REFLECTION CHARACTERISTICS OF 1-D EBG GROUND PLANE AND ITS APPLICATION TO A PLANAR DIPOLE ANTENNA

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Abstract—A one-dimensional electromagnetic bandgap (1-D EBG) ground plane was designed and characterized. The 1-D EBG ground plane, composed of metal patches, metal lines, and a ground plane, has in-phase reflection characteristics when the polarization of the incident electromagnetic waves is parallel to the direction of the EBG unit-cell array. The proposed 1-D EBG ground plane was applied to the design of low-profile directive dipole antennas. The radiators of the designed antennas could be placed very close to the 1-D EBG ground plane without noticeable performance degradation of the antennas. In particular, the dipole antenna designed with the 1-D EBG ground plane and directors has higher directivity and a better F-B (front-to-back) ratio as compared to a conventional dipole antenna backed with a normal ground plane.

1. INTRODUCTION

Specially textured electromagnetic bandgap (EBG) structures exhibit very interesting surface electromagnetic properties. Various EBG structures have been extensively studied and implemented in antennas, microwave components and circuits [1–11]. In particular, high impedance two-dimensional (2-D) EBG surfaces such as mushroomlike structures and uni-planar structures have been applied to the ground plane of low profile directive antennas [12–18]. Because the high impedance EBG surface reflects incident waves in-phase rather than

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out-of-phase in a certain frequency band, a linear antenna designed in this frequency band can efficiently radiate energy even if the radiating element is placed very close to the EBG ground plane. The resulting low profile antenna exhibits directional radiation characteristics, because incident and reflected waves interfere constructively.

These EBG ground planes having a 2-D configuration result in large antenna size, because the 2-D EBG structures require many unit cells to ensure an in-phase reflection property. In addition, the 2-D EBG ground planes are not suitable to realize antennas on a single printed circuit board (PCB). In order to design a directive antenna that operates efficiently near the edge of a ground plane, printed antennas designed on a single PCB with a modified EBG ground plane have been proposed [20, 21]. However, the EBG ground planes used in these antennas still have a 2-D configuration composed of metal patches and vias.

In this paper, a one-dimensional (1-D) EBG structure is designed and the properties of the 1-D EBG ground plane are analyzed. The design principle of the EBG structure is first discussed, and then the reflection property and input-match frequency band of the EBG ground plane are obtained through simulations. The 1-D EBG ground plane is utilized for the design of low-profile dipole antennas. The performances of the designed antennas are evaluated and compared to that of an antenna with an ordinary ground plane.

2. ONE-DIMENSIONAL EBG STRUCTURES

The EBG structure is composed of periodic unit cells, which are made of metallic and dielectric elements. The typical EBG structure is a 2-D mushroom-like EBG structure, as proposed by Sievenpiper et al. [22]. Because of the structural resonance of the EBG structure in the resonant frequency band, the impedance of the 2-D patterned surface is so high that the surface can be regarded as a magnetic conductor. If a radiating element is located near the surface, the image current is in-phase rather than out-of-phase with the original current on the radiator. This means that the surface reflects an incident wave emitting from the radiating element in-phase, rather than out-ofphase. Therefore, a low-profile antenna can be designed above the EBG ground plane without degradation of the radiation efficiency [22, 23].

In order to design an EBG structure that has an in-phase reflection property, on a single PCB, a 1-D configuration is proposed, as illustrated in Figure 1(a). When an x-polarized wave is incident in the y-direction as shown in the figure, the capacitance between the adjacent metal patches and the inductance from the current flowing

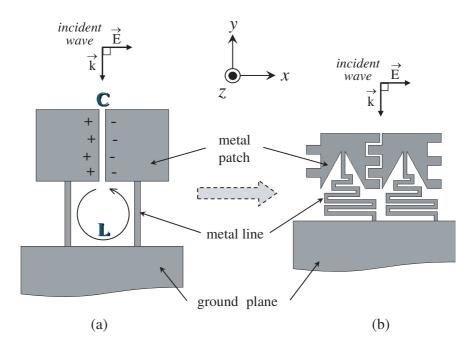


Figure 1. Configuration of the 1-D EBG structures: (a) proposed 1-D EBG structure, (b) modified 1-D EBG structure. (\vec{k} : wave vector, \vec{E} : electric field).

through the long conducting path create resonance at the resonance frequency, f_{res} , as in Equation (1).

