

ELECTROMAGNETIC SCATTERING FROM A CHIRAL-COATED PEMC CYLINDER

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Abstract—An analytical solution for the scattering of an electromagnetic plane wave from a perfect electromagnetic conducting (PEMC) circular cylinder coated with chiral material is derived. The PEMC cylinder as well as coating layer is of infinite length (2-D problem). Parallel polarization of the plane wave is considered for the analysis. The response of the chiral coated geometry has been observed for DPS-chiral, DNG-chiral and chiral-nihility coating layers. Also the behavior of the monostatic echo width for DPS-chiral and DNG-chiral layers has been studied against the admittance parameter. Results of bistatic echo width for the PEMC, PEC and PMC core have been presented. Under special conditions our results are in a very good agreement with the published literature.

1. INTRODUCTION

PEMC material is the generalization of PEC and PMC materials. Perfect electromagnetic conductor (PEMC) was introduced by Lindell and Sihvola [1]. Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures is studied [2]. Losses in PEMC boundary [3] and Possible applications of perfect electromagnetic conductor (PEMC) media is given [4]. Lindell and Sihvola also introduced the PEMC resonator [5]. Ruppin developed an analytical theory for the electromagnetic scattering from PEMC cylinder [6]. PEMC boundary conditions are

$$\vec{n} \times (\vec{H} + M\vec{E}) = 0 \quad (1)$$

$$\vec{n} \cdot (\vec{D} - M\vec{B}) = 0 \quad (2)$$

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where M denotes the admittance of the PEMC boundary. It is obvious from Equations (1) and (2), that PMC corresponds to $M = 0$, while PEC corresponds to $M \rightarrow \pm\infty$. The electrodynamics of substances with simultaneously negative values of ϵ and μ was given by Veselago [7]. Scattering of electromagnetic plane waves by a conducting cylinder coated with DNG materials is studied by Li and Shen [8]. Irci and Ertürk investigated the transparency and scattering maximization with metamaterials-coated conducting cylinder by considering DPS, ENG, MNG, and DNG coating layers in their analysis [9]. Plane wave reflection from a planar interface of air and PEMC medium is studied [10]. Ahmed et al., studied the low contrast circular PEC cylinder buried in metamaterial half space [11]. Ahmed and Naqvi studied the PEMC cylinder buried in a dielectric half space [12]. Ahmed and Naqvi also investigated the PEMC Parallel cylinders using the iterative procedure [13], PEMC strip and strip grating [14], multiple incident plane waves incident on coated PEMC cylinder [15], directive EM-scattering from a coated PEMC cylinder [16]. Ahmed and Naqvi [17] have provided analytical solution for the scattering of electromagnetic waves from coated PEMC circular cylinder in free space, while its coating has negative permittivity and/or permeability. Jaggard et al., studied the electromagnetic waves in chiral media [18]. Engheta and Bassiri studied one- and two-dimensional dyadic Greens functions in chiral media [19]. Lakhtakia introduced the concept of nihility [20,21]. Ahmed and Naqvi studied the directive EM-radiation of a line source in the presence of a coated nihility cylinder [22] and nihility cylinder coated with a chiral layer [23]. Engheta and Jaggard investigated the electromagnetic chirality and its applications [24]. Scattering from chiral cylinders was studied by many investigators [25–27] while Lakhtakia et al., studied nihility cylinder and perfect lenses [28–31]. Tretyakov et al., investigated waves and energy in chiral nihility [32] while Cheng et al., studied waves in planar waveguide containing chiral nihility metamaterial [33]. Electromagnetic scattering from parallel chiral cylinders of circular cross sections using an iterative procedure was studied by Sharkawy and Elsherbeni [34]. Cheng-Wei et al., investigated chiral nihility effects on energy flow in chiral materials [36]. Chiral nihility slab backed by fractional dual interface is addressed by Naqvi [35].

