

## UNCERTAINTY PROPAGATION AND SENSITIVITY ANALYSIS IN RAY-TRACING SIMULATIONS

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**Abstract**—Up to now, ray-tracing simulations are commonly used with a deterministic approach. Given the input parameters, the ray-tracing algorithm computes a value for the electric field. In this paper, we present a method that aims at computing the mean and standard deviation of the electric field. More precisely, we aim to obtain the probabilistic content of the electric field value and direction. We assume that this uncertainty results from input random variables which we consider uniformly distributed. Since ray-tracing computations have a high computational cost, we use spectral methods in order to optimize the number of simulations. We consider 2D electromagnetic propagation for the multi-path components, which can interact with the environment through four processes: transmission, single reflection, double reflection and diffraction. These are modelled using adequate coefficients. In order to calculate the polynomial chaos expansion coefficients, we use the projection method and Gauss-Legendre quadratures. These coefficients can then be used to determine the Sobol indices of input parameters. This is done in order to neglect variables in practical computation of the uncertainties.

### 1. INTRODUCTION

Ray-tracing is a classical high frequency method for calculating propagation through regions of varying propagation velocity, absorption characteristics, and reflecting surfaces. To the authors' knowledge, the uncertainties related to the output of ray-tracing simulations are not

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known. There are two reasons that cause these uncertainties. First, the underlying mathematical model (i.e., GTD) is constructed on simplified hypothesis. Second, uncertainties are also attached to the values of the input parameters. In this paper the uncertainties are supposed to only result from uncertain input variables, and a spectral method is used in order to determine the uncertainty propagation throughout the model. Projection methods are considered to determine generalized polynomial chaos expansion from which the probabilistic content of the electromagnetic field can be obtained. The chaos coefficients are then used to perform a sensitivity analysis and the computation of Sobol indices. Finally, an algorithm that aims at computing the mean and standard deviation of the electric field and its direction, for all multi-path components, is presented.

The computation of the uncertainty on ray-tracing calculations is particularly important in the framework of electromagnetic exposure level assessment. Following guidelines for limiting exposure published by independent scientific organizations [1], governments have established legal levels. Prior to the construction of new antennas, operators have to show that these levels will not be exceeded. This can only be done through numerical computation using ray-tracing. It is then obvious that having a knowledge of the uncertainty on ray-tracing computations could help both the governments and the operators.

## **2. UNCERTAIN INPUTS FOR A RAY-TRACING MODEL**

Ray-tracing softwares allow to compute the electric field impinging on a receive antenna. The waves are modelled as rays, also referred to as the multi-path components (MPC). The electric field at the receiver is calculated by summing all the different multi-path components that draw a path between the transmitter and the receiver. The electric field of each MPC is computed thanks to the ray-tracing model that take into consideration the different interactions undergone by the ray through its propagation. It should be noticed that these expressions give a-different-value for every multi-path component independently. In order to calculate the uncertainty on the output of the ray-tracing model, an uncertainty on each multi-path component is to be computed.

With the ray-tracing model considered, one has first to determine which inputs shall be considered as uncertain variables. Next, since every multi-path component undergo a combination of interactions: transmission, reflection and diffraction, our analysis begins with the study of each interaction taken individually.

### 3. SPECTRAL METHODS

It is known that ray-tracing simulations have an highly important computation cost. For that reason, the use of Monte Carlo simulations in order to derive the mean value and the standard deviation of the electric field strength is practically impossible. Hence, a method that decreases the number of computations is required when calculating the statistics of the output.

Polynomial chaos decomposition of a model responses were introduced by Wiener in 1938 [2]. Twenty years ago [3], it was suggested to use spectral methods in engineering. It has been used recently in mechanics. Let  $\mathcal{M}$  be the stochastic model given by the mathematical representation of the ray-tracing algorithm presented in Section 5. The output random variable  $Y$  is the model response to a random vector  $X$ ; e.g., for the computation of the electric field strength  $Y = \mathcal{M}(X)$ .

