Progress In Electromagnetics Research C, Vol. 21, 257–271, 2011

A FAST DOA ESTIMATION ALGORITHM FOR UNIFORM CIRCULAR ARRAYS IN THE PRESENCE OF UNKNOWN MUTUAL COUPLING

J. L. Xie*, **Z. S. He, and H. Y. Li**

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

Abstract—Based on the beamspace transform and the rank reduction theory (RARE), a fast direction of arrival (DOA) estimation algorithm in the presence of an unknown mutual coupling is proposed for uniform circular arrays (UCAs). Via relying on the circular symmetry and expanding the mutual coupling into a limited number of phase modes, the azimuth estimates are able to be obtained without the exact knowledge of mutual coupling. Then, by using the special structure of mutual coupling matrix and the characteristic of mutual coupling coefficients, the elimination of spurious estimates and estimations of the mutual coupling coefficients are able to be handled simultaneously. The Propagator Method (PM) is used to avoid the eigenvalue decomposition. The RARE matrix of PM allows decreasing the computation cost via using a well known identity for block matrices. Moreover, an implementation of rooting polynomial substitutes the one-dimension search. Therefore, the computation burden is greatly reduced. Numerical examples are presented to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

The direction of arrival (DOA) estimation of multiple narrowband signals is a classic problem in array signal processing. The uniform circular array (UCA) is able to provide 360° of coverage in the azimuth plane and has uniform performance regardless of angle of arrival. Thus, sometimes, UCA is more suitable than uniform linear array (ULA) for applications such as radar, sonar, and wireless communications.

Received 26 April 2011, Accepted 19 May 2011, Scheduled 27 May 2011

* Corresponding author: Julan Xie (xiezhu0415@yahoo.com.cn).

Due to the circular symmetry, the beamspace transformation, based on the phase-mode excitation principle, is usually applied to obtain the desired Vandermode structure for the steering vector in the mode space. This transformation results in the development of several DOA estimation algorithms with low computational cost, such as UCA-RB-MUSIC [1], UCA-ESPRIT [1], UCA-RARE [2] and Sparse UCA Root-MUSIC [3], in the meanwhile dealing with coherent sources via spatial smoothing technique [4, 5].

All the algorithms referred to above ignore the mutual coupling effect, which ultimately destroys the underlying model assumptions needed for their efficient implementations. In fact, most of the algorithms are very sensitive to the array manifold errors due to the mutual coupling fact. Therefore, it is necessary to take the mutual coupling into account. There are several calibration algorithms which have been proposed [6–10]. In [6] and [7], both the proposed algorithms take the mutual coupling into account and employ the UCA-RARE algorithm to estimate the azimuth angle first. With the open-circuit voltages of the antenna elements expanded in spherical mode, a Root-MUSIC algorithm is able to be performed in the elevation space to obtain the elevation estimates in [6]. In [7], a 1D parameter search replaces the implementation of Root-MUSIC algorithm in the elevation space. In the 1D parameter search for elevation estimates, the elevation-dependent mutual coupling effect can be efficiently compensated by the elevation-dependent receiving mutual impedances. In [8], using the maximum likelihood technique optimized by the emperor selective genetic algorithm for UCAs, authors perform the 2D DOA estimation in the presence of mutual coupling. These algorithms are based on the knowledge of mutual coupling. While in [9] and [10], 1D DOA estimation algorithms in the presence of unknown mutual coupling are proposed. Both the algorithms in [9] and [10] estimate the DOAs using a one-dimension search and yield spurious estimates due to the ambiguity of the array manifold in the presence of the mutual coupling. In [11] and [12], the supplements for the ambiguity of the array manifold are presented and the solutions are proposed.

In this paper, we will assume that the noncoherent narrowband sources are located at the same elevation angle. We consider 1-D (azimuth) angular estimation in the unknown mutual coupling. Using a new formulation of the beamspace array manifold in the presence of mutual coupling, the rank reduction theory (RARE) based on the Propagator Method (PM) [13, 14] can be used to estimate the azimuth angle without the knowledge of mutual coupling. We find that the solution to eliminate the spurious estimates proposed in [11] and [12] failed sometimes. This will be illuminated by an example. Based on the special structure of mutual coupling matrix and the characteristic of mutual coupling coefficients, an appropriate solution to eliminate the spurious estimates is presented. Compared with the DOA estimation algorithms mentioned in [9] and [10], our proposed algorithm estimate the azimuth angle by rooting a polynomial instead of a one-dimension search. In addition, the Propagator Method avoids the eigenvalue decomposition and its corresponding process to reduce the dimension of the RARE matrix is introduced. Therefore, the computation burden is greatly reduced.

2. PROBLEM FORMULATION

Consider a UCA consisting of N identical elements uniformly distributed over the circumference of a circle of radius r. Assume that D narrowband sources, centered on wavelength λ, impinge on the array from directions ϕ_i $(i = 1, \ldots, D)$, respectively, where $\phi_i \in [0, 2\pi)$ is the azimuth angle measured from the X-axis counter-clockwise. The $N \times 1$ vector received by the array in the presence of mutual coupling [15] is expressed by