In this paper, an infinite PEMC circular cylinder coated with homogeneous, isotropic and linear chiral layer is considered. Both the co-polarized and cross-polarized components of the scattered field are observed. The electric field vector of incident plane wave is taken parallel to the axis of PEMC cylinder. The eigenfunction expansion method is applied to solve the problem. Appropriate boundary

conditions are applied to solve the unknown scattering coefficients. Using the large argument approximation of Hankel function, the bistatic echo width in the far-zone is calculated. Authors have incorporated the chiral nihility and DNG chiral medium to find out the changes in the scattering behavior due to a coated geometry. For the first time, in case of a chiral-coated PEMC cylinder, the variation in the monostatic echo width for positive and negative values of the admittance parameter $M\eta_1$ has been observed. Also the scattering maximization for a specific selection of the admittance parameter $M\eta_1$ has also been reported, for the first time, in the case of DNG chiral medium coating over the PEMC cylinder. For the verification of analytical formulation and numerical code results under special conditions, are compared with the published work and are found in good agreement. We have used $e^{j\omega t}$ time dependence which is suppressed through out the analysis.

2. ANALYTICAL FORMULATION

2.1. Chiral-coated PEMC Cylinder

The geometry of the problem used for the analysis is shown in Fig. 1. It contains a PEMC circular cylinder which has been coated with a chiral layer of uniform thickness. For simplicity, we assume that axis of the cylinder is coincident with z -axis of the coordinate system. Radius of the cylinder without coating is a while radius of cylinder with coating is b . Region outside the cylinder has been termed as region 0 with $\rho > b$ and has wavenumber $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, while chiral coating layer is termed as region 1 with $a < \rho < b$ and has wavenumber $k_{\pm} = \omega(\sqrt{\mu_1\epsilon_1} \pm \xi)$. Where, ξ is the chirality parameter. Also $\mu_1 = \mu_0\mu_c$ and $\epsilon_1 = \epsilon_0\epsilon_c$, is the permeability and permittivity of the region 1 respectively.

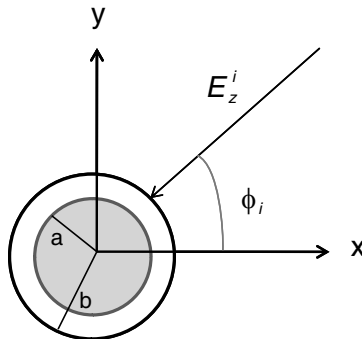


Figure 1. PEMC cylinder coated by a chiral coating layer.

2.2. Parallel Polarization

Consider the case when a parallel polarized plane wave is normally incident on the chiral-coated PEMC cylinder. The incident electromagnetic field in terms of cylindrical coordinates (ρ, ϕ) is given by

$$E_{0z}^i = E_0 e^{jk_0 \rho \cos(\phi)} \quad (3)$$

The incident electric field is written in terms of infinite Fourier-Bessel series as

$$E_{0z}^i = E_0 \sum_{n=-\infty}^{\infty} j^n J_n(k_0 \rho) e^{jn(\phi)} \quad (4)$$

Using Maxwell's equation the corresponding (ϕ) component of the magnetic field is given by

$$H_{0\phi}^i = \frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n J'_n(k_0 \rho) e^{jn(\phi)} \quad (5)$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

is the free space impedance, $J_n(\cdot)$ is the Bessel function of first kind and prime represents the derivative with respect to its whole argument. The inner core material is a PEMC cylinder, so the scattered field will contain a cross-polarized field component in addition to the co-polarized field component. The scattered fields in region 0 are.

$$E_{0z}^s = E_0 \sum_{n=-\infty}^{\infty} j^n a_n H_n^{(2)}(k_0 \rho) e^{jn(\phi)} \quad (6)$$

$$H_{0\phi}^s = \frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n a_n H_n^{(2)'}(k_0 \rho) e^{jn(\phi)} \quad (7)$$

$$H_{0z}^s = -\frac{E_0}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)}(k_0 \rho) e^{jn(\phi)} \quad (8)$$

$$E_{0\phi}^s = E_0 \sum_{n=-\infty}^{\infty} j^n b_n H_n^{(2)'}(k_0 \rho) e^{jn(\phi)} \quad (9)$$