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}\tag{1}$$

In order to make the unit cell much smaller than the operating wavelength, metal patches and metal lines in the unit cells are changed to interdigitated patches and meandered lines, as shown in Figure 1(b). The detailed structure is presented in Figure 2.

The operating bandwidth follows the relation in Equation (2) [22]; thus, if the inductance is increased and the capacitance is decreased, the fractional bandwidth is increased.

$$\frac{\Delta f}{f_{res}} \propto \sqrt{\frac{L}{C}} \tag{2}$$

The 1-D EBG structure in Figure 2 was designed to realize wideband operation by lengthening the current path along the meandered line and reducing the fringing field between the neighboring metal

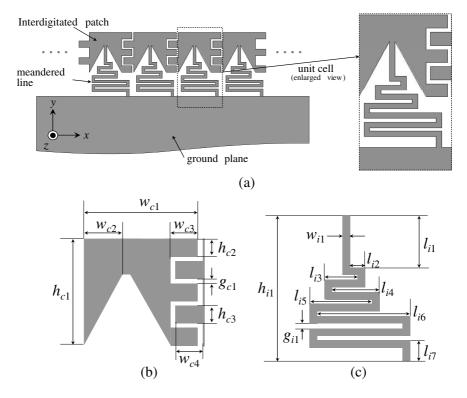


Figure 2. Schematic diagram of the designed 1-D EBG structure: (a) 1-D EBG ground plane and its unit-cell, (b) interdigitated capacitor and (c) meandered line inductor of the 1-D EBG cell.

patches. The 1-D EBG structure was designed on a FR-4 substrate with a relative permittivity of 4.4 and a loss tangent of 0.02. The thicknesses of the substrate and the metal (copper) are 1.2 and 0.035 mm, respectively. The design parameters of the 1-D EBG structure in Figure 2 are as follows: $w_{c1} = 3.15$ mm, $w_{c2} = 1.10$ mm, $w_{c3} = 0.75$ mm, $w_{c4} = 0.75$ mm, $h_{c1} = 3.10$ mm, $h_{c2} = 0.50$ mm, $h_{c3} = 0.50$ mm, $g_{c1} = 0.15$ mm, $w_{i1} = 0.20$ mm, $l_{i1} = 1.30$ mm, $l_{i2} = 0.40$ mm, $l_{i3} = 0.90$ mm, $l_{i4} = 1.30$ mm, $l_{i5} = 1.70$ mm, $l_{i6} = 2.55$ mm, $l_{i7} = 0.60$ mm, $h_{i1} = 4.0$ mm, and $g_{i1} = 0.15$ mm. The resulting unit cell in Figure 2(a) is 3.3-mm-wide and 5.0-mm-high, in total.

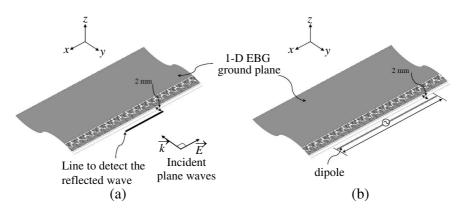


Figure 3. Configuration of the simulated structure and the simulation condition: (a) to obtain the reflection phase, (b) to find the inputmatch frequency band. (\vec{k} : wave vector, \vec{E} : electric field).

3. REFLECTION CHARACTERISTICS OF 1-D EBG GROUND PLANE

The reflection characteristics of the 1-D EBG ground plane were calculated by using a commercial simulation tool, HFSS, under the simulation conditions shown in Figure 3. The conventional simulation method that uses a unit cell and periodic boundary conditions [23–25] is not suitable to evaluate the reflection property of the 1-D EBG structure because the designed structure is arrayed along only one direction and its properties are affected by the ground size. In order to obtain the reflection phase of the designed 1-D EBG ground plane, the 1-D EBG ground plane was placed in a radiation boundary. The phase of the reflected waves from the 1-D EBG ground plane was detected at an imaginary line by using the simulation geometry illustrated in Figure 3(a). In this simulation, the calculated reflection phase is not that of the 1-D EBG structure itself, but that of the 1-D EBG ground plane.

