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A NUMERICAL ANALYSIS OF A DIPOLE ANTENNA IN THE VICINITY OF A HOMOGENEOUS BI-ISOTROPIC OBJECT

H. Zhu[1,*], B.-J. Hu[1], X. Y. Zhang[1,2], and J. Bao[1]

[1]Guangdong Provincial Key Laboratory of Short-range Wireless Detection and Communication, School of Electronic and Information Engineering, and South China University of Technology, Guangzhou 510641, China

[2]State Key Laboratory of Millimeter Waves, Nanjing 210096, China

Abstract—A numerical solution for the dipole antenna with a bi-isotropic object in the vicinity is developed. This solution is based on the combined surface integral equation which could deal with homogeneous situation. A fields splitting scheme is deployed to circumvent the difficulties caused by the complexity of constitutive relationships of bi-isotropic materials. With the aids of MoM, a FORTRAN program can be developed. At the end of this paper, some numerical results are presented.

1. INTRODUCTION

In recent years, novel materials have attracted more and more interests. The main reason is the conflict between the ever-growing demand for wireless usage and limited frequency spectrum resources. Many solutions have been proposed to solve this problem. Among them is bi-isotropic material, a novel material which has a more well-known sub-class, chiral material. Chiral material has found many applications in different areas, such as physics, chemistry, pharmaceutical etc. However, the more general one, bi-isotropic material, has a shorter history of research and has not found so many applications. Nonetheless, with the growing interests, more and more efforts have been put into the research of bi-isotropic material, and new applications keep appearing, such as absorption material [1] and used with microstrip antennas [2]. In such circumstances, a well-developed

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* Corresponding author: Hui Zhu (zhuhui@lifudv.com).

numerical solution would be very helpful for research and application development. In 1970's, solutions for scattering from a chiral body have been developed [3, 4]. With the evolution of computer technology, numerical methods have become affordable. With the accuracy and convenience that they can provide, numerical solutions running on computers are developed in numerous amount for different scattering problems, including those involving bi-isotropic materials [5, 6].

The numerical techniques used to develop these solutions can be categorized into two kinds. One is integral equation method, and the other is finite difference method, both in time or frequency domain. Among these numerical methods, the Method of Moments (MoM) is one major numerical method adopted in the electromagnetic community. Because of good accuracy that it can provide, it has been used to develop many numerical solutions in various scenarios. We can find the MoM based numerical solutions for conducting bodies of revolution [7], dielectric three-dimensional (3-D) bodies [8], and 3-D bodies of revolution [9, 10]. Urged by the requirement of numerical solution for unconventional materials, the solution for electromagnetic scattering from 3-D chiral bodies is developed by Worasawate et al. [11], from 3-D inhomogeneous chiral bodies by Hasanovic et al. [12], and from a chiral body of revolution by Yuccer et al. [13]. The solution for bi-isotropic bodies is developed by Wang et al. [14], and that for bi-isotropic bodies of revolution is developed by Bao et al. [15]. All the numerical solutions mentioned here involve only scatterers. However, bi-isotropic scatterers appear in conjunction with antennas in many instances, such as in the research of electromagnetic compatibility, especially when investigating the SARs. A numerical solution [16] developed for a similar case via the FDTD method can be found. However, using a FDTD method could suffer from longer computation time or the difficulty of convergence. Besides, no publication based on MoM can be found, and it is our aim to develop a numerical solution for such a problem via surface integral equation (SIE) in conjunction with MoM.

Because of the unique constitutive relationship that bi-isotropic materials have, electric and magnetic fields are coupled with each other, and the integral equation generated would be very complex if using this constitutive relationship directly. To circumvent this problem, a scheme of fields splitting is employed [17]. After performing the fields splitting, the Maxwell's equations associated with the fields in bi-isotropic body can be replaced with two sets of Maxwell's equations with propagation in free space. After that, a group of equations will be established by enforcing the boundary condition on the surface of both dipole antenna and bi-isotropic body. Instead of excited with incident

fields, the excitation is located on the dipole antenna. The regular matrix equation is expanded to include the effect of the presentation of dipole antenna. With the feeding usually located at the center of the dipole antenna, the total tangential electric field would vanish on its surface. Unlike the equations for bi-isotropic body, we do not need a combined field integral equation to obtain the solution. The electric field integral equation (EFIE) is enough. The finally resulted matrix equations will be solved numerically with MoM, and the currents will be determined. Then, we can obtain the interested parameters such as RCS, etc.