The spectral method consists in using a polynomial chaos expansion for the response. When the input random variables are standard uniform random variables on  $[-1, 1]$ , the model response may directly be expanded on a basis given by the Legendre polynomials. As we consider here non standard uniform variables, an isoprobabilistic transform  $I$  is to be used, for a model with  $M$  random inputs

$$X = I(\zeta) \quad \zeta \sim U([-1, 1]^M). \quad (1)$$

Thus, any second-order random variable may be expanded in the following way

$$Y = \mathcal{M}(I(\zeta)) \approx \sum_{k=0}^{P-1} y_k \Psi_k(\zeta) \quad (2)$$

where the Legendre polynomial chaos expansion has been truncated. The number of coefficients  $P$  is given by [4]

$$P = \frac{(M+p)!}{M!p!} \quad (3)$$

with  $p$  the maximal degree of unidimensional polynomials. In order to compute the chaos coefficients  $\{y_k; k = 0, \dots, P-1\}$ , we use the projection method which lead to the following expression

$$y_k = \frac{1}{\|\Psi_k\|^2} \int \mathcal{M} \circ I(\zeta) \Psi_k(\zeta) f_U(\zeta) d\zeta \quad (4)$$

where we can simplify the probability density function to  $f_U(\zeta) = (1/2)^M$ , since we assume uniformly distributed independent random

variables. The multidimensional integral is computed using a Gauss-Legendre quadrature [5]

$$y_k = \frac{(1/2)^M}{\|\Psi_k\|^2} \sum_{i_1=1}^K \dots \sum_{i_M=1}^K w_{i_1} \dots w_{i_M} \mathcal{M} \circ I(\zeta_{i_1}, \dots, \zeta_{i_M}) \Psi_k(\zeta_{i_1}, \dots, \zeta_{i_M}) \quad (5)$$

where we use [4] a scheme of order  $K = p + 1$  and  $\{\zeta_{i_j}; i_j = 1, \dots, K\}$  and  $\{w_{i_j}; i_j = 1, \dots, K\}$  in each dimension are the integration points and weights computed using the theory of orthogonal polynomials. Finally, the mean value and the standard deviation are derived from the chaos coefficients using [6]

$$\mu_{PC} = y_0 \quad (6)$$

and

$$\sigma_{PC}^2 = \sum_{k=1}^{P-1} y_k \|\Psi_k\|^2. \quad (7)$$

#### 4. SENSITIVITY ANALYSIS

Sobol indices are a measure of the model sensitivity w.r.t. uncertain inputs. Let the  $M$  input parameters of the ray-tracing algorithm be gathered in a input vector  $x$ . For any function  $f$ , the Sobol decomposition of  $f(x)$  reads [7]

$$\begin{aligned} f(x_1, \dots, x_M) = & f_0 + \sum_{i=1}^M f_i(x_i) + \sum_{1 \leq i < j \leq M} f_{ij}(x_i, x_j) \\ & + \dots + f_{1\dots n}(x_1, \dots, x_M). \end{aligned} \quad (8)$$

The Sobol indices are then defined as follows

$$S_{i_1 \dots i_s} = \frac{D_{i_1 \dots i_s}}{D} \quad (9)$$

where the partial variance

$$\begin{aligned} D_{i_1 \dots i_s} = & \int_{[0,1]^s} f_{i_1 \dots i_s}^2(x_{i_1}, \dots, x_{i_s}) dx_{i_1} \dots dx_{i_s}, \\ & 1 \leq i_1 < \dots < i_s \leq M; s = 1, \dots, M. \end{aligned} \quad (10)$$

The total variance  $D$  can be written as

$$D = \int_{[0,1]^M} f^2(x) dx - f_0^2 \quad (11)$$

$$= \sum_{i=1}^M D_i + \sum_{1 \leq i < j \leq M} D_{ij} + \dots + D_{1\dots M}. \quad (12)$$

It has been shown [4] that approximations for the Sobol indices can be computed using the chaos coefficients presented in Section 3. In order to present that formula, we need to introduce multi-index  $\alpha = (\alpha_1, \dots, \alpha_M)$  to rewrite the chaos polynomials  $\Psi_j(x)$  of (2) in the following way

$$\Psi_\alpha(x) = \prod_{i=1}^M L_{\alpha_i}(x_i), \quad (13)$$

where  $L_k(x)$  is the  $k$ th Legendre polynomial. Then, let us introduce the set of  $\alpha$  multi-indices such that only the indices  $(i_1, \dots, i_s)$  are nonzero

$$\mathcal{I}_{i_1 \dots i_s} = \left\{ \alpha : \begin{array}{l} \alpha_k > 0 \quad k \in (i_1, \dots, i_s) \quad \forall k = 1, \dots, M \\ \alpha_j = 0 \quad j \notin (i_1, \dots, i_s) \quad \forall j = 1, \dots, M \end{array} \right\}. \quad (14)$$