For the comparative simulation study, an ordinary ground plane and a 1-D EBG ground plane having the 46-mm-width and infinite length were used. Eleven unit cells were placed at the edge of the 1-D EBG ground plane. Incident plane waves polarized along the xaxis were applied with the radiation boundary, because the unit cells are arrayed along the x-axis. The detected reflection phases of the infinitely long ground planes with and without the 1-D EBG structure are plotted in Figure 4. The reflection phase of the 1-D EBG ground plane exhibits very strong frequency dependence as compared with

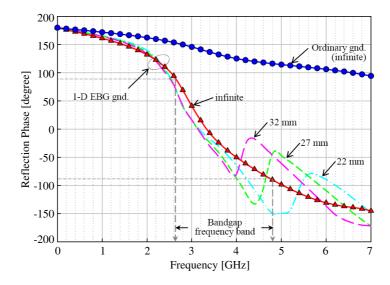


Figure 4. Reflection phases of the ground planes with and without the 1-D EBG structure obtained by the simulation method as shown in Figure 3(a). The reflection phases of the 22-, 27-, 32-mm-long and infinitely-long 1-D EBG ground planes are also compared to examine the effect of the ground length on the reflection characteristics.

that of an ordinary ground plane. Resonance of the 1-D EBG ground plane takes place where the reflection phase crosses over the zero point. The bandgap frequency band of the 1-D EBG ground plane is defined to be the frequency range where the reflection phase is between 90° and -90° [22]. The resonant frequency of the structure was 3.35 GHz and the bandgap frequency band of the 1-D EBG ground plane was between 2.62 GHz to 4.80 GHz, corresponding to fractional bandwidth of 65.1%. The 1-D EBG ground plane demonstrated a much wider bandgap frequency band as compared with the conventional 2-D EBG ground planes, which exhibit bandwidth of 2.5 \sim 33%, as reported in [12–15, 19]. When the 1-D EBG ground plane has a finite length, the reflection phase is distorted due to the interaction between the incident wave and the wave reflected at the clear cut end edge of the ground plane. The incident wave and the reflected wave from the clear cut end edge of the ground plane suffer from out of phase interference at the frequency where the length of the ground plane corresponds to a half-wavelength. The ground lengths of 32 mm, 27 mm, and 22 mm correspond to half-wavelength at 4.5, 5.5, and 6.5 GHz, respectively. Distortion in the reflection phase in Figure 4 was observed at these frequencies.

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To use the 1-D EBG ground plane for low-profile antenna applications, the input-match frequency band should also be estimated, because the dipole antenna cannot be matched well in the entire range of the bandgap frequency of the 1-D EBG ground plane. To find the input-match frequency band, dipoles with various lengths were placed close to the 1-D EBG ground plane, as shown in Figure 3(b). Figure 5 shows the reflection coefficients of dipole antennas with various lengths. The 29-mm-long dipole shows the best input-matching among the various dipoles. The resonant frequency of the 29-mm-long dipole corresponds to the frequency where the 1-D EBG ground plane exhibits a quadratic phase, as shown in Figure 4. The best impedance matching around the quadratic phase was also observed with the 2-D EBG structure [23].

In addition, the in-phase reflection characteristics of the 1-D EBG ground plane were estimated by observing the radiation pattern of an antenna backed with the EBG ground plane. As shown in Figure 6(a), the polarization dependent radiation pattern of the antenna was simulated for the antenna with the length of 29 mm, which exhibited the best input-matching. Figure 6(b) shows the radiation patterns of the antenna with respect to the angle between the EBG ground plane and the dipole. When the dipole is placed parallel to the ground plane

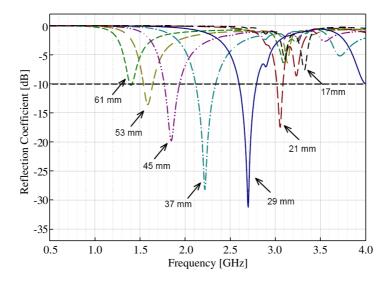


Figure 5. Reflection coefficients of dipoles with various lengths ranging from 17 to 61 mm, which were obtained by using the simulation method shown in Figure 3(b).

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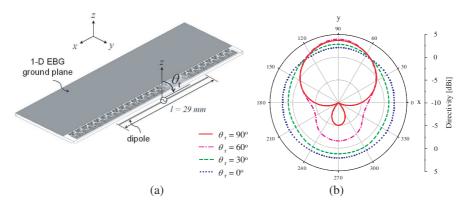


Figure 6. Radiation pattern of the dipole antenna with respect to the angle between the 1-D EBG ground plane and the dipole arms: (a) configuration of the ideal dipole antenna with 1-D EBG ground plane for the simulation study, (b) simulated radiation pattern on the *xy*-plane.

