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AN EXTENSION OF THE KELVIN IMAGE THEORY TO THE CONDUCTING HEAVISIDE ELLIPSOID

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Abstract—The Kelvin image theory for a conducting sphere is extended to the case of a conducting oblate spheroid in uniform motion along its axis of revolution (a Heaviside ellipsoid) using the well-known method provided by Special Relativity. The results derived are checked in various ways.

1. INTRODUCTION

Some time ago, we studied electromagnetic images of conducting spheroids that move uniformly along the axis of revolution with respect to the laboratory frame [1]. (Following Maxwell [2], we define an electromagnetic image as a point charge or system of point charges, or a continuous charge distribution, on one side of a surface which would produce on the other side of that surface the same electromagnetic field which the actual charge of that surface really does produce.) Our argument was based on the well-known method provided by Special Relativity [3]: start from the corresponding electrostatic image solution to the problem of a conducting spheroid in its rest frame. Make a Lorentz boost to the lab frame from the rest frame, taking into account relativistic length contraction, charge invariance and transformation formulae for the electric and magnetic fields. In this way we found that the electromagnetic image of a conducting spheroid in uniform motion along its axis of revolution is a moving line charge or a point charge or a charged disc, depending on the shape of the spheroid. That result was obtained long time ago by Searle [4] using a more cumbersome method, in the framework of the classical, aether-based interpretation of Maxwell's electromagnetic theory.

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Finding the electromagnetic image of a moving spheroid belongs to a few problems of the electrodynamics of moving bodies whose solution was known earlier, before the advent of Special Relativity, and which were solved via the "Frame Hopping Method" in a simple and elegant way. (The famous examples are Einstein's calculation of the radiation pressure of a monochromatic plane linearly polarized electromagnetic wave on a perfect plane mirror in uniform motion [3, 5, 6] and Poincaré's derivation of the electromagnetic field due to a uniformly moving point charge [7, 8].) While the Frame Hopping Method is the standard research tool (cf, e.g., [9–12]), simple illustrations of the method in applications to the as yet unsolved problems seem to be lacking in the literature. So in this paper we present a generalization of the Kelvin image theory for a conducting sphere in the field of a point charge to the case of a moving spheroidal conductor in the field of a comoving charge. Our analysis leads to results whose analytical form is quite simple and thus, hopefully, could be helpful for teaching the electrodynamics of moving bodies.

2. A SUMMARY OF THE KELVIN IMAGE THEORY FOR A CONDUCTING SPHERE

For the convenience of the reader, we give a brief summary of the Kelvin image theory for a conducting sphere.

Consider a perfectly conducting sphere of radius a, charged with an electric charge Q, in the field of a static point charge q at the distance b from the center, all with respect to their common rest frame Σ'. The following results are well documented in almost all textbooks on electromagnetism.

The electrostatic field outside the sphere due to the actual charge on the sphere is the same as that due to two image point charges. One of them is located at the distance a^2/b from the center and it has the charge $-qa/b$; the other image charge is at the center of the sphere and it has the charge $Q + qa/b$. While the result is almost always presented as an application of the uniqueness theorem, it was reached originally due to a fortunate observation that, in a well-known solution to the classical electrostatic problem of a point charge outside a perfectly conducting sphere *at zero potential*, the Legendre series expressing the potential due to the actual charge on the sphere can be interpreted in a simple way as the potential due to a single point charge. This discovery, which "seems to have been reserved" for the young William Thomson, later Lord Kelvin, led him to the principle of images ([13–15], cf also [2, 16]), a powerful method for solving boundary-value problems in electrostatics and elsewhere.