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}(\phi)\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}(\phi) = [\mathbf{a}(\phi_1) \quad \ldots \quad \mathbf{a}(\phi_D)]$ is the $N \times D$ matrix of the steering vectors, $\mathbf{s}(t) = [\ s_1(t) \quad \ldots \quad s_D(t)\ ]^T$ is the $D \times 1$ signal vector, $\mathbf{n}(t) = [\ n_1(t) \quad \ldots \quad n_N(t)\ ]^T$ is the $N \times 1$ noise vector. The signal vector $\mathbf{s}(t)$ and the vector $\mathbf{n}(t)$ of the additive and spatially white noise are assumed to be statistically independent and zero-mean. The $N \times N$ matrix $\mathbf{C}$ is the mutual coupling matrix (MCM). Due to the circular symmetry, a model for the MCM of UCAs [15] can be a complex symmetric circulant matrix. The steering vector with mutual coupling can be modeled as

$$\tilde{\mathbf{a}}(\phi) = \mathbf{C}\mathbf{a}(\phi) \tag{2}$$

The covariance matrix $\mathbf{R}$ of the received data is constructed and an eigendecomposition of $\mathbf{R}$ results in a signal and noise subspace

$$\mathbf{R} = E\left\{\mathbf{x}(t)\,\mathbf{x}^H(t)\right\} = \mathbf{E}_s\Lambda_s\mathbf{E}_s^H + \mathbf{E}_n\Lambda_n\mathbf{E}_n^H \tag{3}$$

where $\mathbf{E}_s$ and $\mathbf{E}_n$ denote the signal and noise subspace eigenvectors and the diagonal matrices Λ_s and Λ_n contain the signal subspace and noise subspace eigenvalues. The MUSIC algorithm estimates the DOAs from the D deepest nulls of the MUSIC function

$$f_{\mathrm{MUSIC}}(\phi) = \tilde{\mathbf{a}}^H(\phi)\,\mathbf{E}_n\mathbf{E}_n^H\tilde{\mathbf{a}}(\phi) \tag{4}$$

This method needs to know the mutual coupling coefficients.

Using the Lemma 2 in [15], Equation (2) can be re-expressed as

$$\tilde{\mathbf{a}}(\phi) = \mathbf{\Gamma}(\phi)\mathbf{c} \tag{5}$$

where $\mathbf{c} = [\ c_1 \quad c_2 \quad \dots \quad c_K\ ]^T$ and the $N \times K$ matrix $\mathbf{\Gamma}(\phi)$ is the sum of the four following matrices

$$\mathbf{\Gamma}(\phi) = \mathbf{\Gamma}_1(\phi) + \mathbf{\Gamma}_2(\phi) + \mathbf{\Gamma}_3(\phi) + \mathbf{\Gamma}_4(\phi) \tag{6}$$

with

$$\begin{aligned}
[\mathbf{\Gamma}_1(\phi)]_{p,q} &= \begin{cases} [\mathbf{a}(\phi)]_{p+q-1}, & p+q \leq N+1 \\ 0, & \text{otherwise} \end{cases} \\
[\mathbf{\Gamma}_2(\phi)]_{p,q} &= \begin{cases} [\mathbf{a}(\phi)]_{p-q+1}, & p \geq q \geq 2 \\ 0, & \text{otherwise} \end{cases} \\
[\mathbf{\Gamma}_3(\phi)]_{p,q} &= \begin{cases} [\mathbf{a}(\phi)]_{N+p-q+1}, & p < q \leq K_s \\ 0, & \text{otherwise} \end{cases} \\
[\mathbf{\Gamma}_4(\phi)]_{p,q} &= \begin{cases} [\mathbf{a}(\phi)]_{p+q-N-1}, & p+q \geq N+2,\ 2 \leq q \leq K_s \\ 0, & \text{otherwise} \end{cases}
\end{aligned} \tag{7}$$

where $K = (N+2)/2$ for N is even and $K = (N+1)/2$ for N is odd. $K_s = N/2$ for N is even and $K_s = (N+1)/2$ for N is odd. $c_n = [\mathbf{C}]_{1,n}$, where $[\bullet]_{i,j}$ denotes the component of ith row and jth column of the matrix. The mutual coupling coefficients are approximated to zeros when the elements are far from each other. If the number of nonzero mutual coupling coefficients is L, Equation (5) is given by

$$\tilde{\mathbf{a}}(\phi) = \mathbf{\Gamma}_L(\phi)\mathbf{c}_L \tag{8}$$

where $\mathbf{\Gamma}_L(\phi)$ represents the first L columns and $\mathbf{c}_L$ denotes the first L elements. Therefore, Equation (4) can be re-expressed as

$$f_{\text{MUSIC}}(\phi) = \mathbf{c}_L^H \mathbf{\Gamma}_L^H(\phi) \mathbf{E}_n \mathbf{E}_n^H \mathbf{\Gamma}_L(\phi) \mathbf{c}_L \tag{9}$$

When $N - D \geq L$, two well-known one-dimension search algorithms [9, 10] estimate the DOAs in the unknown mutual coupling by constructing an analogous MUSIC function:

$$\begin{aligned}
f(\phi) &= \mathbf{w}_o^H \mathbf{P}^{-1}(\phi)\, \mathbf{w}_o \\
\mathbf{c}_L(\phi) &= \frac{\mathbf{P}^{-1}(\phi)\, \mathbf{w}_o}{\mathbf{w}_o^H \mathbf{P}^{-1}(\phi)\, \mathbf{w}_o}
\end{aligned} \tag{10}$$

or

$$\begin{aligned}
f(\phi) &= \frac{1}{\det\{\mathbf{P}(\phi)\}} \quad \text{or} \quad \frac{1}{\lambda_{\min}\{\mathbf{P}(\phi)\}} \\
\mathbf{c}_L &= v_{\min}\{\mathbf{P}(\phi)\} \quad \text{and} \quad c_1 = 1
\end{aligned} \tag{11}$$