Finally, the chaos-based Sobol indices are defined as

$$SU_{i_1 \dots i_s} = \sum_{\alpha \in \mathcal{I}_{i_1 \dots i_s}} \frac{y_\alpha^2 \|\Psi_\alpha\|^2}{\sigma_{PC}^2}, \quad (15)$$

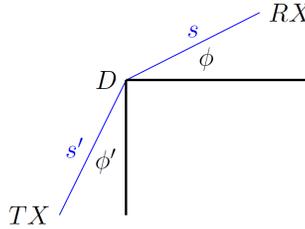
where the models defined on  $[0, 1]^M$  are first mapped onto  $[-1, 1]^M$  by a linear transform of the input parameters.

## 5. RAY-TRACING MODEL

Let a 2D-indoor ray-tracing model at 2.45 GHz be considered. It is obvious that the method presented up to now could easily be adapted to more complex 3D ray-tracing models. Four possible phenomena are taken into consideration: transmission through a wall, reflection (simple or double) on a wall and diffraction on edges. A perpendicular polarization of the electric field is also assumed. With these hypothesis, the electric field of each of the transmitted and reflected rays is given by

$$\underline{E} = T_1 T_2 \dots R_1 R_2 \sqrt{60 P_{TX} G_{TX}} \frac{e^{-j\beta d}}{d}, \quad (16)$$

where  $P_{TX}$  and  $G_{TX}$  are, respectively, the transmitted power and gain,  $\beta$  is the free-space wave number, and  $d$  is the distance travelled by the multi-path component. For our simulations, we use an unlimited number of transmissions combined or not with one (simple or double) reflection or one diffraction. The coefficients  $T$  and  $R$  are computed assuming lossy dielectrics [8–10].



**Figure 1.** Geometrical parameters for an edge-diffraction.

The model adopted to calculate the electric fields of each diffracted ray is given by

$$\underline{E} = \underline{E}_0(s')D\sqrt{\frac{s'}{s(s+s')}}e^{-j\beta s} \quad (17)$$

where  $s'$  and  $s$  are, respectively, the distance between the diffraction point and the transmitter and the diffraction point and the receiver, see Fig. 1 and  $\underline{E}_0$  is the free-space field strength. In order to compute the diffraction coefficient  $D$ , we use the heuristic extension of the UTD for a wedge with impedance faces (assuming vertical polarization) is used [11].

## 6. PROBABILITY DENSITY FUNCTIONS

The uncertain input parameters are determined by inspection of the ray-tracing model. Some parameters appear as multiplicative factors. Therefore, their effect on the output can be easily assessed and should not be taken into consideration. Hence, the uncertain parameters we ought to treat are the following. First, the coordinates of the emitter antenna and the positions of the walls. Second, the dielectric properties of the latter such as the permittivity and the conductivity are also prone to uncertainty. Finally we should also assume that the thickness of these walls should be a random variable. The following variables are taken into consideration:

- the transmitter coordinates  $(x_{TX}, y_{TX})$ ,
- the wall permittivity  $\varepsilon$ ,
- the wall conductivity  $\sigma$ ,
- the wall thickness  $t$ ,
- the position of the wall and
- the diffraction point coordinates  $(x_D, y_D)$ .

Uniformly distributed random variables are chosen in order to model these uncertainties. It should be noticed that we carried out simulations that prove that an uncertainty of 20 MHz on a central value of 2.45 GHz led to variations in the electric field that were negligible compared to the others we introduce later. Hence, we used a 2.45 GHz frequency as an exact parameter.

## 7. NUMERICAL ANALYSIS

A maximal uncertainty of 5 cm has been chosen for the positions, e.g., for a deterministic coordinates of (1, 2) for the emitter antenna, we use the following probability density functions (in m)

$$x_{TX} \sim U(0.95, 1.05) \quad y_{TX} \sim U(1.95, 2.05). \quad (18)$$

This uncertainty is obtained through an analysis of the common sizes of today's electronic devices.

