Progress In Electromagnetics Research, Vol. 119, 381–394, 2011

A DIFFERENTIAL EVOLUTION APPROACH FOR ROBUST ADAPTIVE BEAMFORMING BASED ON JOINT ESTIMATION OF LOOK DIRECTION AND ARRAY GEOMETRY

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Abstract—The performance of traditional beamformers tends to degrade due to inaccurate estimation of covariance matrix and imprecise knowledge of array steering vector. The inaccurate estimation of covariance matrix can be attributed to limited data samples and the presence of desired signal in the training data. The mismatch between the actual and presumed steering vectors can be due to the error in the position (geometry) and/or in the look direction estimate. In this paper, we propose a differential evolution (DE) based robust adaptive beamforming that is able to achieve near optimal performance even in the presence of geometry error. Initially, we estimate an optimal steering vector by maximizing and minimizing the signal power in and out of the desired signal's angular range, respectively. Then, we estimate the look direction and reconstruct the covariance matrix. Based on the obtained steering vector, estimate for look direction and reconstructed covariance matrix, near optimal output SINR, can be obtained with the increase in the input SNR without observing any saturation even in the presence of geometry error. Numerical simulations are presented to demonstrate the efficacy of the proposed algorithm.

Received 22 May 2011, Accepted 10 August 2011, Scheduled 16 August 2011

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1. INTRODUCTION

Adaptive beamforming is a versatile approach to detect and enhance a desired signal while suppressing noise and interferences at the output of a sensor array. Adaptive beamforming has been widely used in mobile satellite communications [1], microphone array speech processing, radar [2], medical imaging [3], antenna arrays [4, 5], and cognitive radio [6]. Traditional adaptive beamforming techniques assume that the training data are free from the desired signal components [7]. Although the assumption of signal-free training snapshots may be true in some areas (such as radar), there are numerous applications (such as speech processing, medical imaging) where the observations are always "contaminated" by the signal component. This leads to the performance degradation in traditional adaptive beamformers. Several approaches have been proposed to improve the robustness of the adaptive beamformers. However, many of the proposed methods are limited to certain types of mismatches. The mismatches can be in the look-direction, gain-phase, array geometry [8–10], incorrect assumption of the signal model, e.g., point-source or scattered-source (either coherent or incoherent scattering) signal model. Besides these mismatches, the performance of the beamformers is also known to degrade when the number of snapshots taken to construct the sample covariance matrix [11] is small, or there are any other effects introduced by the propagation environment.

In [12], an approach based on loading of the diagonal of the sample array covariance matrix is proposed to improve the robustness against more general mismatches. While having the advantage of being invariant to the type of mismatches, the choice of the optimal loading factor is not obvious.

In capon beamforming [13], the uncertainty setting of the steering vectors is delimited by upper bounding the norm of the difference between the actual and presumed steering vectors, i.e., the norm of the mismatch vector. The capon beamformers can provide robustness against the uncertainty in the look direction. However, if any other of the steering vector mismatches (such as geometry error) become dominant, these methods cannot be expected to provide sufficient robustness [14]. Genetic algorithms [15–18], differential evolution [19–21] and swarm intelligence algorithms [22] have been used to solve diverse problems in array signal processing, antenna design and communications. Recently, beamforming algorithms using particle-filter [23], neural networks [24], and global optimizations techniques such as PSO [25] and differential evolution (DE) [29] have been proposed. In [25], the authors assume that information regarding the array geometry, signal to noise ratio, directions of arrival of signal and interferences can be obtained exactly by using the methods presented in [26–28]. In [29], we proposed a DE based adaptive beamformer which performed well for look direction as well as for a considerable amount of geometry error. In [30], we proposed a novel algorithm to estimate the look direction and to reconstruct the covariance matrix so that near optimal performance without the effect of saturation can be achieved as the input SNR increases. In [29, 30], the information about the signal to noise ratio is not used as well as the directions of arrival of the signal of interest, and the interferences are estimated instead of assuming them to be precisely known.

In this paper, we improve the robustness of the algorithm presented in [30] by correcting any error in the originally presumed steering vector by maximizing and minimizing the signal power respectively in and out of the desired signal's angular range. Then we use the approach proposed in [30] to obtain the optimal weight vector.