Let the origin of the Σ' frame coincide with the center of the sphere so that its equation is

$$x'^2 + y'^2 + z'^2 = a^2, \tag{1}$$

and also choose the x'-axis in such a way that the charge q has coordinates $(b, 0, 0)$. Since the electrostatic field $\mathbf{E}'$ near a given point of the surface of a conductor and the surface charge density σ' at the given point are related by the law

$$\mathbf{E}' = (\sigma'/\epsilon_0)\mathbf{n}', \tag{2}$$

where $\mathbf{n}'$ is outward unit normal to the surface, one easily finds that the charge density over the surface of the sphere is given by

$$\sigma'(x', y', z') = \frac{Q + qa/b}{4\pi a^2} - \frac{q}{4\pi a}\frac{(b^2 - a^2)}{(a^2 + b^2 - 2bx')^{3/2}}, \tag{3}$$

where of course x', y', z' satisfy Equation (1).

The force acting on the charge q is found from Coulomb's law

$$\mathbf{f}'_{\rightarrow q} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q + qa/b}{b^2}\mathbf{u}_x + \frac{(-qa/b)}{(b - a^2/b)^2}\mathbf{u}_x\right] q, \tag{4}$$

and the force acting on the sphere is obtained either directly from Newton's third law, or by integrating the electrostatic force on a surface element of area dS' over the sphere [17]; the force on the charged element is $\sigma' dS' \mathbf{E}'/2$, where σ' is given by Equation (3) and $\mathbf{E}'$ is the electrostatic field just outside the surface given by Equation (2) ([18], cf also [2]).

3. IMAGE THEORY FOR THE HEAVISIDE ELLIPSOID

Now introduce an inertial frame Σ which is in a standard configuration with respect to the Σ' frame (Σ' moves relative to Σ with velocity $\mathbf{V} = (V, 0, 0)$ in the positive direction of their common x, x' axes, y and z axes being parallel to the corresponding y' and z' axes, respectively, and the origins of the two frames coincide at $t = t' = 0$). The Σ and Σ' coordinates are related by the standard Lorentz transformation

$$x' = \gamma(x - Vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - Vx/c^2), \tag{5}$$

where $\gamma = (1 - V^2/c^2)^{-1/2}$. Also, from the condition that the so-called source-free Maxwell's equations $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$, $\nabla \cdot \mathbf{B} = 0$ obey the principle of Special Relativity, one finds the relations between the fields $\mathbf{E}'$, $\mathbf{B}'$ and $\mathbf{E}$, $\mathbf{B}$ (as measured in Σ' and Σ, respectively) at the same point:

$$\begin{aligned} \mathbf{E}_\parallel &= \mathbf{E}'_\parallel, \quad \mathbf{E}_\perp = \gamma[\mathbf{E}'_\perp - \mathbf{V} \times \mathbf{B}'], \\ \mathbf{B}_\parallel &= \mathbf{B}'_\parallel, \quad \mathbf{B}_\perp = \gamma[\mathbf{B}'_\perp + (1/c^2)\mathbf{V} \times \mathbf{E}'], \end{aligned} \tag{6}$$

where $\|$ and $\perp$ denote the field components parallel and normal to $\mathbf{V}$, respectively [3, 19].

Consider now the actual system discussed above as observed from the Σ frame, with respect to which the conductor and the point charge q are in uniform motion with the velocity $\mathbf{V}$. In Σ, the conductor has the shape of a moving ellipsoid whose equation is

$$\gamma^2(x - Vt)^2 + y^2 + z^2 = a^2. \tag{7}$$

Introducing the "present position" coordinates x_0, y_0, z_0 given by

$$x_0 = x - Vt, \quad y_0 = y, \quad z_0 = z, \tag{8}$$

Equation (7) becomes

$$\gamma^2 x_0^2 + y_0^2 + z_0^2 = a^2. \tag{9}$$

(Note that the "present position" coordinates represent a continuum of Cartesian coordinate systems, one for each instant of time t, which are all at rest with respect to one and the same *reference system* Σ. By the way, the (tacit) introduction of these coordinates in [3] was a source of some confusion [20].) Consequently, in Σ, our conductor is an oblate spheroid uniformly moving at the speed V whose semi-axes bear the ratio $\gamma^{-1} : 1 : 1$, the shorter semi-axis being parallel to the direction of motion. The moving ellipsoid (9) is known as a "Heaviside ellipsoid", a term introduced by Searle ([4], cf also [21–24]). Thus, in Σ, taking into account charge invariance, the actual system considered is a conducting Heaviside ellipsoid carrying a charge Q, uniformly moving at the speed V, with semi-axes a/γ, a, a, in the field of the comoving point charge q located on its axis of revolution, at the distance b/γ from the center of the ellipsoid.

