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INFLUENCE OF THE POLE NUMBER ON THE MAGNETIC NOISE OF ELECTRICAL AC MACHINES

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Abstract—This paper deals with the influence of the AC machine pole number on vibrations and noise of electromagnetic origin. First, rules of design of AC machines are reminded, pointing out the influence of the pole number. Then, the origin of magnetic vibrations and noise is explained. Analytical mechanical relations are given, allowing to estimate vibrations and noise of a machine. After that, the influence of the pole number is studied: on the machine radius, on the stator deformations, on the mechanical resonance frequencies and on the noise. The conclusions underline that machines with high pole number have stator vibrations of high amplitude. Calculations compare three machines with different pole number and fed at different frequencies. The conclusion is that, at the same speed and working power, machines with high pole number fed at high frequency are noisier than those with low number of poles fed at low frequency. Practical experiments illustrate these theoretical considerations.

1. INTRODUCTION

The noise of electrical machines is an important factor of quality. It has essentially 3 origins:

- Mechanical noise is due to frictions of the bearings. It increases with the square of the speed, thus it is important only for high speed machines.
- Aerodynamic noise is more important. Parts in rotation produce air turbulences. It increases with the fifth power of the speed. This noise is really dominant for high speed machines.

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- Electromagnetic phenomena are at the origin of the rotor rotation, but they create radial forces on the stator, and so vibrations and noise. This origin is generally the most important source of noise for low speed machines.

In a general way, a low speed machine is less noisy than a high speed one. Nevertheless the magnetic noise can be important, particularly on machines with high number of poles.

If a machine is fed by an electronic speed variator, many choices of pole number and voltage frequency can be done to obtain the same rotation speed. This paper deals with the pole number choice.

Firstly, the rules of AC machine design are reminded to establish a relation between the machine dimensions and its pole number.

Secondly, the process of the magnetic noise and vibration generation is explained and analytical relations are given.

Thirdly, the variation of the vibrations and noise with the pole number is studied, at constant or variable frequency.

Finally, an example of a noisy machine with high pole number is given.

Let us precise that the aim of this paper is not to predict exactly the noise of a machine, what is impossible, but to give general tendencies usable by AC machine designers.

2. INFLUENCE OF THE NUMBER OF POLE PAIRS ON THE DESIGN OF AC MACHINES

The RMS value of the E induced electromotive force in a stator phase is given by:

$$E = \frac{\hat{B}}{\sqrt{2}} L v n_1 K_1^s \tag{1}$$

where $\hat{B}$ is the maximum value of the flux density in the air-gap (generally 0.6 to 1 T), L is the length of the magnetic circuit, K_1^s is the winding factor (close to 1), n_1 the number of wires series-connected. v, which is the linear speed of the flux density wave along the stator wires, can be expressed as follow:

$$v = \frac{D}{2}\frac{\omega}{p} \tag{2}$$

where D is the internal diameter of the stator, ω the angular frequency ($\omega = 2\pi f$ with f the electric supply frequency) and p the number of pole pairs.

So, noting $K_1'^s = \frac{K_1^s \pi}{\sqrt{2}}$, it can be written:

$$DL = \frac{E}{\hat{B} K_1'^s n_1} \frac{p}{f} \tag{3}$$

One can observe than the DL product is proportional to p.

Neglecting the voltage drop due to both the conductors resistance and flux leakage, noting I the current RMS value, the apparent power S of a three phase machine is given by: $S = 3EI$.

Let introduce the peripheral load given by $A = 3n_1 I/\pi D$. It depends on the power, the diameter and the cooling factor. It takes generally values from 20000 to 50000 Amp × turns/meter [1]. One can deduce an important relation:

$$D^2 L = \frac{S}{A \pi K_1'^s \hat{B}} \frac{p}{f} \tag{4}$$

$D^2 L$ is proportional to the weight and the volume of the machine. So, for the same power and frequency, a machine with a high number of poles is more voluminous.