Uniform intervals for the non spatial random inputs are still to be found. As the dielectric properties vary much with the composition, concrete walls are chosen with the following properties

$$\varepsilon_r \sim U(4, 9) \quad (19)$$

for the relative permittivity and

$$\sigma \sim U(0.001, 0.1) \quad (20)$$

for the conductivity (in S/m). Finally, walls are supposed to have a thickness of 10 cm, and a maximum uncertainty of 5 cm from this central value is considered; this lead to the probability density function (in m)

$$t \sim U(0.095, 0.105). \quad (21)$$

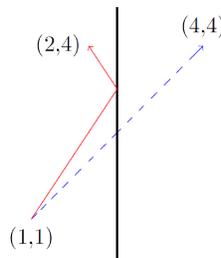
For single and double reflections a 10 cm interval around the central value is used for the coordinate whose direction is normal to the wall. Finally, for diffractions, a maximum spread of 5 cm around the deterministic coordinates  $x_D$  and  $y_D$  is assumed.

It should be noticed that, in the following, the multipath contributions are added incoherently in order to calculate the total strength field. In fact, if a coherent summation were carried out, the total strength field would be very sensitive with respect to the antenna position since constructive and destructive interferences would occur. Conversely, when incoherent summation is used, which is the case in this paper, it is expected that the total field strength is not very sensitive to the antenna position due to the  $1/d$  factor for the amplitude in (16), where  $d$  does not vary a lot.

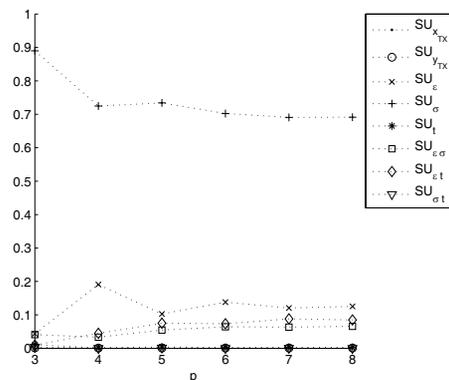
As every multi-path component will undergo combination of single interactions such as transmission, reflection or diffraction, it is interesting to study these interactions individually. We will see that some characteristics are common to all of these interactions. In fact, although we have reduced the computation time using spectral methods, it is still necessary to find a way to decrease the number of simulations with the ray-tracing algorithm. This was done using the Sobol indices that allowed us to determine which inputs have higher influence than others on the output uncertainty. The following results will be presented for different values of  $p$ , the maximum degree of unidimensional Legendre polynomials.

## 8. UNCERTAINTY PROPAGATION THROUGH THE RAY-TRACING MODEL

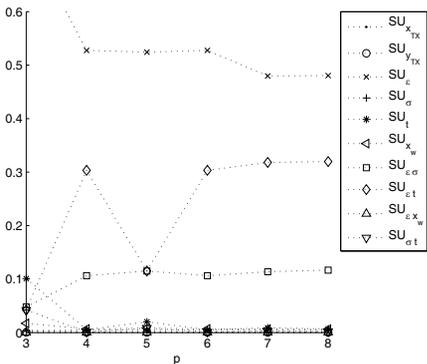
In Fig. 2, for a transmission, we use a position (in m) of the emitter and receiver respectively of  $(1, 1)$  and  $(4, 4)$  whereas for a single reflection they are  $(1, 1)$  and  $(2, 4)$ . In both cases, the horizontal coordinate of the wall is 2.5 m. For a diffraction, we use the situation presented in Fig. 1 with the following deterministic coordinates:  $(0, 0)$  for TX,  $(3, 4)$  for RX and  $(1, 3)$  for the diffraction point. The Sobol indices obtained for a transmission are presented in Fig. 3 where  $p$  is the maximal order of the Legendre unidimensional polynomials. We can observe that the indices start to converge for a chaos order  $p = 5$ . It can also be noticed that the wall conductivity is the most important parameter for the Sobol variance analysis as it explains almost 70% of the total variance when taken alone. If we add the indices  $SU_{\varepsilon}$ ,  $SU_{\sigma}$  and  $SU_{\varepsilon\sigma}$ , it is remarkable that more than 88% of the total variance can be recovered. The effect of uncertainties on the transmitter position is very low (less than 1%) .



