

## A NEW THREE-DIMENSIONAL CONICAL GROUND-PLANE CLOAK WITH HOMOGENEOUS MATERIALS

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**Abstract**—A new three dimensional conical ground plane electromagnetic cloak is proposed and designed based on the coordinate transformation of Maxwell's equations. Material parameters of the conical invisible cloak are derived which have simple form and lesser inhomogeneity compared with other 3-dimensional cloaks. Because of convenient form of the constitutive tensors of the conical cloak, we propose a new strategy for homogeneous approximation of the materials of the cloaks. Numerical simulations confirm that approximation with eight slices, is more than enough and this cloak can hide any object on the ground as well as inhomogeneous ones.

### 1. INTRODUCTION

Invisibility has been one of the most attractive topics for human at all times. Recently, Pendry et al. [1] have proposed an applicable method for controlling electromagnetic fields, which uses a coating made of metamaterials that can bend incident electromagnetic field and inhibits it from penetrating to cloaked region, as the electromagnetic fields outside the cloaked region remain unaltered. This theory is based on the form invariant of Maxwell's equations in coordinate transformation. Then, Schurig and co-workers proposed a practical cloak at microwave frequency [2]. Different mechanisms for cloaking structure have been proposed such as coordinate transformation, anomalous localized resonances [3], scattering cancellation including layered absorbing [4–6] or plasmonic shells [7] and transmission line based cloaking [8]. Two kind of coordinate transformation is used to design invisibility cloaks: point transformation (3-D cloaks), which transform a point into a

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cloaked region; and line transformation (2-D cloaks), which transform a line within a cloaked region.

For line transformed cloak, one tangential component of the material parameters at the inner boundary has a singularity. Based on the coordinate transformation method various cloaking shapes have been proposed, such as cross section case (2-D) circular cylinders [9–11], squares [12], elliptic [13], tessellated and stellated [14], arbitrary N-sided regular polygonal [15], trapeziform [16], and 3-D ellipsoidal, toroids [17], and arbitrary shape cloaks [18–22].

In addition to the cloaking of objects in the free space, it is also interesting to examine a more useful case, the half-space case that is to cloak objects which are usually supported by a half-space ground [23–25]. The design aim of such a cloak is to reduce the abnormal scattering from the objects on the ground plane, residue the total reflection similar to the PEC or dielectric ground [26].

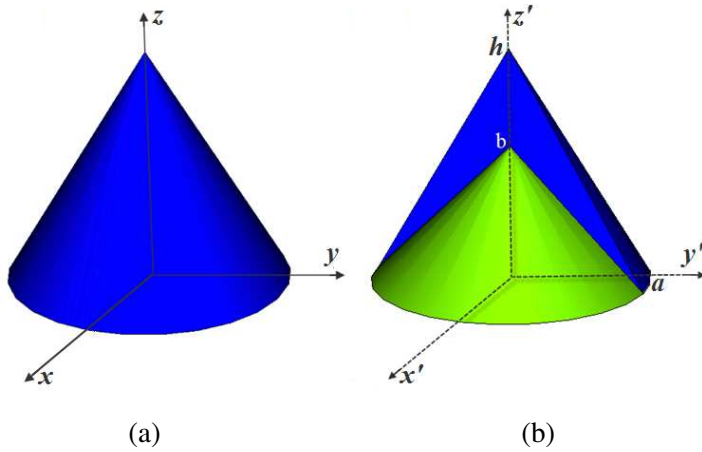
Nowadays, more research on the cloaking shapes focuses on the 2-D (radially symmetric) cases. In this paper, we propose a 3-D cloaking structure that can hide an object on the ground (dielectric-half space). First, we apply coordinate transformation method to derive the material parameters of the conical cloak. Second, electric field distribution in the vicinity of the half-space conical cloak under both incident plane wave and spherical wave with numerical simulation is obtained to verify the cloaking efficacy. Because of the simplicity in the form of the conical cloak parameter tensors we propose a kind of homogeneous approximation of the conical cloak's materials. Finally, backward scattering of the conical cloak with inhomogeneous and approximated homogeneous materials is compared that shows the efficiency of the homogeneous approximated conical cloak is very high.

