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# NUMERICAL INVESTIGATION INTO THE DESIGN OF SHAPED DIELECTRIC LENS ANTENNAS WITH IMPROVED ANGULAR CHARACTERISTICS

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**Abstract**—The feasibility of designing a compact-size beam-switching dielectric lens antenna (DLA) with improved angular (scanning) characteristics is investigated numerically using in-house software based on the Muller boundary integral equations and hybrid genetic algorithm. It is demonstrated that joint optimization of the lens shape and feeding array parameters enables one to minimize the directivity degradation for off-axis feeds which is a well-known drawback of conventional extended hemielliptic DLAs fed by focal arrays. The key to success is found in proper shaping the lens profile and using arrays of non-identical feeds.

## 1. INTRODUCTION

Integrated dielectric lens antennas (DLAs) fed by focal arrays are good candidates for imaging and wireless communication systems [1–6]. The desirable feature of such antennas is a stable beam profile for all feeds (with no directivity degradation for off-axis feeds) and also a high, near the diffraction limit, directivity for all the beams. The former requirement is naturally satisfied in DLAs with radially symmetric lenses (homogeneous or multi-shell) fed by multiple feeds placed along the lens periphery, e.g., [4]. Such DLAs are widely used up to K-bands, but are less attractive at higher frequencies due to bulky feeding structures and integration complexity. To overcome these difficulties, flat-bottom hemielliptic DLAs fed by arrays of planar feeds were proposed [1–3]. The key component of such antennas is

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a homogeneous hemielliptic (or hemispherical) lens extended with a cylindrical extension up to the ellipse geometrical focus. This design provides efficient focusing of the rays propagating parallel to the lens axis and enables direct mounting of the lens on a dielectric substrate supporting the feeding network and detectors. The inherent drawback of such a configuration is that, unlike spherical lenses, elliptical ones have only one focus and therefore their focusing and collimating capabilities degrade in the case of none-symmetrical excitation [2, 3, 7]. As a result a strong beam distortion appears for offaxis feeds. This reduces the antenna scan angle and becomes a bottleneck in the design of multi-beam antennas.

As to our best knowledge, no successful attempt of improving the angular characteristics of flat-bottom integrated DLAs has been reported until now. Being inspired by recent achievements in the design of shaped DLAs for single-beam application [8–11], we decided to approbate this optimization strategy for paving the way for designing a shaped DLA with improved scanning capabilities.

The paper is organized as follows. The outline of the methods of analysis and the antenna model are provided in Sections 2 and 3, respectively. The design procedure and simulation results for conventional hemielliptic and optimized shaped DLAs are given in Section 4, and the findings are summarized in Conclusions.

Compared to the earlier publication [12], this paper provides detailed description of the model and an extension towards the design of 3-D axisymmetric DLAs.

## 2. METHODS OF ANALYSIS

Synthesis of a compact-size shaped DLA fed by a focal array is a challenging task whose fulfillment requires utilization of reliable CAD tools. Indeed, electromagnetic solvers based on the high-frequency approximations (which are fast enough for optimization) cannot accurately reproduce multiple internal reflections and fail as soon as profile radius of curvature becomes comparable with the wavelength, e.g., [13]; whereas standard full-wave frequency- and time-domain solvers are usually too slow for synthesis.

To get a fast and trustable solution of the diffraction problem, we use the in-house CAD tool based on the Muller Boundary Integral Equations (MBIE) and the Method of Analytical Regularization (MAR). This rigorous mathematical approach guarantees exponentially-fast monotonic convergence of the numerical solution and controlled accuracy for any set of the lens and feeding array parameters [13–15]. This software was carefully validated earlier against highfrequency and FDFD solvers when applied for the design of resonant and non-resonant dielectric scatterers [7, 13, 15, 16] and proved to be a reliable engine for synthesis-oriented CAD tools.