'A' varies approximately with $D^{0.4}$ [2]; approximately: $A \approx A' D^{0.4}$ (where A' is a constant close to 50000), then

$$D^{2.4} L \approx \frac{S}{A' \pi K_1'^s \hat{B}} \frac{p}{f} \tag{5}$$

If the slip of induction machines is neglected, calling $\cos(\varphi)$ the power factor and C the torque, considering $\eta S \cos(\varphi) \approx C 2\pi f/p$ (η is the efficiency), it comes:

$$D^{2.4} L \approx \frac{2C}{A' K_1'^s \hat{B} \eta \cos(\varphi)} \tag{6}$$

The machine volume is approximately proportional to the torque.

3. ORIGIN OF MAGNETIC VIBRATIONS AND NOISE

The b flux density in the air gap of AC machines is at the origin of tangential forces and torque. But radial forces acting between stator and rotor exist: the per unit area Maxwell's force are given by the approximate expression:

$$f = \frac{b^2}{2\mu_0} \tag{7}$$

where b is the instantaneous value of the radial air gap flux density, and $\mu_0 = 4\pi 10^{-7}$ H/m the permeability of the free space.

Taking into account the harmonics of flux density (due to the power supply [3], the slots [4], the saturation [5] ...), an infinity of non static forces components appear [6]; their expression is:

$$f(\alpha, t) = \hat{F} \cos(\omega_f t - m\alpha - \psi) \tag{8}$$

where m is the mode number, $\hat{F}$ the magnitude in N/m^2, f_f the force frequency ($\omega_f = 2\pi f_f$). These forces, which rotate at the angular speed ω_f/m, create stator vibrations and noise.

The mode number m is an important factor:

- for $m = 0$, the attraction between stator and rotor, and so the vibration, is uniform along the air gap (Fig. 1).
- the case $m = 1$ is particular and very dangerous : the attraction is maximum in an air gap point and minimum at the opposite. The rotor brings out of center. This case is very rare.
- for $m \geq 2$ there are m maximum attraction points between the stator and the rotor, this leads to turning stator deformations as shown on Fig. 2.

A force can create or not important vibrations and noise. It depends on several parameters: the force magnitude, frequency and mode number but also the mechanical structure of the machine.

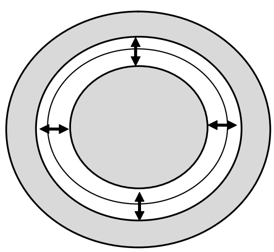

Figure 1. Stator deformations for $m = 0$.

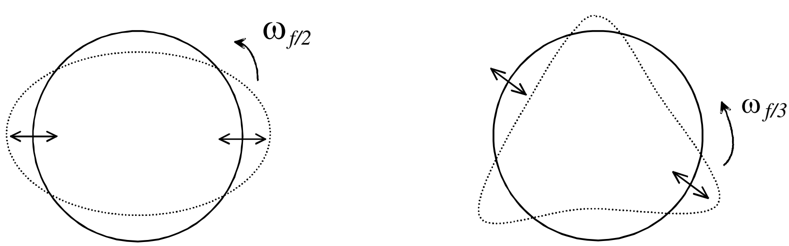

Figure 2. Stator deformations for $m = 2$ and 3.

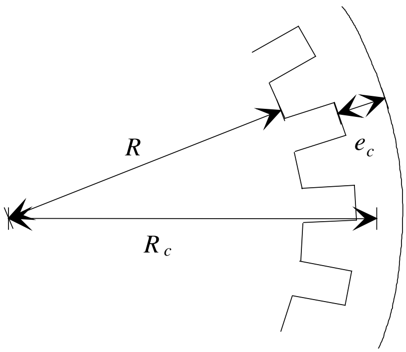

Figure 3. Notations about stator geometry.

4. MECHANICAL RELATIONS

Knowing the parameters of the forces, analytical relations allow to estimate vibration amplitude and noise. They come from the beam theory [7, 8].

First, the static deformations have to be calculated, relatively to a constant force wave. Secondly, the frequencies of mechanical resonance permit to estimate the vibration amplitudes. Then the magnetic noise can be estimated.