 $(\theta_r = 90^\circ)$, the dipole antenna consequently has a radiation pattern with high directivity due to the in-phase reflection by the EBG ground plane. When the angle is changed from 90° to 0°, the radiation pattern of the dipole antenna changes to an omni-directional radiation pattern, as in a conventional dipole antenna without a reflector. When the polarization of the dipole is orthogonal to the EBG ground plane, the EBG structure did not have any effect on the radiation pattern of the dipole antenna.

4. DIRECTIONAL DIPOLE ANTENNA WITH 1-D EBG GROUND PLANE

The designed antenna is depicted in Figure 7. The half-wavelength dipole was designed to be resonant at 2.6 GHz, which is near the inputmatch frequency band of the 1-D EBG ground plane, and the 1-D EBG structure was patterned on the edge of the backside ground plane. Eleven unit cells were arrayed under each arm of the dipole antenna. The planar dipole, which was excited by a simple microstrip balun, was placed 4 mm from the 1-D EBG ground plane, which corresponds to $\lambda_0/28$. The same substrate as used for the design of the 1-D EBG structure was utilized. The design parameters are as follows: $w_{a1} = 93.6 \text{ mm}, w_{a2} = 2.27 \text{ mm}, w_{a3} = 36.3 \text{ mm}, w_{a4} = 5.55 \text{ mm}, h_{a1} = 55.0 \text{ mm}, h_{a2} = 29.0 \text{ mm}, h_{a3} = 4.0 \text{ mm}, l_{a1} = 20 \text{ mm}, g_{a1} = 1.46 \text{ mm}, w_{b1} = 2.27 \text{ mm}, w_{b2} = 1.18 \text{ mm}, \text{ and } w_{b3} = 0.50 \text{ mm}.$

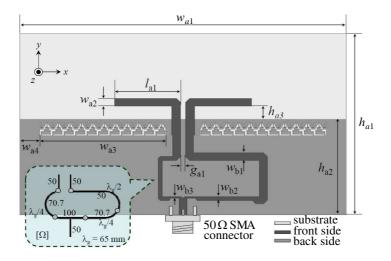


Figure 7. Configuration of the designed planar dipole antenna with the 1-D EBG ground plane.

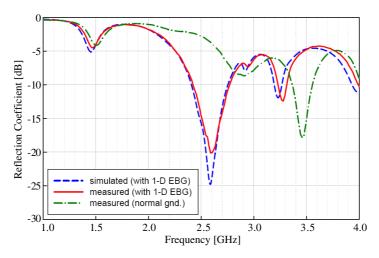


Figure 8. Reflection coefficients of the designed dipole antenna with and without the 1-D EBG ground plane.

The simulated and measured reflection coefficients of the designed antenna are shown in Figure 8. As compared to the antenna with the normal ground plane without the 1-D EBG structure, the antenna with the 1-D EBG ground plane shows good impedance matching at the design frequency. The resonant frequency and the fractional bandwidth of the antenna were measured to be 2.59 GHz and 13.44% (2.42 GHz ~ 2.77 GHz), respectively. Figure 9 illustrates the simulated and measured radiation patterns. The results show that the antenna has directional radiation characteristics with peak realized gain of 3.56 dBi and radiation efficiency of 82.26%. The front (+y) and backward (-y) gains were measured to be 2.95 and -4.13 dBi, respectively. The corresponding F-B (front-to-back) ratio was 7.08 dB.

Directors are widely used to increase the directivity and decrease backward radiation of linear antennas [26–28]. In order to obtain higher gain and a better F-B ratio in the designed antenna, two directors were placed near the radiators of the previously designed antenna, as shown in Figure 10. The distance between the radiating element and the directors, d_{a1} , is 6 mm. The length (l_{a2}) and the width (w_{a5}) of the two directors are 32 mm and 2 mm, respectively.

The simulated and measured reflection coefficients of the dipole antenna with the 1-D EBG ground plane and director are plotted in Figure 11. The resonant characteristics of the antenna are slightly affected by the directors but the resonant frequency is still near the frequency in which the reflected wave from the 1-D EBG ground plane has a quadratic phase. The resonant frequency and the operating bandwidth of the antenna were measured to be 2.68 GHz and 19.59% (2.43 GHz ~ 2.95 GHz), respectively. As predicted, higher gain and a better F-B ratio can be obtained. Specifically, as presented in Figure 12, antenna gain of 5.31 dBi and radiation efficiency of 83.74% were obtained. An F-B ratio of 15.70 dB (the front and backward gains of 5.30 dBi and -10.40 dBi, respectively) was also achieved.