As an optimization routine we use a hybrid genetic algorithm (HGA) developed as a combination of a binary GA and a steepest descent gradient (SDG) algorithms. Here genetic algorithm (GA) performs an exhaustive global exploration and the gradient-based SDG algorithm is used for the straight-forward down-hill local search in the neighborhood of the most promising solution found by GA. Such a two-step strategy enables us to significantly reduce the GA stagnation period observed at the later stage of optimization and was found to be very effective when dealing with multi-parameter and multi-extremum functions typical for electromagnetic synthesis [17, 18]. Merging HGA with MBIE/MAR solver enabled us to build full-wave synthesis-oriented software capable of fast and reliable optimization of arbitrary-shaped dielectric scatterers.

## 3. MODEL DESCRIPTION

To assess the principle possibility of designing a shaped flat-bottom DLA with improved angular characteristics, we perform a test-case study of a DLA with a lens made of Rexolite ($\varepsilon = 2.53$) and having the flat bottom size of approximately $6\lambda_0$ ($\lambda_0$ is the free-space wavelength).

The optimization goal that to be achieved via joint optimization of the lens shape and feeding array parameters is the following: to obtain the best possible main-beam directivity for the central feed and simultaneously to minimize the directivity degradation for the off-axis ones.

The problem is considered in the two-dimensional (2-D) formulation and the antenna is modeled as an arbitrary-shaped homogeneous dielectric lens illuminated by a linear array of feeds positioned as shown in Fig. 1.

The lens contour is defined by a given number of spline nodes connected by cubic splines. The initial lens profile corresponds to the conventional extended hemielliptic DLA whose eccentricity equals the inverse of the material refracting index [2]. During optimization the lens extension and flat bottom remains fixed and the front part is shaped in a way to satisfy the optimization goal (the fixed and floating nodes are highlighted in Fig. 1 by different marks). The number of nodes used for the front-part profile description is 10 (that gives the distance between the neighboring nodes of about one wavelength in dielectric, $\lambda_e$) and the relative increment of the radius vector for each spline node with respect to its neighbors is limited to $1\lambda_0$.

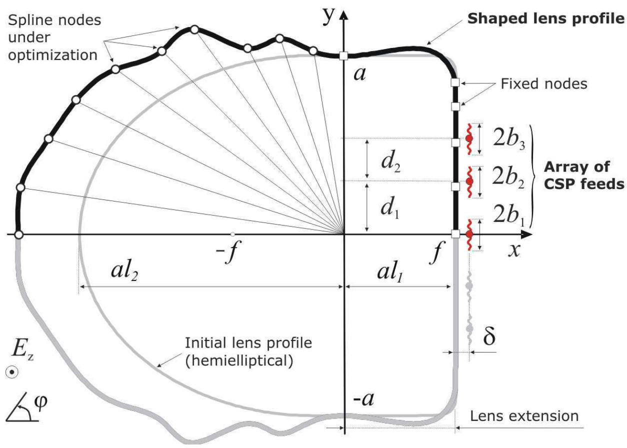



**Figure 1.** Geometry and notations of the 2-D model of a shaped DLA fed by a focal array. The lens is symmetric with respect to $x$-axis (the mirrored part is shown in grey color). Curvy lines associated with CSP feeds indicate the branch cuts in the real space due to CSPs.

The lens is illuminated by a feeding array whose elements are modeled as complex-source point (CSP) beams. As shown in [19], a line current located in a point with a complex coordinate produces in real space a beam whose waist is controlled by the imaginary part of the CSP coordinate. Such a model is very convenient because it preserves the mathematical rigorousity of the problem formulation and enables a compact-form representation of a feed with a directive radiation pattern [18].

