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DIFFRACTION FIELD BEHAVIOR NEAR THE EDGES OF A SLOT AND STRIP

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Abstract—The diffraction field asymptotics on the edges of a slot in the plane conducting screen and of a complementary strip is considered using the exact solutions of corresponding stationary diffraction problems, which have been derived before on the bases of the slot (strip) field expansions into discrete Fourier series. It is shown that as nearing the slot (strip) edges, the fields decrease or increase indefinitely in magnitude by the power law with an exponent of modulus less than unity, so the given exact diffraction solutions yield finite value of electromagnetic energy density in any point of space.

1. INTRODUCTION

Any solution of diffraction problems should satisfy certain boundary conditions, including the conditions at infinity (radiation conditions), conditions at the interfaces of different media and conditions on their fractures (edges) [1–5]. The conditions on extended interfaces are formulated as exact equalities (for the spatial field components) [1–5], but the other conditions are of less stringent character. The energy density of the field, diffracted by a finite obstacle, should vanish at infinity, and field energy must be finite in any bounded space domain [1–6], i.e., the local energy density should be an integrable coordinate function [6]. Thus, the interface conditions are taken into consideration directly at solving the diffraction problem, but the conditions at infinity and on the edges are used for selecting and testing of obtained solutions. Generally, a solution of diffraction problem is constructed as a superposition of an infinite number of different waves, each of which satisfies necessary conditions at the interfaces

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individually, but not at all interfaces. The solving procedure for a diffraction problem is reduced actually to enforce the remaining conditions on the total field. Thus, there is no need to require satisfaction of the conditions at infinity and on the edges initially for each component of the diffraction field. It is enough that their sum, i.e., the total field, satisfies the necessary conditions, although each component can violate these conditions separately. For example, Shevchenko [7] has shown that the sum of infinite number of plane waves can satisfy the infinity condition, whereas these waves satisfy the only condition of bounded amplitudes.

Such an approach has been proved to be helpful for the construction of exact solutions, which describe the electromagnetic wave diffraction by a slot in a plane conducting screen and by a complementary strip [8]. There are a lot of works, where such diffraction problems are studied under various boundary conditions exactly and approximately (see, for example, [9–13]). However, as it is known for us, only in the work of [8] their exact solutions under the classical formulation of boundary conditions are presented for the first time. These solutions are based on the field representation in the form of Fourier integrals and series. That is why they do not admit an explicit form of the field asymptotics near the edges of the conducting surfaces, as it was easily obtained for the fields of Sommerfeld's diffraction by an isolated half-plane [14] (see also [2–4, 15]). In the present work, we study behavior of the rigorous solutions [8] at the edges of a slot and strip, because in paper [8] evaluation of such a behavior was not carry out. Our main goal is to confirm physical authenticity of the approach in [8] in terms of the edge conditions, i.e., to show that this approach provides finite values of the energy density for the field solutions at the edge of a slot and strip.

2. DIFFRACTION FIELDS IN SPACE

In this section, we reproduce some results of [8], which are necessary for the subsequent consideration. The problem under study is formulated as a problem of stationary two-dimensional diffraction of a plane electromagnetic wave

$$u^{(\mathrm{inc})} = \exp[ik(\alpha_0 x + \beta_0 z)] \tag{1}$$

by a slot in the infinitesimally thin perfectly conducting screen, or by a complementary strip (Fig. 1). Here, $i = \sqrt{-1}$ is the imaginary unite; $k = \omega/c$ is the wavenumber, $\alpha_0 = \cos\vartheta$; $\beta_0 = \sin\vartheta$ are the parameters of wave propagation in x and z axes; ϑ is the angle of its incidence. The field temporal dependence is assumed as

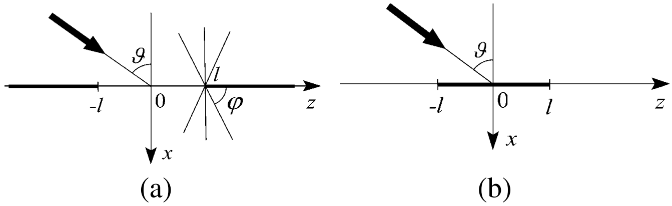

Figure 1. Diffraction by (a) a slot and (b) strip [8].