where $\mathbf{P}(\phi) = \mathbf{\Gamma}_L^H(\phi)\mathbf{E}_n\mathbf{E}_n^H\mathbf{\Gamma}_L(\phi)$ and $\mathbf{w}_o = [\ 1,\ \ 0, \dots 0\ ]^T$. $\det\{\bullet\}$ is the determinant of a matrix. $\lambda_{\min}\{\bullet\}$ is the smallest eigenvalue of a matrix and $v_{\min}\{\bullet\}$ is the corresponding eigenvector. Note that the Equation (10) is based on the following minimization problem

$$(\phi, \mathbf{c}_L) = \arg\min_{(\phi,\mathbf{c})} \mathbf{c}_L^H \mathbf{P}(\phi)\mathbf{c}_L, \quad s.t.\ \mathbf{c}_L^H \mathbf{w}_o = 1 \tag{12}$$

The implementation of one-dimension search results in higher computational load.

3. BEAMSPACE TRANSFORM IN THE PRESENCE OF MUTUAL COUPLING

Here, the beamspace transformation will be introduced. The beamspace steering vector in the presence of mutual coupling is given. These form the theoretical basis of our algorithm.

The nth $(n = 1, \dots N)$ component of the array steering vector $\mathbf{a}(\phi)$ is

$$[\mathbf{a}(\phi)]_n = e^{jkr\cos(\phi-\gamma_n)} = \sum_{m=-\infty}^{\infty} j^m J_m(kr)\, e^{jm(\phi-\gamma_n)} \tag{13}$$

where $\gamma_n = 2\pi(n-1)/N$. Since $J_m(kr)$ decays exponentially, we can assume that, for $m \gg kr$ the higher order Bessel functions are negligible. Therefore, the steering vector can be truncated by considering a finite number of modes. We assume that the truncated order is M. A rule of thumb for determining M is $M = \lceil kr \rceil$, with $\lceil\bullet\rceil$ the ceiling operation. Exciting the array with the weight vector $\mathbf{w}_k = 1/N[\ e^{-jk\gamma_1} \quad \dots \quad e^{-jk\gamma_N}\ ]^H$ results in

$$\mathbf{w}_k^H\mathbf{a}(\phi) = j^k J_k(kr)e^{jk\phi} + \sum_{q=1}^{\infty}\Big[j^{k+qN}J_{k+qN}(kr)e^{j(k+qN)\phi} + j^{k-qN}J_{k-qN}(kr)e^{j(k-qN)\phi}\Big] \tag{14}$$

The first term in (14) becomes dominant if $N \geq 2M + 1$. Then the beamspace steering vector is

$$\mathbf{a}_b(\theta, \phi) = \mathbf{W}^H\mathbf{a}(\phi) = \mathbf{T}(\phi)\mathbf{g} \tag{15}$$

where $\mathbf{W} = \sqrt{N} \begin{bmatrix} \mathbf{w}_{-M} & \dots & \mathbf{w}_0 & \dots & \mathbf{w}_M \end{bmatrix}$ and

$$\begin{aligned} z &= e^{j\phi} \\ \mathbf{T}(\phi) &= \begin{bmatrix} \mathbf{Q}(z) & 0_{M\times 1} \\ 0_{1\times M} & 1 \\ \mathbf{\Pi Q}(1/z) & 0_{M\times 1} \end{bmatrix} \in \mathbb{C}^{(2M+1)\times(M+1)} \\ \mathbf{Q}(z) &= \operatorname{diag}(z^{-M}\ z^{-M+1}\ \dots\ z^{-2}\ z^{-1}) \\ [\mathbf{g}]_m &= j^{M+1-m} J_{M+1-m}(kr), \quad m = 1, 2, \dots M+1 \end{aligned} \tag{16}$$

$\mathbf{\Pi}$ is the $M \times M$ anti-diagonal matrix.

It should notice that [4] :

$$\mathbf{C} = \sum_{n=1}^{K} c_n \mathbf{V}^n + \sum_{n=K+1}^{N} c_{N+2-n} \mathbf{V}^n \tag{17}$$

$\mathbf{V}^1 = \mathbf{I}$ and $\mathbf{V}^n (n > 1)$ is the $(n-1)$th power of the cyclic permutation operator given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{0}_{(N-1)\times 1} & \mathbf{I}_{(N-1)\times(N-1)} \\ 1 & \mathbf{0}_{1\times(N-1)} \end{bmatrix} \tag{18}$$

Rewrite Equation (2) as

$$\tilde{\mathbf{a}}(\phi) = \sum_{n=1}^{K} c_n \mathbf{V}^n \mathbf{a}(\phi) + \sum_{K+1}^{N} c_{N+2-n} \mathbf{V}^n \mathbf{a}(\phi) \tag{19}$$