## 2. THEORY ANALYSIS

Figure 1(a) shows the cone geometry in the original space  $(x, y, z)$ , which  $a$  and  $h$  are the base cone radius and height of the conical cloak, respectively. In Figure 1(b), the cone in the transformed space  $(x', y', z')$  is demonstrated, where  $b$  is the cloaked object height. The geometric transformation that compress the original cone space into the transformed space is

$$x' = x, \quad y' = y, \quad z' = \frac{h-b}{h}z - \frac{b}{a} \left( \sqrt{x^2 + y^2} - a \right). \quad (1)$$

According to the form invariance property of Maxwell's equations during the transformation, the permittivity and permeability tensors



**Figure 1.** The scheme of coordinate transformation from (a) original space to (b) transformed space.

of the medium in the transformed space become

$$\begin{aligned}\varepsilon' &= A\varepsilon A^T / \det(A), \\ \mu' &= A\mu A^T / \det(A),\end{aligned}\quad (2)$$

which  $A_{ij} = \partial x'_i / \partial x_j$  is the Jacobian matrix between the transformed coordinate and the original coordinate.  $\varepsilon$ ,  $\mu$  and  $\varepsilon'$ ,  $\mu'$  represent the permittivity and permeability of the original and transformed region, respectively. In our study, the original space is free space:  $\varepsilon = I\varepsilon$  and  $\mu = I\mu$ . Now, we can obtain the permittivity and permeability tensors of the cloaking region

$$\varepsilon' = \mu' = \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}, \quad (3)$$