In simulations, the lens is illuminated by a symmetrical array of 5 CSP feeds (Fig. 1). The feeds are located outside the lens close to its flat bottom ($\delta = \lambda_0/10$) and oriented to radiate in the negative $x$-axis direction ($\varphi = 180°$). Such a positioning guarantees good approximation of a planar feed put in a direct contact with dielectric lens, but also helps avoid difficulties with numerical integration. Parameters included in optimization are: (i) CSP beam waists controlled by parameters $b_i$ and (ii) spacing between the elements, controlled by parameters $d_i$. The ranges of parameters variation are the following: feed spacing $d_i/\lambda_e = 0.69 \pm 0.18$ and beam-waist parameter $kb_i = 1.5 \pm 0.8$ ($k = 2\pi/\lambda_0$ is the free space wavenumber). The former avoids overlapping between the neighboring feeds, whereas the latter provides a sufficient flexibility in variation of the feeds patterns. Note that the central value of the beam-waist parameter ($kb = 1.5$) corresponds to the optimal edge illumination conditions if defined for the central feed placed in the focus of the classical extended hemielliptic lens [20].

Synthesis of the DLA is first performed at a fixed frequency and for $E$-polarization case only. After that the performance of the optimized DLA is studied for the case of $H$-polarization and within a relative frequency band of $\pm 5\%$.

During optimization the quality of the antenna design is evaluated according to the following cost-function

$$F_{cost} = \sum_{n=1}^{3} \left(D_{\max} - D_n(\gamma_n)\right)^2 \qquad (1)$$

where $D_n$ is the main-beam directivity of the $n$-th feed in the given direction $\gamma_n$ defined to be the same as produced by a conventional hemielliptic DLA fed by a uniformly-spaced array (the reference solution is shown in Section 4.1). Our software is designed for minimum seeking, therefore a maximum accessible value of the directivity is introduced, $D_{\max} = 40$. The cost-function defined by Eq. (1) tends to minimum when directivities of all feeds approach $D_{\max}$.

Finally, it is important to notice that the selected model and methods of analysis enable us to fully account for the effects related to multiple internal reflections, edge diffraction, and directive nature of the primary feed patterns. At the same time, the model does not account for the cross-polarization effects and coupling between neighboring elements of the feeding array. The former effects are usually negligible for DLAs made of low permittivity dielectric materials such as Rexolite, whereas the latter may become critical especially for closely spaced arrays. In this paper we focus our attention on accurate description of the diffraction phenomena and definition of the optimal illumination conditions for lenses with non-symmetrical excitation, whereas the question of building a matched feeding network remains out of scope.

## 4. NUMERICAL RESULTS

### 4.1. Reference Solution: Extended Hemielliptic DLA

As a reference solution we consider a classical DLA with an extended hemielliptic lens whose eccentricity equals to the inverse of the refracting index of its material $e = \varepsilon^{-1/2}$, that gives $l_1 = [\varepsilon - 1]^{-1/2}$, and $l_2 = [\varepsilon/(\varepsilon - 1)]^{-1/2}$ (see Fig. 1). Main-beam directivity and scan angle of the CSP versus feed position are shown in Fig. 2. As seen for both polarizations the main-beam directivity quickly degrades when

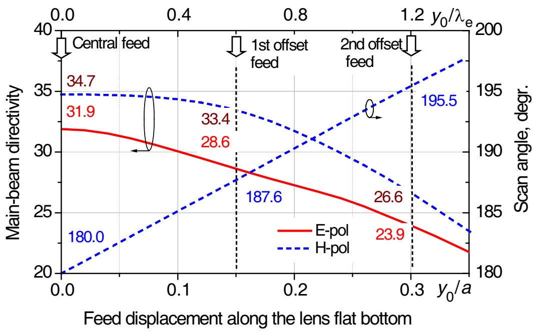



**Figure 2.** Main-beam directivity (left axis) and scan angle (right axis) of the CSP feed (with $kb = 1.5$) illuminating the hemielliptic rexolite lens vs. feed off-axis displacement: normalized by lens radius (bottom axis) and normalized by the wavelength in dielectric (top axis). The arrows indicate initial positions of the feeding array elements used for synthesis.

the feed is displaced from the axis of symmetry and looses $\sim$ 25% of its maximum value for the most offset position. The scan angle varies almost linearly. Both observations are in line with earlier studies [2, 3]. The points that correspond to initial positions of the array elements are indicated with the arrows, and the corresponding values of the directivity for the $E$ and $H$ polarized CSPs ($D_n^e$, $D_n^h$) and scan angles ($\gamma_n$) are shown nearby.