The following notations are used

- Y_{ms} is the static distortion amplitude relating to a force of m modes,
- Y_{md} is the vibration amplitude relating to a force of m modes,
- R is the internal radius of stator; $R = D/2$ (see Fig. 3),
- R_c is the frame average radius (see Fig. 3),
- e_c is the frame radial thickness behind slots (see Fig. 3),
- E is the elasticity coefficient or Young modulus: $E = 2.1\,10^{11}\,\mathrm{N/m^2}$ for iron.

The amplitude of the static deformation of the stator is given by:

$$\text{for } m = 0\colon\ Y_{0s} = \frac{RR_c}{Ee_c}\hat{F}, \tag{9}$$

$$\text{and for } m \geq 2\colon\ Y_{ms} = \frac{12RR_c^3}{Ee_c^3}\frac{\hat{F}}{(m^2-1)^2} \tag{10}$$

The amplitude of static vibrations can be amplified if the force frequency is close to a resonance frequency relative to the same mode number.

For vibrations of mode 0, the resonance frequency F_0 can be calculated by relation (11).

$$F_0 = \frac{837.5}{R_c\sqrt{\Delta}} \tag{11}$$

where $\Delta = \frac{\text{weight of frames+weight of teeth}}{\text{weight of frames}}$.

For $m \geq 2$, we estimate resonance frequencies F_m by relation (12).

$$F_m = F_0 \frac{e_c}{2\sqrt{3}R_c} \frac{m(m^2-1)}{\sqrt{m^2+1}} \tag{12}$$

The amplitude of dynamic vibrations Y_{md} is obtained by multiplying the amplitude of static deformations by a coefficient η_m depending on frequencies.

$$Y_{md} = \eta_m Y_{ms} \tag{13}$$

η_m is estimated by :

$$\eta_m = \frac{1}{\sqrt{\left(1-\left(\frac{f_f}{F_m}\right)^2\right)^2 + \left(2\xi_a \frac{f_f}{F_m}\right)^2}} \tag{14}$$

where ξ_a is an absorption coefficient (generally for an induction motor: $0.01 < \xi_a < 0.04$).

5. VIBRATIONS IN RELATION TO THE POLE NUMBER

5.1. Influence on the Radius

The diameter D is chosen so that all the slots (for the windings) can be placed; then the choice of the length L depends on the relation (4). When p is high, the diameter D is large and the length is small. The following relation has to be respected [2, 9, 10]:

$$0.5 < \frac{2pL}{\pi D} < 1.6$$

Let us consider that, in a general way:

$$\frac{2pL}{\pi D} \approx 1 \tag{15}$$

From the relation (5), it comes:

$$D^{3.4} \approx \frac{2S}{A'\pi^2 K_1'^s \hat{B}} \frac{p^2}{f} \tag{16}$$

Then the radius R can be expressed in relation to p:

$$R \approx 0.247 \left(\frac{S}{A' K_1^s \hat{B} f} \right)^{0.294} p^{0.588} \tag{17}$$

So the ideal radius increases with the number of poles. A pole number twice higher leads to a radius (and diameter) 1.5 times bigger.

5.2. Static Deformations

The machines manufacturers generally use standard frames. So, different kinds of machines (with different number of poles) can have the same dimensions. But, in a general way, the frame width is inversely proportional to the pole pair number [11]. ALGER [12] gave, very roughly, relations between the frame width and the radius:

$$e_c \approx 2R/5p, \quad R_c \approx 1.4R \tag{18}$$

So from the relations (9) and (10), using the relations (17) and (18), it comes:

$$\text{For } m=0: \; Y_{0s} \approx 4.1206\,10^{-12} \left(\frac{S}{A' K_1^s \hat{B} f} \right)^{0.294} p^{1.588} \hat{F} \tag{19}$$

$$\text{and for } m \geq 2: \; Y_{ms} \approx 6.057\,10^{-10} \left(\frac{S}{A' K_1^s \hat{B} f} \right)^{0.294} p^{3.588} \frac{\hat{F}}{(m^2-1)^2} \tag{20}$$

It can be observed that, for a $m \geq 2$ or more, for a given power, force amplitude and frequency, the stator deformation is proportional to $p^{3.588}$. Comparatively to a $p = 1$ machine, a $p = 2$ machine has deformations 12 times more important. For $p = 3$, $p = 4$, $p = 5$ the deformations are respectively 52, 145 and 323 times higher.