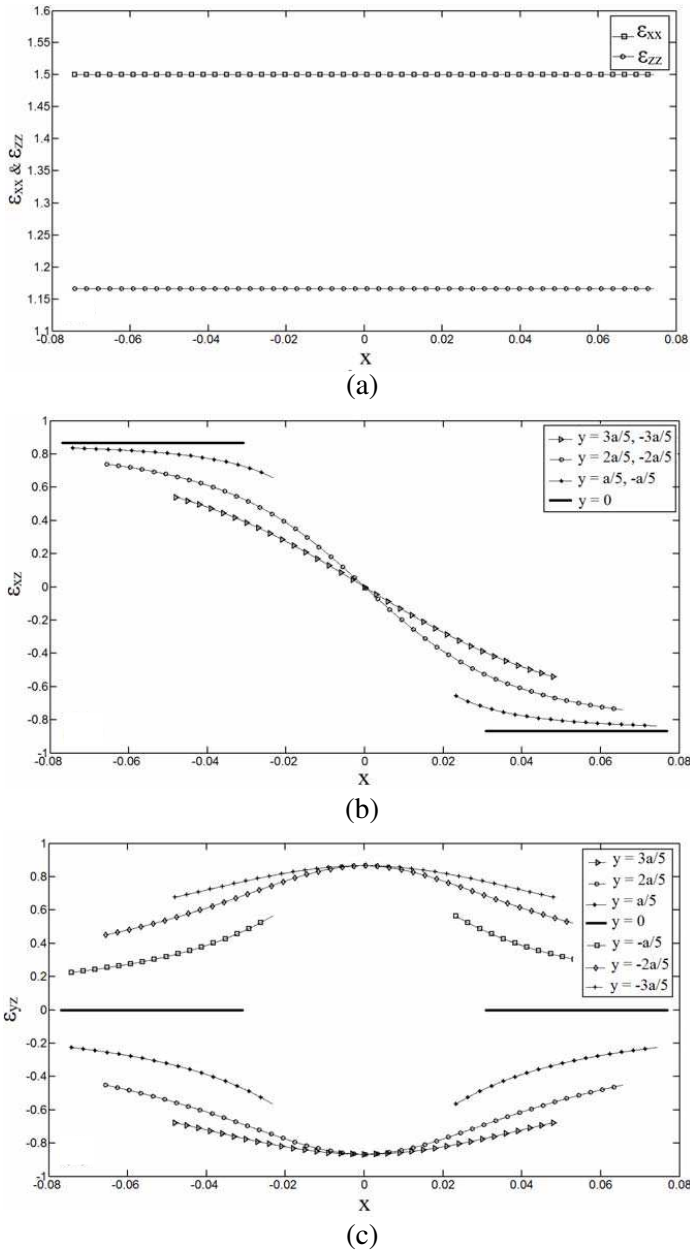
In which

$$\varepsilon_{xx} = \varepsilon_{yy} = \frac{h}{h-b}, \quad (4a)$$

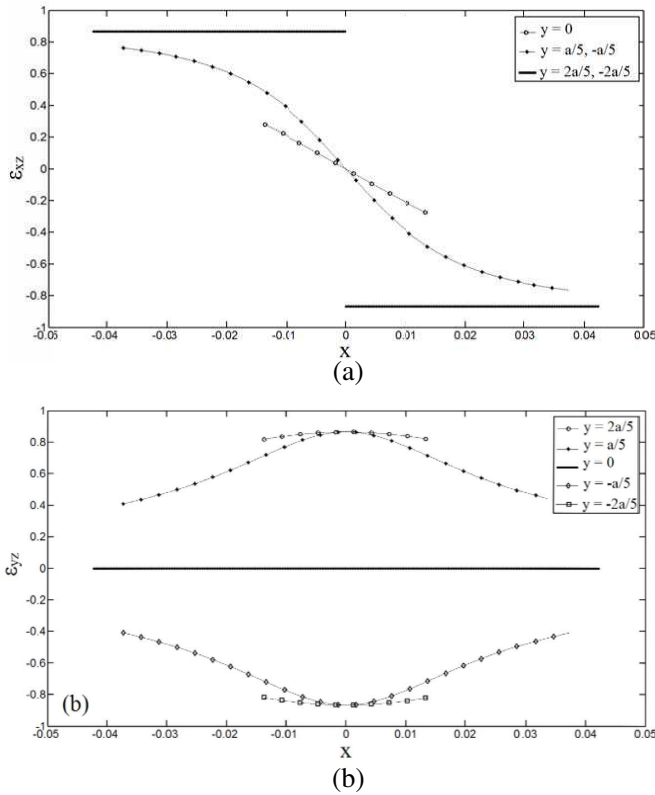
$$\varepsilon_{xz} = \varepsilon_{zx} = \frac{-bh}{a(h-b)} \cdot \frac{x}{\sqrt{x^2 + y^2}}, \quad (4b)$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{-bh}{a(h-b)} \cdot \frac{y}{\sqrt{x^2 + y^2}}, \quad (4c)$$

$$\varepsilon_{zz} = \frac{b^2h}{a^2(h-b)} + \frac{h-b}{h}. \quad (4d)$$



**Figure 2.** Material parameters of the conical cloak for cut-plane in  $z = h/4$ , (a)  $\epsilon_{xx}$  and  $\epsilon_{zz}$ , (b)  $\epsilon_{xz}$ , (c)  $\epsilon_{yz}$ .



**Figure 3.** Material parameters of the conical cloak for cut-plane in  $z = 3h/5$ , (a)  $\epsilon_{xz}$  (b)  $\epsilon_{yz}$  ( $\epsilon_{xx}$  and  $\epsilon_{zz}$  are same as Figure 4(a)).

Equations (4a)–(4d) give us the full parameters of the permittivity and permeability tensors for the conical cloaks. Figures 2 and 3 give better perception for variation of conical cloak material parameters (for  $h = 3b$  and  $a = \sqrt{3}b$  and different cut plane in  $z$  axis) versus space variations.

The wave impedance of the dielectric half space is

$$\eta_d = \eta_0 \sqrt{\frac{\mu_d}{\epsilon_d}}, \quad (5)$$

where  $\epsilon_d$  and  $\mu_d$  represent the relative permittivity and permeability of the dielectric half-space, respectively. when a plane wave obliquely incident on nonmagnetic ( $\mu_d = 1$ ) dielectric half-space, makes an angle of  $\theta_i$  with  $z$  axis, the reflected wave makes an angle of  $\theta_r$  with the  $z$  axis, and the transmitted wave makes an angle of  $\theta_t$  with the negative

z axis.