2. THEORY AND FORMULAE

A dipole antenna and a bi-isotropic body are placed in free space, as shown in Figure 1. The bi-isotropic is characterized by relative permittivity ε_r, relative permeability μ_r, relative chiral parameter κ_r and relative Tellegen parameter χ_r. The constitutive relationship of bi-isotropic material can be written in terms of these four parameters

$$\mathbf{D} = \varepsilon_r\varepsilon_0\mathbf{E} + (\chi_r - j\kappa_r)\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\mathbf{H} \quad (1)$$

$$\mathbf{B} = \mu_r\mu_0\mathbf{H} + (\chi_r + j\kappa_r)\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}\mathbf{E} \quad (2)$$

Without losing generality, the dipole antenna is placed along z-axis, and it is excited at the center.

The boundary condition employed here is that the tangential component of fields on both sides of the boundary surface should be continuous. For the dipole antenna, there are no fields inside it, and the incident field should be an impressed field, so the boundary condition will be rewritten as

$$\mathbf{E}^{inpressed}\Big|_{\tan} + \mathbf{E}^{s}|_{\tan} = 0 \text{ (on surface of antenna)} \quad (3)$$

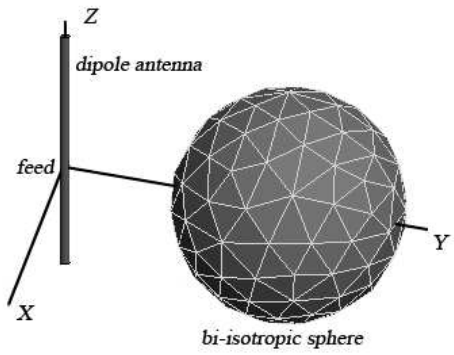

Figure 1. The dipole antenna and bi-isotropic object.

And for bi-isotropic object, there is no impressed field, and the boundary condition is

$$\mathbf{E}^s|_{\tan} = \mathbf{E}^d\Big|_{\tan} \quad \text{(on surface of BI body)} \tag{4}$$

$$\mathbf{H}^s|_{\tan} = \mathbf{H}^d\Big|_{\tan} \quad \text{(on surface of BI body)} \tag{5}$$

It should be noticed that the scattering fields consist of two parts, one from dipole antenna and the other from bi-isotropic objects. They are denoted as $\mathbf{E}_A^s$ and $\mathbf{E}_B^s$, $\mathbf{H}_B^s$, respectively.

The scattering fields can be linked to the surface currents via the Green's function.

$$\mathbf{E}^s(\mathbf{J},\mathbf{M}) = -L(\mathbf{J}) - K(\mathbf{M}) \tag{6}$$

$$\mathbf{H}^s(\mathbf{J},\mathbf{M}) = K(\mathbf{J}) - L(\mathbf{M})/\eta_0^2 \tag{7}$$

The details of the integro-differential operators L and K can be found in [15]. As mentioned above, the fields inside the bi-isotropic object would be very difficult to handle. This problem is solved by introducing the fields splitting scheme [17]. In this scheme, the fields inside the bi-isotropic materials are divided into two elliptically polarized uncoupled groups, denoted as "plus" group, "$\mathbf{E}_+$, $\mathbf{H}_+$", and "minus" group, "$\mathbf{E}_-$, $\mathbf{H}_-$". Each group fulfills Maxwell's equation with corresponding parameters.

$$\mathbf{E}_\pm(\mathbf{J}_\pm,\mathbf{M}_\pm) = -L_\pm(\mathbf{J}_\pm) - K_\pm(\mathbf{M}_\pm) \tag{8}$$

$$\mathbf{H}_\pm(\mathbf{J}_\pm,\mathbf{M}_\pm) = K_\pm(\mathbf{J}_\pm) - \frac{1}{\eta_\pm^2}L_\pm(\mathbf{M}_\pm) \tag{9}$$

The operators $L_\pm$ and $K_\pm$ here are the same with L and K except that ε, μ and k are replaced by $\varepsilon_+(\varepsilon_-)$, $\mu_+(\mu_-)$, and $k_+(k_-)$. And the relations between these two sets of parameters can also be found in [15], so are the relations between $\mathbf{J}$, $\mathbf{M}$ and $\mathbf{J}_\pm$, $\mathbf{M}_\pm$. With these manipulations, (4) and (5) could be rewritten as