On the other hand, the imaginary system discussed above (the actual charge q plus two image charges, at rest in Σ'), when considered from the Σ frame, consists of the actual charge q at the distance b/γ from the center of the ellipsoid plus the charge $Q + qa/b$ at the center and the charge $-qa/b$ at the distance $a^2\gamma^{-1}/b$ from the center, all in uniform motion at the velocity $\mathbf{V}$. The conclusion is a consequence of relativistic length contraction and charge invariance. We shall now investigate whether the last two (imaginary) moving charges are the electromagnetic image of the Heaviside ellipsoid.

The $\mathbf{E}$ and $\mathbf{B}$ fields in Σ are related to the corresponding $\mathbf{E}'$ and $\mathbf{B}'$ fields in Σ' by the transformation formulae (6). Thus the components of the electromagnetic field due to the charged moving spheroid (9) can be obtained from the electrostatic field of the corresponding charged sphere at rest (1). By using the same transformation formulae, one can get the components of the electromagnetic field due to the two moving charges from the electrostatic field of the corresponding two image charges at rest. But the electrostatic field due to the charged sphere at rest is the same as that due to its two image charges at rest. It follows that the **E** and **B** fields due to the conducting Heaviside ellipsoid (9) carrying a charge Q, outside the ellipsoid, are the same as those of two point charges which are moving with the same velocity **V** with respect to Σ, the charge $Q + qa/b$ at the center and the charge $-qa/b$ at the distance $a^2\gamma^{-1}/b$ from the center, under the proviso that there is also a comoving charge q located on the axis of revolution of the spheroid, at the distance b/γ from the center (Figure 1). Obviously, for $q = 0$ our electromagnetic image system reduces to the well-known electromagnetic image of an isolated Heaviside ellipsoid, as it should [1, 4, 22].

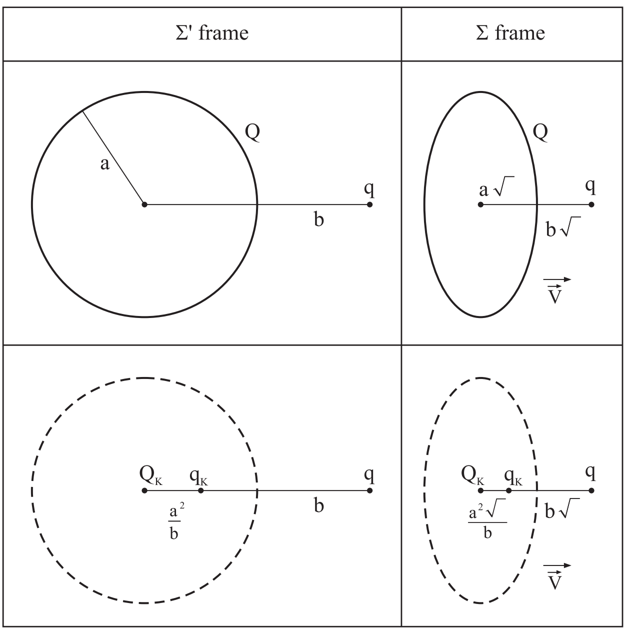

Figure 1. A point charge q in front of a conducting body carrying a charge Q (top) and equivalent electromagnetic image system (bottom) in their rest frame Σ' (left) and in the lab frame Σ (right), in which the body has the shape of a sphere and the Heaviside ellipsoid, respectively; $q_K \equiv -qa/b$, $Q_K \equiv Q - q_K$, and $\sqrt{\ } \equiv (1 - V^2/c^2)^{1/2} \equiv \gamma^{-1}$. Relativistic effects are depicted accurately for the case $b = 2a$ and Σ' moves relative to Σ at a speed V such that $\gamma = 2$.