The rest of this paper is organized as follows. In Section 2, we describe the array signal model and the existing robust adaptive beamformers as well as some background on DE algorithm. In Section 3, we explain the proposed approach that includes the optimization formulation of estimating the steering vector and its implementation with the proposed DE algorithm. Section 4 presents the simulation results that demonstrate the efficacy of the proposed solution. Section 5 concludes the paper.

2. BACKGROUND

2.1. Signal Model

Consider a narrowband beamforming model in which K narrowband plane wave signals, modeled as statistically independent zero-mean random sequence, impinge on an array of M sensors ($K < M$) from directions θ_s and θ_i ($i = 1, 2, \ldots, K-1$). The received signal at the array is given by

$$\mathbf{x}(t) = \mathbf{a}(\theta_s)s_d(t) + \sum_{i=1}^{K-1} \mathbf{a}(\theta_i)s_i(t) + \mathbf{n}(t) \tag{1}$$

where $s_d(t)$, $s_i(t)$ and $\mathbf{n}(t)$ are the desired signal, i-th interference and noise, respectively. $\mathbf{a}(\theta)$ is the steering vector of the plane wave from direction θ.

Conventional adaptive beamforming calculates an optimal weight vector that minimizes the interference-plus-noise output power subjected to a unity response of the desired signal

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{in} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a} = 1 \tag{2}$$

where $\mathbf{w} = [w_1, \ldots, w_M]^T$ is the complex vector of beamformer weights; M is the number of sensors; $\mathbf{R}_{in}$ is the interference-plus-noise covariance matrix; $\mathbf{a}$ is the desired signal steering vector; $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. Solving (2), the optimal beamformer weight vector can be expressed as

$$\mathbf{w}_{mvdr} = (\mathbf{a}^H \mathbf{R}_{in}^{-1} \mathbf{a})^{-1} \mathbf{R}_{in}^{-1} \mathbf{a} \tag{3}$$

However, in practice, only the estimate ($\hat{\mathbf{R}}$) of the matrix $\mathbf{R}_{in}$ can be obtained from the discrete-sampled array signal received

$$\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}(n) \mathbf{x}(n)^H \tag{4}$$

where N is the number of snapshots. The presence of the desired signal components $\mathbf{a}(\theta_s) s_d(t)$ in the array received signal in the computation of $\hat{\mathbf{R}}$ will lead to a substantial performance degradation as measured by the output signal-to-interference-and-noise ratio (SINR). That is, the output SINR of the beamformer saturates as input signal-to-noise ratio (SNR) increases. This problem is further complicated when there is a mismatch between the actual and the presumed steering vectors (denoted as $\bar{\mathbf{a}}$).

2.2. Existing Robust Adaptive Beamformers

A simple-yet-effective approach for robust adaptive beamforming is diagonal loading, which offers robustness by adding a positive value to the diagonal terms of the sample covariance matrix. The weight vector can be obtained using the loaded sample covariance matrix $\mathbf{R}_{dl}$ according to

$$\mathbf{w}_{lsmi} = (\bar{\mathbf{a}}^H \mathbf{R}_{dl}^{-1} \bar{\mathbf{a}})^{-1} \mathbf{R}_{dl}^{-1} \bar{\mathbf{a}} \tag{5}$$

where $\bar{\mathbf{a}}$ denotes the *presumed* steering vector; $\mathbf{R}_{dl} \triangleq \gamma \mathbf{I} + \hat{\mathbf{R}}$ is the diagonally loaded sample covariance matrix; γ denotes the loading factor; $\mathbf{I}$ is the identity matrix. Such an approach is termed as the loaded sample matrix inverse (LSMI) beamformer. Although it has been shown to improve the performance, it is not clear how much loading factor or what is the suitable value for γ is required.

