}$ are given by

$$\|\mathbf{w}_{e_1}\|^2 = \hat{\mathbf{w}}_{mls}^H\mathbf{E}_n\mathbf{E}_n^H\hat{\mathbf{w}}_{mls} = \sum_{i=K+d+1}^{M}\frac{|\beta_i|^2}{\hat{\lambda}_i^2} \tag{23}$$

and

$$\|\mathbf{w}_{e_2}\|^2 = \frac{\gamma_2^2}{\gamma_1} = \frac{\left(\sum_{i=1}^{M} |\beta_i|^2/\hat{\lambda}_i^3\right)^2}{\sum_{i=1}^{M} |\beta_i|^2/\hat{\lambda}_i^4} \tag{24}$$

respectively, where $\beta_i = \mathbf{e}_i^H \hat{\mathbf{r}}_{xz}(\underline{\alpha})$, $\gamma_1 = \sigma^4 \sum_{i=1}^{M} |\beta_i|^2/\hat{\lambda}_i^4$, and $\gamma_2 = \sigma^2 \sum_{i=1}^{M} |\beta_i|^2/\hat{\lambda}_i^3$. Let the eigenvalues of the signal subspace ($\hat{\lambda}_i$ for $i = 1, \ldots, K+d$) be significantly greater than those of the noise subspace ($\hat{\lambda}_i$ for $i = K+d+1, \ldots, M$), then (24) can be approximated by

$$\|\mathbf{w}_{e_2}\|^2 = \frac{\gamma_2^2}{\gamma_1} \approx \frac{\left(\sum_{i=K+d+1}^{M} |\beta_i|^2/\hat{\lambda}_i^3\right)^2}{\sum_{i=K+d+1}^{M} |\beta_i|^2/\hat{\lambda}_i^4} \tag{25}$$

Since the eigenvalues $\hat{\lambda}_i$, $i = K+d+1, \ldots, M$, approach the noise variance $\sigma_n^2$ as the number of data snapshots approaches infinity, it follows that (25) can be further approximated by

$$\|\mathbf{w}_{e_2}\|^2 = \frac{\gamma_2^2}{\gamma_1} \approx \sum_{i=K+d+1}^{M} \frac{|\beta_i|^2}{\sigma_n^4}. \tag{26}$$

We note from (23) that $\|\mathbf{w}_{e_1}\|^2 = \sum_{i=K+d+1}^{M} |\beta_i|^2/\sigma_n^4$ as the number of data snapshots approaches infinity. Therefore, (26) reveals that the performance of the proposed method approaches that of the MLS-SCORE-SP algorithm as the number of data snapshots approaches infinity.

## 4. SIMULATION EXAMPLES

In this section, we present several simulation examples by using the beamforming methods including the conventional LCMV algorithm [1] with both the true covariance matrix $\mathbf{R}_{xx}$ and the direction vectors of the SOIs exactly known, the Cross-SCORE algorithm [3], the C-CAB algorithm [4], the MLS-SCORE algorithm and MLS-SCORE-SP algorithm [8], and the proposed method shown by (22) for comparison. For all simulations, a uniform linear array (ULA) with $M = 10$ and a half-wavelength for inter-element spacing is considered. Assume that the SOIs and SNOIs are binary phase-shift-keying (BPSK) signals with rectangular pulse shape. The SOIs have signal-to-noise ratio (SNR) and baud rate equal to 5 dB and 5/11, respectively. Two SNOIs with cycle frequencies equal to 4.6 and 7.8 impinge on the array from $-20°$ and $40°$ off broadside, respectively. Moreover, the SNOIs have the interference-to-noise ratio (INR) and baud rate equal to 5 dB and 5/11,