Using $\mathbf{V}^n \mathbf{a}(\phi) = \mathbf{a}(\phi - 2\pi(n-1)/N)$ and Equation (15), yields

$$\mathbf{W}^H \mathbf{V}^n \mathbf{a}(\phi) = \mathbf{M}_n \mathbf{T}(\phi)\, \mathbf{g} \tag{20}$$

where $\mathbf{M}_n = \operatorname{diag}\left\{ e^{j2\pi M(n-1)/N} \dots 1 \dots e^{-j2\pi M(n-1)/N} \right\}$. Hence, the beamspace steering vector in the presence of mutual coupling can be expressed as

$$\tilde{\mathbf{a}}_b(\phi) = \mathbf{W}^H \tilde{\mathbf{a}}(\phi) = \mathbf{M}_s \mathbf{T}(\phi)\, \mathbf{g} \tag{21}$$

where $\mathbf{M}_s = \sum_{n=1}^{K} c_n \mathbf{M}_n + \sum_{n=K+1}^{N} c_{N+1-n} \mathbf{M}_n$. Because of the symmetry of the mutual coupling coefficients and the periodicity of $e^{j2\pi M(n-1)/N}$, it is easy to find that the diagonal elements of $\mathbf{M}_s$ is centro-symmetry. Hence, Equation (21) can be modeled as

$$\tilde{\mathbf{a}}_b(\theta, \phi) = \mathbf{T}(\phi)(\mathbf{m} \odot \mathbf{g}(\theta)) = \mathbf{T}\phi)\tilde{\mathbf{g}}(\theta) \tag{22}$$

where $\tilde{\mathbf{g}}(\theta) = \mathbf{m} \odot \mathbf{g}(\theta)$ and $\mathbf{m}$ is the first $M+1$ elements of the diagonal elements of $\mathbf{M}_s$. "$\odot$" denotes the Hadamard product of vectors.

4. NEW BLIND CALIBRATION METHOD AND DOA ESTIMATION

In this section, we will present a new DOA estimation algorithm in the presence of unknown mutual coupling. The algorithm includes two parts: DOA estimation and the elimination of the spurious estimates.

4.1. The Azimuth Angle Estimation

Recalling Equation (1), the corresponding beamspace array signal model is

$$\mathbf{x}_b(t) = \mathbf{W}^H \mathbf{x}(t) \tag{23}$$

The covariance matrix $\mathbf{R}_b$ of the beamspace data is given by

$$\mathbf{R}_b = E\left\{\mathbf{x}_b(t)\,\mathbf{x}_b^H(t)\right\} = \mathbf{W}^H \mathbf{R} \mathbf{W} \tag{24}$$

$\mathbf{R}_b$ can be partitioned as follows

$$\mathbf{R}_b = [\mathbf{B}_1 \mid \mathbf{B}_2] \tag{25}$$

where $\mathbf{B}_1$ and $\mathbf{B}_2$ are $(2M+1) \times D$ and $(2M+1) \times (2M+1-D)$ dimension matrices, respectively. For Propagator Method [13, 14], there is a linear operator $\mathbf{P}_r$ given by

$$\mathbf{P}_r = \left(\mathbf{B}_1^H \mathbf{B}_1\right)^{-1} \mathbf{B}_1^H \mathbf{B}_2 \tag{26}$$

Construct a matrix $\mathbf{E}_p$ as follows

$$\mathbf{E}_p = \begin{bmatrix} \mathbf{P}_r \\ -\mathbf{I}_{(2M+1-D)} \end{bmatrix} \tag{27}$$

According to the principle of Propagator Method, the following equation holds true

$$\tilde{\mathbf{a}}_b^H(\theta, \phi)\mathbf{E}_p = \mathbf{0} \tag{28}$$

Observe the Equation (26), it only requires the inversion of an $D \times D$ dimension matrix. Compared with the computation complexity $O((2M+1)^3)$ for eigenvalue decomposition, its computation complexity is $O(D^3)$.

Obviously, combining Equations (22) and (28), an analogous MUSIC spectrum function can be expressed as

$$f_{PA}(\phi) = \tilde{\mathbf{g}}^H \mathbf{T}^H(\phi)\,\mathbf{E}_p \mathbf{E}_p^H \mathbf{T}(\phi)\,\tilde{\mathbf{g}} \tag{29}$$

When $M \geq D$, this structure allows using a rank reduction algorithm, namely UCA-RARE [2]. Therefore, we can root the sample polynomial

$$P_1(z)\Big|_{|z|=1} = \det\left\{\mathbf{T}^T(1/z)\,\mathbf{E}_p \mathbf{E}_p^H \mathbf{T}(z)\right\} \tag{30}$$

and then find the signal azimuth angle from roots of (30), which are located closest to the unit circle.

The matrix $\mathbf{U} = \mathbf{T}^T(1/z)\,\mathbf{E}_p\mathbf{E}_p^H\mathbf{T}(z)$ can be partitioned into four parts with

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \\ \mathbf{U}_3 & \mathbf{U}_4 \end{bmatrix} \tag{31}$$

where $\mathbf{U}_1$, $\mathbf{U}_2$, $\mathbf{U}_3$ and $\mathbf{U}_4$ are $D\times D$, $D\times(M+1-D)$, $(M+1-D)\times D$ and $(M+1-D)\times(M+1-D)$ matrices, respectively. Due to the particularity of $\mathbf{E}_p$, the following equation always holds true:

$$\mathbf{U}_4 = \begin{pmatrix} 2 & 0 & \dots & 0 & 0 & 0 \\ 0 & 2 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 2 & 0 & 0 \\ 0 & 0 & \dots & 0 & 2 & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \tag{32}$$