$\exp(-i\omega t)$. Usually, two-dimensional diffraction problems are reduced to independent consideration of two different field polarizations [1–5]: the first of which is characterized by orthogonality of the electric vector to the plane of wave incidence (we call that the H polarization), and the second one is defined by orthogonality of the magnetic vector to this plane (the E polarization). They determine various spatial field components

$$E_y = u \quad H_x = \frac{i}{k}\frac{\partial u}{\partial z} \quad H_z = -\frac{i}{k}\frac{\partial u}{\partial x} \tag{2a}$$

for the H polarization, and

$$E_x = -\frac{i}{k}\,\frac{\partial \bar{u}}{\partial z} \quad E_z = \frac{i}{k}\,\frac{\partial \bar{u}}{\partial x} \quad H_y = \bar{u} \tag{2b}$$

for the E polarization, where u and $\bar{u}$ are the scalar complex functions of the coordinates, which should satisfy the Helmholtz equation and different boundary conditions [1–5]. Polarization of the total field (H or E) is determined by the type of the incident wave function (1) (u or $\bar{u}$), which corresponds to the direction of its electric vector (orthogonally or in parallel to the plane of incidence).

The solution of the diffraction problem in [8] is based on the field representation in the finite slot segment $-l \le z \le l$ in the form of the discrete Fourier series

$$E_y(0,z) = \sum_{n=1}^{\infty}\left[\, a_n^{(\mathrm{s})}\cos\left(k\xi_n^{(\mathrm{s})}z\right) + ia_n^{(\mathrm{a})}\sin\left(k\xi_n^{(\mathrm{a})}z\right)\right]\theta\left(l^2 - z^2\right) \tag{3a}$$

for the H polarization, and

$$E_z(0,z) = \sum_{n=1}^{\infty}\left[\, \bar{a}_n^{(\mathrm{s})}\cos\left(k\bar{\xi}_n^{(\mathrm{s})}z\right) + i\bar{a}_n^{(\mathrm{a})}\sin\left(k\bar{\xi}_n^{(\mathrm{a})}z\right)\right]\,\theta\left(l^2 - z^2\right) \tag{3b}$$

for the E polarization, where $a_n^{(\mathrm{s,a})}$ and $\bar{a}_n^{(\mathrm{s,a})}$ are the unknown amplitudes of the symmetric and antisymmetric (in z) Fourier components,

$$\xi_n^{(\mathrm{s})} = \bar{\xi}_n^{(\mathrm{a})} = \frac{\pi}{kl}\left(n - \frac{1}{2}\right) \quad \xi_n^{(\mathrm{a})} = \frac{\pi}{kl} n \quad \bar{\xi}_n^{(\mathrm{s})} = \frac{\pi}{kl}(n-1) \tag{4}$$

are their spatial frequencies, and $\theta(x)$ is the step Heaviside's function: $\theta(x) = 1$ if $x \geq 0$ and $\theta(x) = 0$ if $x < 0$.

On the other hand, out of the plane $x = 0$ containing the screen with a slot, the diffraction field can be represented in terms of a Fourier integral. Its propagation outside this plane can be described with the help of the integral expressions

$$E_y(x, z) = \int_{-\infty}^{+\infty} A(\beta) e^{ik(\alpha x + \beta z)} d\beta \quad \text{for } x \geq 0 \tag{5a}$$

$$E_z(x, z) = -\int_{-\infty}^{+\infty} \bar{A}(\beta) e^{ik(\alpha x + \beta z)} d\beta \quad \text{for } x \geq 0 \tag{5b}$$

where

$$\alpha = \sqrt{1 - \beta^2} \tag{6}$$

(with the nonnegative imaginary part [2–5]), and similarly for the other spatial field components at $x \geq 0$ and $x \leq 0$ [8]. Equating (3) and (5) at $x = 0$, one obtains the coupling formulas between the amplitudes of the field expansions in all space (continuous spectrum) and on the slot (discrete spectrum):