The physical explanation is that machines with high pole number have an important diameter and a small frame width. So the frame is less rigid and can become deformed more easily [13].

Calculations have been done for a machine with the following characteristics: $S = 5000\,\text{VA}$, $A' = 50000\,\text{AT/m}$, $K_1^s = 0.96$, $\hat{B} = 0.6\,\text{T}$, $f = 50\,\text{Hz}$.

A force wave, whose amplitude $\hat{F}$ is $2000\,\text{N/m}^2$, will be considered. This value is normal for an AC machine; it corresponds for example to the pressure due to a radial flux whose density is $0.07\,\text{T}$ (relation (7)).

The Figs. 4 and 5 give the amplitude of static deformation in relation to p, calculated with relations (19) and (20).

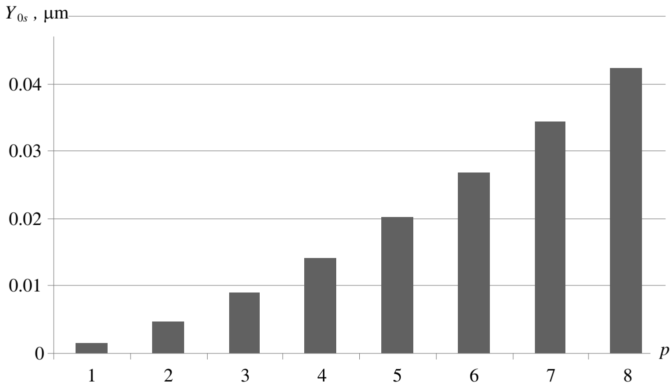

Figure 4. Static deformation for $m = 0$, in relation to p.

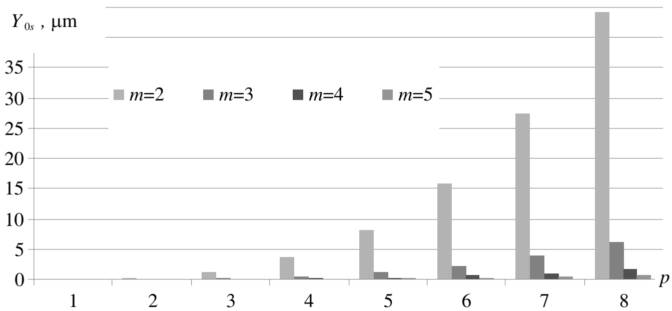

Figure 5. Static deformation for $m = 2, 3, 4$ and 5; in relation to p.

5.3. Dynamic Vibrations

The resonance frequencies are influenced by the size of the machine, and so by the pole pair number. From relations (11) and (12), using relations (17) and (18) it comes:

$$F_0 = \frac{837.5}{R_c\sqrt{\Delta}} = \frac{2420.5}{\sqrt{\Delta}}\left(\frac{A'K_1^s\hat{B}f}{S}\right)^{0.294}\frac{1}{p^{0.588}} \tag{21}$$

$$\begin{aligned} F_m &= F_0\frac{1}{12.124p}\frac{m(m^2-1)}{\sqrt{m^2+1}} \\ &= \frac{199.645}{\sqrt{\Delta}}\left(\frac{A'K_1^s\hat{B}f}{S}\right)^{0.294}\frac{1}{p^{1.588}}\frac{m(m^2-1)}{\sqrt{m^2+1}} \end{aligned} \tag{22}$$

The resonance frequencies have been estimated with the same machine as the previous one (5000 VA). Results are given on Fig. 6.

It appears that the resonance frequencies are inversely proportional to the pole pair number. Nevertheless, in a general way, the influence of the resonance frequencies on the noise depends on chance (it depends on the force wave frequency, often tied to the number of slots or to the PWM harmonics). So in the following discussions, the influence of the resonance frequencies will not be taken into account so that the coefficient η_m is taken equal to 1.

6. NOISE IN RELATION TO THE POLE NUMBER

The level of acoustic power can be estimated, in decibels, by [7, 8]:

$$L_W = 10\log\left(\frac{8200\sigma f_f^2 Y_{md}^2 S_e}{10^{-12}}\right) \tag{23}$$

where S_e is the external vibrating area and σ is the radiation factor which characterizes the capacity of the machine to be a good loudspeaker for the wave length.