Making use of a well known identity for block matrices [16]

$$\det\left\{\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}\right\} = \det\{\mathbf{D}\}\det\left\{\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right\} \tag{33}$$

which holds true for arbitrary matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and nonsingular matrix $\mathbf{D}$. Equation (31) can be given by

$$\begin{aligned} P_1(z)\big|_{|z|=1} &= \det\{\mathbf{U}\} \\ &= \det\{\mathbf{U}_4\}\det\left\{\mathbf{U}_1 - \mathbf{U}_2\mathbf{U}_4^{-1}\mathbf{U}_3\right\} \\ &= 2^{M-D}\det\left\{\mathbf{U}_1 - \mathbf{U}_2\mathbf{U}_4^{-1}\mathbf{U}_3\right\} \end{aligned} \tag{34}$$

Therefore, the roots for $P_1(z)$ are equivalent to these of

$$P_2(z)\big|_{|z|=1} = \det\left\{\mathbf{U}_1 - \mathbf{U}_2\mathbf{U}_4^{-1}\mathbf{U}_3\right\} \tag{35}$$

Notice that the matrix to compute the determinant in Equation (35) is $D \times D$ dimension while in Equation (30) it is $(M+1)\times(M+1)$ dimension. Thus, we prefer using Equation (35) to Equation (30) in order to further reduce the computation cost. Similar to the Root-MUSIC roots, RARE roots enjoy the so called conjugate reciprocity property. That is, if z_0 is a root of $P_2(z)$, then $\tilde{z}_0 = 1/z_0^*$ is also a root of $P_2(z)$. Therefore, besides the spurious estimates $(\phi_i+\phi_j)/2$ [2], there are spurious estimates $\phi_i+\pi$ for $\phi_i < \pi$, $\phi_i-\pi$ for $\phi_i > \pi$, $(\phi_i+\phi_j)/2+\pi$ for $(\phi_i+\phi_j)/2 < \pi$ and $(\phi_i+\phi_j)/2-\pi$ for $(\phi_i+\phi_j)/2 > \pi$.

4.2. The Solution to Eliminate the Spurious Estimate

Here, we only consider the DOA estimation for compact UCA, in which the distance between neighborhood elements is smaller than half-wavelength. Therefore, both algorithms presented in Section 2 also have spurious estimates. In [11], authors suggest adding a rank constraint and judge whether the estimated mutual coupling coefficients abiding the rule:

$$|c_1| > |c_2| > |c_3| > \ldots > |c_L| \tag{36}$$

They consider the DOAs whose corresponding estimated mutual coupling coefficients do not satisfy the Equation (36) as spurious DOAs. However, this method can not be guaranteed to cover all spurious estimates. In [12], authors advise to select the estimates with D biggest values of $f(\phi) = \mathbf{w}_o^H \mathbf{P}^{-1}(\phi)\mathbf{w}_o$ as true estimates or choose the estimates with same estimate mutual coupling coefficients as the real estimates. Whereas, sometimes $f(\phi)$ for the spurious estimates may be larger than it for real estimates. In this paper, we will present another method to eliminate the spurious estimates.

Just as described in Equation (10), for each estimated ϕ, we can compute the estimated mutual coupling coefficients as

$$\mathbf{c}_L(\phi) = \frac{\mathbf{F}^{-1}(\phi)\,\mathbf{w}_o}{\mathbf{w}_o^H \mathbf{F}^{-1}(\phi)\,\mathbf{w}_o} \tag{37}$$

This equation is the optimal solution to the following constraint quadratic minimization problem

$$(\phi, c_L) = \arg\min_{(\phi,c)} \mathbf{c}_L^H \mathbf{F}(\phi)\mathbf{c}_L, \quad s.t.\mathbf{c}_L^H \mathbf{w}_o = 1 \tag{38}$$

where $\mathbf{F}(\phi) = \mathbf{\Gamma}_L^H(\phi)\mathbf{G}_p\mathbf{G}_p^H\mathbf{\Gamma}_L(\phi)$ and $\mathbf{G}_p = [\mathbf{P}^T \mid -\mathbf{I}_{(N-D)}]^T$, $\mathbf{P} = (\mathbf{D}_1^H\mathbf{D}_1)^{-1}\mathbf{D}_1^H\mathbf{D}_2$. The $N \times D$ dimension matrix $\mathbf{D}_1$ and $N \times (N-D)$ dimension matrix $\mathbf{D}_2$ partition the matrix $\mathbf{R}$ as $\mathbf{R} = [\mathbf{D}_1 \mid \mathbf{D}_2]$.

There are four classes of estimated results for our proposed algorithm. We label them as $\mathbf{\Phi}_1 = (\ \psi_1 \ldots \psi_D\ )$, $\mathbf{\Phi}_2 = (\ \alpha_1 \ \ldots \ \alpha_{D_s}\ )$, $\mathbf{\Phi}_3 = (\ \varphi_1 \ \ldots \ \varphi_D\ )$ and $\mathbf{\Phi}_4 = (\ \beta_1 \ \ldots \ \beta_{D_s}\ )$, where $D_s = D(D-1)/2$. $\mathbf{\Phi}_1$ is the gather of the real DOAs $\psi_i\,(i = 1, 2, \ldots, D)$ and $\alpha_t = (\psi_i + \psi_j)/2(t = 1, 2, \ldots, D_s)$, $\varphi_i = \psi_i + \pi$ for $\psi_i < \pi$ or $\varphi_i = \psi_i - \pi$ for $\psi_i > \pi\,(i = 1, 2, \ldots, D)$, $\beta_t = \alpha_t + \pi$ for $\alpha_t < \pi$ or $\beta_t = \alpha_t - \pi$ for $\alpha_t > \pi(t = 1, 2, \ldots, D_s)$. A lot of simulations have led to a rule of thumb that the estimated mutual coupling coefficients corresponding to $\mathbf{\Phi}_3$ and $\mathbf{\Phi}_4$ do not comply with the rule in Equation (36). Therefore, we can using this rule to remove the spurious estimates $\mathbf{\Phi}_3$ and $\mathbf{\Phi}_4$. In fact, we only need to compute the absolute values of the last two elements of each