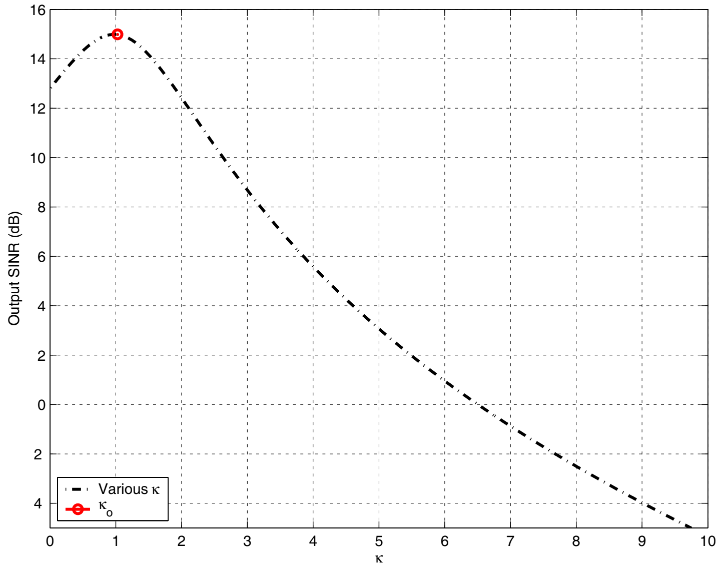


**Figure 1.** The output SINR versus the loading factor $\kappa$ for *Example 1.*

respectively. The control vector $\mathbf{c}$ and the time delay $\tau$ are set to $[1\ 0\ \ldots\ 0]^T$ and 0, respectively.

*Example 1*: Here, we consider the case of two SOIs ($d = 2$). The SOIs have different cycle frequencies equal to 2.0 and 2.6 and impinge on the array from 0° and 60° off broadside, respectively. Figure 1 shows the output signal-to-interference plus noise ratio (SINR) of the proposed method with various $\kappa$, where $1 \times 10^3$ data snapshots are used. We note from Figure 1 that the best choice of $\kappa$ in this case is approximately equal to 1.0 and the corresponding SINR is approximately equal to 14.987 dB. Meanwhile, the $\kappa$ calculated by the analytical formula of (21) is equal to 1.03 and the corresponding SINR is 14.984 dB. As expected, the analytical formula of (21) can find an appropriate loading factor that almost maximizes the array output SINR. Next, we present the output SINR versus the number of data snapshots for comparison. From Figure 2, we can see that the proposed method effectively alleviates the performance degradation due to the estimation error and outperforms the MLS-SCORE algorithm. It is slightly worse than the MLS-SCORE-SP algorithm in the circumstance that the dimension of the signal subspace is low and correctly known. However, the proposed method avoids the computational burden for solving and evaluating the eigenvalue problem. Moreover, we observe

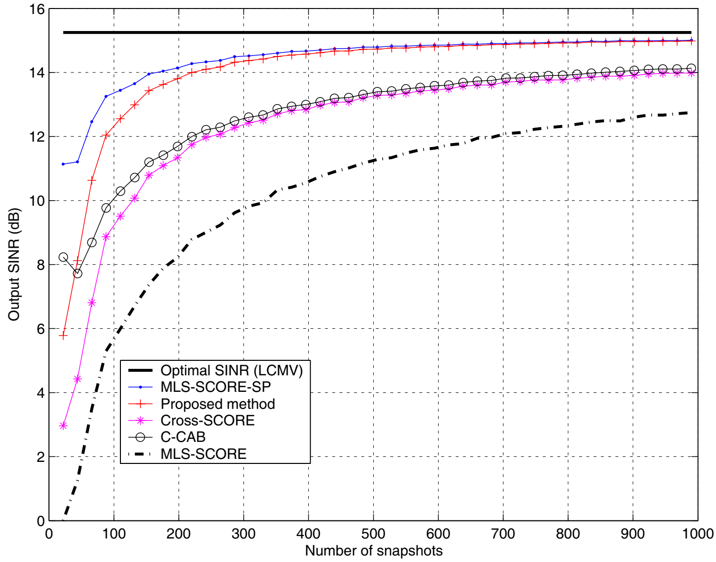


**Figure 2.** The output SINR versus the number of data snapshots for *Example 1*.

that the Cross-SCORE algorithm and the C-CAB algorithm have similar performance but they can not perform as well as the proposed method. The reason is that the required dominant eigenvectors or singular vectors of these algorithms for computing the weight vectors may no longer be able to converge to the desired solutions in the situation with multiple SOIs (see [8] and [4] for more details). Hence, these two algorithms inevitably suffer from performance degradation in the presence of multiple SOIs.

*Example 2*: We present the output SINR versus the number of SOIs for comparison. In the example, $M$ is 15. The cycle frequencies and direction angles of the SOIs are set to $\underline{\alpha} = \{2.0, 2.6, 3.4, 5.0, 5.8, 6.6, 7.2, 8.4, 9.0, 9.6, 10.2, 10.8\}$ and $\Theta = \{0°, 60°, -60°, 20°, -40°, 80°, -80°, -10°, 10°, 30°, -30°, 50°\}$, respectively. Moreover, $1 \times 10^3$ data snapshots are used. Figure 3 illustrates the performances of the MLS-SCORE-SP algorithm with the dimension of the signal subspace correctly and incorrectly known. We observe from Figure 3 that the proposed method can provide performance very close to the MLS-SCORE-SP algorithm when the dimension of the signal subspace is correctly known and the number $d$ of SOIs is less than 6. However, the proposed method achieves better performance than the MLS-SCORE-SP algorithm when $d$ is larger than 6. Furthermore,