S_e can be estimated by: $S_e = L2\pi(R_c + e_c/2)$.

Considering the relations (17) and (18), it comes:

$$S_e \approx 0.5376 p^{0.176}\left(\frac{S}{A' K_1^s \hat{B} f}\right)^{0.588} \tag{24}$$

Thus the influence of the pole pair number on the vibrating surface is limited (it varies with $p^{0.176}$).

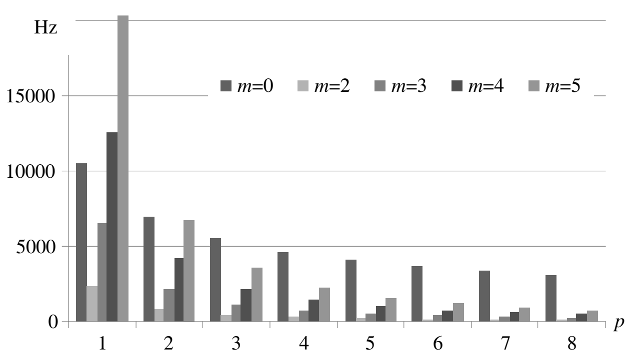

Figure 6. Resonance frequencies estimated for a 5000 VA machine, versus p.

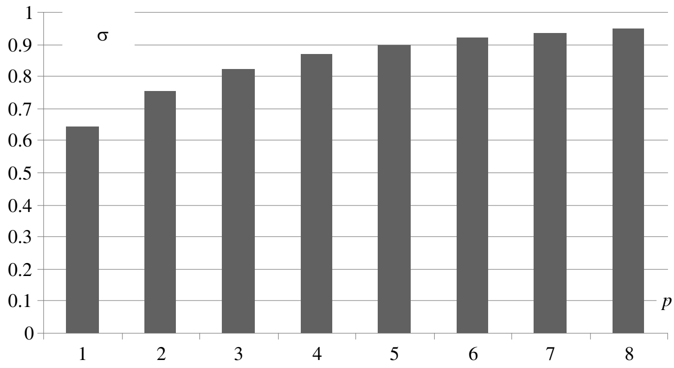

Figure 7. Variation of the radiation factor σ with p.

The radiation factor σ is close to 1 when the circumference of the machine is larger than the wave length. Authors estimate σ by considering a vibrating cylinder or sphere [7, 8, 12]. Here σ will be approximated by relation (25) where λ is the wave length of the noise frequency : $\lambda = 344/f_f$.

$$\sigma \approx 1 - e^{\frac{-\pi D_e}{\lambda}} \tag{25}$$

Calculations have been done with the same machine as previous and considering a force wave frequency (and noise component frequency) $f_f = 1000\,\text{Hz}$. The results (Fig. 7) show that a high pole number machine, which is larger, is a better loud speaker than a small pole number machine.

Finally the acoustic power level due to magnetic phenomena can then be estimated in relation to p, using relations (19), (20), (23), (24), and (25). The calculations concern always the same machine (5000 VA) and force wave ($\hat{F} = 2000\,\text{N/m}^2$, $f_f = 1000\,\text{Hz}$).

The results (Fig. 8) permit to observe that, for the same power and the same force wave characteristics, a high pole number is noisier than an small pole number. For example, with the previous machine and force, neglecting the influence of resonances:

- for $m = 0$, the acoustic level varies from 30 dB with $p = 1$ (silence) to 61 dB with $p = 8$ (noise).
- for $m = 2$, the acoustic level varies from 54 dB with $p = 1$ (low noise) to 121 dB with $p = 8$ (very high noise).

Let us remind than those calculations only concern noise of electromagnetic origin. In a general way, the noise of aerodynamic and mechanical origin is more important with a high speed machine.