## 4.2. Synthesis of a Shaped BS-DLA

A representative optimization run is shown in Fig. 3. Here the optimization process is described both in terms of the cost-function and main-beam directivity values. After first 20 iterations performed by GA, the best found solution is delivered to the SDG optimization loop which works until no further improvement is achieved. Keeping in mind the total number of unknowns (i.e., 15, including 10 parameters for the lens profile, 2 for the feeds spacing, and 3 for beam-waists) and the size of the GA population (which is 50 individuals), the total number of iterations is much smaller than would be needed if GA works along. Nevertheless, stability in hitting this extremum observed in multiple independent runs (skipped for brevity) confirms the reliability of the found solution as well as the efficiency of the developed software.

The advantage in the antenna performance characteristics is also remarkable. Compared to reference solution the relative directivity

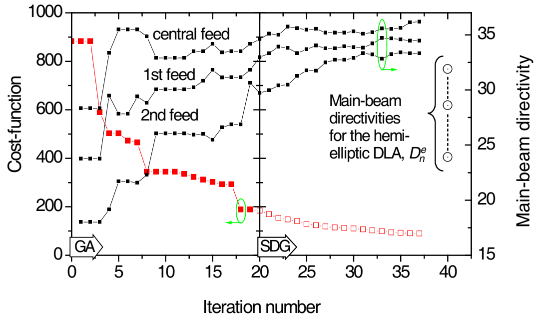



**Figure 3.** Cost-function (left axis) and directivity (right axis) values vs. number of iteration of the optimization routine. The family of three curves associated with the right axis indicates directivities of three independent feeds of the focal array when assisted by the shaped lens. The reference values of the main-beam directivities for the conventional hemielliptic DLA are indicated by hollow circles on the right side.

**Table 1.** Optimal parameters of the feeding array.

| Spacing, $d_i$ | Beam-waist parameters, $kb_i$ (HPBW, full angle) | | |
|---|---|---|---|
| $d_1 = d_2 = 0.8\lambda_e$ | $kb_1 = 1.5$ (60°) | $kb_2 = 1.3$ (66°) | $kb_3 = 1.1$ (74°) |

degradation for the most offset feed is reduced to less than 10% (with respect to the central feed) in contrast to more than 25% difference observed for the conventional design. Moreover, the relative advantage in absolute values of directivity for each feed achieves roughly 12%, 30%, and 38% for the central and offset feeds, respectively. For convenience the values of the main-beam directivities for the shaped (optimized) and two hemielliptic lenses (which are the initial hemielliptic lens and the one whose aperture equals to that of the optimized lens) are shown in Fig. 4. As seen in Fig. 5 the improved performance of the shaped DLA is achieved thanks to suppression of the side-lobe level and reduction of the off-axis beams distortion.

The shaped lens whose profile corresponds the best found solution is shown by a thick black line in Fig. 1 and the optimal feeding array parameters are given in Table 1 (the half-power beam width (HPBW) is indicated for the CSP feed radiating in free space).