The region 1 has two interfaces at $\rho = a$ and $\rho = b$. The total field in region 1 is expressed in terms of oppositely travelling cylindrical

waves as under.

$$E_{1z} = E_0 \sum_{n=-\infty}^{\infty} j^n [c_n J_n(k_+\rho) + d_n J_n(k_-\rho) + e_n Y_n(k_+\rho) + f_n Y_n(k_-\rho)] e^{jn(\phi)} \quad (10)$$

$$H_{1\phi} = \frac{E_0}{j\eta_1} \sum_{n=-\infty}^{\infty} j^n [c_n J'_n(k_+\rho) + d_n J'_n(k_-\rho) + e_n Y'_n(k_+\rho) + f_n Y'_n(k_-\rho)] e^{jn(\phi)} \quad (11)$$

$$H_{1z} = -\frac{E_0}{j\eta_1} \sum_{n=-\infty}^{\infty} j^n [c_n J_n(k_+\rho) - d_n J_n(k_-\rho) + e_n Y_n(k_+\rho) - f_n Y_n(k_-\rho)] e^{jn(\phi)} \quad (12)$$

$$E_{1\phi} = E_0 \sum_{n=-\infty}^{\infty} j^n [c_n J'_n(k_+\rho) - d_n J'_n(k_-\rho) + e_n Y'_n(k_+\rho) - f_n Y'_n(k_-\rho)] e^{jn(\phi)} \quad (13)$$

where a_n, b_n, c_n, d_n, e_n and f_n are the unknown scattering coefficients which can be found using appropriate boundary conditions at the interfaces ($\rho = a$ and $\rho = b$), separating the layers. The boundary conditions at $\rho = a$ are given by

$$H_{1z} + ME_{1z} = 0 \quad \rho = a, \quad 0 \leq \phi \leq 2\pi \quad (14)$$

$$H_{1\phi} + ME_{1\phi} = 0 \quad \rho = a, \quad 0 \leq \phi \leq 2\pi \quad (15)$$

The boundary conditions at $\rho = b$ are given by

$$E_{0z} = E_{1z} \quad \rho = b, \quad 0 \leq \phi \leq 2\pi \quad (16)$$

$$H_{0\phi} = H_{1\phi} \quad \rho = b, \quad 0 \leq \phi \leq 2\pi \quad (17)$$

$$H_{0z}^s = H_{1z} \quad \rho = b, \quad 0 \leq \phi \leq 2\pi \quad (18)$$

$$E_{0\phi}^s = E_{1\phi}, \quad \rho = b, \quad 0 \leq \phi \leq 2\pi \quad (19)$$

where

$$E_{0z} = E_{0z}^i + E_{0z}^s \quad (20)$$

$$H_{0\phi} = H_{0\phi}^i + H_{0\phi}^s \quad (21)$$

Application of the above boundary conditions at $\rho = a$ and $\rho = b$, yields a linear Matrix in terms of the unknown scattering coefficients. The solution of this linear Matrix gives the unknown coefficients. Putting the values of a_n and b_n in Equations (6)–(9), the scattered co-polarized and cross-polarized fields are obtained for chiral-coated PEMC cylinder. Using the asymptotic form of Hankel functions, the far-zone scattered field is obtained.

2.3. Echo Width (σ)

Echo width is the ratio of the total power scattered by the scatterer to the incident power per unit area on the scatterer

$$\sigma = 2\pi\rho \frac{W^s}{W^i} = 2\pi\rho \frac{|E^s|^2}{|E^i|^2}$$

For the parallel polarization case, the normalized bistatic echo width (RCS) of the co-polarized and cross-polarized field components is given by

$$\sigma_{co}/\lambda_0 = \frac{2}{\pi} \left| \sum_{n=-\infty}^{\infty} a_n e^{jn(\phi)} \right|^2 \quad (22)$$

$$\sigma_{cross}/\lambda_0 = \frac{4}{\pi} \left| \sum_{n=-\infty}^{\infty} b_n e^{jn(\phi)} \right|^2 \quad (23)$$

where a_n is the scattering co-efficient of Co-polarized field and b_n is the scattering coefficient of Cross-polarized field. Using the duality principle the perpendicular polarization case can be formulated.