$$\begin{aligned} A(\beta) &= \frac{kl}{2\pi} \sum_{n=1}^{\infty} \left[a_n^{(\mathrm{s})} Q_n^{(\mathrm{s})}(\beta) + a_n^{(\mathrm{a})} Q_n^{(\mathrm{a})}(\beta) \right] \\ Q_n^{(\mathrm{s})}(\beta) &= \operatorname{sinc}\left[kl \left(\beta - \xi_n^{(\mathrm{s})} \right) \right] + \operatorname{sinc}\left[kl \left(\beta + \xi_n^{(\mathrm{s})} \right) \right] \\ Q_n^{(\mathrm{a})}(\beta) &= \operatorname{sinc}\left[kl \left(\beta - \xi_n^{(\mathrm{a})} \right) \right] - \operatorname{sinc}\left[kl \left(\beta + \xi_n^{(\mathrm{a})} \right) \right] \end{aligned} \tag{7}$$

and similarly for the amplitudes of the E polarization [8], where sinc is the conventional notation for the function $\operatorname{sinc} x = \sin x / x$. By this way, the solution of diffraction problem is reduced to finding the amplitudes $a_n^{(\mathrm{s,a})}$ and $\bar{a}_n^{(\mathrm{s,a})}$ of the discrete Fourier series (3), which are determined as the solutions of infinite-dimensional systems of linear algebraic equations [8].

The amplitudes of the field diffraction by a strip (Fig. 1(b)) satisfy the same equations (Babinet's principle [2, 3]), so that the expressions for the H diffraction fields in the case of a strip coincide with the expressions for the E diffraction fields in the case of a slot, and inversely [8].

3. FIELD ASYMPTOTICS ON THE EDGES OF A SLOT AND STRIP

The field behavior near the edges of a slot (strip) can be studied by direct calculation of the diffraction field components (5) at small distances from an edge ρ [5, 6]

$$E(\rho, \varphi) \approx \rho^{\tau} (b_0 + b_1 \rho + b_2 \rho^2 + \ldots) \tag{8}$$

where τ is the exponent determining the field asymptotics at $\rho \to 0$. For its computation, it is convenient to use the following expansion

$$E(\rho,\varphi) \approx \exp\left(\tau \ln\rho + c_0 + c_1\rho + c_2\rho^2 + \ldots\right) \tag{9}$$

whose coefficients are related to the coefficients of expansion (8) by the equations

$$b_0 = \exp(c_0); \quad b_1 = b_0 c_1; \quad b_2 = b_0 \left(c_2 + c_1^2/2\right); \ldots$$

Then, the exponent τ can be determined as a solution of the linear approximation problem for the logarithm of the complex value E (9) by the method of least squares [16] for several small values ρ. Here, one can use various values of angle φ as nearing the slot edges in various directions (Fig. 1(a)), because for them the field asymtotics should be the same. Fig. 2 displays the values of the real part of τ for various components of the electric and magnetic fields [8] near the edges of a slot and strip, computed by this method. Under computation, we have taken into account from 100 to 200 modes of the discrete spectrum (3) and from 10000 to 20000 modes of the continuous spectrum (5) on the

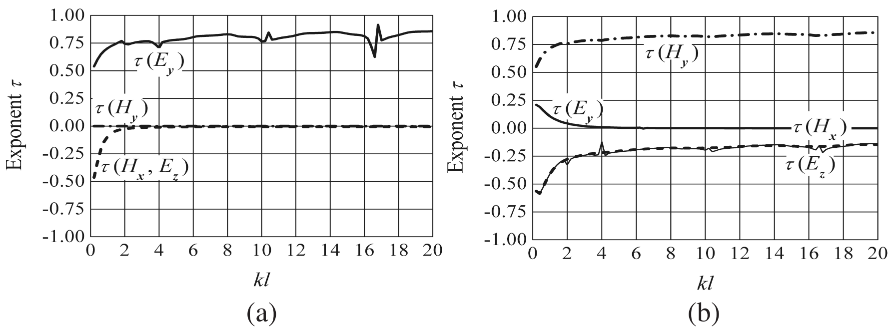

Figure 2. Exponent τ, determining the power field asymptotics on the edge of (a) a slot and (b) strip, and computed by the method of direct evaluation of various diffraction field components, as a function of the slot (strip) half-width l.

grid of the integration argument β of the step $\Delta\beta = 0.1$. The obtained values for the imaginary part of τ in magnitude are not greater than 0.001. This is the maximum calculation error for the fields on the conducting screen, which was used as a criterion for the determination of finite number of included modes (3), (5) [8]. Hence, the imaginary part of the exponent τ may be neglected, and this exponent can be considered as a real value.