$\mathbf{c}_L(\phi)$ and compare them with each other and 1. Then, the spurious estimates can be fixed on. Thereafter, we compare the rest estimated mutual coupling coefficients corresponding to $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ and select the estimates with nearly same $\mathbf{c}_L(\phi)$ as the true estimates. Compared with the second method proposed in [12], which compares all estimated mutual coupling coefficients to find the nearly same mutual coupling coefficients $\mathbf{c}_L(\phi)$, our method reduce the computation time.

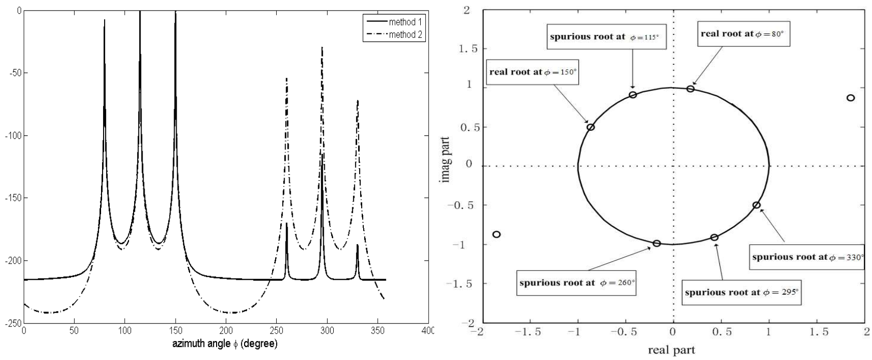
The steps involved in the proposed algorithm can be summarized below:

1) Compute the sample covariance matrix $\hat{\mathbf{R}} = (1/Q)\sum_{q=1}^{Q}\mathbf{x}(q)\mathbf{x}^H(q)$ by averaging over Q data snapshots. Compute the beamspace covariance matrix $\hat{\mathbf{R}}_b = \mathbf{W}^H\hat{\mathbf{R}}\mathbf{W}$.
2) Construct matrices $\hat{\mathbf{E}}_p$ from $\hat{\mathbf{R}}_\mathbf{b}$ and $\hat{\mathbf{G}}_p$ from $\hat{\mathbf{R}}$, respectively.
3) Obtain the azimuth angle estimates with Equation (35).
4) Eliminate the spurious estimates.

Although we construct two matrices $\mathbf{E}_p$ and $\mathbf{G}_p$ which span the noise space in beamspace and element-space, respectively, their computation complexity is $O(D^3)$ and it is smaller than the computation complexity $O(N^3)$ for eigenvalue decomposition in element-space. Moreover, the matrix $\mathbf{P}(\phi)$, used in [9] to compute the inverse matrix and in [10] to calculate the determinant or to perform the eigenvalue decomposition, is $N \times N$ dimension, while the matrix to calculate the determinant in our proposed algorithm is $D \times D$ dimension. Therefore, the computation cost is greatly reduced. Finally, we use rooting a polynomial instead of the implementation of one-dimension search. Thus, the computation burden is further decreased.

5. SIMULATIONS

In this example, we will show how our proposed algorithm works. A UCA of radius $r = 0.5\lambda$ with $N = 11$ is employed. The signals and noise in our simulations are assumed to be stationary, zero mean, and uncorrelated Gaussian random processes. Noise is both spatially and temporally white. The mutual coupling vector is $\mathbf{c} = [1, 0.6237 + j0.3875,\ 0.3658 + j0.2316,\ 0.1643 + j0.1089,\ 0.0978 + j0.0147,\ 0.0135 - j0.0087]^T$. The truncated order is $M = \lceil kr \rceil = 4$. Two signals arrive at the array from directions $\phi_1 = 80^\circ$ and $\phi_2 = 150^\circ$, respectively. The $SNR = 20\,\mathrm{dB}$ is quoted per source per array element. Fig. 1(a) shows the spatial



(a) (b)

Figure 1. (a) The special spectrum. (b) The roots of our proposed algorithm.

spectra for the algorithms presented in Section 2. Here we label the algorithm using Equation (10) as method 1 and the one using Equation (11) as method 2. There are six peak values located at azimuth $\phi = (\ 80^\circ \quad 115^\circ \quad 150^\circ \quad 260^\circ \quad 295^\circ \quad 330^\circ\ )$. Apparently, the peak value in spurious estimate $\phi = (80^\circ + 150^\circ)/2 = 115^\circ$ is larger than it for the real estimate $\phi = 80^\circ$. Therefore, we may eliminate the real DOA $\phi = 80^\circ$ if we select the estimates with D biggest values of $f(\phi) = \mathbf{w}_o^H \mathbf{P}^{-1}(\phi)\mathbf{w}_o$ as true estimates. Fig. 1(b) presents the roots obtained by our proposed algorithm. Table 1 is the mean value of estimated mutual coupling coefficients $\hat{\mathbf{c}}$ for 300 independent experiments. It is clear that the estimated mutual coupling coefficients corresponding to $\phi = (\ 260^\circ \quad 295^\circ \quad 330^\circ\ )$ do not satisfy the rule in Equation (36). Although the estimated mutual coupling coefficients corresponding to spurious estimate $\phi = 115^\circ$ comply with the rule in Equation (36), it is quite different from the estimated mutual coupling coefficients corresponding to $\phi = 80^\circ$ and $\phi = 150^\circ$. However, the discrepancy between the estimated mutual coupling coefficients corresponding to $\phi = 80^\circ$ and $\phi = 150^\circ$ is quite small. Thus, we consider these two DOAs as real DOAs.