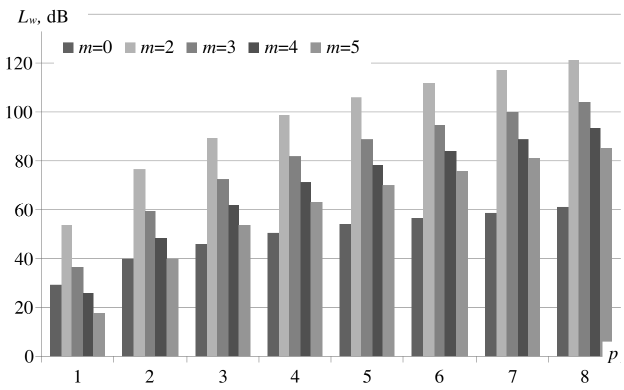

Figure 8. Variation of the acoustic power level (due to magnetic phenomena) with p and f.

7. MACHINE FED BY ELECTRONIC AT VARIABLE FREQUENCY

When a machine is fed by a converter at variable frequency, it is better to limit the noise to choose a low pole number machine fed by a low frequency rather than a high pole number machine fed by a high frequency.

For example, a machine $p = 4$–50 Hz–5000 VA is fed at 50 Hz by an electronic drive. The synchronous speed is 750 rd/mn. Another possibility is to fed at 25 Hz a $p = 2$–50 Hz–10000 VA machine or at 12.5 Hz a $p = 1$–50 Hz–20000 VA machine (the rated power has to increase if p decreases because the voltage, and so the power, decreases with the frequency in order to keep a constant flux density in the air-gap).

If the noise frequency is the same in the three case (due, for example, to the PWM switching frequency), then the noise is lower with the third machine ($p = 1$) and higher with the first one ($p = 4$).

Calculations have been done with the three machines, considering $A' = 50000\,\mathrm{AT/m}$, $K_1^s = 0.96$, $\hat{B} = 0.6\,\mathrm{T}$, $f = 50\,\mathrm{Hz}$. The results are given in the Table 1 for a noise frequency at 1000 Hz.

Regarding the relation (4), it can be surprising that the volume of the machine (D^2L) change in the Table 1. This is due to the peripheral load A, which increases with D. Nevertheless the considered power was the apparent power and not the active one. It is known that the power factor is better (near to 1) when p is small. In reality, the apparent power S needs to be smaller than 10000 in the second case and largely

Table 1. Characteristics, vibrations and noise due to different machines with ($\hat{F} = 2000\,\mathrm{N/m^2}$, $f_f = 1000\,\mathrm{Hz}$).

p	S (VA)	R (cm)	L (cm)	D^2L (cm^3)	Y_0 (µm)	Y_2 (µm)	σ	Se (cm^2)	L_{W0} (dB)	L_{W2} (dB)
4	5000	10.57	8.3	37.12	0.0143	3.682	0.87	80	50.5	98.9
2	10000	8.62	13.5	40.27	0.0057	0.375	0.82	110	43.9	80.2
1	20000	7.03	22.1	43.69	0.0023	0.038	0.79	156	37.4	61.7

smaller than 20000 in the third case. Finally the volume of the three machines is approximately the same.

The previous remark intensifies the conclusion about the calculations presented in Table 1: the noise is largely smaller when the pole number is small, although the power is higher.

8. EXPERIMENTATIONS

A synchronous $p = 4$–16 kW machine is studied. It is fed with an electronic inverter whose PWM frequency is 3 kHz. When the fundamental frequency is 50 Hz, this machine rotates approximately at 750 rd/mn. The PWM supply generates current harmonics, and so flux density harmonics at 5950 Hz and 6050 Hz. Their combinations with the flux density fundamental (at 50 Hz) generate force waves, and noise, at 5900 Hz ($m = 0$), 6000 Hz ($m = 0$ and $m = 8$) and 6100 Hz ($m = 8$).

An important noise can be heard at 5900 Hz ($m = 0$) and 6100 Hz ($m = 8$), respectively 78 dB and 79 dB measured with a microphone at 1 meter, as shown on Fig. 9.