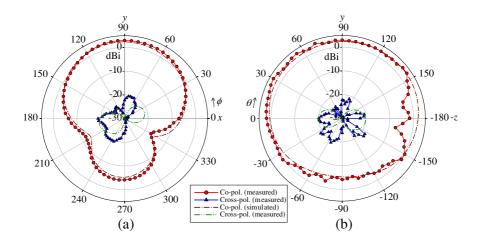


Figure 9. Radiation patterns (realized gain) of the designed dipole antenna with the 1-D EBG ground plane: (a) *xy*-plane, (b) *yz*-plane.

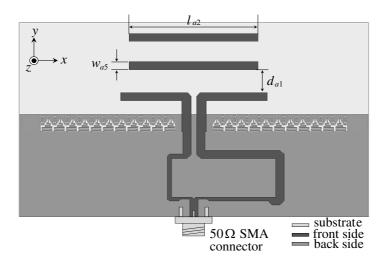


Figure 10. Configuration of the director SMA added dipole antenna with the 1-D EBG ground plane.

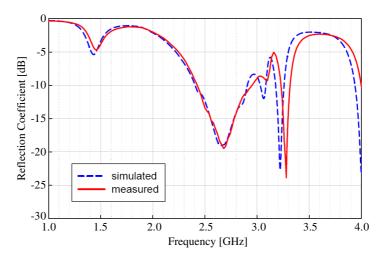


Figure 11. Reflection coefficients of the director added dipole antenna with the 1-D EBG ground plane.

For a comparative study, a conventional dipole antenna with a normal ground plane, illustrated in the inset of Table 1, was also fabricated. The radiator of the antenna is placed a quarter-wavelength apart from the ground plane so that the total area of the antenna is a little bit larger than that of the antennas backed with the 1-D EBG

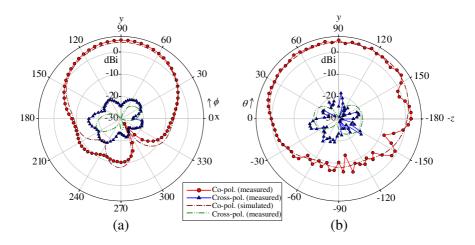


Figure 12. Radiation patterns (realized gain) of the dipole antenna with the 1-D EBG ground plane and directors: (a) xy-plane, (b) yz-plane.

	Designed dipole antenna with 1-D EBG ground plane (in Figure 7)	Designed dipole antenna with 1-D EBG ground plane and directors (in Figure 10)	Conventional dipole antenna placed $\lambda/4$ apart from normal ground plane
Structure			
Distance, D	4.00 mm $\approx 0.04 \lambda_0$	20.27 mm $\approx 0.18 \lambda_0$	$26.00 \text{ mm} \approx 0.23 \lambda_0$
Resonant frequency	2.59 GHz	2.68 GHz	2.62 GHz
Peak directivity	4.27 dBi	5.96 dBi	5.72 dBi
FB ratio	7.08 dB	15.70 dB	9.28 dB
HPBW on xy-plane	95°	83°	94°
Radiation efficiency	82.26%	83.74%	86.28%

Table 1. Performance compar	ison of the antennas.
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ground plane. Antenna performances of three cases are summarized in Table 1. All the antennas are resonant at the frequency around 2.6 GHz and show good radiation efficiency due to the well matched input-impedance. The dipole antenna with the 1-D EBG ground plane

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and directors has a more directive radiation property than the other antennas. In the case of the antenna with directors, there is an increment in the frontward (+y) radiation and a decrement in the backward (-y) radiation in comparison with the EBG backed antenna without directors, and there is a reduction in backward (-y) radiation in comparison with the conventional antenna.

5. CONCLUSION

A one-dimensional EBG structure was designed and integrated with a printed dipole antenna. The 1-D EBG structure was realized at the edge of a ground plane so that an antenna together with directors can be easily integrated with the 1-D EBG ground plane on a single PCB. The 1-D EBG ground plane exhibited much wider bandgap frequency band relative to the conventional 2-D EBG ground planes, and a the dipole antenna integrated with the 1-D EBG ground plane thereby also achieved a wide operating frequency band. The dipole antenna placed very close to the 1-D EBG ground plane efficiently radiates energy and exhibits directional radiation patterns. To achieve a more directive radiation pattern, directors can also be integrated with the 1-D EBG backed dipole antenna on a single PCB.

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