It is interesting to notice that the optimal performance of the antenna is achieved when the shaped lens is illuminated by a uniformly-spaced array of non-identical feeds: with a central feed providing the optimal edge illumination [20], and the offset feeds with patterns that widen versus the off-axis displacement. The latter recommendation is

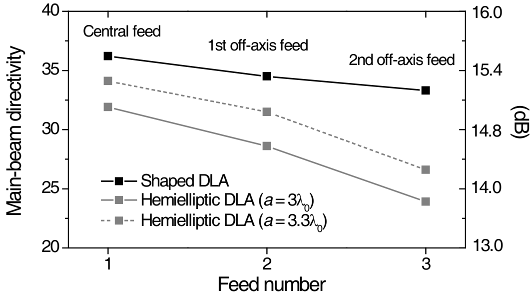
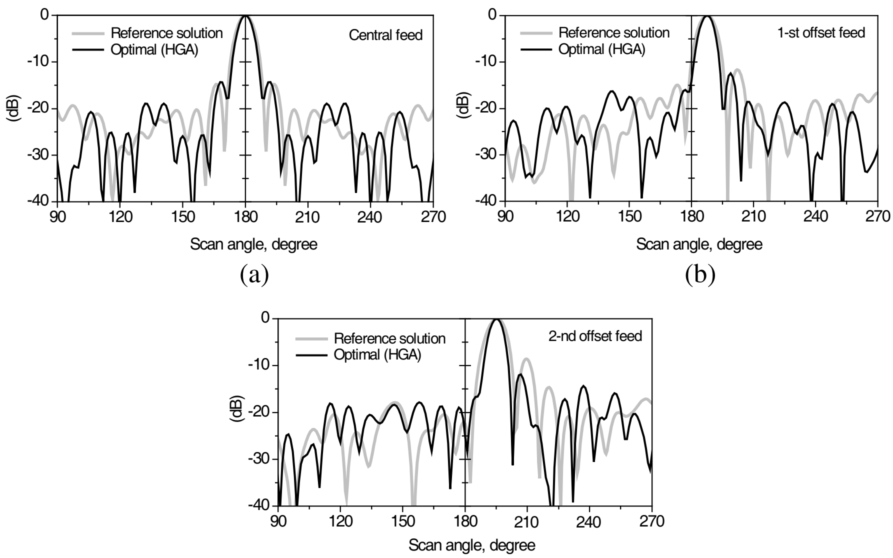



**Figure 4.** Main-beam directivities of the shaped DLA and two hemielliptic DLAs with dimensions $a = 3\lambda_0$ (the initial design) and $a = 3.3\lambda_0$ (same aperture size as the shaped DLA).




(a) (b) (c)

**Figure 5.** Normalized radiation patterns for each feed of the focal CSP array assisted by the shaped (black line) and hemielliptic (grey line) lenses.

contrary to all the previously reported designs where arrays of identical feeds were used, e.g., [2, 3, 6].

To provide a physical interpretation for this phenomenon, we propose to refer to the schematic ray-tracing picture shown in Fig. 6. As it is highlighted in the inset, the surface areas involved in focusing

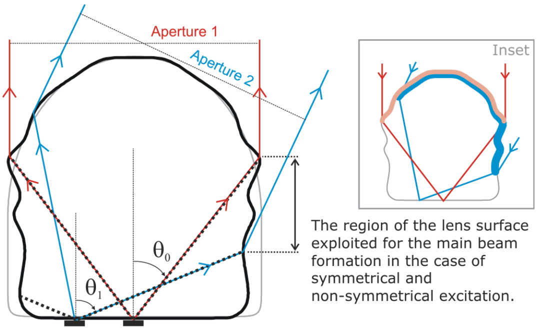



**Figure 6.** Schematic drawing of the main beam formation for the case of a shaped lens illuminated in the symmetrical and non-symmetrical manner. The inset depicts the areas of the lens surface involved in the focusing the incident plane wave.