The  $x^o$  and  $z^o$  axes are orthogonal, the  $x^o$  axis oriented along the incident wave. It is seen that

$$x^o = x \sin \theta_i + z \cos \theta_i \quad (6)$$

and that a unit vector in the  $z'$  direction can be expressed as

$$\hat{z}^o = -\hat{x} \cos \theta_i + \hat{z} \sin \theta_i \quad (7)$$

$$E_i = \hat{y} E_0 \exp[i\beta_1(x \sin \theta_i + z \cos \theta_i)] \exp(-i\omega t) \quad (8)$$

The reflected wave can be expressed as

$$E_r = \hat{y} \rho E_0 \exp[i\beta_1(x \sin \theta_r - z \cos \theta_r)] \exp(-i\omega t) \quad (9)$$

where  $\rho$  is the reflection coefficient

$$\rho = \frac{\cos \theta_i - \sqrt{\varepsilon_d - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_d - \sin^2 \theta_i}} \quad (10)$$

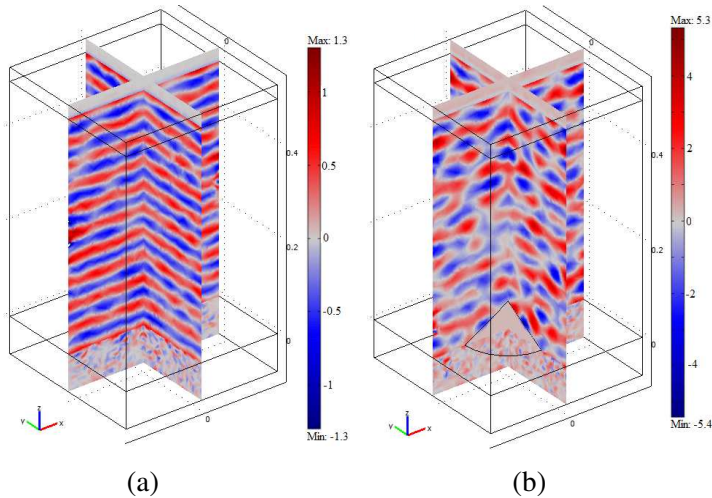
We carried out full-wave simulation of the conical cloak with constitutive tensors to approve the (4a)–(4d) designed formulate.

### 3. NUMERICAL SIMULATIONS AND DISCUSSION

Several Full-wave simulations are performed to confirm the validity of the efficiency of the cloak. The commercial software COMSOL Multiphysics based on the finite-element method is used to simulate the 3-D conical electromagnetic cloak. In the simulations the cloaking materials are assumed to be lossless and the interior region and internal boundaries of the cloak set as perfect electric conductor (PEC) that assuring the fields inside it are zero, and also showing the cloaking performance. Perfectly matched layers (PMLs) are applied at the top of the computational domain to reduce the backscattering at this boundary. Bottom layer of the computational domain is dielectric layer. We assume the dielectric half-space is nonmagnetic  $\mu_d = 1$  and its dielectric constant  $\varepsilon_d = 10$  that is the typical dielectric constant of the earth at microwave frequencies. In the first case, we consider the plane wave irradiation at frequency  $f_0 = 6$  GHz.

Setting  $a = 0.1$  m,  $b = 0.1/\sqrt{3}$  m and  $h = \sqrt{0.03}$  m, material parameters of the cloak are extracted from (4a)–(4d) which are:  $\varepsilon_{xx} = \varepsilon_{yy} = 1.5$ ,  $\varepsilon_{xy} = \varepsilon_{yx} = 0$ ,  $\varepsilon_{xz} = \varepsilon_{zx} = -0.866x/\sqrt{x^2 + y^2}$ ,  $\varepsilon_{yz} = \varepsilon_{zy} = -0.866y/\sqrt{x^2 + y^2}$ ,  $\varepsilon_{zz} = 1.1667$  and  $\mu = \varepsilon$ . Figure 4(a), shows the electric field distribution of the computational domain under time harmonic plane wave incident from air to dielectric half-space. Figure 4(b) shows the electric field distribution when a cone shaped

object with PEC boundaries is placed on a dielectric half-space. It can be seen that the total reflection coefficient of dielectric half-space (Figure 4(a)) is very small in comparing with a condition that an object is placed on a dielectric half space (Figure 4(b)).