3.1. The Surface Charge Density on the Heaviside Ellipsoid

One can determine the charge density σ on the surface of the Heaviside ellipsoid (9) by using relation

$$\sigma dS = \sigma' dS', \tag{10}$$

where dS and dS' is area of a surface element of the body in Σ and Σ', respectively, and σ' is given by Equation (3); Equation (10) is a consequence of charge invariance. A little calculus reveals that area of an elementary ring of the sphere (1) between two planes orthogonal to the x' axis is

$$dS' = 2\pi a dx', \tag{11}$$

where dx' is infinitesimal distance between the planes; area of the corresponding Lorentz-transformed elementary ring in Σ of the ellipsoid (9) is

$$dS = 2\pi\sqrt{a^2 + \beta^2\gamma^4 x_0^2}\, dx_0, \tag{12}$$

where $\beta = V/c$ and $dx_0 = \gamma^{-1}dx'$. From Equations (10)–(12) and (3) one gets

$$\begin{aligned}\sigma(x_0, y_0, z_0) \ =\ & \frac{\gamma}{\sqrt{a^2 + \beta^2\gamma^4 x_0^2}} \frac{Q + qa/b}{4\pi a} \\ & - \frac{\gamma q(b^2 - a^2)}{\sqrt{a^2 + \beta^2\gamma^4 x_0^2} 4\pi (a^2 + b^2 - 2b\gamma x_0)^{3/2}},\end{aligned} \tag{13}$$

where x_0, y_0, z_0 satisfy Equation (9). A little reflection, making use of the uniqueness theorem in the Σ' frame, reveals the physical meaning of the second term in expression for σ (13): it is the surface charge distribution which shields the interior of the Heaviside ellipsoid from the electromagnetic field of the comoving charge q. Consequently, the first term in (13) is the surface charge density on an *isolated* Heaviside ellipsoid (9) carrying a charge $Q + qa/b$; as it is well known, it is the same charge density one would have if an isolated conducting ellipsoid with the same total charge and of the same geometric shape were at rest in Σ [4, 25, 26].

Alternatively, one can find the charge density (13) using the general relationship between the electric field $\mathbf{E}$ near a given point of the surface of a uniformly moving conductor and the surface charge density σ at the given point. Namely, for a system of conducting bodies at rest in Σ' in the electrostatic equilibrium, $\mathbf{B}'$ vanishes everywhere and $\mathbf{E}' = 0$ inside the conductors; then Equation (6) imply that in Σ both $\mathbf{E}$ and $\mathbf{B}$ vanish inside the conductors and also that at every point outside the conductors $\mathbf{E}$ and $\mathbf{B}$ must be functions of x, y, z and t of the form

$$\mathbf{E} = \mathbf{E}(x_0, y_0, z_0), \quad \mathbf{B} = \mathbf{B}(x_0, y_0, z_0). \tag{14}$$

Now applying Gauss's law to the proper Gaussian pillbox we obtain, in the same way as in the electrostatic case, that the required relationship between $\mathbf{E}$ and σ reads [26]:

$$\mathbf{E} \cdot \mathbf{n} = \sigma/\epsilon_0, \tag{15}$$

where $\mathbf{n}$ is the outward unit normal to the surface at the given point, since $\mathbf{E} = 0$ inside the moving conductor.