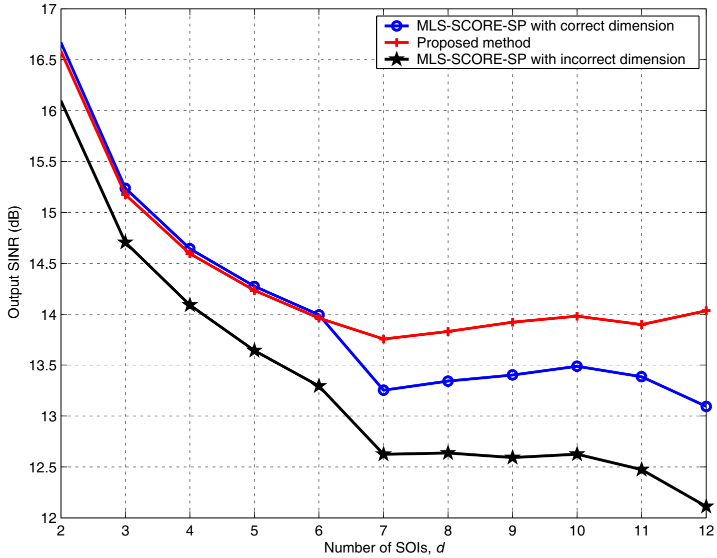


**Figure 3.** The output SINR versus the number of SOIs for *Example 2*.

the MLS-SCORE-SP algorithm suffers from significant performance degradation when the dimension of the signal subspace is incorrectly known, where the incorrect dimension is assumed to be $d + 3$ (the correct one should be $d + 2$) in this example.

## 5. CONCLUSION

An efficient method has been presented to overcome the performance deterioration under only finite sample data available for cyclic beamforming. When the estimation error vector associated with the cyclic correlation vector is a white Gaussian random vector, the proposed method provides a weight vector with a diagonal loading data covariance matrix. An analytical formula for determining the loading factor has also been derived. Several results have confirmed the validity of the proposed method and shown the effectiveness of the proposed method.

## ACKNOWLEDGMENT

This work was supported by the National Science Council of TAIWAN under Grant NSC97-2221-E002-174-MY3.

## REFERENCES

1. Van Trees, H. L., *Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory*, John & Wiley Sons, Inc., New York, USA, 2002.
2. Gardner, W. A., A. Napolitano, and L. Paura, "Cyclostationarity: Half a century of research," *Signal Processing*, Vol. 86, 639–697, 2006.
3. Agee, B. G., S. V. Schell, and W. A. Gardner, "Spectral self-coherence restoral: A new approach to blind adaptive signal extraction using antenna arrays," *Proc. IEEE*, Vol. 78, 753–767, Apr. 1990.
4. Wu, Q. and K. M. Wong, "Blind adaptive beamforming for cyclostationary signals," *IEEE Trans. Signal Processing*, Vol. 44, 2757–2767, Nov. 1996.
5. Du, K.-L. and M. N. S. Swamy, "A class of adaptive cyclostationary beamforming algorithms," *Circuits, Systems, and Signal Processing*, Vol. 27, No. 1, 35–63, Jan. 2008.
6. Mayhan, J. T., "Area coverage adaptive nulling from geosynchronous satellites: Phased arrays versus multiple-beam antennas," *IEEE Trans. Antenn. Propag.*, Vol. 34, 410–419, Mar. 1986.
7. Yu, K.-B., "Adaptive beamforming for satellite communication with selective earth coverage and jammer nulling capability," *IEEE Trans. Signal Processing*, Vol. 44, No. 12, 3162–3166, Dec. 1996.
8. Lee, J.-H. and C.-C. Huang, "Blind adaptive beamforming for cyclostationary signals: A subspace projection approach," *IEEE Antenn. and Wirele. Propag. Lett.*, Vol. 8, 1406–1409, Dec. 2009.
9. Dandawaté, A. V. and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Trans. Signal Processing*, Vol. 42, No. 9, 2355–2369, Sep. 1994.
10. Li, Y., Y.-J. Gu, Z.-G. Shi, and K. S. Chen, "Robust adaptive beamforming based on particle filter with noise unknown," *Progress In Electromagnetics Research*, Vol. 90, 151–169, 2009.
11. Wang, W., R. Wu, and J. Liang, "A novel diagonal loading method for robust adaptive beamforming," *Progress In Electromagnetics Research C*, Vol. 18, 245–255, 2011.