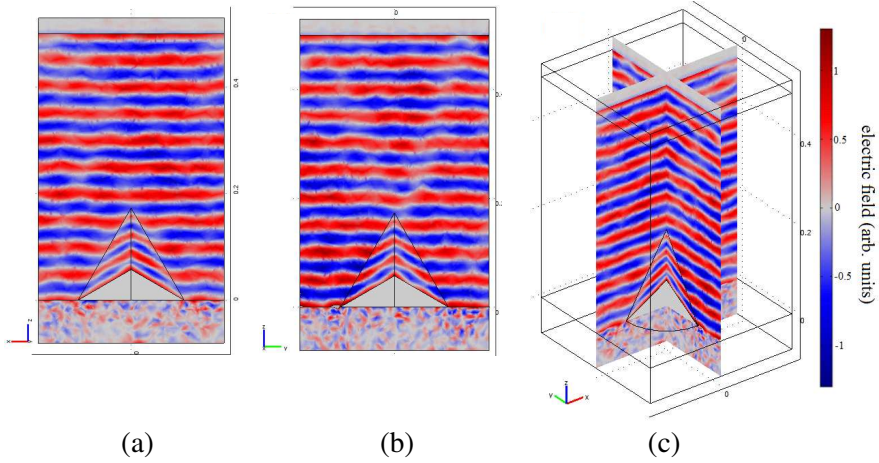
**Figure 4.** The electric field distribution in the computational domain when (a) wave incident from air on the dielectric half-space, (b) cone shaped object is placed on the dielectric half-space.

Then, the conical cloak is applied to hide the PEC object in Figure 5. In the computational domain, The inner boundary of the cloak is PEC boundary condition and the outer boundary of the cloak is continuous boundary conditions. The time harmonic plane wave similar to previous case is incident from top to bottom. The total reflection coefficient is similar to Figure 4(a) and incident wave is planar in the whole of the propagating path. The wave doesn't enter through the inner region of the conical cloak and object inside it can't be detected by the observer in air.

In the next case, the conical cloak is considered under spherical wave irradiation. The result is shown in Figure 6 The point source is located at  $(0, 0, 0.43)$ .

The inconsiderable scattered waves in Figure 6(a) and Figure 6(b) clearly verifies the invisibility of the whole system. Hence, conical cloak doesn't change the incident wave form and can hide any object from detection.

The electromagnetic wave propagates nearby the object which is protected by the conical cloak with some scattering. By decreasing the mesh size in the computation procedure, one can reduce the scattering



**Figure 5.** The electric field distribution of conical cloak under an incident plane wave (a) The electric field in the  $x$ - $z$  plane, (b) The electric field in the  $y$ - $z$  plane, (c) in two orthogonal planes when the plane wave is incident along  $z$  direction.

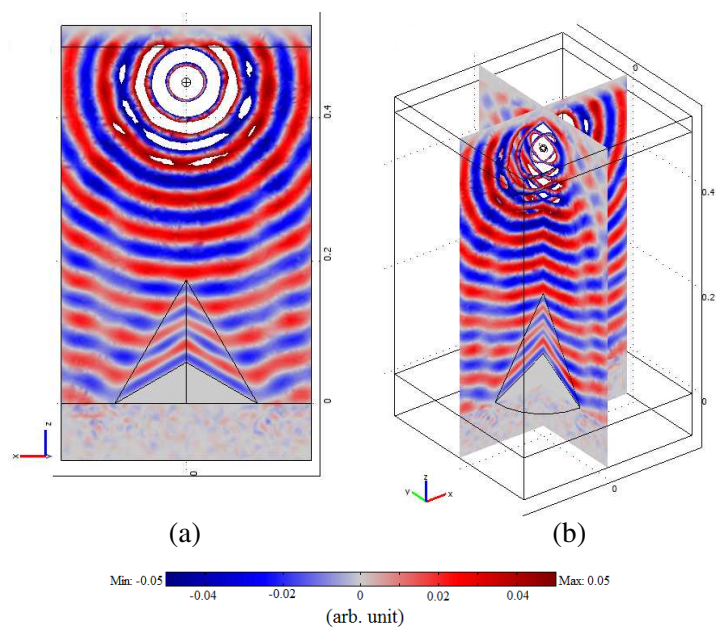
of the cloak that requires further computer memory.

In the next case study, we consider homogeneous approximation of the conical cloak. When, the parameters of permittivity and permeability are just homogeneously anisotropic and are not equal to zero, that could be realized easily in actual applications. Since the three parameter tensors in (4a)–(4d) equations and two zero tensors in (3), totally five parameter tensors are constant, we can approximate the other four tensors and investigate the cloak efficiency. In this way, we investigate the division of the conical cloak in  $N$  slices that whose parameters (4b) and (4c) tensors are change stepwise. Figure 7 shows stepwise approximation of the conical cloak.