(Note that, in Σ, the tangential component of $\mathbf{E}$ does not, in general, vanish just outside the surface of a uniformly moving conductor, i.e., the tangential component of $\mathbf{E}$ is discontinuous across the surface of the moving conductor (contrary to what happens in the electrostatic case or in non-stationary situations across a fixed boundary between two media). The discontinuity of the tangential component of $\mathbf{E}$ is due to the fact that $\partial\mathbf{B}/\partial t$ is, in general, infinite at a point just outside the surface of a uniformly moving conductor. It is perhaps worthwhile to recall that the problem of boundary conditions at a moving boundary was somewhat tricky for the pioneers in the field [27–29]. Note also that some interesting properties of the electromagnetic field due to a uniformly moving conductor (in the electrostatic equilibrium in its rest frame) are pointed out in [30].)

One can easily find the outward unit normal $\mathbf{n}$ at a given point of the ellipsoidal surface (9). Namely, introducing $f(x_0, y_0, z_0) = \gamma^2 x_0^2 + y_0^2 + z_0^2 - a^2$, equation of the Heaviside ellipsoid becomes $f(x_0, y_0, z_0) = 0$. Then $\mathbf{n} = (\nabla f/|\nabla f|)_{f=0}$, which gives

$$\mathbf{n} = \left(\gamma^2 x_0 \mathbf{u}_x + y_0 \mathbf{u}_y + z_0 \mathbf{u}_z\right) \left(\gamma^4 x_0^2 + y_0^2 + z_0^2\right)^{-1/2}, \tag{16}$$

where x_0, y_0, z_0 satisfy Equation (9). On the other hand, the electric field outside the moving spheroid (9) is the sum of the electric fields $\mathbf{E}_q$, $\mathbf{E}_{Q+qa/b}$ and $\mathbf{E}_{-qa/b}$ due to the uniformly moving charges q, $Q + qa/b$ and $-qa/b$, respectively. Using the well-known expression for the electric field of a point charge $\tilde{Q}$ uniformly moving at velocity $\mathbf{V}$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{\tilde{Q}\mathbf{R}}{R^3} \frac{\left(1 - \frac{V^2}{c^2}\right)}{\left(1 - \frac{V^2}{c^2}\sin^2\alpha\right)^{3/2}}, \tag{17}$$

where $\mathbf{R}$ is the position vector of the point of observation relative to the present location of the charge and α is the angle between $\mathbf{R}$ and $\mathbf{V}$, and also using the law of sines, a little calculation reveals that the electric fields due to q, $Q+qa/b$ and $-qa/b$ at a given point *just outside* the Heaviside ellipsoid (9) are given by relations

$$\mathbf{E}_q = \frac{1}{4\pi\epsilon_0}\frac{q\gamma\left[\left(x_0 - b\gamma^{-1}\right)\mathbf{u}_x + y_0\mathbf{u}_y + z_0\mathbf{u}_z\right]}{(a^2+b^2-2b\gamma x_0)^{3/2}}, \tag{18}$$

$$\mathbf{E}_{Q+qa/b} = \frac{1}{4\pi\epsilon_0}\frac{(Q+qa/b)\gamma(x_0\mathbf{u}_x + y_0\mathbf{u}_y + z_0\mathbf{u}_z)}{a^3}, \tag{19}$$

$$\mathbf{E}_{-qa/b} = -\frac{1}{4\pi\epsilon_0}\frac{qb^2\gamma\left[\left(x_0 - a^2\gamma^{-1}/b\right)\mathbf{u}_x + y_0\mathbf{u}_y + z_0\mathbf{u}_z\right]}{(a^2+b^2-2b\gamma x_0)^{3/2}}, \tag{20}$$

where x_0, y_0, z_0 satisfy Equation (9). Now replacing $\mathbf{n}$ and $\mathbf{E}$ in Equation (15) by expression (16) and the sum of the fields (18)–(20), one obtains that σ is given by Equation (13), as it should be.