As shown below, such an approach to determine the parameter τ is not quite reliable when considering Fourier components of the continuous spectrum (5) in any finite range of the propagation parameter β. Now, we apply another technique for the evaluation of an exponent of power field asymtotics. Let us pay attention to the discrete field expansions (3) in the slot plane $x = 0$. When computing these sums, one is also limited by the finite number of the included discrete modes (3). We shall evaluate the asymptotic behavior of their coefficients and the magnitude of remainders of the corresponding finite sums at $n \to \infty$. For that, assume the functions in the left-hand side of (3) as

$$E_{y,z}^{(s)} = \left(l^2 - z^2\right)^\tau \exp(ik\beta_0 z) \quad E_{y,z}^{(a)} = i(z/l)\left(l^2 - z^2\right)^\tau \exp(ik\beta_0 z) \quad (10)$$

Within constant factors, they describe behavior of the symmetric and antisymmetric parts of the fields E_y (3a) and E_z (3b) with the edge asymptotics ρ^τ at $\rho \to 0$, where $\rho = |l - z|$ or $\rho = |l + z|$. For our consideration, we can suppose that the functions (10) describe the field behavior over the whole slot region $-l \le z \le l$, because the fields on a slot display roughly similar coordinate dependences [8]. Besides, in (10), we have introduced additional exponential factors, which describe the incident field distribution (1) on a slot and which should be taken into account. For functions (10), let us calculate the amplitude coefficients of the Fourier series (3). Here, one can use routine formulas for the spatial frequencies (4) at great n, when $(\pi n/kl) \gg \beta_0$ and $\pi n \gg 1$. Within coefficients, independent of n and τ, we get

$$a_n^{(s)} \sim l^{2\tau}\Gamma(\tau+1)\cos\left(\frac{\pi\tau}{2}\right)(-1)^{n+1}\left(\frac{2}{\pi(n-1/2)}\right)^{\tau+1} \quad (11a)$$

$$a_n^{(a)} \sim l^{2\tau}\Gamma(\tau+1)\cos\left(\frac{\pi\tau}{2}\right)(-1)^{n+1}\left(\frac{2}{\pi n}\right)^{\tau+1} \quad (11b)$$

$$\bar{a}_n^{(s)} \sim l^{2\tau}\Gamma(\tau+1)\sin\left(\frac{\pi\tau}{2}\right)(-1)^{n+1}\left(\frac{2}{\pi(n-1)}\right)^{\tau+1} \quad (11c)$$

$$\bar{a}_n^{(a)} \sim l^{2\tau}\Gamma(\tau+1)\sin\left(\frac{\pi\tau}{2}\right)(-1)^{n+1}\left(\frac{2}{\pi(n-1/2)}\right)^{\tau+1} \quad (11d)$$

where Γ is Euler's gamma function [17]. As seen from here, the ratio of two next coefficients, for each of the four series, depends only on the parameter τ

$$a_n/a_{n+1} \approx -[1 + (\tau + 1)/n]$$

where a_n is $a_n^{(\mathrm{s,a})}$ or $\bar{a}_n^{(\mathrm{s,a})}$. Therefore, parameter τ can be calculated with the help of a given ratio at sufficiently great n

$$\tau \approx -n\left[(a_n/a_{n+1}) + 1\right] - 1 \tag{12}$$

The obtained Equation (11) also provides the opportunity to evaluate the residual sum magnitude for the Fourier series (3). Assuming that the sums of these series up to the Nth term are computed, where N is a sufficiently great number ($(\pi N/kl) \gg \beta_0$, $\pi N \gg 1$). Let $n = N + m$, then, for the residual sums of these series S_N, we obtain from (11) within constant coefficients