3. NUMERICAL RESULTS AND DISCUSSION

The numerical results are based on the above analytical formulations for a PEMC circular cylinder coated with chiral material. The radius of the PEMC cylinder is taken as $a = 0.5$ m while radius of the coated PEMC cylinder is $b = 1.0$ m and the chirality of the coating layer is taken to be $\xi = 0.002$, in all the plots. To check the validity of the analytical formulation and numerical code, we have compared the results of chiral-coated PEMC cylinder with those for a chiral-coated PEC cylinder [23], i.e., when $M\eta_1 \rightarrow \infty$. For Figs. 2 and 3, the core of the geometry has been taken to be PEMC, PEC and PMC whereas the coating layer is considered of DPS-chiral material with $\epsilon_c = 3$, $\mu_c = 2$. Fig. 2 shows the co-polarized components of the bistatic echo width of the chiral coated geometry with different cores. While Fig. 3 presents the cross-polarized components of the far-zone bistatic echo width DPS-chiral coated geometry. The difference between the co- and cross-polarized components for this type of coating layer is obvious from these two figures.

Figures 4 and 5 represent the plots when coating layer is taken to be DNG-chiral, i.e., $\epsilon_c = -3$, $\mu_c = -2$. Again the core of the geometry has been taken to be PEMC, PEC and PMC. Fig. 4 shows the co-polarized components of the far-zone bistatic echo width of the chiral-

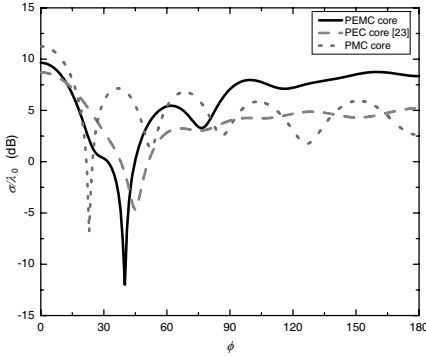


Figure 2. Co-pol component of bistatic echo width of a chiral coated geometry ($\mu_c = 2$, $\varepsilon_c = 3$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

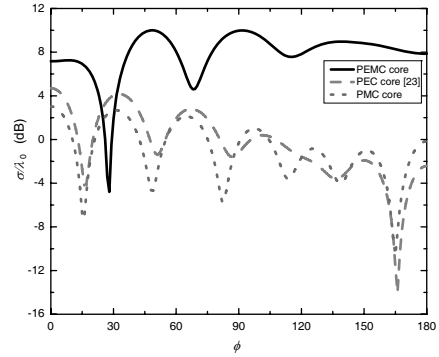


Figure 3. Cross-pol component of bistatic echo width of a chiral coated geometry ($\mu_c = 2$, $\varepsilon_c = 3$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

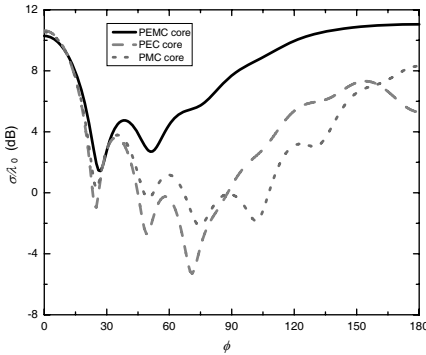


Figure 4. Co-pol component of bistatic echo width of a chiral coated geometry ($\mu_c = -2$, $\varepsilon_c = -3$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

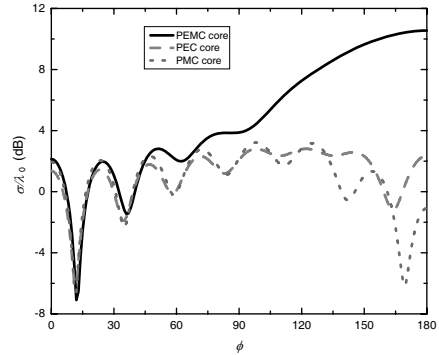


Figure 5. Cross-pol component of bistatic echo width of a chiral coated geometry ($\mu_c = -2$, $\varepsilon_c = -3$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

coated geometry with different cores. While Fig. 5 presents the cross-polarized components of the far-zone bistatic echo width DPS-chiral coated geometry. The difference between the co- and cross-polarized components in the sense of forward and backward echo width for this type of coating layer is obvious from these two figures.