To explicitly relate the amount of loading factor to the uncertainties in the desired signal steering vector, the authors in [13] proposed a different optimization formulation for solving the beamformer's weight. The formulation is based on a quadratic optimization problem with a multi-dimensional spherical constraint that models the uncertainty as the square-norm of the mismatch vector:

$$\min_{\mathbf{a}} \quad \mathbf{a}^H \hat{\mathbf{R}}^{-1} \mathbf{a} \qquad \text{subject to} \quad \|\mathbf{a} - \bar{\mathbf{a}}\|^2 \leq \varepsilon. \tag{6}$$

where ε quantifies the level of the uncertainty between the nominal and actual steering vector. By imposing a quadratic equality constraint on (1) and using the Lagrange multiplier method, the estimated steering vector is given by:

$$\hat{\mathbf{a}} = \bar{\mathbf{a}} - (\mathbf{I} + \lambda \hat{\mathbf{R}})^{-1} \bar{\mathbf{a}} \tag{7}$$

and the Lagrange multiplier λ is obtained by solving the following constraint equation:

$$g(\lambda) \triangleq \left\| (\mathbf{I} + \lambda \hat{\mathbf{R}})^{-1} \bar{\mathbf{a}} \right\|^2 = \varepsilon. \tag{8}$$

The estimated steering vector is later used to formulate the weight vector

$$\mathbf{w}_{rcb} = (\hat{\mathbf{a}}^H \hat{\mathbf{R}}^{-1} \hat{\mathbf{a}})^{-1} \hat{\mathbf{R}}^{-1} \hat{\mathbf{a}} \tag{9}$$

To avoid the need to know the loading factor, the authors in [31] proposed a beamformer that can be formulated as a ridge regression problem. As a result, the design of the weight vector does not require any preset parameter even though the steering vector used is inaccurate. At the end of the formulation, the authors in [31] also show that this parameter-free robust beamformer is in fact a diagonal loading approach with the loading parameter

$$\begin{aligned} \rho &= (M-1)\hat{\sigma}_{LS}^2 / \|\boldsymbol{\eta}_{LS}\|^2 \\ \hat{\sigma}_{LS}^2 &= \left\| \hat{\mathbf{R}}^{1/2} \mathbf{B} \boldsymbol{\eta}_{LS} - \hat{\mathbf{R}}^{1/2} \bar{\mathbf{a}} / M \right\|^2 \\ \boldsymbol{\eta}_{LS} &= (\mathbf{B}^H \hat{\mathbf{B}} \mathbf{B})^{-1} \mathbf{B}^H \hat{\mathbf{R}} \bar{\mathbf{a}} / M \end{aligned} \tag{10}$$

where $\mathbf{B}$ is an $M \times (M-1)$ semi-unitary matrix orthogonal to the nominal steering vector $\bar{\mathbf{a}}$. The beamformer weight is the same as that in LSMI beamformer with the diagonally-loaded covariance matrix $\mathbf{R}_{rr} \triangleq \rho \mathbf{I} + \hat{\mathbf{R}}$:

$$\mathbf{w}_{rr} = (\bar{\mathbf{a}}^H \mathbf{R}_{rr}^{-1} \bar{\mathbf{a}})^{-1} \mathbf{R}_{rr}^{-1} \bar{\mathbf{a}} \tag{11}$$

2.3. Differential Evolution

Evolutionary algorithms (EAs) are population-based stochastic algorithms that can effectively handle real-world optimization problems which are non-continuous and/or non-differentiable and characterized by chaotic disturbances, randomness and complex non-linear dynamics. Differential evolution (DE) [32], a simple and powerful global optimization algorithm, has attracted much attention due to its simplicity and less number of parameters to tune.

The effectiveness of conventional DE in solving a numerical optimization problem depends on the selected mutation and crossover strategies and their associated parameter values [33–35]. Motivated by the observation that each optimization is unique and to solve a specific problem, different mutation strategies with different parameter settings may be better during different stages of the evolution than a single mutation strategy with unique parameter settings as in the conventional DE, an ensemble of mutation and crossover strategies and parameter values for DE (EPSDE) [36] was proposed.

EPSDE consists of a pool of mutation and crossover strategies along with a pool of values for each of the associated control parameters. Each member in the initial population is randomly assigned with a mutation strategy and associated parameter values taken from the respective pools. The population members (target vectors) produce offspring (trial vectors) using the assigned mutation strategy and parameter values. If the generated trial vector is better than the target vector, the mutation strategy and parameter values are retained with trial vector which becomes the parent (target vector) in the next generation. The combination of the mutation strategy and the parameter values that produced a better offspring than the parent are stored. If the target vector is better than the trial vector, then the target vector is assigned with a re-initialized mutation strategy and associated parameter values from the respective pools or from the successful combinations stored with equal probability. This leads to an increased probability of production of offspring by the better combination of mutation strategy and the associated control parameters in the future generations.