$$\begin{aligned}
S_N^{(\mathrm{s})} &= \sum_{m=1}^{\infty} a_{N+m}^{(\mathrm{s})} \cos\left(k\xi_{N+m}^{(\mathrm{s})} z\right) \sim \sum_{m=1}^{\infty} \frac{\sin[c(N+m-1/2)]}{(N+m-1/2)^{\tau+1}} \\
S_N^{(\mathrm{a})} &= i\sum_{m=1}^{\infty} a_{N+m}^{(\mathrm{a})} \sin\left(k\xi_{N+m}^{(\mathrm{a})} z\right) \sim -\sum_{m=1}^{\infty} \frac{\sin[c(N+m)]}{(N+m)^{\tau+1}} \\
\bar{S}_N^{(\mathrm{s})} &= \sum_{m=1}^{\infty} \bar{a}_{N+m}^{(\mathrm{s})} \cos\left(k\bar{\xi}_{N+m}^{(\mathrm{s})} z\right) \sim -\sum_{m=1}^{\infty} \frac{\cos[c(N+m-1)]}{(N+m-1)^{\tau+1}} \\
\bar{S}_N^{(\mathrm{a})} &= i\sum_{m=1}^{\infty} \bar{a}_{N+m}^{(\mathrm{a})} \sin\left(k\bar{\xi}_{N+m}^{(\mathrm{a})} z\right) \sim \sum_{m=1}^{\infty} \frac{\cos[c(N+m-1/2)]}{(N+m-1/2)^{\tau+1}}
\end{aligned}$$

where $c = \pi(1 + z/l)$. To calculate these sums, one can use equations [18]

$$\begin{aligned}
\sum_{m=1}^{\infty} \frac{\cos cm}{(m+D)^{\tau+1}} &= \frac{1}{\Gamma(\tau+1)} \int_0^{+\infty} t^\tau \frac{e^{-(D-1)t} - e^{-Dt}\cos c}{e^t + e^{-t} - 2\cos c} dt \approx \frac{1}{2D^{\tau+1}} \\
\sum_{m=1}^{\infty} \frac{\sin cm}{(m+D)^{\tau+1}} &= \frac{\sin c}{\Gamma(\tau+1)} \int_0^{+\infty} t^\tau \frac{e^{-Dt}}{e^t + e^{-t} - 2\cos c} dt \approx \frac{\cot(c/2)}{2D^{\tau+1}}
\end{aligned}$$

($D = N - 1/2$; N; $N - 1$). Because of great magnitude of parameter D, the main contributions to these integrals are produced by a small vicinity of the point $t = 0$. This feature allows us to put $\exp(t) + \exp(-t) \approx 2$ in denominators of the integrands and yields to simple expressions, written after the signs of approximate equality. Then, we obtain the following simple evaluation for all four residual sums

$$S_N \sim \frac{1}{N^{\tau+1}\cos(\pi z/2l)} \tag{13}$$

It follows from here that the magnitude of remainder of every Fourier series (3) tends to zero under increase of the number N of included series terms. This takes place at sufficiently great distance from the slot edges, when $\cos(\pi z/2l)$ noticeably differs from zero. However, if z is close to the slot edges, such a deduction is not true: for any finite N, one cannot achieve small magnitude of the residual sums (13) at $z \to \pm l$. In other words, terms of the series with low spatial frequencies (small numbers n) determine the behavior of their sum (3) in the distance of the slot edges, whereas the field asymptotics at these edges will be determined by higher-order Fourier modes. Thus, the standard technique of infinite sum truncation makes possible correct simulation of diffraction fields in the distance of the slot edges, but it does not provide adequate evaluation of their edge asymptotic behavior.