(or collimation) of the arriving plane wave (or primary feed radiation) for the case of a symmetrical and non-symmetrical excitation are different: in the former case, only the front part is involved whereas sides of the lens remain in shadow. Because of this a wider beam for an off-axis feed is needed to illuminate (and thus to effectively exploit) the whole area responsible for collimation of a tilted beam. It is difficult to provide a quantitative description of the trade-off between the illumination and spillover losses for a shaped lens excited by an off-axis feed. Nevertheless it is intuitively clear that shaping the lens profile helps one to obtain a uniform distribution in the antenna aperture but does not itself guarantees the best antenna performance because the losses associated with non-uniform illumination and the power that misses the lens aperture are determined by the antenna aperture illumination conditions [20]. This explains the need for joint optimization of the lens shape and feeding system. Simulation results obtained in multiple independent runs (skipped for brevity) performed for 2-D lenses of different sizes and fed by arrays with different number of feeds clearly show that the best performance is achieved when the lens is illuminated by the array of non-identical feeds with HPBW increasing proportional to the feed displacement. Verification of this recommendation for 3-D DLAs requires additional studies and will be reported in a forthcoming paper.

### 4.3. Towards the Design of 3-D Axisymmetric DLAs

In this subsection, we assess the validity of the aforementioned recommendations for the design of 3-D axisymmetric DLAs.

Unfortunately the selected approach does not enable us to directly synthesize a 3-D DLA. Nevertheless we can accurately study the performance of the optimally-shaped 2-D lens for the case of a feeding array operating in the $E$ and $H$ polarization modes that is to some extent equivalent to description of an axisymmetric DLA in both principle planes.

To perform such an assessment, we consider the same antenna as was synthesized in Section 4.2 but illuminate it with an array of $H$-polarized feeds (spacing and beam waist parameter values are the same as given in Table 1). This design is used as the initial guess for the SDG optimization algorithm which is used to locally manipulate with the DLA parameters aiming at the same optimization goal, i.e., maximum directivity with minimum degradation for off-axis feeds. The optimization process in terms of cost-function and main-beam directivity values is depicted in Figs. 7 and 8, respectively. Note that here optimization is carried out only for the $H$-case, whereas the curves for the $E$-case are plotted simply to illustrate the trade-off between the designs quality with respect to both polarizations. As it is seen, the design that has been recognized as the best one for the $E$-polarization is not optimal for the $H$-one, although it can be adjusted to provide better performance by means of local optimization.

The values of the parameters that correspond to the optimal designs found for the $E$- and $H$-polarizations are given in Fig. 9. Note that optimal $E$ and $H$ designs differ only by the lens shape, whereas the array parameters remain unchanged. This means that (i)

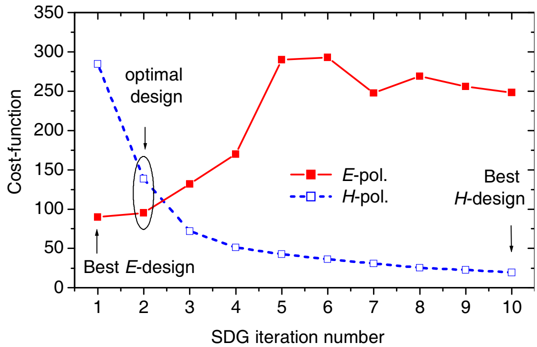



**Figure 7.** Cost-function value vs. iteration number of the SDG algorithm. The initial guess corresponds to the optimal $E$-design. Optimization is performed only for the $H$-case (hollow marks) whereas the curve for the E-case (filled marks) is plotted to illustrate the trade-off between the designs quality for another polarization.

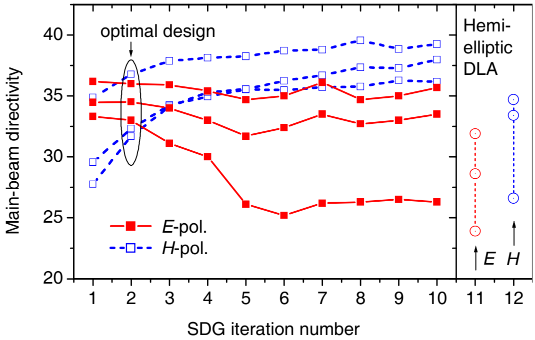



**Figure 8.** Main-beam directivity vs. iteration number of the SDG algorithm. The two families of three curves each represent directivities of the feeds of the $E$-polarized (filled marks) and $H$-polarized (hollow marks) arrays. The corresponding cost-functions values are given in Fig. 7. The reference values of the directivities for the hemielliptic DLA fed by $E$- and $H$-polarized focal arrays are shown by hollow circles on the right side.