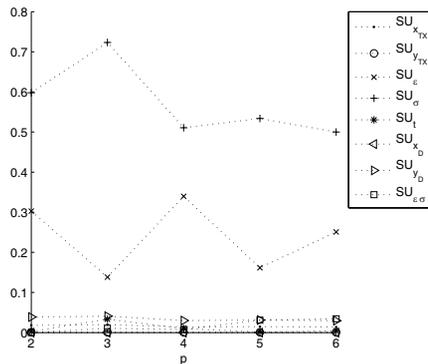
**Figure 2.** Deterministic coordinates (in m) for a transmission (dashed) and a single reflection.



**Figure 3.** Sobol indices for a transmission. Not shown indices are negligible.



**Figure 4.** Sobol indices for a reflection. Not shown indices are negligible.



**Figure 5.** Sobol indices for a diffraction. Not shown indices are negligible.

Using again spectral methods to determine the Sobol indices for a reflection, we obtain the results of Fig. 4. In this case, it is no longer the conductivity but the permittivity that has the major influence on the electric strength uncertainty. Next, the second order index  $SU_{\epsilon t}$  which has an effect superior to 30%. If we consider  $\epsilon$ ,  $\sigma$  and  $t$ , the sensitivity analysis shows that more than 92% of the total variance is kept. It should be noticed that very similar results are obtained when studying a double reflection: the variance explained by the three latter inputs accounts for more than 95%.

The last case we studied is a diffraction. As seen in Fig. 5, the two most important inputs for that interaction are the conductivity

and the permittivity with the first one clearly the more important. Once again, we observe that the position uncertainties do not affect much the output variance. Moreover, similarly to a transmission, if we consider a simplified model only keeping the wall properties as uncertain, 90% of the output variance is kept. It should be noted that for this kind of interaction, it was not possible to expand the response with Legendre polynomials of order 7 or more. This is due to the complexity of the  $D$  coefficient and the high number (seven) of inputs. As we already mentioned, the Sobol indices presented here are obtained for the specific geometries exposed above. Nevertheless, when varying relative positions of the emitter and the walls for these single interactions, the indices undergo very little modifications, at most 5%.

## 9. OUTPUT UNCERTAINTY COMPUTATION

The data presented in Section 8 allow to suggest a method to compute the output uncertainties due to imprecise inputs when considering a 2D ray-tracing software.

### 9.1. Electric Field Strength Module

As presented earlier, each of the three basic interactions have similar characteristics in terms of sensitivity analysis related to the electric field strength. More precisely, the Sobol indices have shown that considering only the three wall properties (permittivity, conductivity and thickness), more than 90% of the total variance is kept. Hence, we propose to use a simplified model whose inputs are given by only these three parameters. In order to validate this assumption, we still have to consider combinations of basic interactions. In fact, the 90% of the output variance is still observed when making sensitivity analysis for more complex paths, such as combinations of several transmissions with one (single or double) reflection or one diffraction. Finally, comparisons to Monte Carlo simulations suggest to use Legendre polynomials of maximal order of 8.

### 9.2. Reception Angle Module

The incidence angle (on the receiver) uncertainty computation is easier. Only position uncertainties have to be considered, since wall properties do not affect the path in the ray-tracing model. We notice that these inputs cause linear variations of the incidence angle, for all interactions. Finally, sensitivity analysis show that Sobol indices are similar for different inputs. This leads us to consider all the uncertain

inputs in order to determine the angle uncertainty, without neglecting parameters as we have done for the electric strength. Moreover, the linear variations allows us to use Legendre polynomials of order 1.

### 9.3. Example

In this example, the mean electric field in a local zone round the receiver is

$$E = \sqrt{\sum_i |E_i|^2} = 0.051 \text{ V/m}, \tag{22}$$

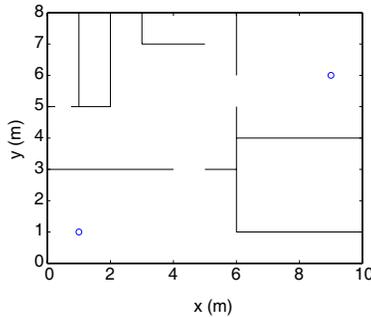
with an uncertainty

$$\sigma = \sqrt{\sum_i \sigma_i^2} = 0.015 \text{ V/m}. \tag{23}$$

In order to illustrate the proposed method, simulations were carried out using the indoor environment presented in Fig. 6. The

**Table 1.** Output of the uncertainty computation algorithm when analysing the indoor environment of Fig. 6:  $\mu$ ,  $\sigma$  and  $VC = \mu/\sigma$  are related to the electric field  $E$  and the incidence angle  $\theta$ ; MPC with  $\mu_E < 0.001 \text{ V/m}$  are not presented.