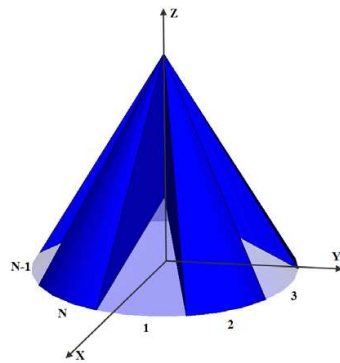
In this step, we investigate the efficiency of the cloak which divided in eight equal slices with constant parameters. Material parameters of the approximated conical cloak are:  $\varepsilon_{xx} = \varepsilon_{yy} = 1.5$ ,  $\varepsilon_{xy} = \varepsilon_{yx} = 0$ ,  $\varepsilon_{xz}^4 = \varepsilon_{xz}^5 = -\varepsilon_{xz}^1 = -\varepsilon_{xz}^8 = 0.804$ ,  $\varepsilon_{xz}^3 = \varepsilon_{xz}^6 = -\varepsilon_{xz}^2 = -\varepsilon_{xz}^7 = 0.321$ ,  $\varepsilon_{zx}^n = \varepsilon_{xz}^n$  and  $\varepsilon_{yz}^5 = \varepsilon_{yz}^7 = -\varepsilon_{yz}^2 = -\varepsilon_{yz}^3 = 0.804$ ,  $\varepsilon_{yz}^6 = \varepsilon_{yz}^8 = -\varepsilon_{yz}^4 = -\varepsilon_{yz}^1 = 0.321$ ,  $\varepsilon_{zy}^n = \varepsilon_{yz}^n$ ,  $\varepsilon_{zz} = 1.1667$  and  $\mu = \varepsilon$ . Figure 8 shows the electric distribution in the vicinity of the eight slices approximated conical cloak. One sees clearly in Figure 8 that total scattering from cloaked region is very small. Figure 9 shows comparison between the reflection coefficient  $S_{11}$  of the conical cloak with inhomogeneous material and eight homogeneous material equal slices with stepwise approximation.



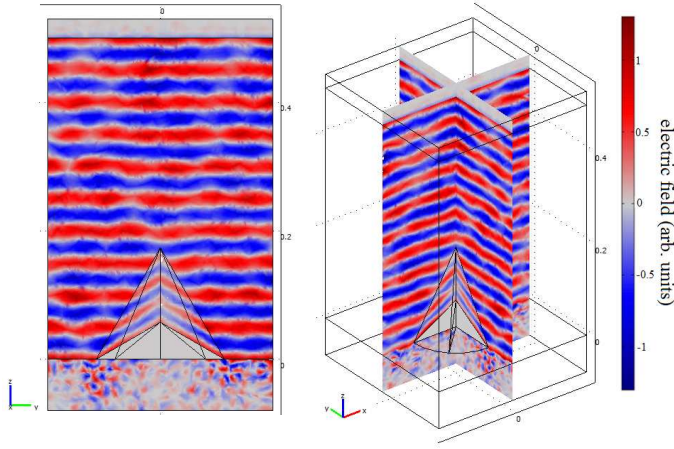
In real applications, artificial metamaterials are lossy. Hence, it is significant to consider the effect of loss in ground plane conical cloak on the invisibility and backscattering efficacy. We investigate the efficacy of the cloak when it's material has loss tangent of 0.01 and 0.1 in both permittivity and permeability that can be easily realized by real



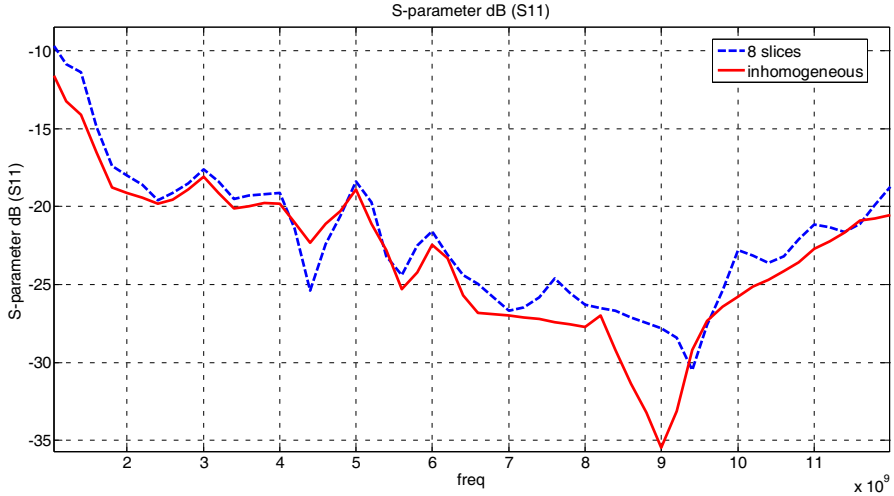
**Figure 6.** The electric field distribution under an spherical wave irradiation, when object is hide under conical cloak (a)  $x$ - $z$  plane. (b) in two orthogonal planes.