If a machine 50 Hz–$p = 2$ is used instead of the presented machine, supplied at 25 Hz with the same PWM frequency (3 kHz), the frequencies of the flux density harmonics would be 5975 Hz and 6025 Hz; and it would create force waves at 5950 Hz, 6000 Hz and 6050 Hz, their amplitude would be the same, but the mechanical response of the frame would be different: a machine with $p = 2$ and a power twice higher would have a smaller diameter (approximately 28.5 cm instead of 33 cm) and a biggest frame width (2.6 cm instead of 1.6 cm). The static deformation, and so the risk of magnetic noise, would be lower. The calculations with the previous relations show that the noise reduction would be 6 dB for a $m = 0$ mode number, and 19 dB for the other mode numbers. As a result the noise line at 78 dB would decrease at 72 dB and the component at 79 dB would become 60 dB.

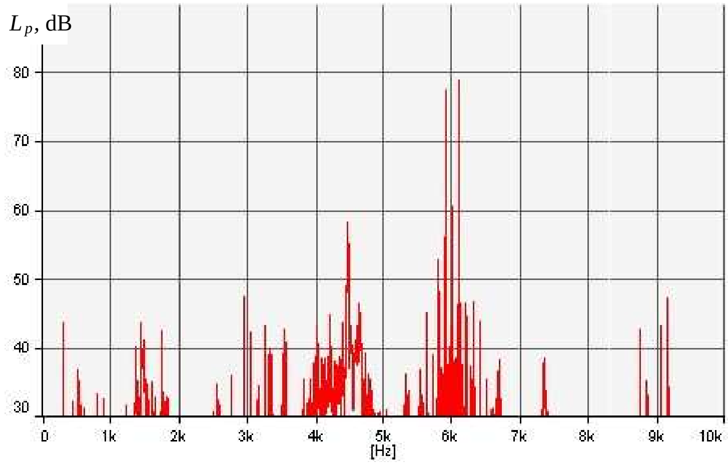

Figure 9. Acoustic pressure level at 1 meter of a $p = 4$ machine, fundamental at 50 Hz.

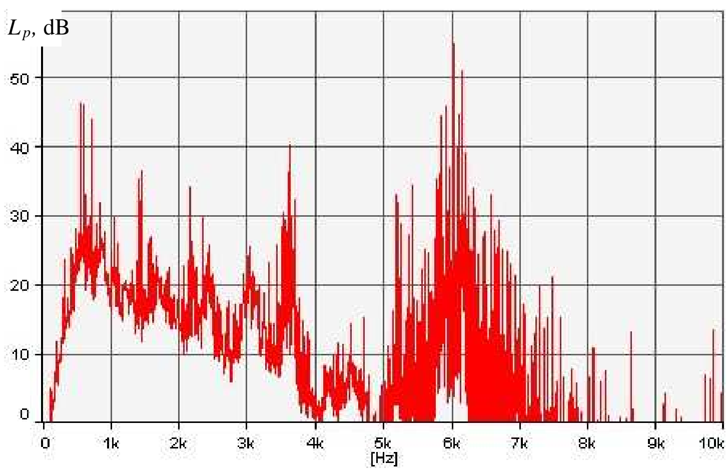

Figure 10. Acoustic pressure level at 1 meter of a $p = 2$ machine, fundamental at 30 Hz.

Let us remark than the speed is the same in the two cases, so the aerodynamic and mechanical noises would approximately be the same.

As an illustration, Fig. 10 shows the Sound Pressure Level measured with a microphone at 1 meter of a $p = 2$ induction machine fed by the same PWM inverter. The fundamental frequency was 30 Hz which is worse than 25 Hz. The noise is low, the highest line is at 55 dB (the scales of the Figs. 9 and 10 are different).

9. CONCLUSIONS

The rules of AC machines design show that the volume is proportional to the power and to the pole number. The radius of the machines is approximately in relation to the pole pair number. A machine with a high pole number has a large radius and a small frame width. The stator vibrations and so the magnetic noise, due to flux density harmonics in the air-gap of the machines, are higher when the radius is large and the frame width is small. The consequence is that the vibrations amplitude are in relation to the pole pair number at power 3.6. Then the risk of noise of magnetic origin is more important on high pole number machines.

When the supply frequency can be chosen, a low pole number machine fed by a low frequency is less noisy than a high pole number machine fed by a high frequency.

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