The second example shows the performance of the algorithms for different SNR levels. A UCA of radius $r = 0.7\lambda$ with $N = 13$ is employed. The mutual coupling vector is $\mathbf{c} = [1, 0.6278 - j0.3974, 0.4943 + j0.2659, 0.4039 + j0.1563, 0.3045 - j0.0963, 0.1278 + j0.1470, 0.0889 + j0.0412]^T$ and truncated order is $M = 5$. Two signals arrive at the array from directions $\phi_1 = 20^\circ$ and $\phi_2 = 100^\circ$, respectively. The SNR level is varied from 0 dB to 40 dB and is quoted per source per array element. The results are based on 300 independent experiments. Note that the solution to eliminate the spurious estimates for the method 1 and the method 2 employs the one proposed in our paper. The root-mean-square-error (RMSE) plots of estimate DOAs for different algorithms and Cramer-Rao bound (CRB) are shown in Fig. 2. Apparently, the estimated RMSEs for all algorithms followed the trend of the CRB. It shows that all algorithms can achieve high resolution without knowing the mutual coupling. The angle search step for the two search methods is 0.1°. The performances for the two search methods are nearly the same for all SNR levels. However, they are a little inferior to the performance for the proposed algorithm when the SNR level is smaller than 30 dB. In fact, the estimated RMSEs for all algorithms are quite close to each other. Note that compared with other two search algorithms, the proposed algorithm achieves comparable performance with low computation cost (see the discussion of the computation cost in Section 4).

In the last example, we demonstrate the performance of the proposed algorithm with angle resolution. A UCA of radius $r = \lambda$ with $N = 19$ is considered. The mutual coupling vector is $\mathbf{c} = [1, 0.7286 + j0.5473,\ 0.6435 + j0.4877,\ 0.4768 + j0.4126,\ 0.3687 + j0.2385,\ 0.2513 - j0.2086,\ 0.1475 + j0.1389,\ 0.1013 + j0.0872,\ 0.0946 + j0.0151,\ 0.0045 + j0.0086]^T$ and truncated order is $M = 7$. The $SNR = 20\,\text{dB}$ is quoted per source per array element. Two signals arrive at the array from directions $\phi_1 = 200°$ and $\phi_2 = 200° + \delta$, respectively. The azimuth angle of the second source is varied as δincreases from 4° to 50° in steps of 2°. For each angle separation, all algorithms are applied to

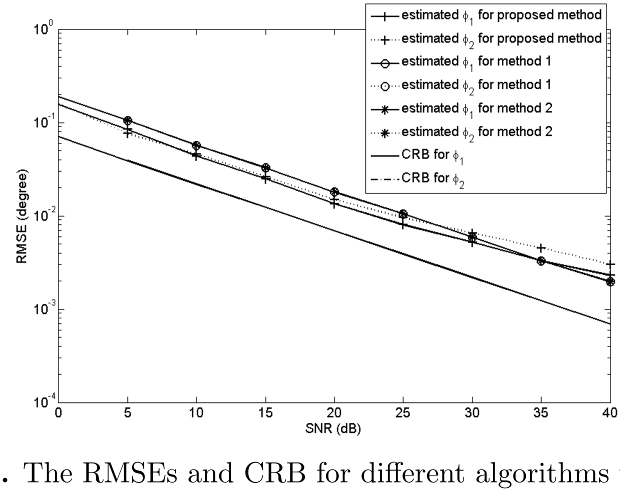


Figure 2. The RMSEs and CRB for different algorithms versus SNR level.

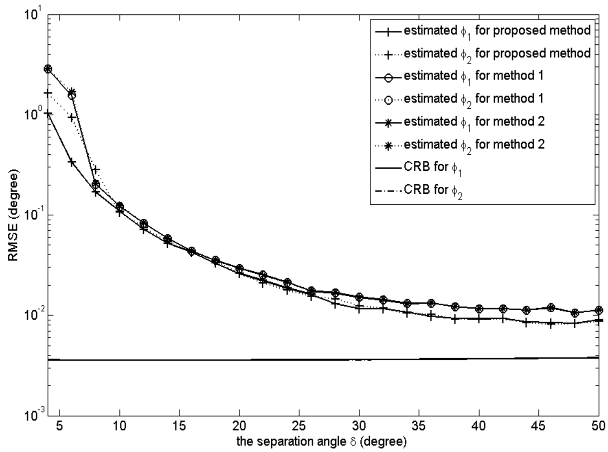


Figure 3. The RMSEs and CRB for different algorithms versus the separation angle δ.

Table 1. The mean value of the estimated mutual coupling coefficients for all estimated azimuth angle.