**Figure 7.** Construction of the homogeneous approximated conical cloak.



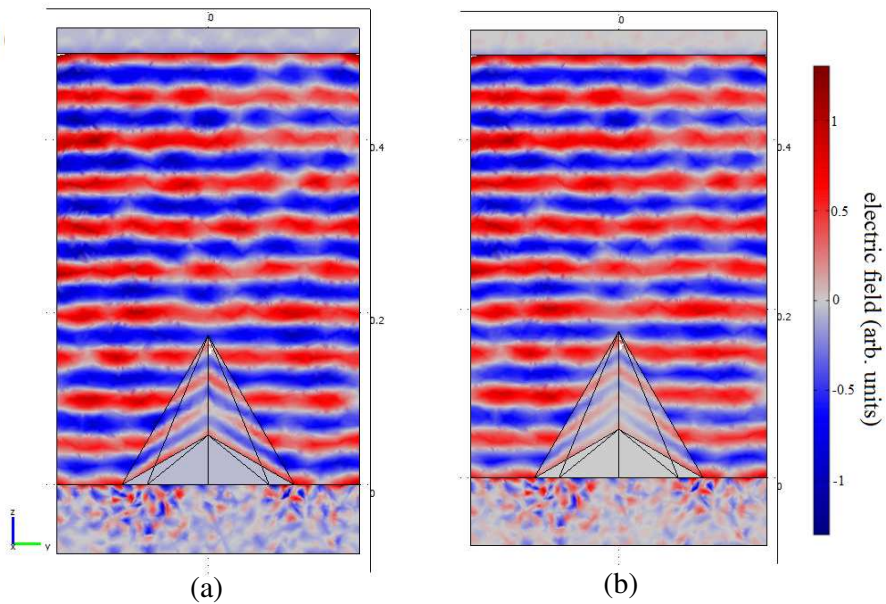
**Figure 8.** The electric field distribution of eight slices approximated conical cloak under an incident plane wave (a) in the  $y$ - $z$  plane. (b) in two orthogonal planes when the plane wave is incident along  $z$  direction.



**Figure 9.** The reflection coefficient of inhomogeneous and homogeneous approximated conical cloak.

metamaterial structures [22]. Figure 10 shows the effect of loss tangent in performance of homogeneous approximated conical cloak.

Since the loss deteriorates the efficiency of invisible cloaking in



**Figure 10.** The electric field distribution in the computational domain of eight slices homogeneous approximated conical cloak with (a) loss tangent of 0.01, (b) loss tangent of 0.1.

the forward-scattering region [27,28], for the back-scattering region and other regions, the electromagnetic fields remain unchanged, approximately. Thus, as we see in Figure 10, performance of the cloak is still very good and the effect of loss is negligible.

#### 4. CONCLUSION

In summary, based on the coordinate transformation in the Cartesian coordinate system, the material parameters of the 3-Dimensional conical electromagnetic cloak are extracted. Behavior of the conical cloak on the ground is considered. Full wave simulation with both planar and spherical electromagnetic waves irradiation is performed for studying the performance of the designed cloak. Then, homogeneous approximation of the conical cloak is considered with eight slices. Finally, backward scattering of the conical cloak with and without approximation is obtained that shows difference between the scattering of inhomogeneous material parameters conical cloak and eight homogeneous slices approximated ones, is small. All of the simulation results confirm that efficiency of the 3-dimensional inhomogeneous and

homogeneous approximated conical cloak is very high and is suitable for hiding any objects with arbitrary shape and size under ground. In addition, the parameters in our proposed homogeneous approximated conical cloak are very simple for realization in practical applications.

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