Figure 6 shows the co-polarized components of the far-zone bistatic echo width of the chiral-coated geometry when the coating layer is taken to be chiral nihility, i.e., for and $\mu_c \rightarrow 0$, $\varepsilon_c \rightarrow 0$. The

cross-polarized components for this type of the geometry is zero for all the three types of the cores, i.e., PEMC, PEC and PMC.

In Figs. 7 and 8, co- and cross-polarized components of monostatic echo width for the cases of DPS-chiral $\epsilon_c = 3$, $\mu_c = 2$, $\xi_c = 0.002$ and DNG-chiral $\epsilon_c = -3$, $\mu_c = -2$, $\xi_c = 0.002$ coating layers have been plotted against the admittance parameter $M\eta_1$ respectively. It is observed that the behavior of monostatic echo width is different for positive and negative values of the parameter $M\eta_1$ which is entirely different situation from achiral coating layers previously studied.

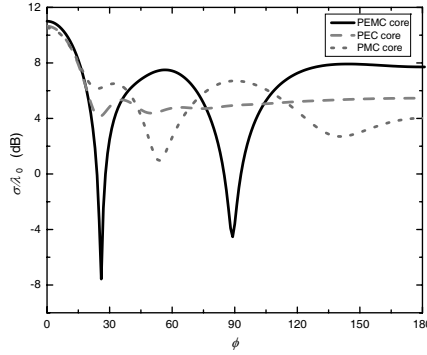


Figure 6. Co-pol component of bistatic echo width of a chiral-nihility coated geometry ($\mu_c = 0$, $\epsilon_c = 0$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

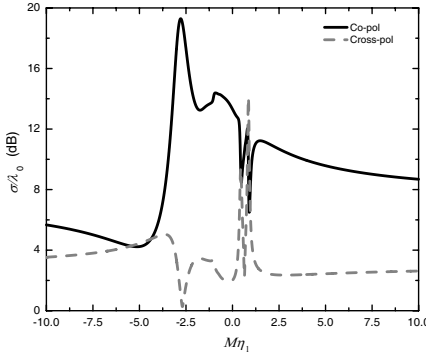


Figure 7. Monostatic echo width of a chiral coated geometry versus admittance parameter ($\mu_c = 2$, $\epsilon_c = 3$, $\xi_c = 0.002$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

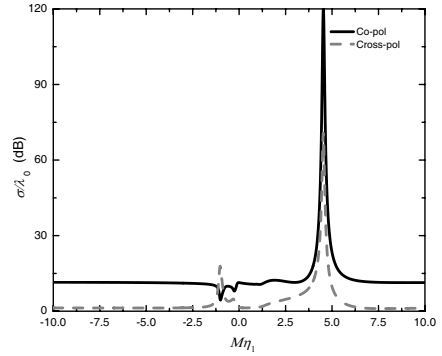


Figure 8. Monostatic echo width of a chiral coated geometry versus admittance parameter ($\mu_c = -2$, $\epsilon_c = -3$, $\xi_c = 0.002$, $a = 0.5$ m, $b = 1.0$ m, $\lambda_0 = 1$ m).

4. CONCLUSION

Different coating layers have been discussed to illustrate the scattering properties of a chiral-coated PEMC cylinder. It is observed that the behavior of the co- and cross-polarized components of bistatic echo is different for different coating layers. One of the important features of this work is the variation in the co- and cross-polarized components of the monostatic echo width, for the positive and negative values of $M\eta_1$. Which has been observed for the first time for any coated and un-coated PEMC cylindrical geometry. Secondly it can be observed in the case of DNG-chiral coating layer that there is a possibility of achieving a strong scattering maximization with an appropriate choice of the $M\eta_1$.

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