3.2. The Force Acting on q and the Charge Distribution Stability on the Heaviside Ellipsoid

The preceding considerations imply that the electromagnetic force acting on the comoving charge q in the field of the Heaviside ellipsoid (9) is simply the Lorentz force by which the electromagnetic field due to the two comoving charges $Q + qa/b$ and $-qa/b$ acts on q. Using the fact that the electric field $\mathbf{E}$ of a uniformly moving point charge $\tilde{Q}$ is given by expression (17), and its magnetic field is

$$\mathbf{B} = (1/c^2)\mathbf{V} \times \mathbf{E}, \tag{21}$$

a simple calculation reveals that the Lorentz force on q, $\mathbf{f}_{\to q}$, is

$$\mathbf{f}_{\to q} = \mathbf{f}'_{\to q}, \tag{22}$$

where $\mathbf{f}'_{\to q}$ is given by Equation (4). Alternatively, the same result is obtained applying the relativistic force transformation equations (cf, e.g., [31, 32]) to $\mathbf{f}'_{\to q}$. (Note that application of the relativistic force transformation equations can be somewhat tricky [33, 34].)

As can be seen, the validity of Newton's third law for the electrostatic interaction in Σ' implies the validity of that law for the corresponding electromagnetic interaction in Σ. Thus the electromagnetic force acting on the Heaviside ellipsoid (9) is simply $-\mathbf{f}_{\to q}$. (By the way, it is perhaps worthwhile to point out that momentum of the electromagnetic field of the system considered (the Heaviside ellipsoid and the comoving charge q) is time independent; also, the system considered does *not* radiate.)

The preceding considerations relate to the problem of how to calculate the electromagnetic force on an element of surface charge σdS of a uniformly moving conductor [25, 26]. The solution is simple:

in Σ', where the conductor is at rest, the electrostatic force on the *same* element of charge $\sigma' dS'$ (charge invariance) is $d\mathbf{F}' = (\sigma' dS'/2)\mathbf{E}'$, where $\mathbf{E}'$ is the electrostatic field just outside the surface. Applying the relativistic force transformation equations for the Lorentz force expression [31, 32] to $d\mathbf{F}'$, one gets that the electromagnetic force on the surface charge element as measured in Σ is given by

$$d\mathbf{F} = (\sigma dS/2)\mathbf{E}^*, \tag{23}$$

where $\mathbf{E}^* \equiv \mathbf{E} + \mathbf{V} \times \mathbf{B}$, and $\mathbf{E}$ and $\mathbf{B}$ are the corresponding fields just outside the surface of the moving conductor.

Now, the $\mathbf{E}^*$ field at a point arbitrarily close to the surface of a uniformly moving conductor is orthogonal to the surface [26, 35]. For the convenience of the reader, we give a short proof of this result based on special relativistic transformations [26].

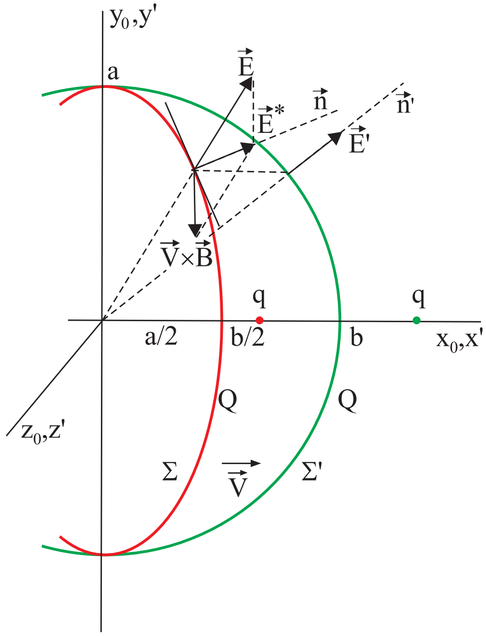

Figure 2. Conducting sphere charged with an electric charge Q in the field of a point charge q in their rest frame Σ' and the same system as observed from the Σ frame as an oblate spheroid in uniform motion at a speed V in the field of the comoving charge q; the corresponding $\mathbf{E}'$, $\mathbf{E}$ and $\mathbf{E}^*$ fields at the *same point* just outside the surface are depicted accurately for the case $b/a = 4/3$ and $\gamma = 2$.