$$\begin{aligned}&(\mathbf{E}_A^s(\mathbf{J}_A) + \mathbf{E}_B^s(\mathbf{J}_B,\mathbf{M}_B))|_{\tan}\\ =\ &\Big(\mathbf{E}_+^d(\mathbf{J}_B,\mathbf{M}_B) + \mathbf{E}_-^d(\mathbf{J}_B,\mathbf{M}_B)\Big)\Big|_{\tan} \quad \text{(on surface of BI body)}\end{aligned} \tag{10}$$

$$\begin{aligned}&(\mathbf{H}_A^s(\mathbf{J}_A) + \mathbf{H}_B^s(\mathbf{J}_B,\mathbf{M}_B))|_{\tan}\\ =\ &\Big(\mathbf{H}_+^d(\mathbf{J}_B,\mathbf{M}_B) + \mathbf{H}_-^d(\mathbf{J}_B,\mathbf{M}_B)\Big)\Big|_{\tan} \quad \text{(on surface of BI body)}\end{aligned} \tag{11}$$

$\mathbf{E}_A^s$ and $\mathbf{H}_A^s$ represent the scattering fields from antenna while $\mathbf{E}_B^s$ and $\mathbf{H}_B^s$ represent the scattering fields from bi-isotropic object. Equation (3) is rewritten here exhibiting the dependencies on the

currents:

$$\mathbf{E}^{inpressed}\Big|_{\tan} = -\left(\mathbf{E}_A^s(\mathbf{J}_A) + \mathbf{E}_B^s(\mathbf{J}_B, \mathbf{M}_B)\right)\big|_{\tan} \text{ (on surface of antenna)} \tag{12}$$

It can be noticed that the only unknowns in (10)–(12) are the surface currents $\mathbf{J}_A$, $\mathbf{J}_B$, and $\mathbf{M}_B$. The subscripts A and B indicate that the currents are on the antenna or on the bi-isotropic object.

3. NUMERICAL PROCESSING

In order to solve Equations (10)–(12) and determine surface currents, we need to transform the Equations (10)–(12) into numerical form and solve them with MoM [18]. The discretization begins with the expansion of surface currents. The triangulated patch basis function is adopted to expand the electric and magnetic surface currents on bi-isotropic object.

$$\mathbf{J}_B = \sum\nolimits_m a_m f_m(\mathbf{r}) \tag{13}$$

$$\mathbf{M}_B = \sum\nolimits_B b_m f_m(\mathbf{r}) \tag{14}$$

f is the mentioned triangulated patch basis function which is the well-known RWG function as detailed in [19]

$$f_m(\mathbf{r}) = \begin{cases} \frac{l_m}{2A_m^+}\rho_m^+, & \mathbf{r} \text{ in } \mathrm{T}_m^+ \\ \frac{l_m}{2A_m^-}\rho_m^-, & \mathbf{r} \text{ in } \mathrm{T}_m^- \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

The discretization of the electric surface current on the dipole antenna is quite simple. We approximate the dipole antenna with a cylinder, on which the current is concentrated on the axis. Then, the axis is divided into N segments, and the current is represented in terms of basis function. The expansion is shown in Figure 2, which is rotated 90° to save the space.

$$\mathbf{J}_A = \sum\nolimits_n c_n \mathbf{J}_n \tag{16}$$

where

$$\mathbf{J}_n = \hat{z} f_n(z) \tag{17}$$

The basis function used in both expansions is

$$f_i(t) = \begin{cases} 1 - |t - t_i|, & |t - t_i| \le 1 \\ 0 & |t - t_i| > 1 \end{cases} \tag{18}$$

here t_i is the ith point on the axis of dipole antenna; t is the normalized arc parameter along the dipole antenna; $\hat{z}$ is the unit vector in z-direction.

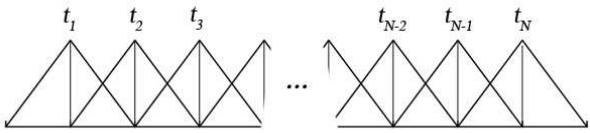

Figure 2. The expansion of current on dipole antenna.

Substituting (13), (14) and (16) into (10)–(12), and testing them by applying Gelarkin method, a set of matrix equation can be generated.

$$\begin{bmatrix} [Z_{mm}^{JJ}] & [Z_{mm}^{JM}] & [Z_{mn}^{JA}] \\ [Z_{mm}^{MJ}] & [Z_{mm}^{MM}] & [Z_{mm}^{MA}] \\ [Z_{nm}^{AJ}] & [Z_{nm}^{AM}] & [Z_{nn}^{AA}] \end{bmatrix} \begin{bmatrix} [a_m] \\ [b_m] \\ [c_n] \end{bmatrix} = \begin{bmatrix} [0] \\ [0] \\ [V^{impress}] \end{bmatrix} \tag{19}$$

Matrix $[Z]$ is the impedance matrix, which requires most of the storage resources and computing time. $[\ [a_m]\ \ [b_m]\ \ [c_n]\ ]^T$ is the unknown matrix which is to be found out by solving this matrix equation. And the matrix on the right hand side of the equation is the excitation matrix, which, in this case, is the voltage excitation applied to the dipole antenna. The details of derivation can be found in [15, 20].