These conclusions hold also true for the representation of fields outside the slot plane in the form of Fourier integrals (4). Really, the substitution (7) and (8) into (5) yields

$$E_y(x,z) = \sum_{n=1}^{\infty} \left[a_n^{(\mathrm{s})} B_n^{(\mathrm{s})}(x,z) + a_n^{(\mathrm{a})} B_n^{(\mathrm{a})}(x,z) \right] \tag{14}$$

where

$$\begin{aligned} B_n^{(\mathrm{s,a})}(x,z) &= \frac{kl}{2\pi} \int_{-\infty}^{+\infty} Q_n^{(\mathrm{s,a})}(\beta) e^{ik(\alpha x + \beta z)} d\beta \\ B_n^{(\mathrm{s})}(x,z) &= C_n^{(\mathrm{s})}(x,z) + C_n^{(\mathrm{s})}(x,-z) \\ B_n^{(\mathrm{a})}(x,z) &= C_n^{(\mathrm{a})}(x,z) - C_n^{(\mathrm{a})}(x,-z) \\ C_n^{(\mathrm{s,a})}(x,z) &= \frac{kl}{2\pi} \int_{-\infty}^{+\infty} \mathrm{sinc}[kl(\beta - \xi_n^{(\mathrm{s,a})})]\, e^{ik(\alpha x + \beta z)} d\beta \end{aligned} \tag{15}$$

The function $\mathrm{sinc}[kl(\beta - \xi)]$ appreciably differs from zero only in a certain vicinity of the point $\beta = \xi$. Therefore, for evaluation, one can replace this function approximately with the finite function

$$f(\beta) = \begin{cases} \cos\left(\frac{kl}{2}\left(\beta - \xi_n^{(\mathrm{s,a})}\right)\right) & \text{for } \xi_n^{(\mathrm{s,a})} - \frac{\pi}{kl} \le \beta \le \xi_n^{(\mathrm{s,a})} + \frac{\pi}{kl} \\ 0 & \text{for } \beta < \xi_n^{(\mathrm{s,a})} - \frac{\pi}{kl} \text{ or } \beta > \xi_n^{(\mathrm{s,a})} + \frac{\pi}{kl} \end{cases}$$

because this function covers the leading lobe of the sinc function. We evaluate the integrals (15) at great n, when the parameters ξ_n (4) amount to sufficiently great values. That allows us to employ the linear approximation on the argument β for the parameter α (6) in the integrands

$$\alpha \approx i\xi_n^{(\mathrm{s,a})} + i\left(\beta - \xi_n^{(\mathrm{s,a})}\right)$$

Then, we get

$$C_n^{(\mathrm{s,a})} \approx \frac{1}{2}\exp\left[ik\xi_n^{(\mathrm{s,a})}(z+ix)\right]F(x,z)$$
$$F(x,z) = \mathrm{sinc}\left[\pi\left(\frac{z+ix}{l}+\frac{1}{2}\right)\right]+\mathrm{sinc}\left[\pi\left(\frac{z+ix}{l}-\frac{1}{2}\right)\right]$$

and residual sums of the series (14) take the form

$$S_n \approx \frac{1}{2}\left[F(x,z)+F(x,-z)\right]\sum_{n=N}^{\infty}a_n^{(\mathrm{s})}\exp\left[ik\xi_n^{(\mathrm{s})}(z+ix)\right]$$
$$-\frac{1}{2}\left[F(-x,z)+F(-x,-z)\right]\sum_{n=N}^{\infty}a_n^{(\mathrm{a})}\exp\left[ik\xi_n^{(\mathrm{a})}(z+ix)\right]$$

Since

$$\left|\sum_{n=N}^{\infty}a_n^{(\mathrm{s,a})}\exp\left[ik\xi_n^{(\mathrm{s,a})}(z+ix)\right]\right| \leq \left|\sum_{n=N}^{\infty}a_n^{(\mathrm{s,a})}\exp\left(ik\xi_n^{(\mathrm{s,a})}z\right)\right|$$

evaluation of residual sums for the integrals on the continuous spectrum (14) is reduced to the evaluation procedure for residual sums of the discrete spectrum (3).

Note that Equation (12) provides an opportunity to study the field asymptotics on the slot edges. Let the field values (3) near the edges be proportional to $(l^2-z^2)^\tau$ with several exponent τ. Then, Equation (12) allows us to evaluate this exponent by computation of the ratio of two next amplitudes of the Fourier expansions (3) at great n. Fig. 3 shows the values of τ determined by such a procedure for the field components E_y (3a) (H polarization) and E_z (3b) (E polarization), which were calculated as rigorous solutions of diffraction problems [8] in the slot plane. Here, the imaginary part magnitude is smaller than 0.001, i.e., smaller than the amplitude calculation error, so the parameter τ can be considered as a real one. For the other spatial field components, the character of asymptotic behavior on the edges can be determined by Equations (2). For E_x, the exponent of power asymptotics is equal to that of E_z (τ_E), for H_x, H_z, and for H_y it is equal to $\tau_H - 1$ and $\tau_E + 1$, respectively [5, 6].