The implementation of the EPSDE algorithm is presented in [36]. The outline of the algorithm is presented in Section 3.

3. PROPOSED APPROACH

In adaptive beamforming, the steering vector ($\mathbf{a}(\theta_s)$) depends on the position of the sensors and the DOA of the desired signal and can be expressed as:

$$\mathbf{a}(\theta_s) = \exp(j\pi P \sin(\theta_s)) \tag{12}$$

where $P = [p_1, p_2, \ldots, p_M]$ is the position vector. To estimate the steering vector, we try to obtain a vector $\mathbf{x} = [P, \theta_s]$ to maximize and minimize the signal power in and out of the desired signal's angular range ($\theta_L < \theta_s < \theta_U$, where θ_L and θ_U are the lower and upper bounds respectively) by using the following expression

$$f(\mathbf{x}) = \frac{\bar{\mathbf{a}}(\theta_s)^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}(\theta_s)}{\bar{\mathbf{a}}(\theta_{in,q})^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}(\theta_{in,q})} \tag{13}$$

where θ_s is the possible look-direction from the desired signal's angular range. Likewise, $\{\theta_{in,q}\}_{q=1}^{L_{in}}$ are derived from the interference-plus-noise angular range. L_{in} is the number of interferences. Note that the directions defined in $\theta_{in,q}$ do not overlap with those in $[\theta_L, \theta_U]$.

After obtaining a proper estimate for the steering vector, we use the approach proposed in [30] to solve for the robust adaptive beamforming problem.

STEP 1: Set the generation count $G = 0$, and randomly initialize a population of Np individuals $P_G = \{\mathbf{x}_{1,G}, \ldots, \mathbf{x}_{Np,G}\}$ with $\mathbf{x}_{i,G} = \{\mathbf{x}_{i,G}^1, \ldots, \mathbf{x}_{i,G}^D\}$, $i = 1, \ldots, Np$ uniformly distributed in the range $[\mathbf{x}_{\min}, \mathbf{x}_{\max}]$. Here $D = M + 1$.

STEP 2: Select a pool of mutation strategies and a pool of values for each associated parameters corresponding to each mutation strategy.

STEP 3: Each population member is randomly assigned with one of the mutation strategy from the pool and the associated parameter values are chosen randomly from the corresponding pool of values.

STEP 4: WHILE stopping criterion is not satisfied, DO

FOR $i = 1$ to Np

Mutation Step

Generate a mutated vector $\mathbf{v}_{i,G} = \{v_{i,G}^1, \ldots, v_{i,G}^D\}$, $i = 1, \ldots, Np$ corresponding to the target vector $\mathbf{x}_{i,G}$

$$\mathbf{v}_{i,G} = \mathbf{x}_{r_1^i,G} + F(\mathbf{x}_{r_2^i,G} - \mathbf{x}_{r_3^i,G})$$

The indices r_1, r_2, r_3 are mutually exclusive integers randomly generated anew for each mutant vector within the range $[1, Np]$, which are also different from the index i.

Crossover Step

Generate a trial vector $\mathbf{u}_{i,G} = \{u_{i,G}^1, \ldots, u_{i,G}^D\}$, $i = 1, \ldots, Np$ for each target vector $\mathbf{x}_{i,G}$

$$u_{i,G}^j = \begin{cases} v_{i,G}^j, & rand^j(0,1) \le CR \text{ or } j = j_{rand} \\ x_{i,G}^j, & \text{otherwise} \end{cases}$$

where $j = 1, \ldots, D$

Selection Step

Evaluate the trial vector $\mathbf{u}_{i,G}$

$$\mathbf{x}_{i,G+1} = \begin{cases} \mathbf{u}_{i,G}, & f(\mathbf{u}_{i,G}) < f(\mathbf{x}_{i,G}) \\ \mathbf{x}_{i,G}, & \text{otherwise} \end{cases}$$

Increment the generation count $G = G + 1$.

STEP 5: END WHILE.