4. NUMERICAL RESULTS

Some examples are demonstrated in this section to validate the developed solution. In this example, there is a bi-isotropic sphere in the vicinity of a dipole antenna. The length of the dipole antenna is $0.5\lambda_0$, and the radius of the bi-isotropic sphere is $0.25\lambda_0$. The distance between the middle point of the dipole antenna and the center of bi-isotropic sphere is $0.26\lambda_0$ with a relative permittivity $\varepsilon_r = 4$ and relative permeability $\mu_r = 1$. The center frequency used here is 300 MHz, corresponding to $\lambda_0 = 1$ m. The antenna is divided into $N = 49$ segments, and the number of the triangle patches used on the surface of bi-isotropic sphere is 849. By running the program written in FORTRAN, the numerical data were generated.

The S_{11} of the dipole is shown in Figure 3. We can see the trend that the resonance shifts upwards. And the difference is very significant when the sphere becomes a bi-isotropic.

The current distributions on the dipole antenna can be found in Figure 4. The trend mention above is found again in this figure.

In the second example, the center frequency is still 300 MHz, but the length of the antenna is changed to $1\lambda_0$, and so is the diameter of the bi-isotropic sphere. With other parameters unmodified, one of the far fields is shown in Figure 5.

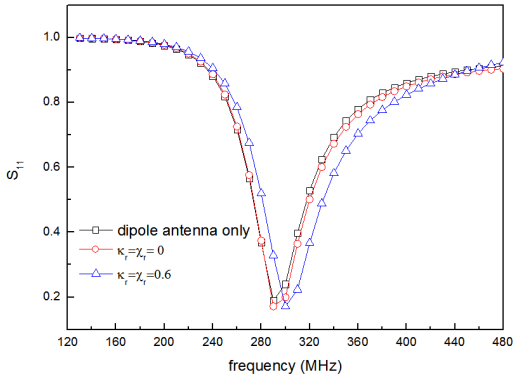

Figure 3. The S_{11} of the dipole antenna.

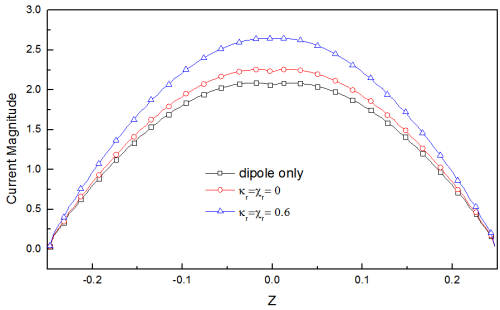

Figure 4. Current distribution of dipole antenna.

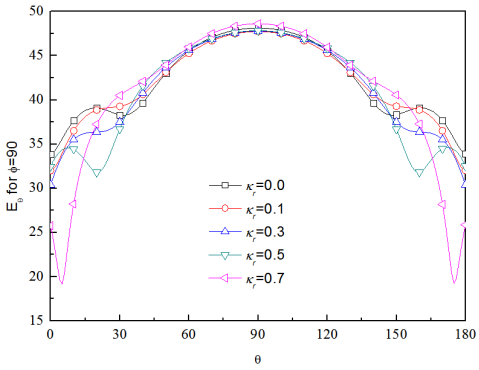

Figure 5. The θ-component electric field on the $\phi = 90$ plane. The chiral parameter of the sphere is varying from 0 to 0.7, while Tellegen parameter chi = 0.3.

5. CONCLUSION

In this paper, a numerical solution for a dipole antenna with a bi-isotropic object in the vicinity is developed. This solution is based on the previous research on chiral and bi-isotropic materials. However, it extends the numerical solution to deal with the case that one object is radiating while the other is scattering. A fields splitting scheme is deployed to circumvent the difficulties caused by the complexity of constitutive relationships of bi-isotropic material. With the aids of MoM, numerical results are generated. Future work could implement the volume integral equation to the bi-isotropic part, resulting in the so called volume-surface integral equation (VSIE), which has the capability of solving inhomogeneous bi-isotropic objects.

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