	80°	230°	115°	195°	150°	330°
c_1	$0.6250 + j0.3960$	$-0.8396 - j0.9093$	$0.8589 - j0.4127$	$-0.4842 + j1.6940$	$0.6283 + j0.3832$	$-0.6207 - j0.8431$
c_2	$0.3619 + j0.2314$	$2.6127 - j1.3242$	$0.6493 - j0.0468$	$0.7570 + j1.9350$	$0.3654 + j0.2327$	$2.7121 - j0.8524$
c_3	$0.1671 + j0.1109$	$2.8398 - j3.2022$	$0.3759 - j0.0363$	$-1.2447 + j5.7813$	$0.1669 + j0.1074$	$3.2576 - j2.3763$
c_4	$0.0950 + j0.0156$	$8.9051 - j4.4197$	$0.1453 - j0.0132$	$2.6807 + j7.8118$	$0.0967 + j0.0159$	$9.1629 - j2.5410$
c_5	$0.0145 - j0.0095$	$7.2758 - j12.0571$	$0.0736 - j0.0400$	$0.8151 + j10.1235$	$0.0142 - j0.0084$	$9.0846 - j9.9107$

obtain the DOA estimates of the two impinging signals. The results are computed from 300 independent trials. The estimated RMSEs and CRB for different algorithms are shown in Fig. 3. For all algorithms, the estimated RMSEs decrease as the separation angle δ increases. The performances for the two search methods are nearly the same whatever value the separation angle δ is. However, they are inferior to the performance for the proposed algorithm when the separation angle δ is larger than 28°. Actually, the performances of all algorithms are quite close to each other when the separation angle δ is larger than 8°. As we know, there is the spurious estimate $(\phi_1+\phi_2)/2$, which is quite close to the real DOAs ϕ_1 and ϕ_2 when the separation angle is small. Thus, all algorithms work worse then.

6. CONCLUSION

In this paper, we propose a fast DOA estimation algorithm for UCAs in the unknown mutual coupling. Based on the beamspace transformation and RARE, the azimuth estimates can be obtained without the knowledge of the mutual coupling. The PM is applied here to avoid the eigenvalue decomposition. A post-process of PM is introduced to reduce the dimension of the matrix used to calculate the determinant in RARE. Furthermore, we use the polynomial rooting instead of the implementation of one-dimension search. Thus, the computation burden is greatly reduced. Using the special structure of mutual coupling matrix and the characteristic of mutual coupling coefficients, an amended solution to eliminate the spurious estimates is presented. Simulation results demonstrate that the method is correct and effective.

REFERENCES

1. Mathews, C. P. and M. D. Zoltowski, "Eigenstructure techniques for 2-D angle estimation with uniform circular arrays," *IEEE Trans. Signal Process.*, Vol. 42, No. 9, 2395–2407, 1994.
2. Pesaventoand, M. and J. F. Böhme, "Direction of arrival estimation in uniform circular arrays composed of directional elements," *Proc. Sensor Array and Multichannel Signal Processing Workshop*, 503–507, Aug. 2002.
3. Goossens, R., H. Rogier, and S. Werbrouck, "UCA Root-MUSIC with sparse uniform circular arrays," *IEEE Trans. Signal Process.*, Vol. 56, 4095–4099, 2008.
4. Mati, W. and S. Jacob, "Direction finding of coherent signals via spatial smoothing for uniform circular arrays," *IEEE Trans. Antennas Propag.*, Vol. 42, No. 5, 613–620, 1994.
5. Jakobsson, A. and P. Stoica, "On the forward-backward spatial APES," *Signal Processing*, Vol. 86, 710–715, 2006.
6. Goossens, R. and H. Rogier, "A hybrid UCA-RARE/Root-MUSIC approach for 2-D direction of arrival estimation in uniform circular arrays in the presence of mutual coupling," *IEEE Trans. Antennas Propag.*, Vol. 43, 841–849, 2007.

7. Buhong, W., H. Hontat, and L. Mookseng, "Decoupled 2D direction of arrival estination using compact uniform circular arrays in the presence of elevation-dependent mutual coupling," *IEEE Trans. Antennas Propag.*, Vol. 58, No. 3, 747–755, 2010.
8. Zhang, T. T., Y. L. Lu, and H. T. Hui, "Compensation for the mutual coupling effect in uniform circular arrays for 2D DOA estimations employing the maximum likelihood technique," *IEEE Trans. Aerosp. Electron. Sys.*, Vol. 44, No. 3, 1215–1221, 2008.
9. QI, C., Y. Wang, Y. Zhang, et al., "DOA estimation and self-calibration algorithm for uniform circular array," *Electronics Letters*, Vol. 41, No. 20, 1092–1094, 2005.
10. Lin, M. and L. X. Yang, "Blind calibration and DOA estimation with uniform circular arrays in the presence of mutual coupling," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, 315–318, 2006.
11. Gao, D. Y., B. Wang, and Y. Guo, "Comments on 'blind calibration and DOA estimation with uniform circular arrays in the presence of mutual coupling'," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, 566–569, 2006.
12. Lin, M. and L. X. Yang, "Reply to the comments on 'blind calibration and DOA estimation with uniform circular arrays in the presence of mutual coupling'," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, 568–569, 2006.
13. Marcos, S., A. Marsal, and M. Benidir, "Propagator method for source bearing estimation," *Signal Processing*, Vol. 42, No. 2, 121–138, 1995.
14. Marcos, S. and M. Benidir, "On a high resolution array processing method non-based on the eigenanlysis approach," *1990 International Conference on Acoustics, Speech, and Signal Processing*, Vol. 5, 2955–2958, 1990.
15. Friedlander, B. and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Trans. Antennas Propag.*, Vol. 39, 273–284, 1991.
16. Lükepohl, H., *Handbook of Matrices*, Wiley, Chichester, New York, 1996.