4. SIMULATION RESULTS

In our simulations, we consider a 10-element ULA with half-wavelength spacing receiving nine Gaussian signals: the SOI from $\theta_s = 4^\circ$ and eight interferences from $\theta_i = \{-70^\circ, -50^\circ, -30^\circ, 20^\circ, 30^\circ, 50^\circ, 60^\circ, 70^\circ\}$. The eight interferences are of equal power (20 dB). A white Gaussian distributed random variable (0 dB) is considered as the additive noise. Also, $\hat{\mathbf{R}}$ calculated from 100 snapshots is used to implement all the beamformers discussed here. The array geometry error is modeled as a uniform random variable according to $\mathcal{U}(-c\lambda, c\lambda)$, where λ and c are the signal wavelength and percentage errors. All the simulations include a look direction error of 4°.

In EPSDE algorithm the only parameter that has to be tuned is the population size (Np). In our experimentation we tried different values for Np (for example 10, 20, 30, 40 and 50). The population size $Np = 20$ gives the best values for maximum function evaluations of 50000 and maximum generations of 2500. The performance of the proposed algorithm is compared with algorithms in the literature such as RR [31], SQP [38, 39], RCB [13] and IRCB [37].

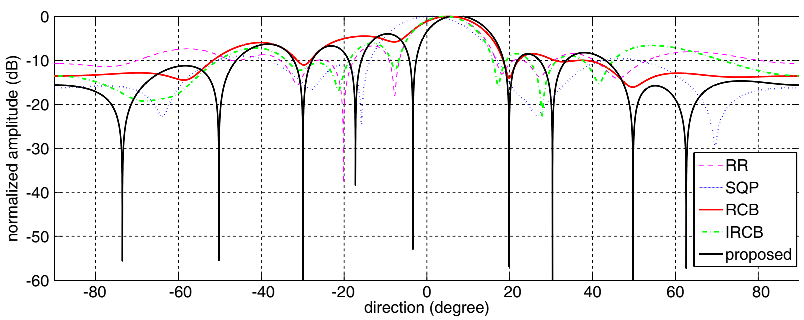

Figure 1. Beampattern of the best results 50% Geometry Error at 0 dB input SNR.

The proposed algorithm is evaluated with no error in geometry and 50% uniform error in the geometry and compared with RR, SQP, RCB and IRCB algorithms. 50% uniform error in geometry implies that the position of a particular sensor element can be located within 50% of the distance between two sensor elements. From the beampattern presented in Figures 1 and 2, it can be observed that the proposed approach is able to estimate most of the interference directions and achieves deeper nulls in the direction of interferences in addition to the lower side lobe levels. Figures 3 and 4 present the performance of the

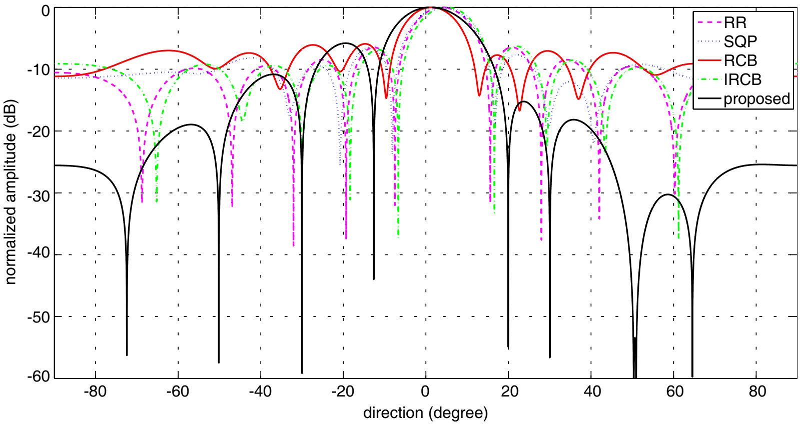

Figure 2. Beampattern of the best results 50% Geometry Error at 30 dB input SNR.

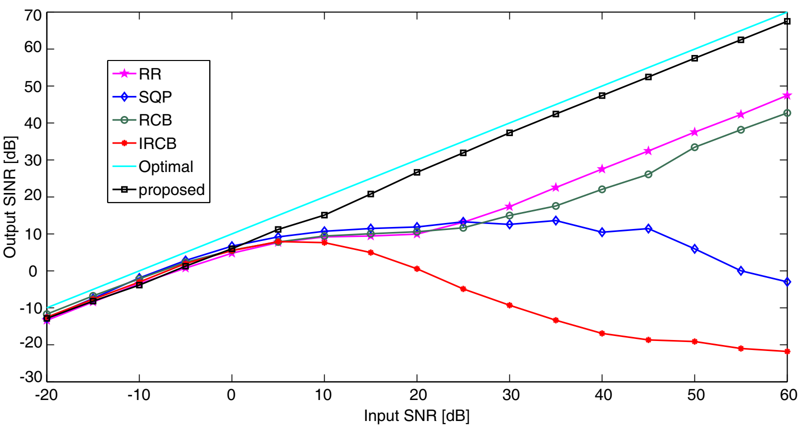

Figure 3. Median output SINR versus SNR for 0% geometry error.