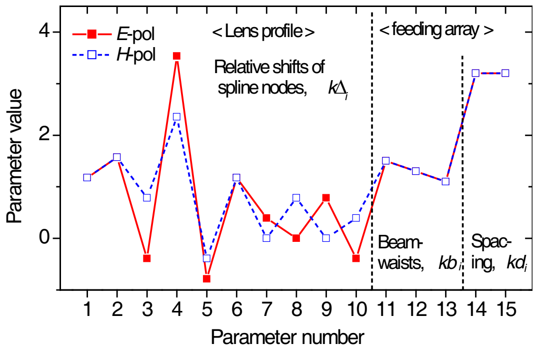



**Figure 9.** Values of the parameters under optimization that correspond to the best designs obtained for both polarizations: (filled marks) are for the $E$-case and (hollow marks) are for the $H$-case, respectively.

the recommendation of using an array of non-identical feeds is still valid for 3-D antennas, (ii) a shaped 3D axisymmetric lens can be designed in a way to radiate nearly-symmetrical beams in both principal planes. Such a design can be found on a trade-off basis at some intermediate step of optimization, e.g., the one marked in Fig. 7 as the "optimal design."

Finally, the performance of the optimized DLA (see the mark

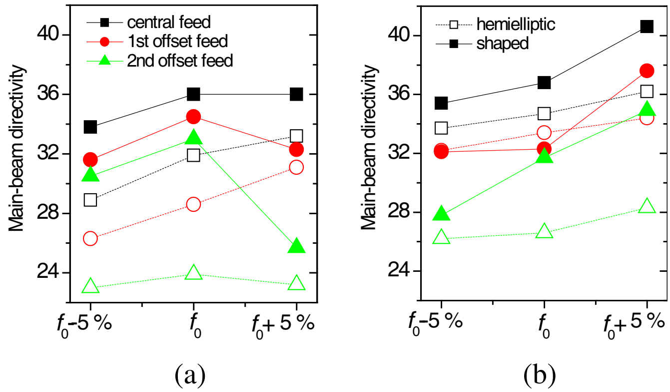



**Figure 10.** Variation of the main-beam directivities of the shaped DLA over a $\pm 5\%$ relative frequency band: (a) $E$-polarization, (b) $H$-polarization. The two families of three curves each depict mainbeam directivities of each feed for the shaped (filled marks) and hemielliptic (hollow marks) DLAs. The central frequency ($f_0$) corresponds to the lens flat bottom size of $6\lambda_0$.

"optimal design" in Fig. 7) is assessed in a relative frequency band of $\pm 5\%$ from the design frequency. The main-beam directivities for each feed at each frequency are depicted in Fig. 10. As seen the improved performance is preserved within the whole frequency band although the tendency in the behavior for $E$ and $H$ polarizations are different. For the $E$-case, the minimum directivity degradation is observed at lower frequencies, whereas for the $H$-case the converse behaviour is observed. Nevertheless, at each frequency point the absolute values for the main-beam directivities for off-axis feeds are better than that of the classical hemielliptic DLA. This evidences a potential possibility for designing shaped BS-DLAs with improved angular characteristics.

## 5. CONCLUSION

The feasibility of designing a shaped flat-bottom lens antenna with improved angular characteristics has been numerically demonstrated by example of a compact-size Rexolite lens fed by a symmetrical linear array of five independent feeds. The trustable solution has been obtained using a reliable in-house CAD tool based on rigorous mathematical approaches.

The outlined recommendations as to the optimal lens profiles and parameters of the feeding array can be considered as a guideline for the design of axisymmetric DLAs for beam switching applications.

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