Taking into account that $\mathbf{B} = (1/c^2)\mathbf{V} \times \mathbf{E}$ and $\mathbf{V} = V\mathbf{u}_x$, and using Equation (6), one finds that

$$E_x^* = E_x', \quad E_y^* = \gamma^{-1}E_y', \quad E_z^* = \gamma^{-1}E_z'; \tag{24}$$

also, if $\mathbf{N} = (N_x, N_y, N_z)$ is a vector perpendicular to the conductor surface in Σ, then, as can be seen, the corresponding normal vector in Σ' is $\mathbf{N}' = (N_x/\gamma, N_y, N_z)$, due to relativistic length contraction. Equation (24) and the fact that $\mathbf{E}' \times \mathbf{N}' = 0$ imply that $\mathbf{E}^* \times \mathbf{N} = 0$. (Alternatively, using the Hertz-Helmholtz identity (cf, e.g., [36]) and Faraday's law $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$, one can prove that the tangential component of $\mathbf{E}^*$ is continuous across the surface of a uniformly moving conductor, and then use the fact that $\mathbf{E}^*$ equals zero inside the conductor [26].)

Thus, the $\mathbf{E}^*$ field just outside the surface of a uniformly moving conductor is orthogonal to the surface and, consequently, the electromagnetic force (23) acting on the surface charge element is orthogonal to the surface, which accounts for the stability of the surface charge distribution on the conductor. This of course applies to our conducting Heaviside ellipsoid as well (Figure 2). Namely, taking into account relation

$$\mathbf{E}^* = \mathbf{E} + (\mathbf{V}/c^2) \times (\mathbf{V} \times \mathbf{E}) = E_x\mathbf{u}_x + \gamma^{-2}E_y\mathbf{u}_y + \gamma^{-2}E_z\mathbf{u}_z, \tag{25}$$

and Equations (18)–(20) and (16), one gets $\mathbf{E}^* \times \mathbf{n} = 0$, as it should be.

4. CONCLUDING REMARKS

In this paper, we have presented image theory for the conducting Heaviside ellipsoid, obtained by Lorentz-transforming the well-known image solution to the corresponding electrostatic problem. In the same way, using the Frame Hopping Method provided by Special Relativity, one can develop image theory for uniformly moving prolate and oblate (but *less* oblate than the Heaviside ellipsoid) conducting spheroids, starting from image solution to the corresponding electrostatic problem. Unfortunately, the electrostatic image of a prolate conducting spheroid in the field of a point charge on its axis is not very simple [37]. Analogous remarks apply to image theory for a dielectric Heaviside ellipsoid and, more generally, for any uniformly moving prolate and oblate (but less oblate than the Heaviside ellipsoid) dielectric spheroid; the corresponding electrostatic images are somewhat cumbersome [38–41].

Another point is that the form of our results presented above for the Heaviside ellipsoid is determined by the formulation of the corresponding source problem (a conducting sphere at rest in the field of a stationary q). It is perhaps worthwhile to recast the results in a more natural form, corresponding to the problem of the Heaviside ellipsoid with semi-axes $a\gamma^{-1}$, a, a in the field of the comoving charge q at a distance b^* from the center. The electromagnetic image of the Heaviside ellipsoid now consists of two comoving charges, the charge $Q + qa\gamma^{-1}/b^*$ at the center, and the charge $-qa\gamma^{-1}/b^*$ at the distance $a^2\gamma^{-2}/b^*$ from the center. Equations (13) and (18)–(20) are recast in the same way, replacing b by $b^*\gamma$ in them.

Lastly, while our conclusions are reached in the framework of the special theory of relativity, it should be stressed that Special Relativity is not actually essential to them. Namely, as can be seen, assuming the validity of Maxwell's equations in the Σ frame, our conclusions follow from a *purely mathematical fact* that Maxwell's equations are Lorentz covariant, in the sense that the primed quantities are considered as convenient mathematical variables that need not have the familiar relativistic interpretation

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