algorithms with increasing SNR values for different levels of geometry error. From the graphs, it can be observed that the performance of the proposed algorithm is consistent and is able to achieve near optimal mean performance over the entire range of the input SNR values. It can also be observed that even though the error in the geometry increases the output SINR remains closer to the optimal values indicating the robustness of the proposed algorithm to the errors in the geometry.

In the proposed algorithm, we try to obtain optimal geometry positions by using DE algorithm. In Figure 5, we present the average mean square error values over the 100 realizations for different input SNR values, to show how well the proposed algorithm is able to recover the optimal geometry positions.

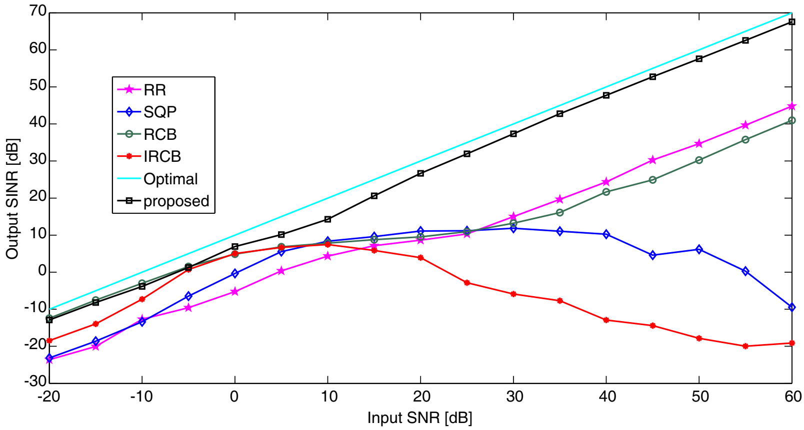

Figure 4. Median output SINR versus SNR for 50% geometry error.

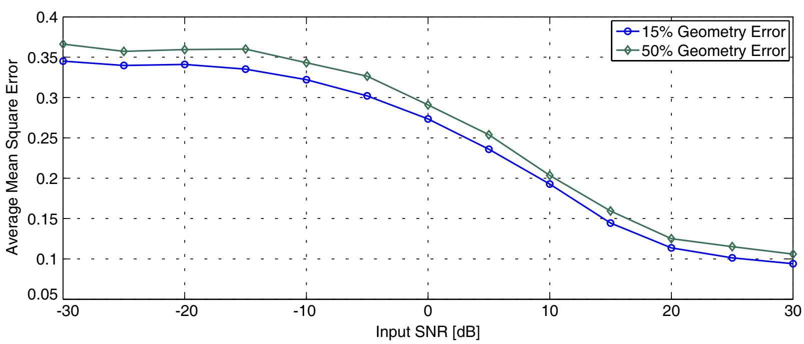

Figure 5. Average mean square error.

The time taken per one realization by each of the algorithms, RR, SQP, RCB, IRCB and proposed, is $5.60 * 10^{-2}$, $7.12 * 10^{-1}$, $2.16 * 10^{-2}$, $3.02 * 10^{-2}$ and $1.08 * 10^{2}$ seconds respectively. The computational complexity of the proposed algorithm is due to the estimation of the optimal steering vector, which takes a cpu time of $1.00 * 10^{2}$ seconds. However, in real-world applications, the estimation of optimal steering vector (geometry) is done only once. Once the optimal steering vector is obtained, the beamforming operation takes $8 * 10^{-2}$ seconds.

5. CONCLUSION

This paper proposes a DE based adaptive beamforming algorithm addressing the issues such as inaccurate estimation of the covariance matrix and mismatch between the actual and presumed steering vectors, which degrade the performance of traditional beamforming approaches. From the simulation results it can be observed that the proposed algorithm is robust to the inaccurate estimation of covariance matrix as well as the geometry errors.

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