In the case of a strip, the rigorous solutions are complementary to the solutions for a slot and determined by the same amplitudes of Fourier expansions [8]. Therefore, in this case, one obtains the same values of exponent τ.

Proceeding from the Sommerfeld's solution of the wave diffraction by a solitary half-plane [2, 3, 14, 15], one could expect that $\tau_H = 0.5$ and $\tau_E = -0.5$ in the limit $kl \to \infty$. In our case, the parameter τ slightly

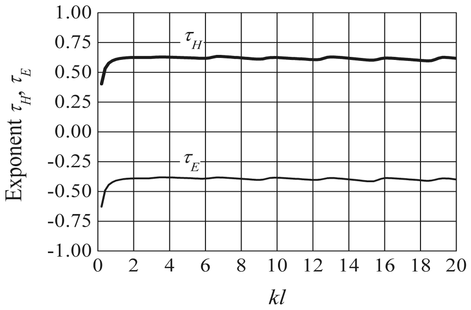

Figure 3. Exponent τ, determining the power field asymptotics on the edges of a slot and strip for two different field polarizations (H and E), and computed using higher order terms of Fourier series, as a function of the slot (strip) half-width l.

varies near the values $\tau_H \approx 0.61$ and $\tau_E \approx -0.39$, differing from the pointed Sommerfeld's values even at $kl > 20$. Such a distinction can be explained by inaccuracy of used evaluation technique for the parameter τ at great kl in view of a limited number of the included modes (3). Here, under increased kl, one should take into consideration higher modes with very great values of the spatial frequencies (4), and for that, our technique needs regular increase of their maximum order n, which significantly increases the amount of computations.

Let us evaluate specifically the value of the slot half-width l, for which the field asymptotics becomes the Sommerfeld's one. In this case, the total diffraction field should be represented by two independent solutions for every edge of a slot separately. In other words, the diffraction field components, arising at two different edges, should not overlap one another in space. It is known that the Sommerfeld's solution is determined by the Fresnel's integrals in the argument $(2k\rho)^{1/2}$ [2, 3, 14, 15], which pass asymptotically to the geometrical-optics solution at great values of this argument. These integrals differ from their asymptotic limits at infinity on the values of the order of the reciprocal of the argument [17]. Setting $\rho = l$ (the middle of a slot), then the Sommerfeld's solution will coincide with the geometrical-optics one, accurate to 0.001 at $4\pi kl \geq 10^4$, or at $kl \geq 1000$.

One should pay attention to appreciable discordance of the results displayed in Figs. 3 and 2, that confirms incorrectness of exponent τ computation using spatial field components. However, in any case, the value of exponent τ, determining the field asymptotics on the edges of a slot (strip), in magnitude is less than unity, so the local energy density of diffraction field is an integrable coordinate function [5, 6].

4. CONCLUSION

The main result of this study is that the rigorous solutions of diffraction problems for a slot and strip [8] correspond to power-type character of the field asymptotics at the edges of a slot (strip) with an exponent of modulus less than unity. It means that these solutions provide finite value of diffraction field energy density in any point of space. Thereby, we have shown once again that an infinite sum of Fourier field components, each of which has the linear and square asymptotics on the edges of an obstacle, determines a power-type asymptotics of the total field, having integrable local energy density.

This result has been established by considering amplitudes of the field Fourier expansions at various distances from the edges. We have ascertained that field behavior in the distance of the slot (strip) edges is determined mainly by sums of Fourier components with low spatial frequencies, whereas the field asymptotics directly on the edges is caused by higher-order Fourier modes. An exponent of such power-type asymptotics has been computed with the help of the ratio of higher-order Fourier amplitudes. The proposed technique can be applied with some modifications to study of fields at rigorous and approximate solving of various diffraction problems.

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