| Components         | $\mu_E$ (V/m) | $\sigma_E$ (V/m) | $VC_E$ (%) | $\mu_\theta$ (°) | $\sigma_\theta$ (°) | $VC_\theta$ (%) |
|--------------------|---------------|------------------|------------|------------------|---------------------|-----------------|
| Direct             | 0.045         | 0.008            | 17.9       | 212.0            | 0.1                 | 0.0             |
| Single Reflections | 0.005         | 0.004            | 76.9       | 221.1            | 0.2                 | 0.1             |
|                    | 0.007         | 0.003            | 51.3       | 206.5            | 0.1                 | 0.0             |
|                    | 0.009         | 0.005            | 54.3       | 131.6            | 0.2                 | 0.1             |
|                    | 0.002         | 0.002            | 75.0       | 333.4            | 0.1                 | 0.0             |
| Double Reflections | 0.010         | 0.005            | 49.5       | 126.0            | 0.2                 | 0.1             |
|                    | 0.005         | 0.004            | 68.9       | 325.0            | 0.3                 | 0.0             |
|                    | 0.005         | 0.004            | 68.9       | 214.9            | 0.3                 | 0.1             |
|                    | 0.005         | 0.003            | 69.6       | 138.0            | 0.2                 | 0.1             |
|                    | 0.001         | 0.000            | 10.0       | 337.3            | 0.1                 | 0.0             |
|                    | 0.011         | 0.005            | 50.4       | 41.9             | 0.2                 | 0.6             |
|                    | 0.002         | 0.001            | 71.4       | 190.1            | 0.0                 | 0.0             |
|                    | 0.002         | 0.001            | 43.4       | 194.0            | 0.0                 | 0.0             |
|                    | 0.004         | 0.003            | 69.5       | 233.9            | 0.2                 | 0.1             |
| Diffractions       | 0.001         | 0.000            | 50.0       | 165.9            | 0.4                 | 0.2             |
|                    | 0.010         | 0.002            | 22.6       | 210.9            | 0.2                 | 0.1             |
|                    | 0.001         | 0.000            | 34.3       | 216.8            | 0.3                 | 0.1             |



**Figure 6.** Example indoor environment. The coordinates for the emitter and receptor antennas are respectively  $(1, 1)$  and  $(9, 6)$ .

deterministic positions of the emitter and receiver are respectively of  $(1, 1)$  and  $(9, 6)$ . The considered geometry leads to 28 multi-path components: the direct one, 4 single reflections, 14 double reflections and 9 diffractions. The outputs of our software are given in Table 1.

## 10. CONCLUSION

An algorithm that allows to quantify the uncertainty on ray-tracing simulations outputs has been implemented: the electric field strength and the receptor angle. A methodology that gives the mean value and the standard deviation for both quantities, for all multi-path components is then suggested. Usually, a ray-tracing software has a highly important computation cost, the presented method optimises the number of ray-tracing simulations by using spectral methods. The output uncertainty considered are only due to uncertain inputs and random uniform variables are used. The uncertainty is propagated throughout the model, using a projection method and Gauss-Legendre quadratures in order to compute the coefficients of the polynomial chaos expansion.

The uncertainty on the electric strength requires more attention than that of the angle. The chaos expansion allows to derive sensitivity indices, i.e., the Sobol indices related to the uncertain input parameters. In order to illustrate the method, a 2D-model for the ray-tracing is considered. Obviously, the methodology could easily be adapted for more complex ray-tracing simulations. Three basic interactions are taken into consideration: a transmission, a reflection and a diffraction. It has been observed that for the three interactions, more than 90% of the total variance is due to the wall properties, i.e., permittivity, conductivity and thickness. Afterwards,

it has been shown that this is also the case for more complex multi-path components. In all situations, the uncertainties on position of the emitter and of the walls can be neglected. This has led to suggest to use a three-inputs reduced model to estimate the uncertainty on the electric strength. The angle uncertainty is easier to obtain, as the computation costs are much reduced. All the uncertain inputs can be kept: position of the emitter and of walls for a reflected multi-path components and position of the emitter and the diffraction point for a diffraction. As the proposed method is non-intrusive, it can be applied to any ray-tracing code without any modification.

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